Inconsolata

Metodo de la Trasformada Inversa

Curso: Temas Selectos I: O25 LAT4032 1

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Abstract

This document provides a template for reports in the ”AI in Financial Services” course, using EB Garamond for prose and Libertinus Math for formulas. It includes a cover page, abstract, table of contents, and sample sections for math and text. Additional content demonstrates tables, code, and references.

Contents

Overview

This document is a minimal example using EB Garamond for prose and libertinust1math for formulas. Links lik are active.

[1]:
```
import math
import random
from collections import Counter

import numpy as np
import matplotlib.pyplot as plt
import seaborn as sns
```

[2]:
```
color = sns.color_palette("muted")
np.random.shuffle(color)
sns.set(style="whitegrid", context="paper", palette=color)
sns.color_palette()
```

[2]:
```
[(0.5843137254901961, 0.4235294117647059, 0.7058823529411765),
 (0.5490196078431373, 0.3803921568627451, 0.23529411764705882),
 (0.8627450980392157, 0.49411764705882355, 0.7529411764705882),
 (0.2823529411764706, 0.47058823529411764, 0.8156862745098039),
 (0.41568627450980394, 0.8, 0.39215686274509803),
 (0.4745098039215686, 0.4745098039215686, 0.4745098039215686),
 (0.8392156862745098, 0.37254901960784315, 0.37254901960784315),
 (0.9333333333333333, 0.5215686274509804, 0.2901960784313726),
 (0.8352941176470589, 0.7333333333333333, 0.403921568627451),
 (0.5098039215686274, 0.7764705882352941, 0.8862745098039215)]
```

Distribución Uniforme (a, b)

Sea $U \sim \text{Unif}(0, 1)$. Si $X \sim \text{Unif}(a, b)$, entonces su funcion de distribucion acumulada es:

$$F_X(x) = \frac{x-a}{b-a}\,\mathbf{1}_{[a,b]}(x) + \mathbf{1}_{(b,\infty)}(x)$$

Encontrando la inversa:

$$\begin{aligned} F_X(x) = u \quad &\iff \quad \frac{x-a}{b-a} = u, \\ &\iff \quad x - a = (b-a)\,u, \\ &\iff \quad x = a + (b-a)\,u. \end{aligned}$$

Entonces:

$$F_X^{-1}(u) = a + (b-a)\,u.$$

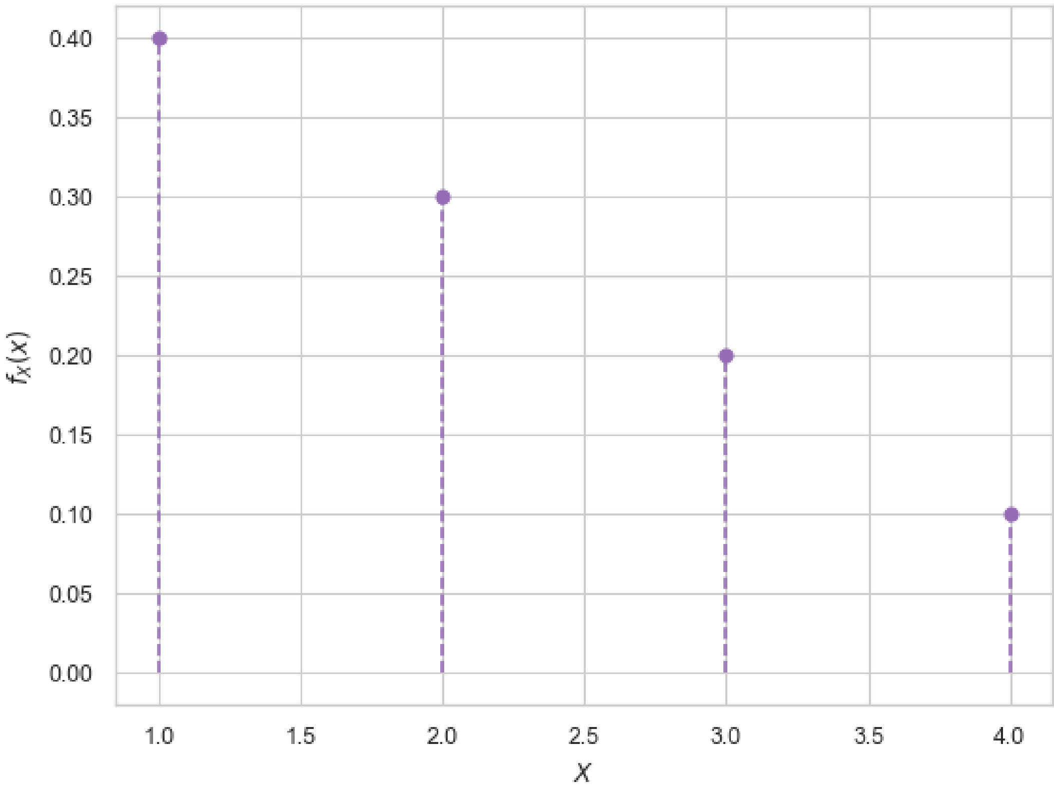
Distribución Discreta

a)

$$f_X(x) = \mathbb{P}(X = x) = \begin{cases} 0.4, & x = 1, \\ 0.3, & x = 2, \\ 0.2, & x = 3, \\ 0.1, & x = 4, \\ 0, & \text{en otro caso} \end{cases}$$

[3]:
```
x_vals = [1, 2, 3, 4]
pmf = [0.4, 0.3, 0.2, 0.1]

plt.vlines(x_vals, 0, pmf, linestyles='--')
plt.plot(x_vals, pmf, 'o')
plt.xlabel('$X$')
plt.ylabel('$f_X(x)$')
plt.show()
```

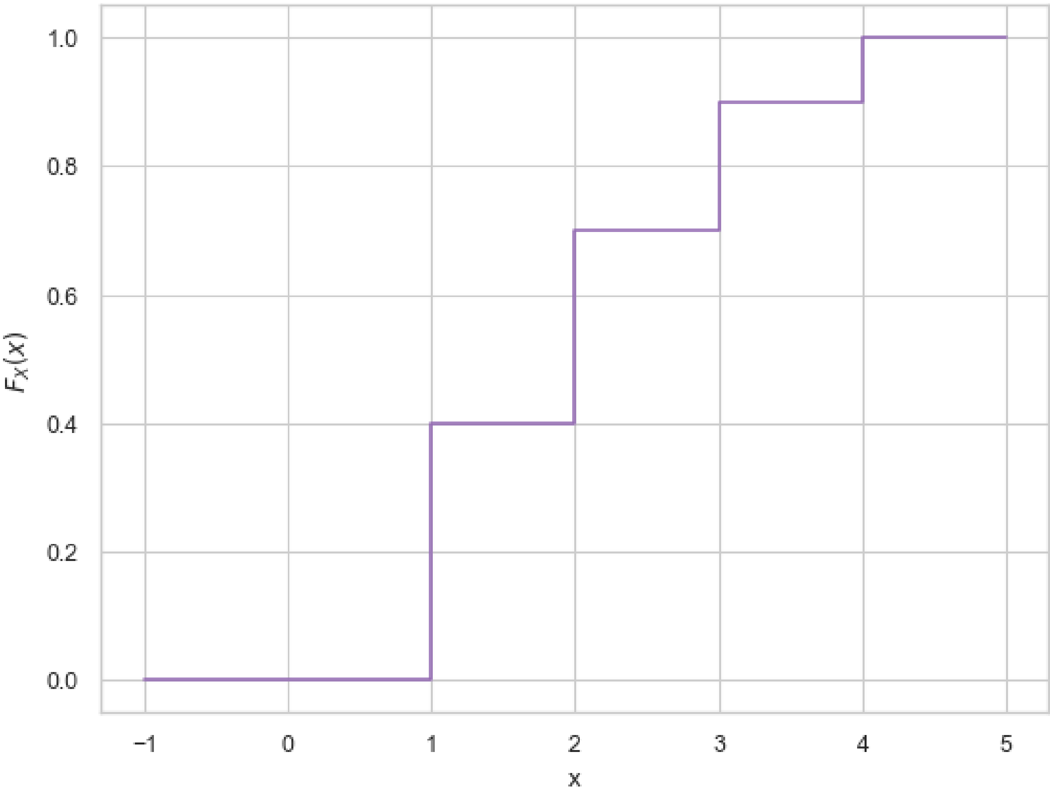


b)

$$F_X(x) = \mathbb{P}(X \leq x) = \begin{cases} 0, & x < 1, \\ 0.4, & 1 \leq x < 2, \\ 0.7, & 2 \leq x < 3, \\ 0.9, & 3 \leq x < 4, \\ 1, & x \geq 4. \end{cases}$$

[4]:
```
x_cdf = [-1, 1, 2, 3, 4, 5]
F_cdf = [0, 0.4, 0.7, 0.9, 1, 1]

plt.step(x_cdf, F_cdf, where='post')
plt.xlabel('x')
plt.ylabel('$F_X(x)$')
plt.show()
```



c)

Sea $U \sim \text{Unif}(0,1)$.

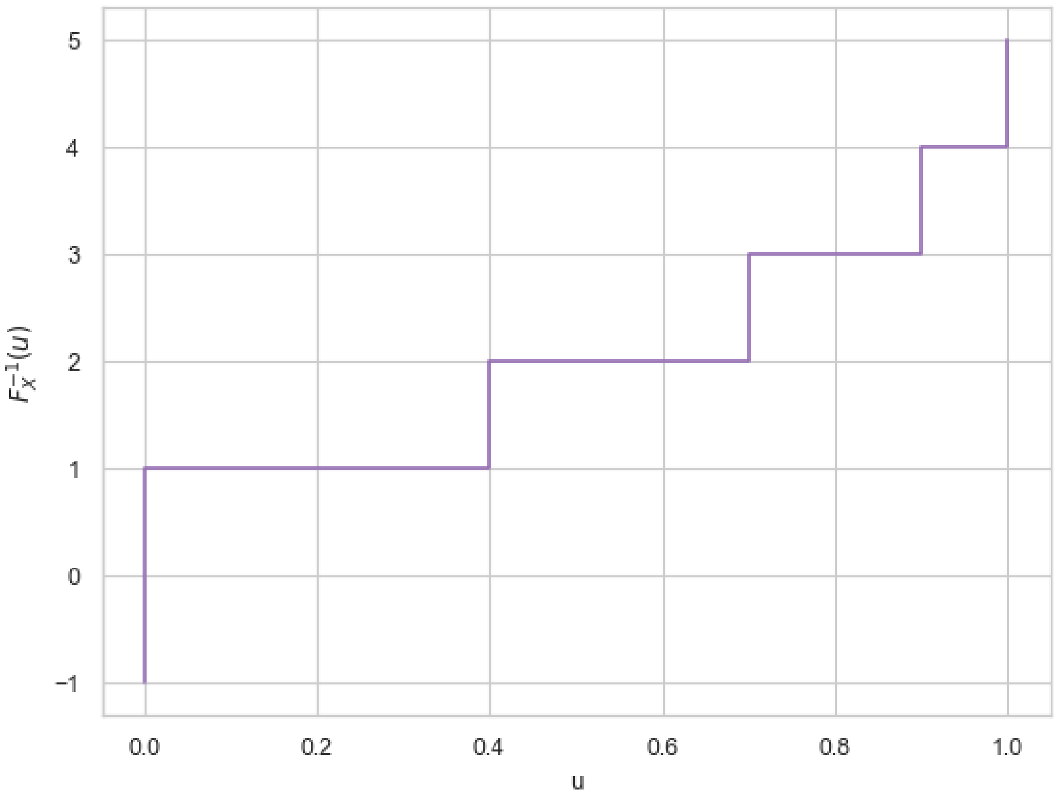
$$
\begin{aligned}
F_X(x) = u \quad &\iff \quad u \in (0, 0.4] \;\Rightarrow\; x = 1,\\
&\iff \quad u \in (0.4, 0.7] \;\Rightarrow\; x = 2,\\
&\iff \quad u \in (0.7, 0.9] \;\Rightarrow\; x = 3,\\
&\iff \quad u \in (0.9, 1] \;\Rightarrow\; x = 4.
\end{aligned}
$$

Entonces:

$$
F_X^{-1}(u) = \begin{cases} 1, & 0 < u \leqslant 0.4,\\ 2, & 0.4 < u \leqslant 0.7,\\ 3, & 0.7 < u \leqslant 0.9,\\ 4, & 0.9 < u \leqslant 1. \end{cases}
$$

[5]:
```python
u_vals = [0, 0, 0.4, 0.7, 0.9, 1]
F_inv_vals = [-1, 1, 2, 3, 4, 5]

plt.step(u_vals, F_inv_vals, where="post")
plt.xlabel("u")
plt.ylabel("$F_X^{-1}(u)$")
plt.show()
```



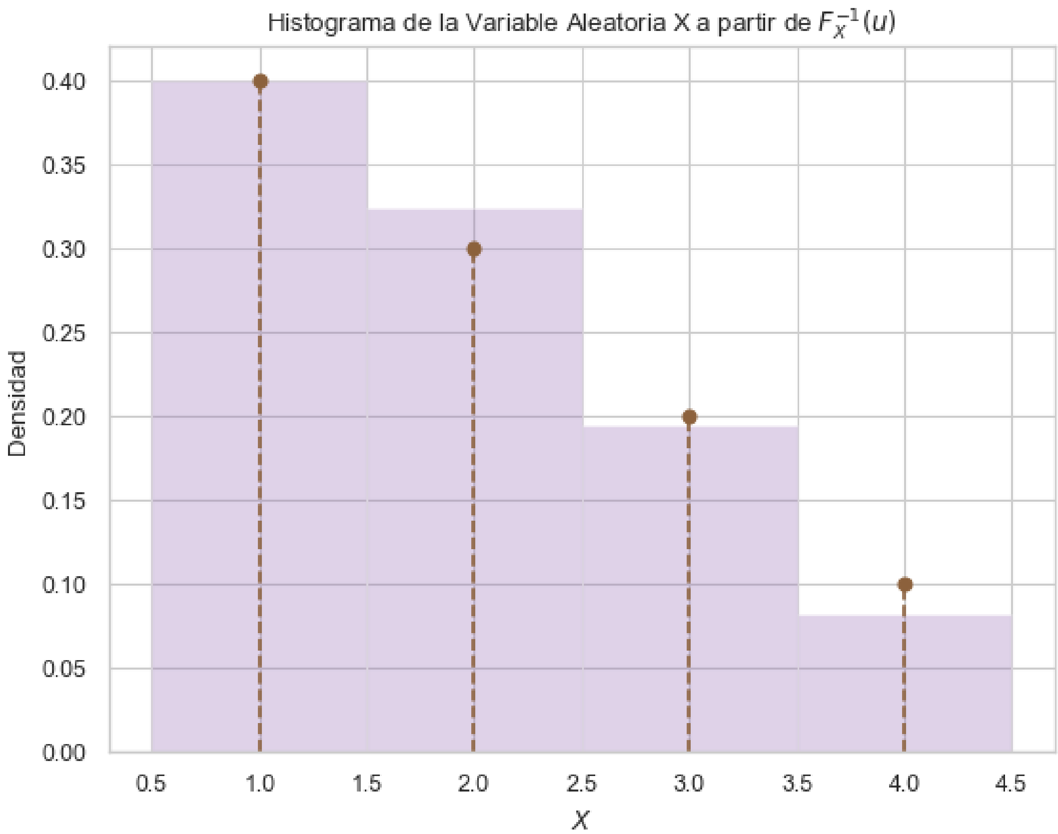
d)

[6]:
```
def F_inv(u):
    if u <= 0.4:
        return 1
    elif u <= 0.7:
        return 2
    elif u <= 0.9:
        return 3
    else:
        return 4

u_samples = np.random.uniform(0, 1, 500)
x_samples = [F_inv(u) for u in u_samples]
```

e)

[7]:
```
plt.hist(x_samples, bins=np.arange(0.5, 5.5, 1), density=True, alpha=0.3)
plt.vlines(x_vals, 0, pmf, linestyles='--', color=color[1])
plt.plot(x_vals, pmf, 'o', color=color[1])
plt.xlabel("$X$")
plt.ylabel("Densidad")
plt.title("Histograma de la Variable Aleatoria X a partir de $F_X^{-1}(u)$")
plt.show()
```



Exponencial Exp(λ)

Sea $U \sim \text{Unif}(0,1)$. Si $X \sim \text{Exp}(\lambda)$, entonces su funcion de distribucion acumulada es:

$$F_X(x) = (1 - e^{-\lambda x})\,\mathbf{1}_{[0,\infty)}(x)$$

Encontrando la inversa:

$$\begin{aligned} F_X(x) = u \iff & 1 - e^{-\lambda x} = u, \\ \iff & e^{-\lambda x} = 1 - u, \\ \iff & -\lambda x = \ln(1-u), \\ \iff & x = -\frac{1}{\lambda}\ln(1-u). \end{aligned}$$

Entonces:

$$F_X^{-1}(u) = -\frac{1}{\lambda}\ln(1-u).$$

Por lo tanto, con $U \sim \text{Unif}(0,1)$,

$$X = F_X^{-1}(U) = -\frac{1}{\lambda}\ln(1-U) \sim \text{Exp}(\lambda).$$

Weibull (r, λ)

Sea $U \sim \text{Unif}(0,1)$, con $r > 0$ y $\lambda > 0$. Si $X \sim \text{Weibull}(r, \lambda)$, entonces su funcion de distribucion acumulada es:

$$F_X(x) = \left(1 - e^{-(\lambda x)^r}\right) \mathbf{1}_{[0,\infty)}(x)$$

Encontrando la inversa:

$$\begin{aligned}
F_X(x) = u \quad &\iff \quad 1 - e^{-(\lambda x)^r} = u, \\
&\iff \quad e^{-(\lambda x)^r} = 1 - u, \\
&\iff \quad -(\lambda x)^r = \ln(1-u), \\
&\iff \quad (\lambda x)^r = -\ln(1-u), \\
&\iff \quad x = \frac{1}{\lambda}\left[-\ln(1-u)\right]^{1/r}.
\end{aligned}$$

Entonces:

$$F_X^{-1}(u) = \frac{1}{\lambda}\left[-\ln(1-u)\right]^{1/r}.$$

Por lo tanto, con $U \sim \text{Unif}(0,1)$:

$$X = F_X^{-1}(U) = \frac{1}{\lambda}\left[-\ln(1-U)\right]^{1/r} \sim \text{Weibull}(r, \lambda).$$

Cauchy (a, b)

Sea $U \sim \text{Unif}(0, 1)$. Si $X \sim \text{Cauchy}(a, b)$, entonces su funcion de distribucion acumulada es:

$$F_X(x) = \frac{1}{\pi} \arctan\Big(\frac{x-a}{b}\Big) + \frac{1}{2}, \qquad x \in \mathbb{R},\ b > 0.$$

Encontrando la inversa:

$$\begin{aligned}
F_X(x) = u \iff & \frac{1}{\pi} \arctan\Big(\frac{x-a}{b}\Big) + \frac{1}{2} = u, \\
\iff & \arctan\Big(\frac{x-a}{b}\Big) = \pi\big(u - \tfrac{1}{2}\big), \\
\iff & \frac{x-a}{b} = \tan\big(\pi(u - \tfrac{1}{2})\big), \\
\iff & x = a + b\ \tan\big(\pi(u - \tfrac{1}{2})\big).
\end{aligned}$$

Entonces:

$$F_X^{-1}(u) = a + b\ \tan\big(\pi(u - \tfrac{1}{2})\big).$$

Por lo tanto, $X = F_X^{-1}(U) = a + b\ \tan(\pi(U - \frac{1}{2})) \sim \text{Cauchy}(a, b)$.

Pareto I (a, b)

Sea $U \sim \text{Unif}(0,1)$, con $a > 0$ y $b > 0$. Si $X \sim$ Pareto I(a, b), entonces su funcion de distribucion acumulada es:

$$F_X(x) = \left(1 - (b/x)^a\right) \mathbf{1}_{[b,\infty)}(x)$$

Encontrando la inversa:

$$\begin{aligned} F_X(x) = u \quad &\Longleftrightarrow \quad 1 - \left(\frac{b}{x}\right)^a = u, \\ &\Longleftrightarrow \quad \left(\frac{b}{x}\right)^a = 1 - u, \\ &\Longleftrightarrow \quad \frac{b}{x} = (1-u)^{1/a}, \\ &\Longleftrightarrow \quad x = b\,(1-u)^{-1/a}. \end{aligned}$$

Entonces:

$$F_X^{-1}(u) = b\,(1-u)^{-1/a}.$$

Por lo tanto,

$$X = F_X^{-1}(U) = b\,(1-U)^{-1/a} \sim \text{Pareto I}(a, b).$$

Mínimo $X_{(1)} = \min\{X_1, \dots, X_n\}$

Sea $U \sim \mathrm{Unif}(0, 1)$ y $X_{(1)} := \min\{X_1, \dots, X_n\}$ con X_i i.i.d. de CDF F.

$$F_{X_{(1)}}(x) = \mathbb{P}(X_{(1)} \mathrm{~w~} x) = 1 - \mathbb{P}(X_1 > x, \dots, X_n > x) = 1 - \big(1 - F(x)\big)^n.$$

Encontrando la inversa:

$$\begin{aligned} F_{X_{(1)}}(x) = u \quad &\iff \quad 1 - \big(1 - F(x)\big)^n = u, \\ &\iff \quad \big(1 - F(x)\big)^n = 1 - u, \\ &\iff \quad 1 - F(x) = (1 - u)^{1/n}, \\ &\iff \quad F(x) = 1 - (1 - u)^{1/n}, \\ &\iff \quad x = F^{-1}\big(1 - (1 - u)^{1/n}\big). \end{aligned}$$

Entonces:

$$F_{X_{(1)}}^{-1}(u) = F^{-1}\big(1 - (1 - u)^{1/n}\big), \quad 0 < u < 1.$$

Mixta $X = \min\{Y, M\}$ con $Y \sim \mathrm{Exp}(\lambda)$

Caracteriza la cdf, localiza la masa en $x = M$ y construye F^{-1} con caso discreto/continuo. Implementa `sample_min_exp_M(lam, M, N)`.

[8]:
```
# TODO: implementa aquí
```

Mixta $X = \max\{Y, M\}$ con $Y \sim \mathrm{Exp}(\lambda)$

Sea $Y \sim \mathrm{Exp}(\lambda)$ y $M > 0$. Defina $X = \max\{Y, M\}$.

CDF

$$F_X(x) = \mathbb{P}(X \text{ w } x) = \begin{cases} 0, & x < M, \\ 1 - e^{-\lambda x}, & x \text{ x } M, \end{cases}$$

con salto en $x = M$ de tamaño $1 - e^{-\lambda M}$.

Inversa (cuantil generalizado $F_X^{-1}(u) = \inf\{x : F_X(x) \text{ x } u\}$)

Sea $U \sim \mathrm{Unif}(0, 1)$.

$$F_X^{-1}(u) = \begin{cases} M, & 0 < u \text{ w } 1 - e^{-\lambda M}, \\ -\dfrac{1}{\lambda} \ln(1 - u), & 1 - e^{-\lambda M} < u < 1, \end{cases}$$

y además $F_X^{-1}(0^+) = M$ y $F_X^{-1}(1) = +\infty$.

Para muestrear: si $U \text{ w } 1 - e^{-\lambda M}$ devuelve M; en caso contrario devuelve $-\frac{1}{\lambda} \ln(1 - U)$.

c) **Probar $F(F^{-1}(u)) \text{ x } u$, $0 < u < 1$.** Defina el cuantil generalizado $F^{-1}(u) := \inf\{x : F(x) \text{ x } u\}$. Sea $S_u = \{x : F(x) \text{ x } u\}$. Para todo $\varepsilon > 0$ existe $x_\varepsilon \in S_u$ con $x_\varepsilon \text{ w } F^{-1}(u) + \varepsilon$. Entonces

$$F\big(F^{-1}(u) + \varepsilon\big) \text{ x } u.$$

Por derecha-continuidad,

$$F\big(F^{-1}(u)\big) = \lim_{\varepsilon \downarrow 0} F\big(F^{-1}(u) + \varepsilon\big) \text{ x } u.$$

Verificación para $X = \max\{Y, M\}$. Con $p_0 := 1 - e^{-\lambda M}$,

$$F^{-1}(u) = \begin{cases} M, & 0 < u \text{ w } p_0, \\ -\frac{1}{\lambda} \ln(1 - u), & p_0 < u < 1. \end{cases}$$

Luego $F(F^{-1}(u)) = F(M) = p_0 \text{ x } u$ si $u \text{ w } p_0$, y $F(-\frac{1}{\lambda} \ln(1 - u)) = u$ si $u > p_0$.

d) **Probar $F^{-1}(F(x)) \text{ w } x$ cuando $0 < F(x) < 1$.** Tome $u = F(x)$. El conjunto $S_u = \{t : F(t) \text{ x } u\}$ contiene a x (trivialmente $F(x) \text{ x } u$). Por lo tanto

$$F^{-1}(F(x)) = \inf S_u \text{ w } x.$$

La igualdad se da cuando F es continua en x.

Verificación para $X = \max\{Y, M\}$. Si $x > M$,

$$F^{-1}(F(x)) = -\frac{1}{\lambda} \ln\big(1 - (1 - e^{-\lambda x})\big) = x.$$

Si $x = M$, $F^{-1}(F(M)) = F^{-1}(p_0) = M = x$.

e) **Generación por transformada inversa.** Con $p_0 := 1 - e^{-\lambda M}$ y $U \sim \mathrm{Unif}(0, 1)$:

$$X = \begin{cases} M, & U \text{ w } p_0, \\ -\dfrac{1}{\lambda} \ln(1 - U), & U > p_0. \end{cases}$$

Es todo.

Variable con CDF por tramos

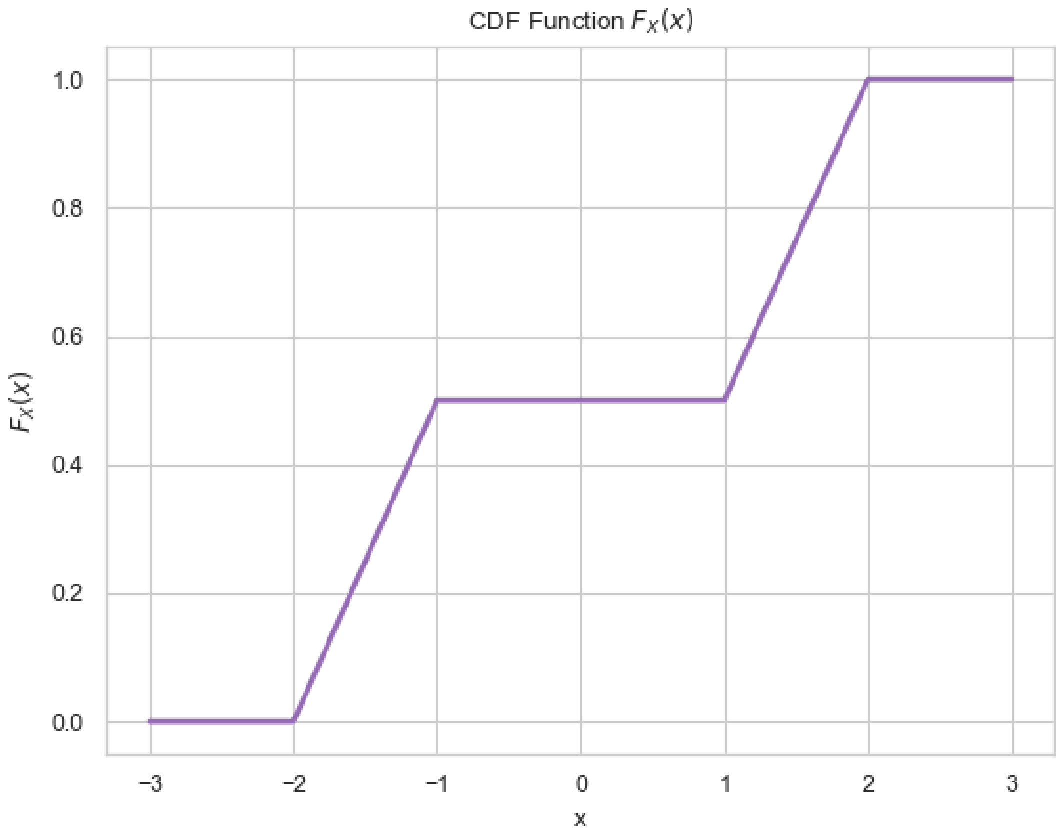
a)

$$F_X(x) = \begin{cases} 0, & x \text{ w } -2, \\ \dfrac{x+2}{2}, & -2 < x < -1, \\ \dfrac{1}{2}, & -1 \text{ w } x < 1, \\ \dfrac{x}{2}, & 1 \text{ w } x < 2, \\ 1, & x \text{ x } 2. \end{cases}$$

[23]:
```python
# Define a fine grid of x values
x = np.linspace(-3, 3, 400)

# Define the piecewise function for F_X(x)
F_x = np.piecewise(x,
                   [x <= -2, (x > -2) & (x < -1), (x >= -1) & (x < 1), (x >= 1) & (x < 2), x >= 2],
                   [0, lambda x: (x + 2) / 2, 0.5, lambda x: x / 2, 1])

# Plot the CDF
plt.plot(x, F_x, lw=2, color=color[0])
plt.xlabel('x')
plt.ylabel('$F_X(x)$')
plt.title('CDF Function $F_X(x)$')
plt.grid(True)
plt.show()
```



CDF Function $F_X(x)$
$F_X(x)$
x
1.0
0.8
0.6
0.4
0.2
0.0
−3
−2
−1
0
1
2
3

b)

Sea $U \sim \mathrm{Unif}(0,1)$.

Encontrando la inversa:

- Para $x \le -2$, $u \in \{0\}$: $F_X^{-1}(0) = -2$ *(convención en el extremo).*
- Para $-2 < x < -1$, $u \in (0, \frac{1}{2})$:

$$\begin{aligned} F_X(x) = u \iff \frac{x+2}{2} = u, \\ \iff x = 2u - 2. \end{aligned}$$

- Para $-1 \le x < 1$, $u \in \{\frac{1}{2}\}$: meseta $\Rightarrow F_X^{-1}(\frac{1}{2}) = -1$ (borde izquierdo).
- Para $1 \le x < 2$, $u \in (\frac{1}{2}, 1)$:

$$\begin{aligned} F_X(x) = u \iff \frac{x}{2} = u, \\ \iff x = 2u. \end{aligned}$$

- Para $x \ge 2$, $u \in \{1\}$: $F_X^{-1}(1) = 2$.

Entonces:

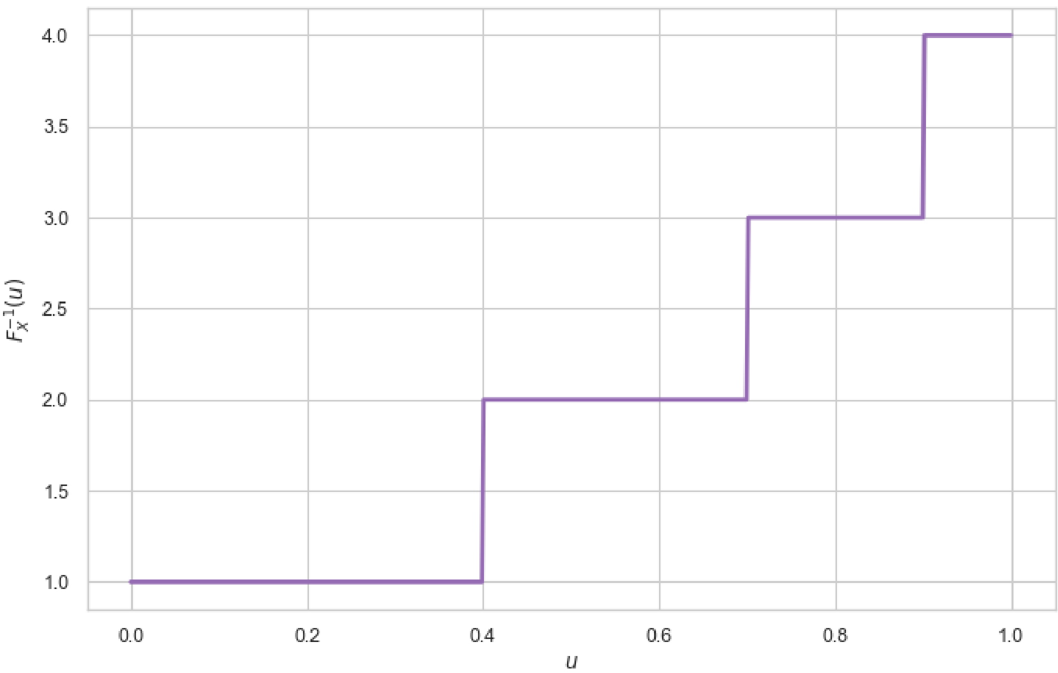
$$F_X^{-1}(u) = \begin{cases} -2, & u = 0, \\ 2u - 2, & 0 < u < \frac{1}{2}, \\ -1, & u = \frac{1}{2}, \\ 2u, & \frac{1}{2} < u < 1, \\ 2, & u = 1. \end{cases}$$

[11]:
```python
# Create a fine grid of u values in [0,1] ensuring u=0 and u=1 are included
u = np.linspace(0, 1, 500)

def F_inv_func(u):
    if u <= 0.4:
        return 1
    elif u <= 0.7:
        return 2
    elif u <= 0.9:
        return 3
    else:
        return 4

F_inv = np.vectorize(F_inv_func)(u)

plt.figure(figsize=(8, 5))
plt.plot(u, F_inv, color=color[0], lw=2)
plt.xlabel('$u$')
plt.ylabel('$F_X^{-1}(u)$')
plt.show()
```


4.0
3.5
3.0
2.5
2.0
1.5
1.0
$F_X^{-1}(u)$
0.0
0.2
0.4
0.6
0.8
1.0
u

Bernoulli (p) desde $U(0,1)$

Sea $U \sim \text{Unif}(0,1)$ y $0 < p < 1$. Defina

$$X = \mathbf{1}_{(0,p]}(U) = \begin{cases} 1, & U \text{ w } p, \\ 0, & U > p. \end{cases}$$

$$\mathbb{P}(X = 1) = \mathbb{P}(U \text{ w } p) = p, \qquad \mathbb{P}(X = 0) = \mathbb{P}(U > p) = 1 - p,$$

usando que $\mathbb{P}(U = p) = 0$. Por tanto $X \sim \text{Bernoulli}(p)$.

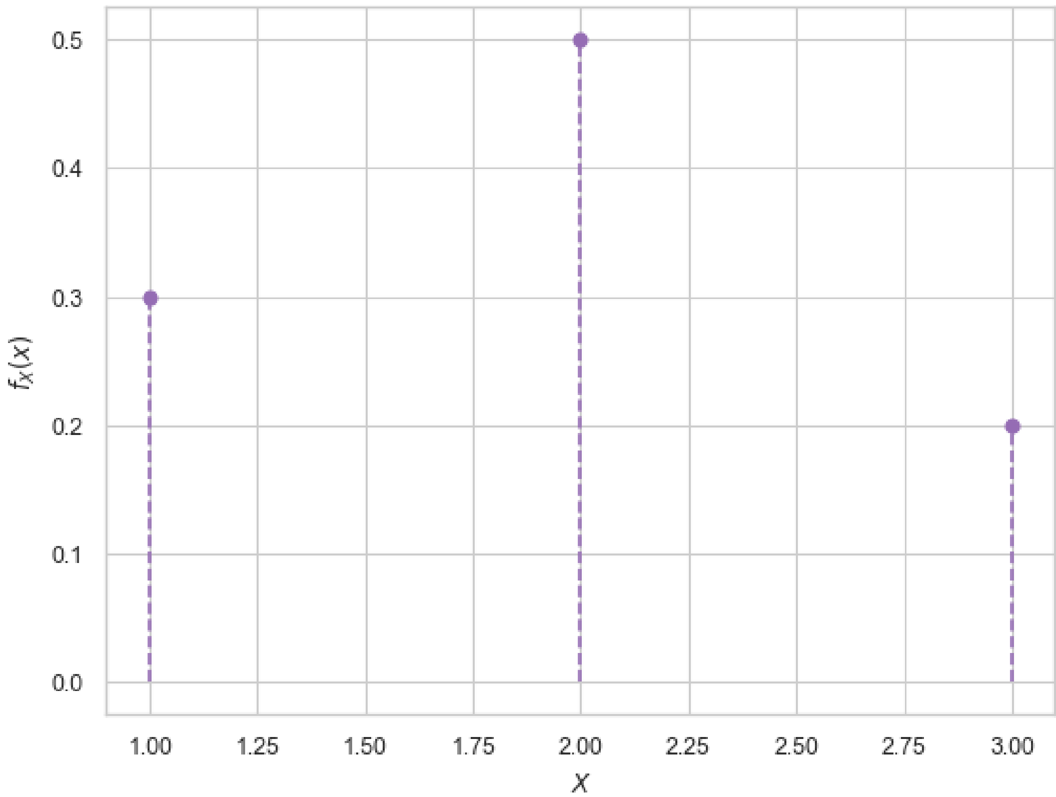
Variable aleatoria discreta

a)

$$f_X(x) = \begin{cases} 0.3 & x = 1, \\ 0.5, & x = 2, \\ 0.2, & x = 3, \\ 0, & \text{en otro caso.} \end{cases}$$

[12]:
```
x_vals = [1, 2, 3]
pmf = [0.3, 0.5, 0.2]

plt.vlines(x_vals, 0, pmf, linestyle='--')
plt.plot(x_vals, pmf, 'o')
plt.xlabel('$X$')
plt.ylabel('$f_X(x)$')
plt.show()
```

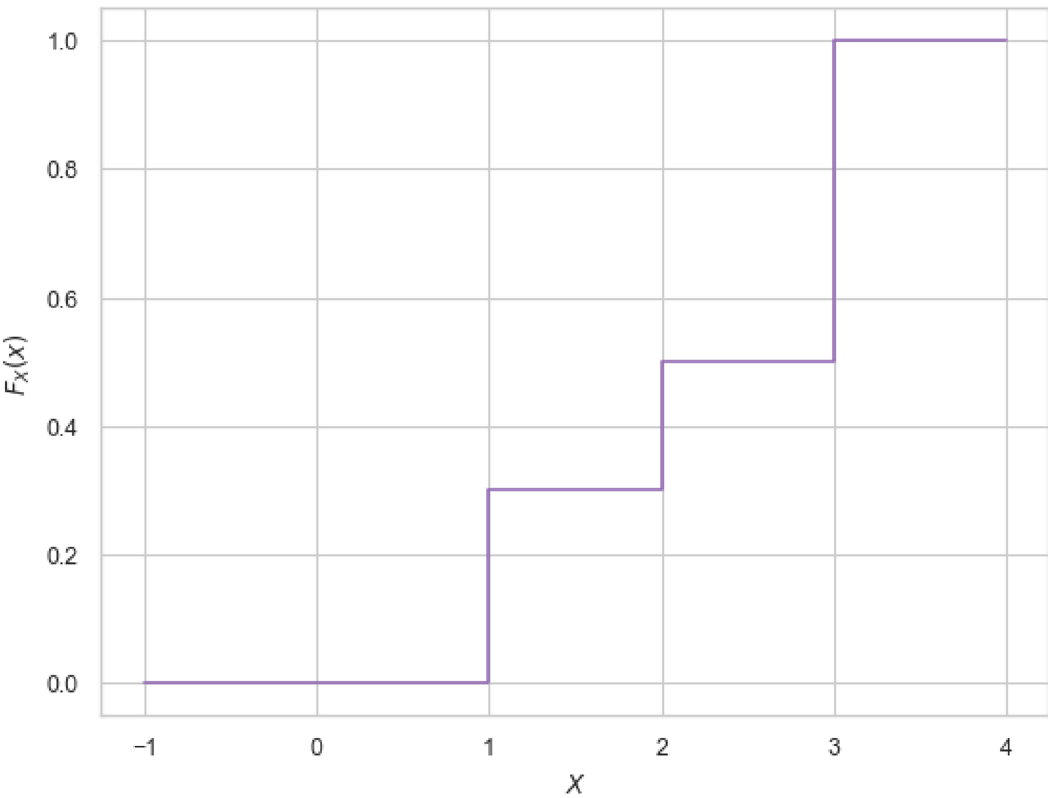


b)

$$F_X(x) = \mathbb{P}(X \text{ w } x) = \begin{cases} 0, & x < 1, \\ 0.3, & 1 \text{ w } x < 2, \\ 0.8, & 2 \text{ w } x < 3, \\ 1, & x \text{ x } 3. \end{cases}$$

[13]:
```python
x_cdf = [-1, 1, 2, 3, 4]
F_cdf = [0, 0.3, 0.5, 1, 1]

plt.step(x_cdf, F_cdf, where='post', linestyle='-')
plt.xlabel('$X$')
plt.ylabel('$F_X(x)$')
plt.show()
```



c)

Sea $U \sim \text{Unif}(0,1)$.

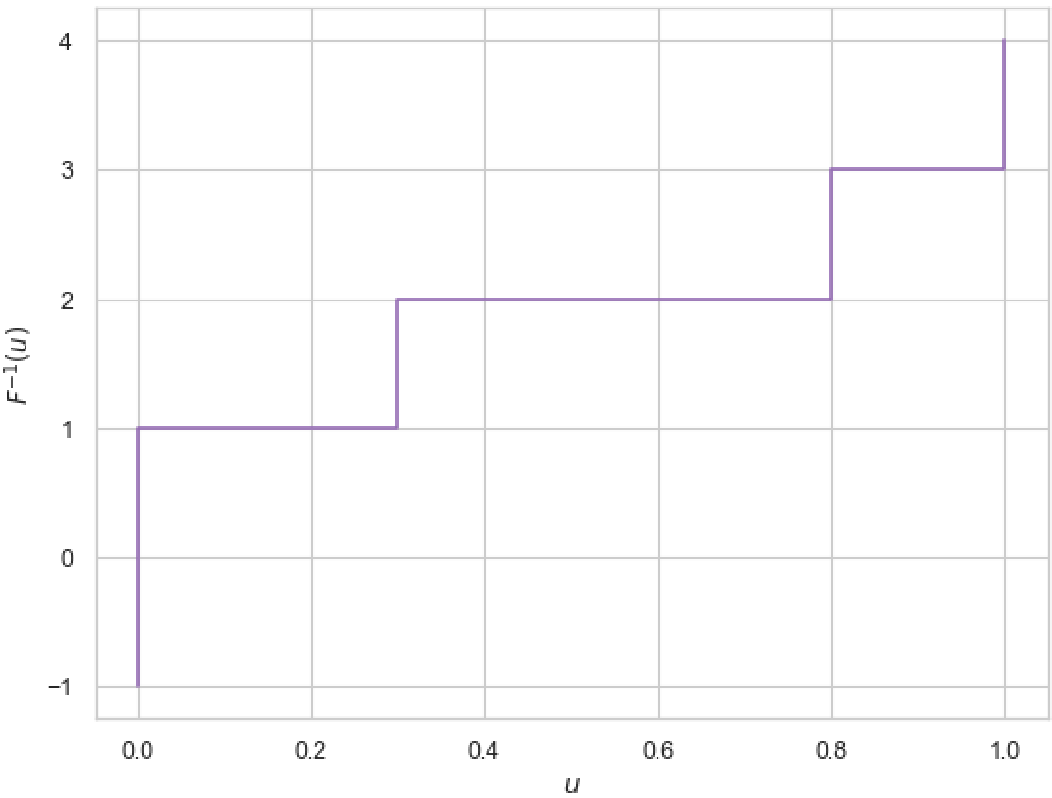
$$
\begin{aligned}
F_X(x) = u \quad &\iff \quad u \in (0, 0.3] \;\Rightarrow\; x = 1, \\
&\iff \quad u \in (0.3, 0.8] \;\Rightarrow\; x = 2, \\
&\iff \quad u \in (0.8, 1] \;\Rightarrow\; x = 3.
\end{aligned}
$$

Entonces:

$$
F_X^{-1}(u) = \begin{cases} 1, & 0 < u \text{ w } 0.3, \\ 2, & 0.3 < u \text{ w } 0.8, \\ 3, & 0.8 < u \text{ w } 1. \end{cases}
$$

[14]:
```
u_vals = [0, 0, 0.3, 0.8, 1.0]
F_inv_values = [-1, 1, 2, 3, 4]

plt.step(u_vals, F_inv_values, where='post')
plt.xlabel('$u$')
plt.ylabel('$F^{-1}(u)$')
plt.show()
```



d) Valor esperado

$$\mathbb{E}[X] = \sum_{x=1}^{3} x \cdot f_X(x) = 1 \cdot 0.3 + 2 \cdot 0.5 + 3 \cdot 0.2 = 1.9.$$

e) Varianza

$$\mathbb{V}[X] = \sum_{x=1}^{3} (x - \mathbb{E}[X])^2 \cdot f_X(x) = (1 - 1.9)^2 \cdot 0.3 + (2 - 1.9)^2 \cdot 0.5 + (3 - 1.9)^2 \cdot 0.2 = 0.49.$$

$$\mathbb{V}[X] = \mathbb{E}[X^2] - (\mathbb{E}[X])^2 = 0.3 \cdot 1^2 + 0.5 \cdot 2^2 + 0.2 \cdot 3^2 - (1.9)^2 = 2.79 - (1.9)^2 = 0.49.$$

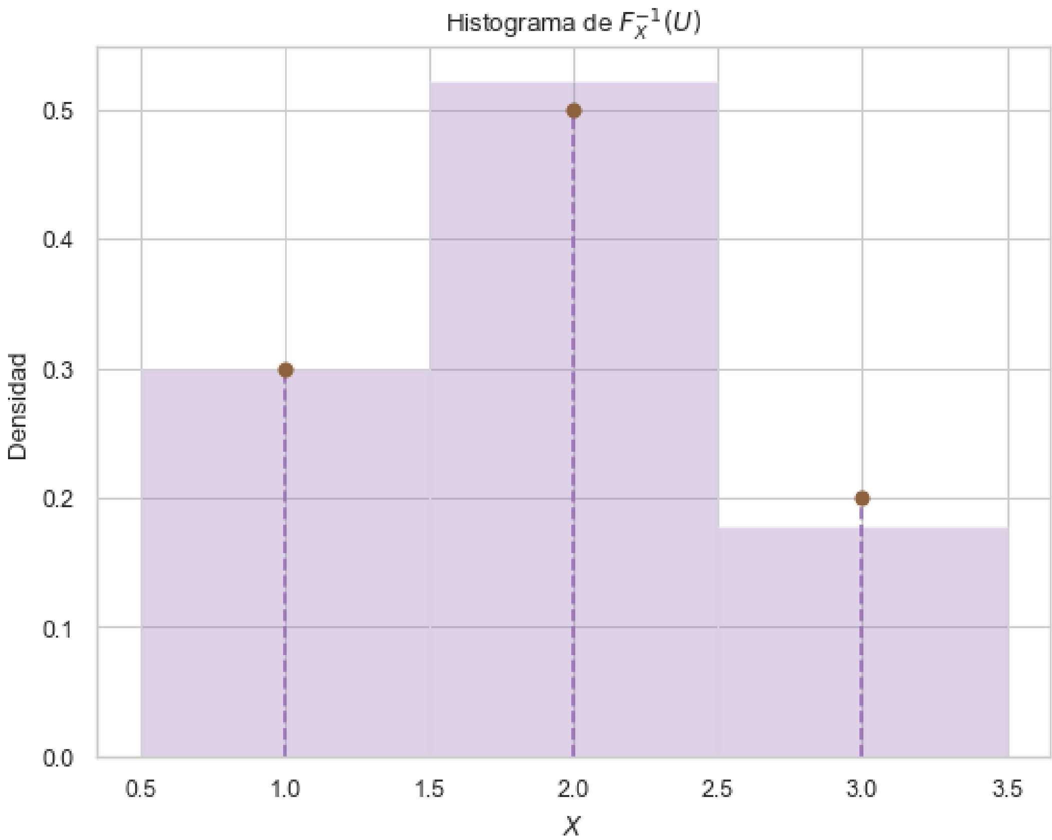
f)

[15]:
```
def F_inv(u):
    if 0.3 < u <= 0.8:
        return 2
    elif u <= 0.3:
        return 1
    else:
        return 3
```

[16]:
```
u_samples = np.random.uniform(0, 1, 500)
x_samples = [F_inv(u) for u in u_samples]
```

g)

[17]:
```
plt.hist(x_samples, bins=np.arange(0.5, 4.5, 1), density=True, alpha=0.3)
plt.vlines(x_vals, 0, pmf, linestyle='--')
plt.plot(x_vals, pmf, 'o')
plt.xlabel("$X$")
plt.ylabel("Densidad")
plt.title("Histograma de $F_X^{-1}(U)$")
plt.show()
```



h)

[18]:
```
x_mean = np.mean(x_samples)
print(f"Media muestral de X: {x_mean}")
print(f"Media teórica de X: 1.9")
print(f"Diferencia: {x_mean - 1.9:.4f}")
```

```
Media muestral de X: 1.878
Media teórica de X: 1.9
Diferencia: -0.0220
```

i)

[19]:
```
x_var = np.var(x_samples)
print(f"Varianza muestral de X: {x_var}")
print(f"Varianza teórica de X: 0.49")
print(f"Diferencia: {x_var - 0.49:.4f}")
```

```
Varianza muestral de X: 0.463116
Varianza teórica de X: 0.49
Diferencia: -0.0269
```

Binomial, Geométrica y Poisson

Implementa generadores por inversión para: - $X \sim \text{Bin}(m, p)$ con $m = 10$ y $p = 1/3$ - $X \sim \text{Geo}(p)$ con $\mathbb{P}(X = k) = p(1-p)^{k-1}$, k x 1 con $p = 3/4$ - $X \sim \text{Poisson}(\lambda)$ con $\lambda = 2$

a) Binomial

[20]:
```python
def rbinom_inv(ntrials: int, p: float, size: int):
    """Binomial(ntrials, p) por transformada inversa."""
    if not (0 <= p <= 1):
        raise ValueError("p debe estar en [0,1].")
    if p == 0:
        return [0]*size
    if p == 1:
        return [ntrials]*size

    c = p/(1.0 - p)                  # paso 2: c
    out = []
    for _ in range(size):
        U  = random.random()         # paso 1
        i  = 0                       # paso 2
        pr = (1.0 - p)**ntrials      # pr = P(X=0)
        F  = pr
        if U < F:                    # paso 3-5
            out.append(i); continue
        while True:                  # paso 6
            pr *= c * (ntrials - i) / (i + 1)  # paso 7
            F  += pr
            i  += 1
            if U < F or i == ntrials:          # paso 8-11 (con tope)
                out.append(i)
                break
    return out

m = rbinom_inv(ntrials=10, p=0.3, size=100)
print(m[:20])
```

```
[4, 3, 2, 3, 3, 2, 3, 2, 3, 3, 4, 0, 5, 6, 2, 3, 3, 3, 4, 1]
```

b) Geometrica

[21]:
```
def rgeom_inv_trials(p: float, n: int):
    """Geom(p) en {1,2,...} por inversión: floor(log(U)/log(1-p))+1."""
    out = []
    for _ in range(n):
        u = random.random()
        x = math.ceil(math.log(u) / math.log(1.0 - p))
        out.append(x)
    return out

# Ejemplo: p=0.3, n=100
muestras = rgeom_inv_trials(0.3, 100)
print(muestras[:20])
```

```
[3, 8, 3, 7, 4, 1, 1, 1, 8, 1, 9, 3, 3, 4, 1, 4, 3, 2, 1, 3]
```

c) Poisson

[22]:
```
def rpois_inv(lam: float, n: int):
    """Poisson(lam) por transformada inversa, siguiendo el algoritmo dado."""
    if lam <= 0:
        raise ValueError("lam debe ser > 0")

    out = []
    for _ in range(n):
        # 1) U ~ U(0,1)
        U = random.random()
        # 2) i=0, p=e^{-lam}, F=p
        i = 0
        p = math.exp(-lam)
        F = p
        # 3) if U < F: X=i
        if U < F:
            out.append(i)
            continue
        # 6-11) loop: p = (lam/(i+1))*p; F = F + p; i = i + 1; if U < F: X=i
        while True:
            i += 1
            p = p * lam / i
            F = F + p
            if U < F:
                out.append(i)
                break
    return out

muestras = rpois_inv(lam=2.0, n=100)
print(muestras[:20])
```

```
[3, 2, 2, 4, 0, 4, 1, 3, 2, 4, 1, 2, 3, 1, 3, 5, 1, 1, 2, 0]
```