Derivatives

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Introduction to Derivatives

Most simple types…

Forward Contract:

- An agreement to buy or sell an asset at a certain **time in the future** and for a certain **price.**
- This type of derivatives are traded on the OTC market.
- The party that agrees to buy the asset at some point in the future for a pre specified price is said to assume a **long position.**
- The party that agrees to sell the asset at some point in the future for a pre specified price is said to assume a **short position.**

Example:

- Today is August 15
- A food company (party A) agrees to buy **1,000 bushels of wheat** from a farmer (party B**) 3 months from now**.
- They both agree a price of **$5 per bushel (forward price)**.
- Three months after, the current price of the bushel at that time is **$6 (spot price)**.
- The farmer delivers 1,000 bushels and receives $5,000 in total from the food company.
- **Who won and who lose money in this situation?**

Example:

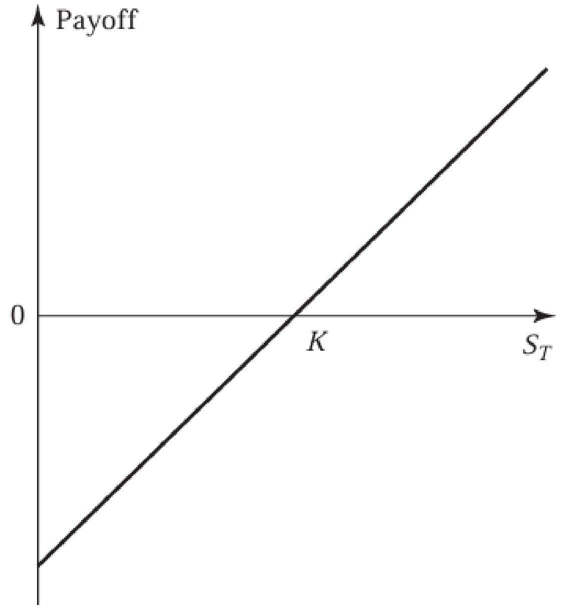
- The buyer saved $1,000 because if it wasn't for the forward contract, he would've pay $6,000 for the entire lot but he only paid $5,000.

- As the price of the wheat would've keep going up, the food company would've saved more and more.

- The difference between the price of the wheat at expiration of the contract S_T, and the agreed price (delivery price) K, represents the **payoff of the forward contract** at that time F_T for the counterparty with the **long position.**

- **Forward Contract Long Position:**
  $F_T = S_T - K$

Payoff Function:



Example:

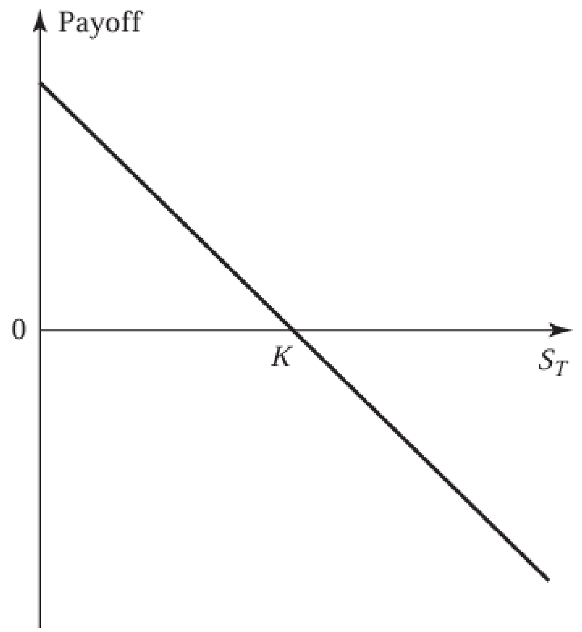
- The farmer lost the opportunity of selling at $6 and he instead agreed on selling its product on a lower price, thus losing an extra $1000.

- As the price of the wheat would've keep going up, the farmer would've lost the opportunity of charging more.

- The difference between the agreed price (delivery price) $\boldsymbol{K}$ and the price of the what at expiration of the contract $\boldsymbol{S_T}$ represents the **payoff of the forward contract** at that time $\boldsymbol{F_T}$ for the counterparty with the **short position.**

- **Forward Contract Long Position:**
  $\boldsymbol{F_T = K - S_T}$

Payoff Function:



Derivatives
Pricing Forwards

- **Forward Price of a stock that pays no dividends:**

$F_{t,T} :=$ Forward price of the underlying at time t with expiration date on T $(t < T)$.

$S_t :=$ Spot price of the underlying at time t.

$r :=$ Risk-free rate.

Two Methods for Owning Non-Dividend Paying Stock at Time T

	Method #1: *Outright purchase: Buy stock at time 0 and hold it to time T*	Method #2: *Forward contract: Buy forward on stock at time 0 and hold it to time T*
Payment at time 0	S_0	0
Payment at time T	0	$F_{0,T}$

$$F_{t,T} = S_t e^{r(T-t)}$$

- **Forward Price of a stock that pays descrete dividends:**

$\boldsymbol{F_{t,T}}$:= Forward price of the underlying at time $\boldsymbol{t}$ with expiration date on $\boldsymbol{T}$ ($\boldsymbol{t} < \boldsymbol{T}$).

$\boldsymbol{S_t}$:= Spot price of the underlying at time $\boldsymbol{t}$.

$\boldsymbol{r}$:= Risk-free rate.

Two Methods for Owning Non-Dividend Paying Stock at Time T

	Method #1: *Outright purchase: Buy stock at time 0 and hold it to time T*	Method #2: *Forward contract: Buy forward on stock at time 0 and hold it to time T*
Payment at time 0	S_0	0
Payment at time T	0	$F_{0,T}$

$$\boldsymbol{F_{t,T} = S_t e^{r(T-t)} - CumValue(Dividends)}$$

• Forward Price of a stock that pays continuous dividends:

$F_{t,T} :=$ Forward price of the underlying at time t with expiration date on T ($t < T$).

$S_t :=$ Spot price of the underlying at time t.

$r :=$ Risk-free rate.

$\mathbf{d} :=$ Dividend yield.

Two Methods for Owning $e^{\delta T}$ Shares of Continuous Dividend Paying Stock Index at Time T

	Method #1: *Outright purchase: Buy stock index at time 0 and hold it to time T*	Method #2: *Forward contract: Buy $e^{\delta T}$ forwards on stock index at time 0 and hold it to time T*
Payment at time 0	S_0	0
Payment at time T	0	$e^{\delta T} F_{0,T}$

$$F_{t,T} = S_t e^{(r-d)(T-t)}$$

Forward Price of currencies:

- Each currency has its own risk-free rate.
- There is a local currency and a foreign currency.
- The risk-free rate of the foreign currency r_f plays the role of a continuously compounded dividend on a stock.

$x_t :=$ The exchange rate (local/foreign. Ex. USD/EUR).

$r :=$ The risk-free rate in dollars.

$r_f :=$ The risk-free rate in euros.

Two methods to have, for example, euros at T:

1. Buy $e^{-r_f(T-t)}$ euros at t and accumulate them to time time T at r_f
2. Buy a forward for 1 euro at time t and pay $F_{t,T}$ at T

$$F_{t,T} = x_t e^{(r-r_f)(T-t)}$$

Derivatives
Synthetic Forwards

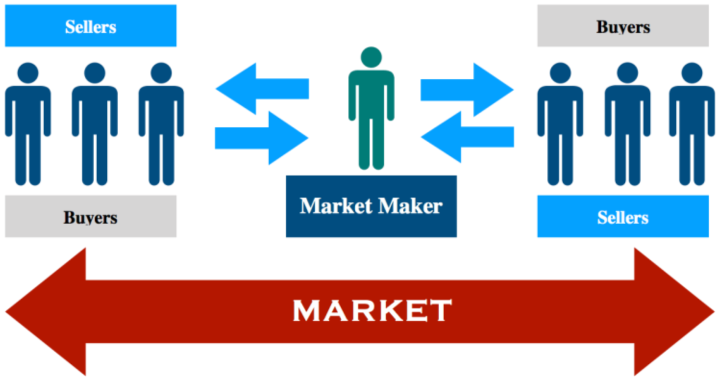
- It is possible to use stock and bonds to create a combination that has the same payments as a forward.

Synthetic long forward: Buy a stock and borrow the price of the stock.

Note: **Borrowing money is the same as selling a zero-coupon bond** whose price is the same as the price of the stock and with the same maturity or expiration as the forward (**lending money is the same as buying that bond**)

Synthetic short forward: Sell the stock and buys the same zero-coupon bond.

- **A market maker** is a firm or individual that provides liquidity to a financial market by actively quoting both buy (bid) and sell (ask) prices for a security, ensuring a continuous two-way market and enabling investors to trade quickly at a fair price. They profit from the difference between the buy and sell prices (the bid-ask spread)
- When an investor goes long (short) on a forward, usually, he or she buys (sell) the instrument from (to) a market maker.
- So, the market maker is going short (long). The contrary position.



Options:

- A contract giving the **buyer the right, but not the obligation**, to **buy** or **sell** an asset at a predetermined price **(strike price)** on or before a certain date **(expiration date)**.
- The **seller (writer) has the obligation** to fulfill the contract if the buyer chooses to exercise the option.
- Traded both on exchanges (standardized) and OTC markets (customized).

Types of Options:

1. **Call Option** → Right to **buy** the underlying asset.
2. **Put Option** → Right to **sell** the underlying asset.

Differences from Futures and Forwards:

- Buyer is not obliged to transact (has a choice).
- Involves an **upfront premium** paid by the buyer to the seller.

Derivatives
Calls and Puts Options

Call:

- A call option allows one to put an upper bound on a future price for buying an asset.

- A call option provides that, at a future date, called **the expiry date**, the owner of the option may; but is not required to, purchase the underlying asset at a fixed price agreed upon in advance **(strike price)**.

Example:

1. **An investor purchases a 6-month call option on a stock with strike price 50. The investor pays 3.35 as premium (option price). At the end of 6 months, the price of the stock is 60.**

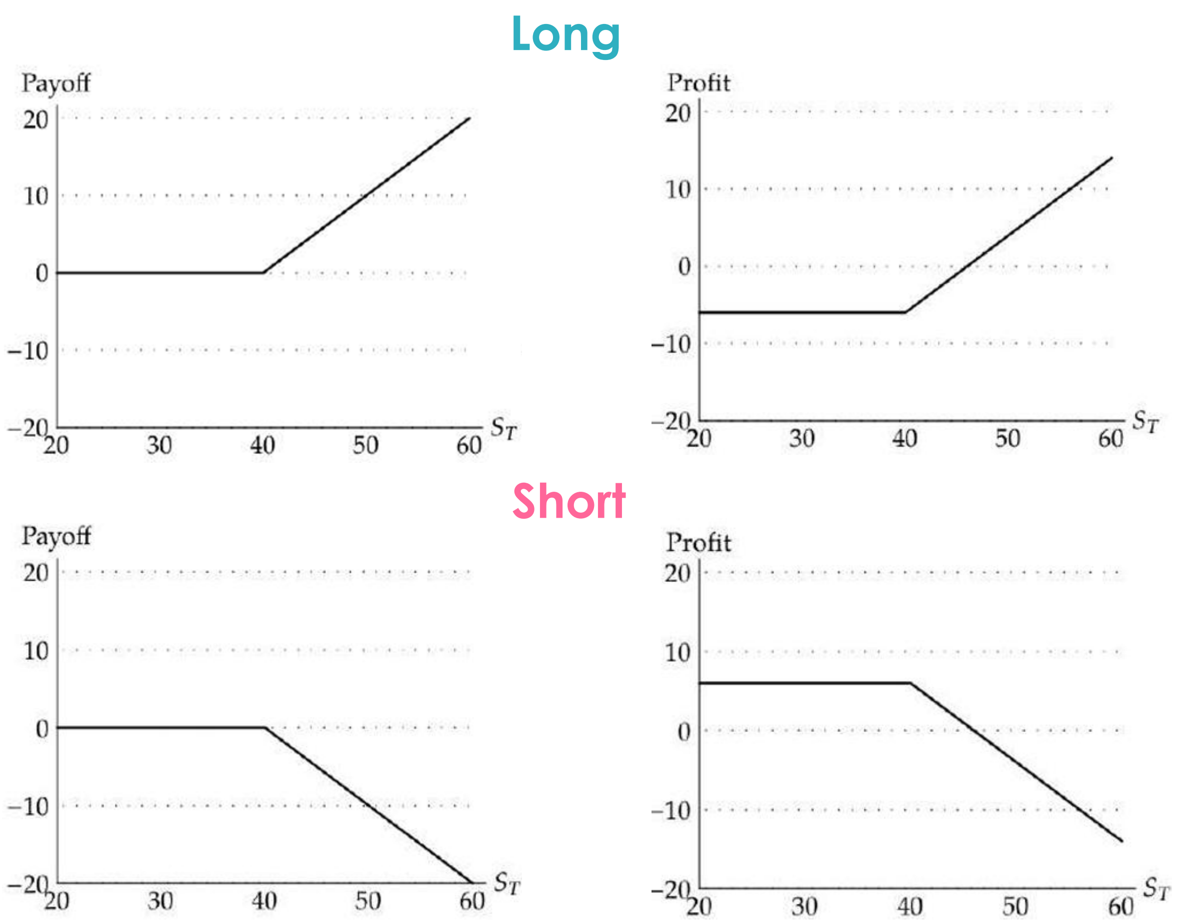
2. **An investor purchases a 6-month call option on a stock with strike price 50. The investor pays 3.35 as premium (option price). At the end of 6 months, the price of the stock is 40.**

Call:

- There are two parties when a call option is traded.

- The buyer (long on a call)
- The seller/writer (short on a call)

Payoff and Profit:



Put:

- A put option allows one to put a lower bound on the sales proceeds of an asset.
- A put option is an agreement that, at a future date, called **the expiry date**, the owner of the option may; but is not required to, sell an asset at a fixed price agreed upon in advance **(strike price)**.

Example:

1. **An investor purchases a 6-month put option on a stock with strike price 50. The investor pays 3.35 as premium (option price). At the end of 6 months, the price of the stock is 40.**

2. **An investor purchases a 6-month put option on a stock with strike price 50. The investor pays 3.35 as premium (option price). At the end of 6 months, the price of the stock is 40.**

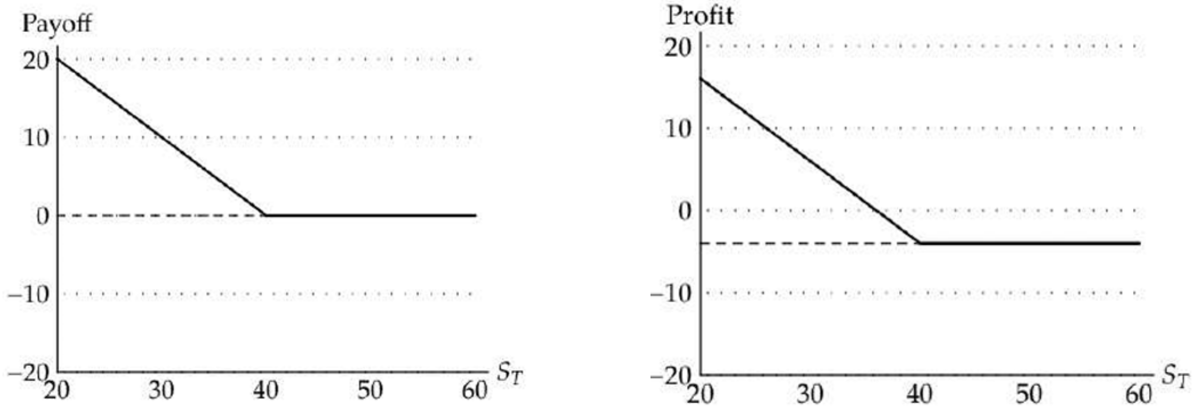
Put:

- There are two parties when a put option is traded.

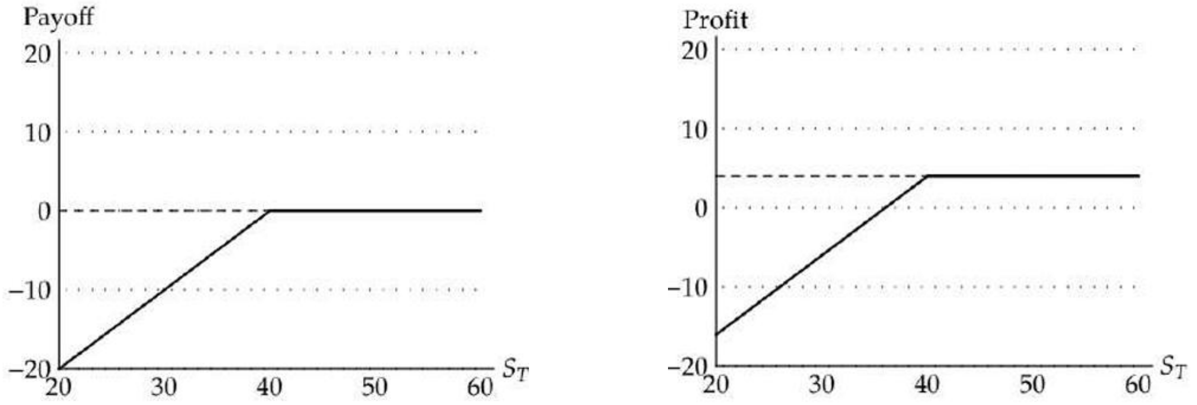
- The buyer (long on a put)
- The seller/writer (short on a put)

Payoff and Profit:

Long



Short

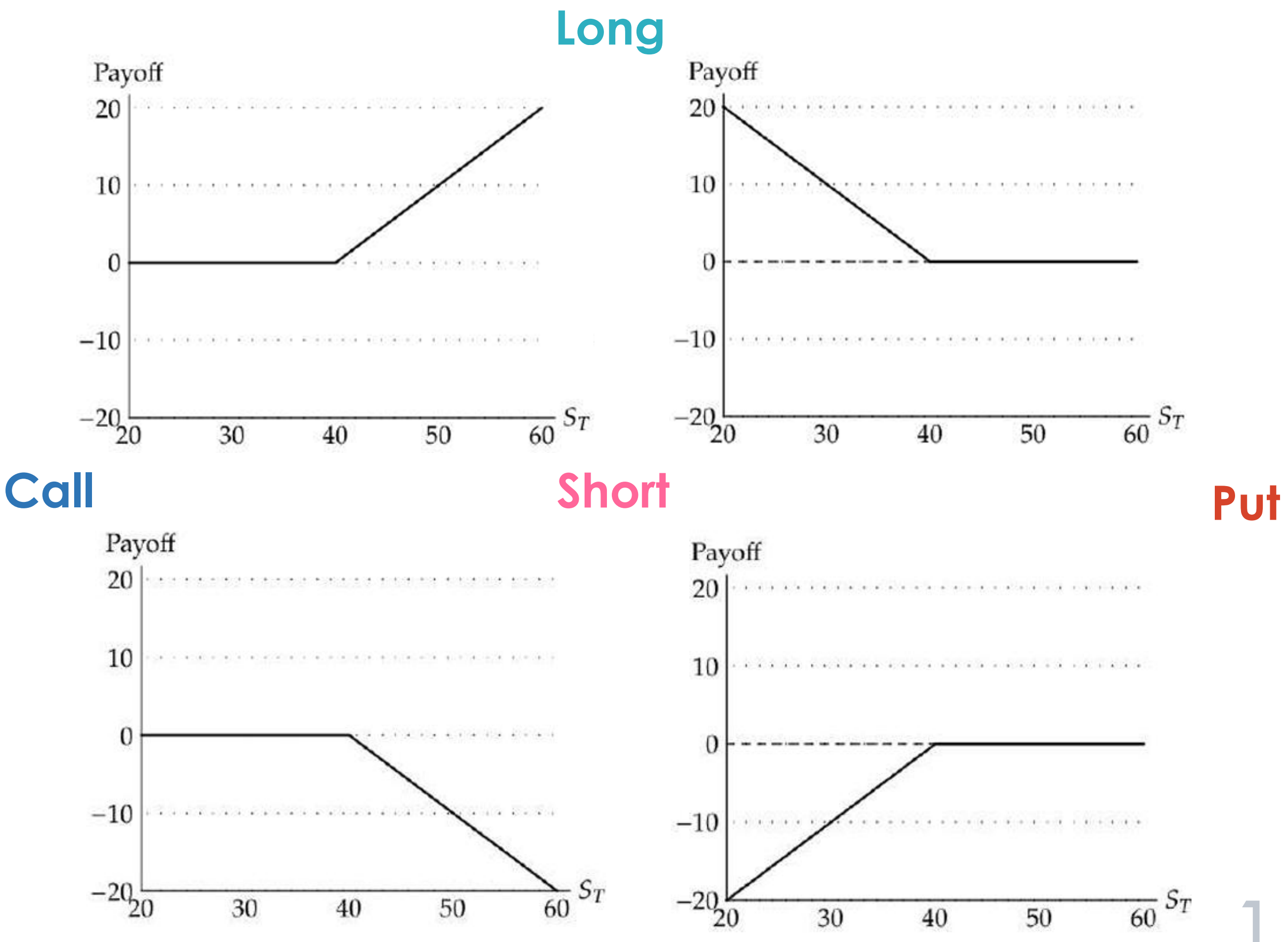


Profit & Payoff:

- **Long Call Option Pay-Off:**
  $$max(S_T - K, 0)$$
- **Long Put Option Pay-Off:**
  $$max(K - S_T, 0)$$

- **Long Call Option Profi:**
  $$max(S_T - K, 0)\ - Call\ Price$$
- **Long Put Option Profit:**
  $$max(K - S_T, 0) - Put\ Price$$

Diagram:



Specifications &Terminology:

With respect to the strike price $\boldsymbol{K}$:

- An option is said to be **at-the-money** If at a given point, the underlying price is equal to the strike price ($\boldsymbol{K} = \boldsymbol{S_t}$).
- If at a given point, exercising the option would result in a positive cash flow, it is said that the option is **in-the-money.**
- If at a given point, exercising the option would result in a positive cash flow, it is said that the option is **out-of-the-money.**

With respect to the expiration moment $\boldsymbol{T}$:

- ***European Option***: An option that can only be exercised at maturity/expiration date.
- ***American Option***: An option that can be exercised at any time until maturity.
- ***Bermudan Option***: An option that can be exercised only at specified periods.

Option Strategies

- Assuming the underlying is a stock price, there are 4 potential direct strategies in which an investor can buy/sell options and stocks.

1. Long put, long stock **<- *Floor***
2. Long call, short stock **<- *Cap***
3. Short call, long stock **<- *Covered Call***
4. Short put, short stock **<- *Covered Put***

- In each strategy, the option offsets, the stock:
  - If we buy the stock, the option is short in the stock
  - If we sell the stock the option is long in the stock

- First two are insurance strategies.

- Writing an option without protection is called ***naked writing***

Option Strategies

- Assuming the underlying is a stock price, there are 4 potential direct strategies in which an investor can buy/sell options and stocks.

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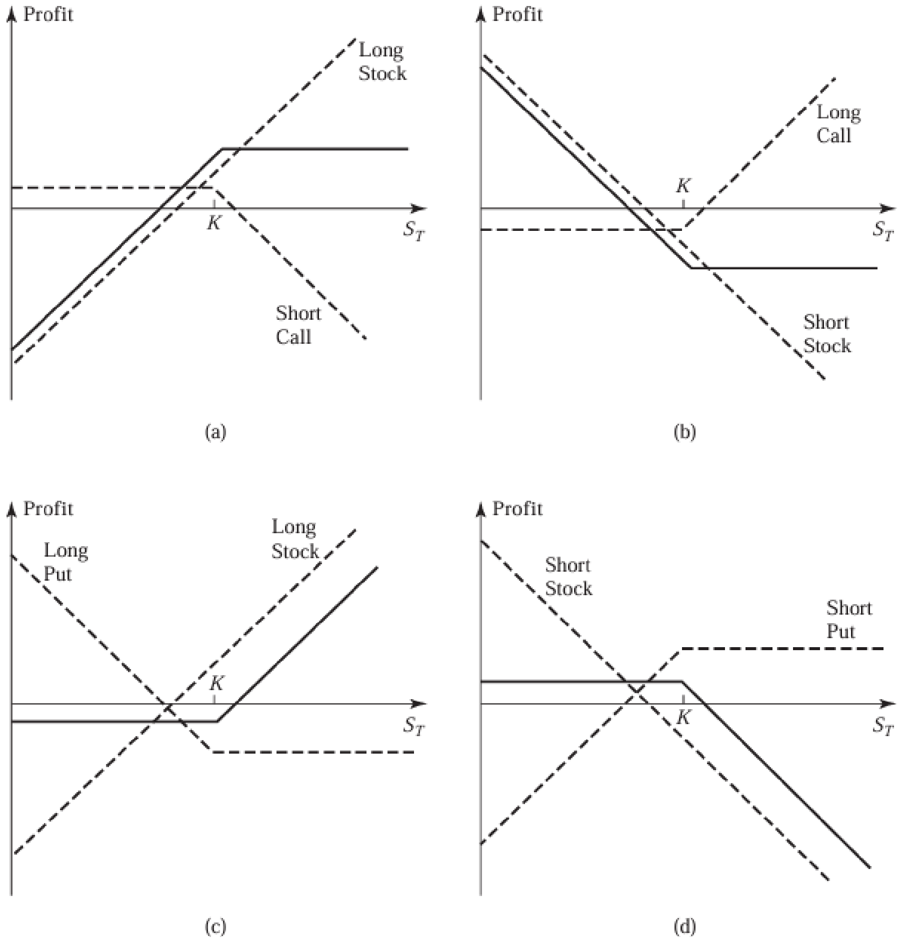
Figure 12 .1 Profit patterns (a) long position in a stock combined with short position in a call; (b) short position in a stock combined with long position in a call; (c) long position in a put combined with long position in a stock; (d) short position in a put combined with short position in a stock.



Option Strategies

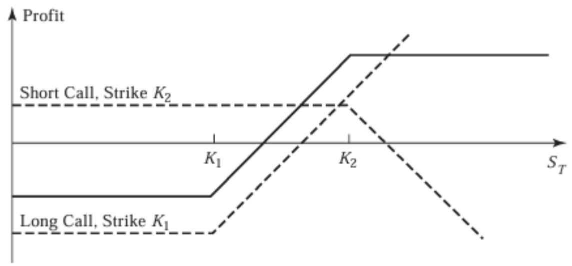
1. *Bull Spreads*
2. *Bear Spreads*
3. *Collars*

- Option strategies involving two options may involve:
    1) Buying an option and selling an option of the same kind (both calls or both puts).
    2) Buying an option of one kind and selling one of the other kind (buy/sell call sell/buy put).
    3) Buying or selling two options of different kind.

Spreads: *Buying and selling an option of the same kind but with different strikes:*

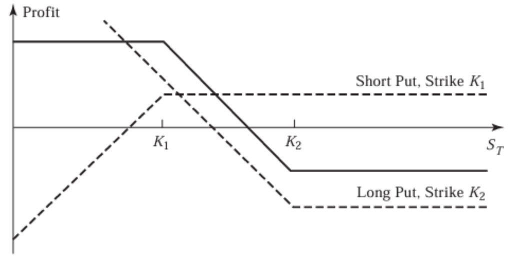
Bull Spread (pays if the stock goes up) :

Figure 12.2 Profit from bull spread created using call options.



Bear Spread (pays if the stock goes down) :

Figure 12.4 Profit from bear spread created using put options.



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- ## Put-Call parity :

Portfolio 1: $\boldsymbol{C(S,K,T) + Ke^{-rT} = Call\ option + Present\ Value\ of\ K}$

Portfolio 2: $\boldsymbol{P(S,K,T) + S_0 = Put\ option + One\ Share\ of\ stock\ S\ at\ t_0}$

Both portfolios end with same payoff at maturiy:

$\boldsymbol{S_T > K}$:

Portfolio 1: Investing $\boldsymbol{Ke^{-rT}}$ will generate $\boldsymbol{K}$ after $\boldsymbol{T}$ and Call will be exercise: $\boldsymbol{max\{S_T - K, 0\} + K = S_T}$

Portfolio 2: Owning stock $\boldsymbol{S}$ gives an investor $\boldsymbol{S_T}$ after $\boldsymbol{T}$ and Put will NOT be exercise: $\boldsymbol{max\{K - S_T, 0\} + S_T = S_T}$

$\boldsymbol{K > S_T}$:

Portfolio 1: Investing $\boldsymbol{Ke^{-rT}}$ will generate $\boldsymbol{K}$ after $\boldsymbol{T}$ and Call will NOT be exercise: $\boldsymbol{max\{S_T - K, 0\} + K = K}$

Portfolio 2: Owning stock $\boldsymbol{S}$ gives an investor $\boldsymbol{S_T}$ after $\boldsymbol{T}$ and Put will be exercise: $\boldsymbol{max\{K - S_T, 0\} + S_T = K}$

- ## Put-Call parity :

Because both portfolio have the same value at the end, no matter what, they should be worth the same at t_0

$$\boldsymbol{C(S,K,T) + Ke^{-rT} = P(S,K,T) + S_0}$$
or
$$\boldsymbol{C(S,K,T) - P(S,K,T) = S_0 - Ke^{-rT}}$$

This is called the Put- Call Parity and it shows the relationship between european call and put prices.

Another way to derive the formula is:

1. Buying a call and selling a put and time 0 and pay $\boldsymbol{K}$ at time $\boldsymbol{T}$. No matter what, you pay strike and receive stock.
2. Enter a forward agreement to buy $\boldsymbol{S}$ at $\boldsymbol{T}$ and pay $\boldsymbol{F_{0,T}}$ (the price of the forward agreement).

So….

$$\boldsymbol{C(S,K,T) - P(S,K,T) = e^{-rT}(F_{0,T} - K)}$$

$* \boldsymbol{e^{-rT}F_{0,T} = Prepaid\ Forward}$

- **On Non-Dividend Stocks**

$F_{t,T} := S_0 e^{rT}$

So Put-Call parity reads:

$$C(S,K,T) - P(S,K,T) = e^{-rT}(S_0 e^{rT} - K)$$

$$C(S,K,T) - P(S,K,T) = S_0 - Ke^{-rT}$$

- **On Dividend Stocks**

$F_{t,T} := S_0 e^{(r-\delta)T}$

So Put-Call parity reads:

$$C(S,K,T) - P(S,K,T) = e^{-rT}(S_0 e^{(r-\delta)T} - K)$$

$$C(S,K,T) - P(S,K,T) = S_0 e^{-\delta T} - Ke^{-rT}$$

- A non dividend paying stock has a price of 40. A European call option allows buying the stock for 45 at the end of 9 months. The continuously compounded risk-free rate is 5%. The premium of the call option is 2.84. Determine the premium of a European put option allowing selling the stock for 45 at the end of 9 months.

Derivatives

Synthetic Stocks and Treasuries

- **Since the put-call parity includes terms for stock S_0 and cash K, we can create a synthetic stock with an appropriate cmbination of options and lending.**

- **With continuous dividends the formula reads:**

$$C(S,K,T) - P(S,K,T) = S_0 e^{-\delta T} - K e^{-rT}$$

$$S_0 = e^{\delta T}(C(S,K,T) - P(S,K,T) + -K e^{-rT})$$

For example, suppose the risk-free rate is 5%. We want to create an investment equivalent to a stock with continuous dividend rate of 2%. We can use any strike price and any expiry; let's say 40 and 1 year…

Derivatives
Synthetic Stocks and Treasuries

- **To create a treasury we can rearrange the equation:**

$$Ke^{-rT} = S_0 e^{-\delta T} - C(S, K, T) + P(S, K, T)$$

For example, suppose the risk-free rate is 5%. We want to create an investment equivalent to a stock with continuous dividend rate of 2%. We can use any strike price and any expiry; let's say 40 and 1 year…

Derivatives
Option Bounds

- The following properties are satisfied by standard American and European call and put options:

1. An American option can be exercised at any time, whereas a European option can only be exercised at expiry. Since the American option gives you all the rights a European option gives you, and more, it must be worth at least as much as a European option with the same strike price and expiry.

2. A call option cannot be worth more than the underlying stock. At the very best, the option allows you to buy the stock for the strike price. If you buy the stock instead of buying the call, you'll have the stock without paying anything. So given the choice of buying a call option or buying the stock at the same price, you'd clearly prefer the latter.

3. A European call option cannot be worth more than the prepaid forward price of the stock, since the buyer cannot get the stock until expiry. If the stock pays continuous dividends, then the upper bound for the price of a European call option is Se^{-dt}

4. Similarly, a put option cannot be worth more than the strike price, and a European put option cannot be worth more than Ke^{-rt}.An option must be worth at least 0, since no negative payoff is possible.

5. A European option is worth at least as much as implied by put-call parity assuming the other option is worth 0.Therefore, a European call option is worth at least as much as implied by put-call parity to a put option that is worth 0.

6. A European put option is worth at least as much as implied by put-call parity to a call option that is worth 0.