Actividad 1: Simulación Estocástica

Curso: TEMAS SELECTOS 1 (O25-LAT4032-1)
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Universidad: Universidad de las Américas Puebla
Fecha: 2025-08-15

Export this notebook to PDF with LaTeX using the provided `amsart_template.tplx` for a Times-like, AMS-style layout.

Command:

```
jupyter nbconvert --to pdf --template amsart_template.tplx actividad1_template.ipynb
```

Selection of exercises: *Indicate here whether you solved* ***evens*** *or* ***odds*** *only (teams max 3).*

0.1 Ejercicio 1

Enunciado

Si $x_0 = 5$ y $x_n = 2x_{n-1} \bmod 150$. Encontrar $x_1, \dots, x_{10}$.

Mathematical justification / reasoning

Write your derivations and explanations here.

[1]:
```
# Ejercicio 1 - Python code
# Write clean, commented, and reproducible code here.
# Use numpy, scipy, matplotlib as needed.
```

0.2 Ejercicio 2

Enunciado

Si $x_0 = 3$ y $x_n = (5x_{n-1} + 7) \bmod 200$. Encontrar $x_1, \dots, x_{10}$.

Mathematical justification / reasoning

Write your derivations and explanations here.

[2]:
```
# Ejercicio 2 - Python code
# Write clean, commented, and reproducible code here.
# Use numpy, scipy, matplotlib as needed.
```

0.3 Ejercicio 3

Enunciado

Aproximar por simulación:

$$\int_0^1 \exp(e^x)\, dx$$

Comparar con la respuesta exacta si es conocida.

Mathematical justification / reasoning

Write your derivations and explanations here.

[3]:
```
# Ejercicio 3 - Python code
# Write clean, commented, and reproducible code here.
# Use numpy, scipy, matplotlib as needed.
```

0.4 Ejercicio 4

Enunciado

Aproximar por simulación:

$$\int_0^1 (1-x^2)^{3/2}\,dx$$

Comparar con la respuesta exacta si es conocida.

Mathematical justification / reasoning

Write your derivations and explanations here.

[4]:
```
# Ejercicio 4 - Python code
# Write clean, commented, and reproducible code here.
# Use numpy, scipy, matplotlib as needed.
```

0.5 Ejercicio 5

Enunciado

Aproximar por simulación:

$$\int_{-2}^{2} e^{x+x^2}\, dx$$

Comparar con la respuesta exacta si es conocida.

Mathematical justification / reasoning

Write your derivations and explanations here.

[5]:
```
# Ejercicio 5 - Python code
# Write clean, commented, and reproducible code here.
# Use numpy, scipy, matplotlib as needed.
```

0.6 Ejercicio 6

Enunciado

Aproximar por simulación:

$$\int_0^\infty e^{-x}\,dx$$

Mathematical justification / reasoning

Write your derivations and explanations here.

[6]:
```
# Ejercicio 6 - Python code
# Write clean, commented, and reproducible code here.
# Use numpy, scipy, matplotlib as needed.
```

0.7 Ejercicio 7

Enunciado

Aproximar por simulación:

$$\int_0^{\infty} \frac{x}{(1+x^2)^2}\, dx$$

Mathematical justification / reasoning

Write your derivations and explanations here.

[7]:
```
# Ejercicio 7 - Python code
# Write clean, commented, and reproducible code here.
# Use numpy, scipy, matplotlib as needed.
```

0.8 Ejercicio 8

Enunciado

Aproximar por simulación:

$$\int_{-\infty}^{\infty} e^{-x^2}\, dx$$

Mathematical justification / reasoning

Write your derivations and explanations here.

[8]:
```
# Ejercicio 8 - Python code
# Write clean, commented, and reproducible code here.
# Use numpy, scipy, matplotlib as needed.
```

0.9 Ejercicio 9

Enunciado

Aproximar por simulación:

$$\int_0^1 \int_0^1 e^{(x+y)^2}\, dy\, dx$$

Mathematical justification / reasoning

Write your derivations and explanations here.

[9]:
```
# Ejercicio 9 - Python code
# Write clean, commented, and reproducible code here.
# Use numpy, scipy, matplotlib as needed.
```

0.10 Ejercicio 10

Enunciado

Aproximar por simulación:

$$\int_0^{\infty} \int_0^{x} e^{-(x+y)}\, dy\, dx$$

Mathematical justification / reasoning

Write your derivations and explanations here.

[10]:
```
# Ejercicio 10 - Python code
# Write clean, commented, and reproducible code here.
# Use numpy, scipy, matplotlib as needed.
```

0.11 Ejercicio 11

Enunciado

Usar simulación para aproximar $\mathrm{Cov}(U, e^U)$, donde $U \sim \mathcal{U}(0,1)$. Comparar con la respuesta exacta.

Mathematical justification / reasoning

Write your derivations and explanations here.

[11]:
```
# Ejercicio 11 - Python code
# Write clean, commented, and reproducible code here.
# Use numpy, scipy, matplotlib as needed.
```

0.12 Ejercicio 12

Enunciado

Sea $U \sim \mathcal{U}(0,1)$. Aproximar por simulación:

(a)

$$\text{Corr}\left(U, \sqrt{1-U^2}\right)$$

(b)

$$\text{Corr}\left(U^2, \sqrt{1-U^2}\right)$$

Mathematical justification / reasoning

Write your derivations and explanations here.

Notes: For subparts, create separate code cells as needed.

[12]:
```
# Ejercicio 12 - Python code
# Write clean, commented, and reproducible code here.
# Use numpy, scipy, matplotlib as needed.
```

0.13 Ejercicio 13

Enunciado

Para variables aleatorias uniformes $U_1, U_2, \ldots$ definir

$$N = \min \left\{ n : \sum_{i=1}^{n} U_i > 1 \right\}.$$

Estimar $\mathbb{E}[N]$ por simulación con: a) 100 valores, b) 1000 valores, c) 10000 valores, d) Discutir el valor esperado.

Mathematical justification / reasoning

Write your derivations and explanations here.

Notes: For subparts, create separate code cells as needed.

[13]:
```
# Ejercicio 13 - Python code
# Write clean, commented, and reproducible code here.
# Use numpy, scipy, matplotlib as needed.
```

0.14 Ejercicio 14

Enunciado

Sea $U_i,\ i \geq 1$ i.i.d. uniformes $(0,1)$. Definir

$$N = \max\left\{ n : \prod_{i=1}^{n} U_i \geq e^{-3} \right\}, \quad \text{con } \prod_{i=0}^{0} U_i = 1.$$

a) Encontrar $\mathbb{E}[N]$ por simulación. b) Encontrar $\mathbb{P}[N = i]$ para $i = 0, 1, 2, 3, 4, 5, 6$.

Mathematical justification / reasoning

Write your derivations and explanations here.

Notes: For subparts, create separate code cells as needed.

[14]:
```
# Ejercicio 14 - Python code
# Write clean, commented, and reproducible code here.
# Use numpy, scipy, matplotlib as needed.
```

0.14.1 Notas

- Incluye justificaciones matemáticas claras y comenta tu código.
- Mantén reproducibilidad: fija semillas cuando apliquen.
- No incluyas capturas si ejecutas todo en el notebook; el PDF exportado mostrará salidas.