

Pricing Forwards & Futures

Saturday, August 30, 2025 10:19 AM

$F_{t,T}$ = Forward Price of an underlying at time t with maturity or expiration date $= T$ ($T > t$)

S_t = The price of the underlying at time t (spot price)

$$F_{t,T} \sim S_t$$

1. It is possible to borrow or lend any amount of money at the risk-free rate

Px. i := annual effective rate (after 1 unit of time we are going to receive $i \cdot$ of our init. investment)

$$\frac{K(1+i \cdot)}{I}$$

Initial Investment Final amount

2. There are no transaction costs

3. Arbitrage is impossible

- **Arbitrage**: set of transactions that represents 0 cost, no possibility of loss, and a possibility of making profit.

Forwards on Stocks (no dividends)

• There are 2 ways to own a stock at time T .

Outright purchase:

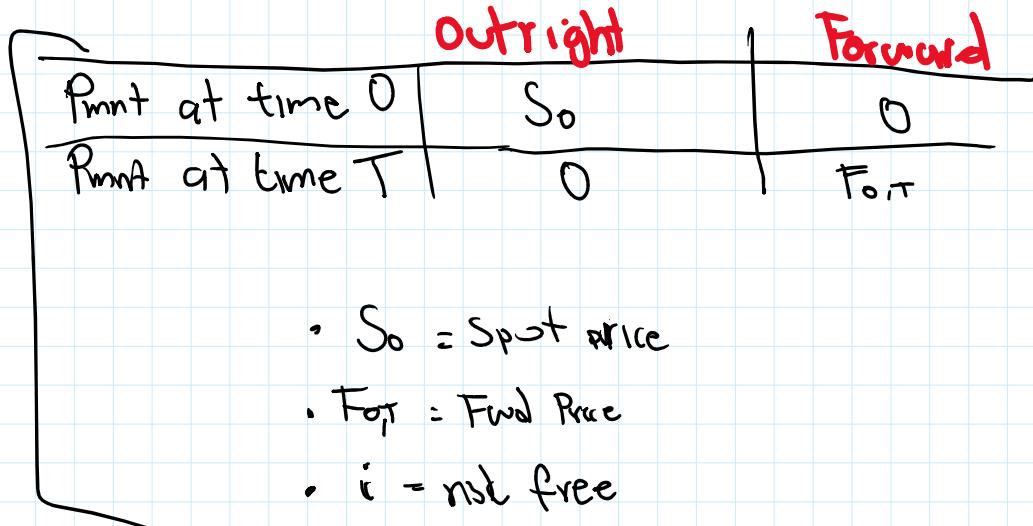
The buy and own the asset

Outright purchase:

To pay and receive the stock immediately and hold it up to time T

Forward Contract You enter at time 0 into a fwd. contract pay for the stock at time T

$$F_{0,T}$$



- S_0 = Spot price
- $F_{0,T}$ = Fwd Price
- i = risk free

- I know I will have to pay $F_{0,T}$ at T
- So I can invest a specific amount of money at time 0 so at time T I pass $F_{0,T}$

$$x \longrightarrow x(1+i)$$

$$x(1+i)^{-1} \longrightarrow x(1+i)(1+i) = x$$

$$F_{0,T}(1+i)^{-1} = S_0$$

$$F_{0,T} = S_0(1+i)$$

$$\ln(1+i) = r := \text{continuously compounded rate}$$

$\ln(1+r) = r$:= continuously compounded rate

$$F_0 e^r = S_0$$

$$\hookrightarrow F_T = S_0 e^{rT}$$

$$S_0 = 100$$

$$T = 1 \text{ year} = 1$$

$$r = 5\%$$

Assume this formula does not apply.

$$\overline{F_T = 107}$$

$$\overline{S_0 e^{rT} = 105.13}$$

$$\hookrightarrow F_T > S_0 e^{rT}$$

- borrow \$100 at r , at T you will owe $100e^{rT}$
- Let's buy the stock and pay \$100

\Rightarrow you have the stock at time 0

- let's enter a short position on the F_T contract
 \hookrightarrow that means that at time T

I will deliver stock to the party

I will receive $F_T = 107$

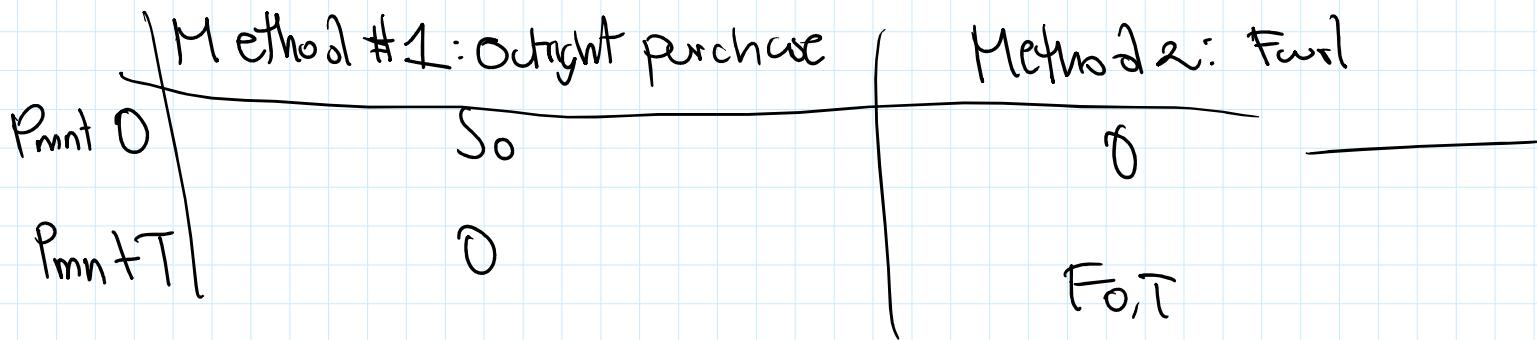
I need to pay my loan of $100 e^{rT} = 105.13$

$$\text{Net profit: } 107 - 105.13 = 1.87$$

$$F_T = S_0 e^{rT}$$



Forward on a stock with discrete dividend



- The stock will pay a dividend of \$1 at T
- I can invest the dividend into r from $\frac{T}{2}$ + $0 \rightarrow T$

$$qT + S_T + \text{Div } e^{rT/2}$$

→ On the 2nd Method you will not receive the dividend of

$$F_{0T} = S_0 e^{rT} - \text{CumValue(Div)}$$

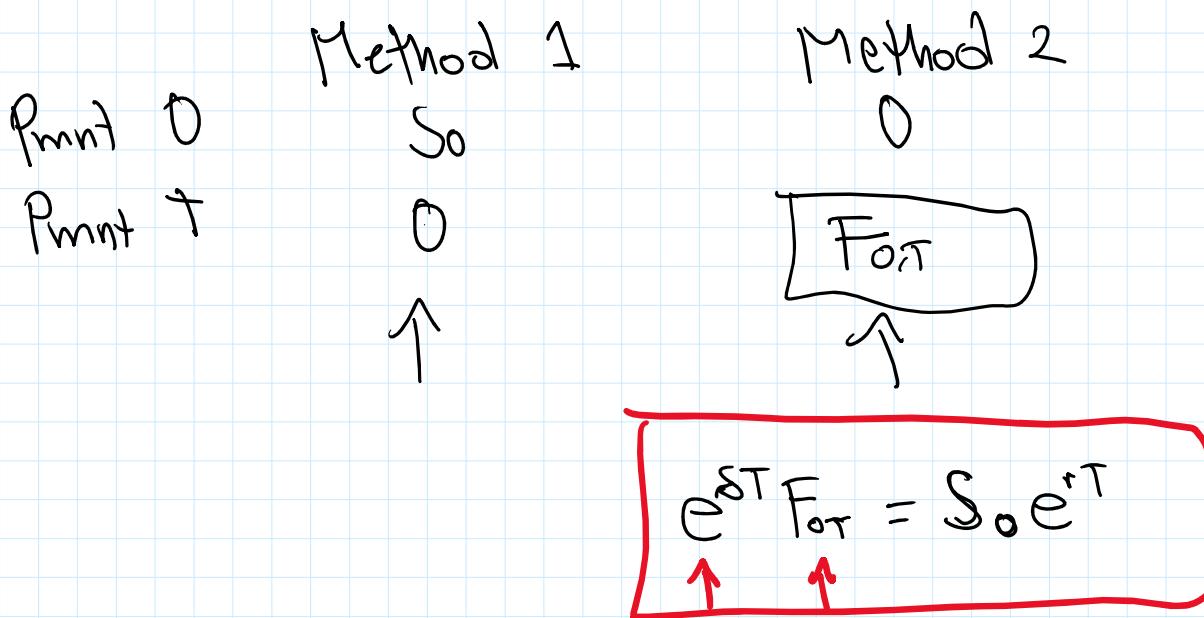
$$F_{0T} = S_0 e^{rT} - \$1 e^{rT/2}$$

Forward on stocks / stock index with continuous dividends

Discrete case \rightarrow Continuous Case

- which rate should I earn on the stock so the final amount equals to the value of the stock plus the dividend invested?

- Continuous dividend rate: δ
- Assume you can reinvest the continuous dividend rate δ on the stock to buy more shares of it.
- After T the investor should have more shares of the stock
- $S_0 e^{\delta T}$
 \uparrow # of shares at T
 after reinvesting



$$F_{0T} = S_0 e^{rT} e^{-\delta T}$$

$$= S_0 e^{(r-s)\tau}$$