Exam

Saturday, October 25, 2025 7:34 AM

Q2. Forwards: multi-case valuation

Data: $S_0 = 95$. Horizon $T = 9\ months$. (Assume 1 year means T = 1)
Compute $F_{0,T}$ for:

i) **No dividends** $r = 6\%$(continuous compounding)
ii) **Continuous dividends**, $r = 6\%$(cont. comp.), $d = 2\%$
iii) **FX forward** USD/MXN: $x_0 = 18.20$, $r_{MXN} = 10\%$ (cont.), $r_{USD} = 4\%$(cont.), $T = 0.5$.
iv) If in (i) the market forward is $F^{mkt} = 100.6$, is there an **arbitrage**? Describe the trade.

i) $F_{0,T} = S_0 e^{rT} = 95 e^{0.06 \times \frac{3}{4}} \approx \underline{99.37}$

ii) $F_{0,T} = S_0 e^{(r-\delta)T} = 95 e^{(0.06-0.02)\frac{3}{4}} = 97.89$

iii) $F_{0,T} = X_0 e^{(r_d - r_f)T} = 18.20 e^{(0.10-0.04)\frac{1}{2}} = 18.75$

iv) $F_{0,T}^{mkt} = 100.6$

the "fair" value of the forward is $F^* = 99.37$ so

$F^{mkt} > F^*$

Today: Borrow 95 to buy asset S_0
and enter a short forward agreement
at $F^{mkt} = 100.6$

At time T: Deliver the asset S_0, we recieve 100.6
and we pay the loan $95e^{0.06(\frac{3}{4})} = 99.37$
we end up with a profit of $100.6 - 99.37 = 1.22$

Q3. Forwards: payoff and profit

Long forward with $K = 100$; maturity in 6 months. Compute **payoff and profit** for **long** and **short** when $S_T \in \{90, 100, 115\}$. Present in a table.

	Long Forward Payoff: $S_T - K$	Short Forward Payoff $K - S_T$
90	$90 - 100 = -10$	$+10$
100	0	0
115	$115 - 100 = +15$	-15

Q4. Put–call parity

Non-dividend-paying stock: $S_0 = 40$. **European call:** $K = 45, T = 0.75$ years, premium $C_0 = 2.84$. **Rate** $r = 5\%$(cont.).
a) Use **put–call parity** to **compute** P_0.
b) If the put trades at 6.80, is parity **violated**? What **arbitrage** would you implement?

$C - P = S_0 - Ke^{-rT}$

$P = C - S_0 + Ke^{-rT}$

a) $= 2.84 - 40 + 45e^{-(0.05)\frac{3}{4}} = \underline{6.18}$

b) $P^{mkt} = 6.8$

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(Sell the expensive put and buy the cheap replicate)

Today: Sell put for 6.8 and

buy a call (-2.84), short stock (40), buy bond $(-45e^{-0.05(\frac{3}{4})} \approx 43.34)$

At T: S_T, Bond, Call will net out against the put

so $6.8 - 6.18 = 0.62$

Q5. Option strategies

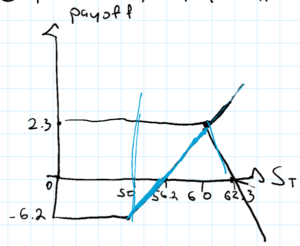
a) **Bull call spread:** Buy $C(K_1 = 50)$ at 6.20, sell $C(K_2 = 60)$ at 2.30.

- Draw the **payoff at maturity** and report **max gain, max loss, and breakeven.**

b) **Bear put spread (debit spread):** Let $S_0 = 55$. Buy **put** $K_H = 60$ at **4.50** and sell **put** $K_L = 50$ at **1.80** (same maturity).

- i) **Net cost** today.
- ii) **Max gain, max loss,** and **breakeven.**
- iii) Describe the **payoff shape** at maturity.

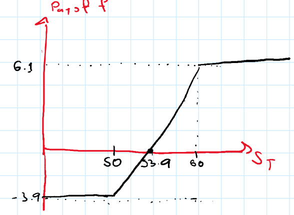
a) Buy $C(50)$ and sell $C(60)$



$\max\{62.3 - 60\} + 2.3 = 0$

we pay this

we recieve



$S_T \geq 60$

$\max\{60 - 50, 0\} - 6.2$

$-\cancel{\max\{60 - 60, 0\}}_{=0} + 2.3 = 60 - 50 - 3.9 = 6.1$

$S_T \leq 50$

$\max\{50 - 50, 0\} - 6.2 = 0 - 0 - 3.9 = -3.9$

$-\max\{50 - 50, 0\} + 2.3$

Which stock price S_T will end up with a payoff $= 0$

$S_T = 50 + 3.9$

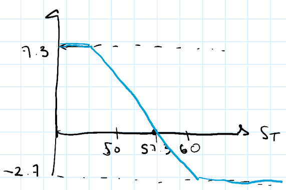
$\max\{53.9 - 50, 0\} - \max\{53.9 - 60, 0\} - 3.9 = 3.9 - 0 - 3.9 = 0$

Bear Spread with put (long put, short put)

- We buy $P(60)$ and sell $P(50)$

we recieve 1.8 and pay 4.5

Net cost $= 2.7$



$S_T = 0$

$\max\{60 - 0, 0\} - \max\{50 - 0, 0\} - 2.7 = 60 - 50 - 2.7 = 7.3$

$S_T = 50$

$\max\{60 - 50, 0\} - \max\{50 - 50, 0\} - 2.7 = 60 - 50 - 2.7 = 7.3$

Breakeven point: $\max\{60 - S_T, 0\} - \max\{50 - S_T, 0\} - 2.7 = 0$

if $S_T = 57.3$ $\underbrace{\max\{60 - 57.3, 0\}}_{2.7} - \underbrace{\max\{50 - 57.3, 0\}}_{0} - 2.7 = 0$

$S_T \geq 60$

$\max\{60 - S_T, 0\} - \max\{50 - S_T, 0\} - 2.7 = -2.7$

Q6. Synthetics & Parity

a) Show how to **replicate a stock or a bond** with options and cash using **put–call parity** (state the identity and explain each term).

b) **Parity check/no-arbitrage:** $S_0 = 52$, $K = 50$, $T = 1$ year, $r = 4\%$ (cont.), $C_0 = 9$, $P_0 = 7$.

- Does parity hold? If **not**, design an **arbitrage** (what to buy/sell **today** and how it closes at T).

$C - P = S - Ke^{-rT} \Rightarrow$

replicate stock:

$S = C - P + Ke^{-rT}$

long stock = long call, short a put and long zero-coupon bond with face value K

replicate bond:

$Ke^{-rT} = S - C + P$

b) $S_0 = 52$, $K = 50$, $T = 1$, $r = 4\%$, $C = 9$, $P = 7$

$9 - 7$ $(C - P) = 2$

$52 - 50e^{-0.04} = 3.96$ $(S - Ke^{-rT})$

Since $C - P < S - Ke^{-rT}$

Today: · long call (-9), short put (7)

· short stock (52), long bond (-48.039)

net inflow = 1.96