Financial Engineering and Risk Management

Futures

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Problems with forward contracts

- Not organized through an exchange.
- Consequently, no price transparency!
- Double-coincidence-of-wants: need someone to take the opposite side!
- Default risk of the counterparty.

Futures contract

- Solves the problem of a multitude of prices for the same maturity by marking-to-market
    - disbursing profits/losses at the end of each day
- Now contracts can be organized through an exchange.
- Can be written on any underlying security with a settlement price
    - Commodities
    - Broad based indicies, e.g. S & P 500, Russel 2000, etc.
    - Volatility of the market, e.g. VIX futures
- http://www.cmegroup.com/market-data/delayed-quotes/commodities.html

Mechanics of a futures contract

- Individuals open a margin account with a broker
- Enter into N futures contracts with price F_0
- Deposit initial margin into the account $\approx 5 - 10\%$ of contract value
- All profit/loss settled using margin account
- Margin call if balance is low

Simulation of the mechanics of corn futures

Contract value	5000.00	Interest rate	0.25
Initial margin	1688.00	volatility	15.73344864
Maintenance margin	1250.00		

http://www.ccstrade.com/futures/corn-futures-c/margin/

		Futures Contracts				Forward Contract	
Date	Price	Position	Profit	Margin Account	Margin Call	Spot Price	Forward price
Feb 22nd	690.25	1		1,688		690	690.36
Feb 25th	697.05	1	340	2,028	0		
Feb 26th	679.31	1	-887	1,688	547		
Feb 27th	686.86	1	377	2,065	0		
Feb 28th	692.54	1	284	2,350	0		
March 1	685.66	1	-344	2,006	0		
March 4	668.22	1	-872	1,688	555		
March 5	648.32	1	-995	1,688	995		
March 6	640.77	1	-377	1,311	0		
March 7	661.03	1	1,013	2,324	0		
March 8	661.95	1	46	2,370	0		
March 11	668.02	1	303	2,673	0		
March 12	661.34	1	-334	2,339	0		
March 13	678.78	1	872	3,211	0		
March 14	687.58	1	440	3,651	0		
March 15	697.10	1	476	4,127	0		
Total Profit/Loss			343				337

Corn | Simulated data | Sheet1

Pros/cons of futures

Pros:

- High leverage: high profit
- Very liquid
- Can be written on a wide variety of underlying assets

Cons:

- High leverage: high risk
- Futures prices are approximately linear function of the underlying – only linear payoffs can be hedged
- May not be flexible enough; back to Forwards!

Pricing futures

- Need martingale pricing formalism
- Deterministic interest rates: forward price $=$ futures price
- At maturity futures price $F_T =$ price of underlying S_T

Hedging using Futures: Long hedge

Today is Sept. 1st. A baker needs $500,000$ bushels of wheat on December 1st. So, the baker faces the risk of an uncertain price on Dec. 1st.

Hedging strategy: buy 100 futures contracts maturing on Dec. 1st – each for 5000 bushels

Cash flow on Dec. 1st

- Futures position at maturity: $F_T - F_0 = S_T - F_0$
- Buy in the spot market: S_T
- Effective cash flow: $S_T - F_0 - S_T = -F_0$

Price fixed at F_0!

Did this cost anything? Cash flows associated with margin calls.

Perfect hedges are not always possible

Why?

- The date T may not be a futures expiration date.
- P_T may not correspond to an integer number of futures contracts
- A futures contract on the underlying may not be available
- The futures contract might not be liquid
- The payoff P_T may be nonlinear in the underlying

Basis $=$ Spot price of underlying - futures price

- Perfect hedge: basis $= 0$ at time T
- Basis risk: basis $\neq 0$ at time T
- Basis risk arises because the futures contract is on a related but different asset, or expires at a different time.

Hedging problem with basis risk

Today is Sept. 1st. A taco company needs $500,000$ bushels of kidney beans on December 1st. So, the taco company faces the risk of an uncertain price.

Problem: No kidney bean futures available. Basis risk inevitable.

Hedge: Go long y soybean futures each for 5000 bushels of soybeans

Cash Flow in 90 days

- Futures position at maturity: $(F_T - F_0)y$
- Buy kidney beans in the spot market: P_T
- Effective cash flow: $C_T = y(F_T - F_0) - P_T$

$P_T \neq yF_T$ for any y: Perfect hedge impossible!

Minimum variance hedging

Variance of the cash flow

$$\begin{aligned}\mathbf{var}(C_T) &= \mathbf{var}(P_T) + \mathbf{var}\big(y(F_T - F_0)\big) \\ &\quad - 2\mathbf{cov}\big(y(F_T - F_0), P_T\big) \\ &= \mathbf{var}(P_T) + y^2\mathbf{var}(F_T) - 2y\mathbf{cov}(F_T, P_T)\end{aligned}$$

Set the derivative with respect to y to zero:

$$\frac{d\mathbf{var}(C_T(y))}{dy} = 2y\mathbf{var}(F_T) - 2\mathbf{cov}(F_T, P_T) = 0$$

Optimal number of Futures contracts:

$$y^* = + \frac{\mathbf{cov}(F_T, P_T)}{\mathbf{var}(F_T)}$$