

# Pricing Forwards & Futures

Saturday, August 30, 2025 10:19 AM

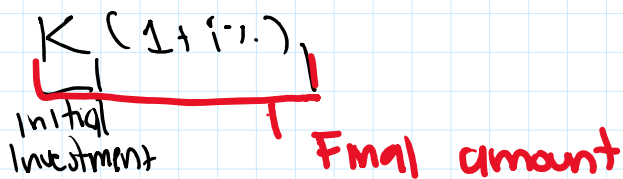
$F_{t,T}$  = Forward Price of and underlying at time  $t$  with maturity or expiration date  $= T$  ( $T > t$ )

$S_t$  = The price of the underlying at time  $t$  (spot price)

$$F_{t,T} \sim S_t$$

1. It is possible to borrow or lend any amount of money at the risk-free rate

ex.  $i :=$  annual effective rate (after 1 unit of time we are going to receive  $i\%$  of our init. investment)



2. There are no transaction costs

3. Arbitrage is impossible

- **Arbitrage**: set of transactions that represents 0 cost, no possibility of loss, and a possibility of making profit.

## Forwards on Stocks (no dividends)

- There are 2 ways to own a stock at time  $T$ .

**Outright purchase:**

you buy and receive the stock

## Outright purchase:

To pay and receive the stock immediately and hold it up to time  $T$

**Forward Contract** You enter at time 0 into a fwd. contract pay for the stock at time  $T$   
 $F_{0,T}$

|                 | Outright | Forward   |
|-----------------|----------|-----------|
| Pay at time 0   | $S_0$    | 0         |
| Pay at time $T$ | 0        | $F_{0,T}$ |

- $S_0$  = Spot price
- $F_{0,T}$  = Fwd Price
- $i$  = risk free

- I know I will have to pay  $F_{0,T}$  at  $T$
- so I can invest a specific amount of money at time 0 so at time  $T$  I pay  $F_{0,T}$

$$X \longrightarrow X(1+i)$$

$$X(1+i)^{-1} \longrightarrow X(1+i)^{-1}(1+i) = X$$

$$F_{0,T}(1+i)^{-1} = S_0$$

$$F_{0,T} = S_0(1+i)$$

$\ln(1+i) = r :=$  continuously compounded rate

$\ln(1+r) = r :=$  continuously compounded rate

$$F_0 e^{-r} = S_0$$

$$\hookrightarrow F_{0T} = S_0 e^{rT}$$

$$S_0 = 100$$

$$T = 1 \text{ year} = 1$$

$$r = 5\%$$

Assume this formula does not apply..

$$F_{0T} = 107$$

$$S_0 e^{rT} = 105.13$$

$$F_{0T} > S_0 e^{rT}$$

- borrow \$100 at  $r$ , at  $T$  year will owe  $100e^{rT}$
  - Let's buy the stock and pay \$100
- $\Rightarrow$  you have the stock at time 0

- let's enter a short position on the  $F_{0T}$  contract

$\hookrightarrow$  that means that at time  $T$

I will deliver stock to the party

I will receive  $F_{0T} = 107$

I need to pay my loan of  $100 e^{5\%} = 105.13$

$$\text{Net profit} = 107 - 105.13 = 1.87$$

$$F_{0T} = S_0 e^{rT}$$

# Forward on a stock with discrete dividend as

|          | Method #1: Outright purchase | Method 2: Forward |
|----------|------------------------------|-------------------|
| Pay at 0 | $S_0$                        | 0                 |
| Pay at T | 0                            | $F_{0,T}$         |

- The stock will pay a dividend of \$1 at  $T/2$
- I can invest the dividend into  $r$  from  $T/2$  to  $T$

at  $T$   $S_T + \text{Div} e^{rT/2}$

- On the 2nd Method you will not receive the dividend

$$F_{0,T} = S_0 e^{rT} - \text{Cum Value (Div)}$$

$$F_{0,T} = S_0 e^{rT} - \$1 e^{rT/2}$$

## Forward on stocks / stock index with continuous dividends

Discrete case  $\longrightarrow$  Continuous Case

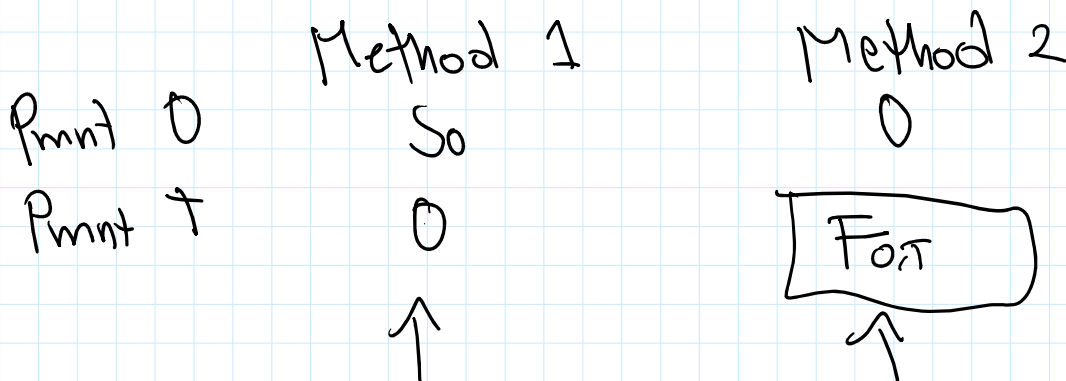
- which rate should I earn on the stock so the final amount equals to the value of the stock plus the dividend invested?

- Continuous dividend rate:  $\delta$

- Assume you can reinvest the continuous dividend rate  $\delta$  on the stock to buy more shares of it.

- After  $T$  the investor should have more shares of the stock

-  $S_0 e^{\delta T}$   
 $\uparrow$  # of shares at  $T$   
 after reinvesting



$$e^{\delta T} F_{0,T} = S_0 e^{rT}$$

$$F_{0,T} = S_0 e^{rT} e^{-\delta T}$$

$$= \underbrace{S_0 e^{(r-s)T}}_1$$