# Pricing Forwards & Futures

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$F_{t,T}$ = Forward Price of and underlying at time $t$ with maturity or expiration date $= T$ $(T > t)$

$S_t$ = The price of the underlying at time $t$ (spot price)

$$F_{t,T} \sim S_t$$

1. It is possible to borrow or lend any amount of money at the risk-free rate

ex. $i :=$ anual effective rate (after 1 unit of time we are going to receive $i\%$ of our init. investment)

$$\underbrace{K}_{\text{Initial Investment}} \underbrace{(1 + i\%)}_{}$$

Initial Investment

Final amount

2. There are no transaction costs

3. Arbitrage is impossible

- **Arbitrage**: set of transactions that represents 0 cost, no possibility of Loss, and a possibility of making profit.

## Forwards on Stocks (no dividends)

- There are 2 ways to own a stock at time T.

**Outright purchase**:

To buy and receive the stock

## Outright purchase:

To pay and receive the stock immediately and hold it up to time $T$

## Forward Contract

You enter at time 0 into a fwd. contract pay for the stock at time $T$

$F_{0,T}$

| | Outright | Forward |
|---|---|---|
| Pmnt at time 0 | $S_0$ | 0 |
| Pmnt at time T | 0 | $F_{0,T}$ |

- $S_0$ = Spot price
- $F_{0,T}$ = Fwd Price
- $i$ = risk free

- I know I will have to pay $F_{0,T}$ at $T$
- so I can invest a specific amount of money at time 0 so at time $T$ I posses $F_{0,T}$

$$x \longrightarrow x(1+i)$$

$$x(1+i)^{-1} \longrightarrow x(1+i)^{-1}(1+i) = x$$

$$\boxed{\begin{array}{c} F_{0,T}(1+i)^{-1} = S_0 \\ F_{0,T} = S_0(1+i) \end{array}}$$

$\ln(1+i) = r$ := continuously compounded rate

$\ln(1+i) = r :=$ continuously compounded rate

$\rightarrow F_0 e^{-r} = S_0$

↳ $F_{0,T} = S_0 e^{rT}$

$S_0 = 100$

$T = 1 \text{ year} = 1$

$r = 5\%$

Assume this formula does not apply..

$F_{0,T} = 107$

$S_0 e^{rT} = 105.13$

$F_{0,T} > S_0 e^{rT}$

- borrow \$100 at $r$, at $T$ you will owe $100 e^{rT}$
- Let's buy the stock and pay \$100

$\Rightarrow$ y have the stock at time 0

- let's enter a short position on the $F_{0,T}$ contract

↳ that means that at time $T$

I will deliver stock to the party

I will receive $F_{0,T} = 107$

I need to pay my loan of $100 e^{5\%} = 105.13$

Net profit: $107 - 105.13 = 1.87$

$$\boxed{F_{0,T} = S_0 e^{rT}}$$

# Forward on a stock with discrete dividends

| | Method #1: Outright purchase | Method 2: Fwrl |
|---|---|---|
| Pmnt 0 | $S_0$ | 0 |
| Pmnt T | 0 | $F_{0,T}$ |

- The stock will pay a dividend of \$1 at $T/2$
- I can invest the dividend into r from $\frac{T}{2}$ to T

0 T

at t $S_T + Div\, e^{rT/2}$

→ On the 2nd Method you will not receive the dividend

$$F_{0,T} = S_0 e^{rT} - \text{CumValue}(Div)$$

$$F_{0,T} = S_0 e^{rT} - \$1 e^{rT/2}$$

# Forward on stocks / stock index with continuous dividends

Discrete case ⟶ Continuous Case

- which rate should I earn on the stock so the final amount equates to the value of the stock plus the dividend invested?

- Continuous dividend rate: $\delta$
- Assume you can reinvest the continuous dividend rate $\delta$ on the stock to buy more shares of it.

- After T the investor should have more shares of the stock

- $S_0 e^{\delta T}$

  # of shares at T after reinvesting

| | Method 1 | Method 2 |
|---|---|---|
| Pmnt 0 | $S_0$ | 0 |
| Pmnt T | 0 | $\boxed{F_{0,T}}$ |

$$\boxed{e^{\delta T} F_{0,T} = S_0 e^{rT}}$$

$$F_{0,T} = S_0 e^{rT} e^{-\delta T}$$

$$= S_0 e^{(r-\delta)T}$$