Temas Selectos I

U1 Juegos Competitivos

Héctor Maravillo

Section 1

Introduction

Strategic settings

Situation of **interdependence** are called **strategic settings** because, in order for a person to decide how **best** to behave, he must **consider** (**guess?**) how other around him choose their actions.

Game Theory provides a systematic and rigorous general theory of **strategic situation** (**competition** or **competition**) among multiple parties, using mathematical models and methods.

"Games"

The term **"game"** generally connotes a situation in which two or more **opponent compete** in wits. Games inherently entail **interdependence** in that one person's (optimal) behavior depends on what he or she **believes** the others will do.

We also normally associate games with **sets of rules** that must be followed by the players.

Rules are synonymous with our ability to agree on exactly what game is actually being played.

Non-cooperative games

The **noncooperative** framework treats all of the agents' actions as **individual actions**.

An individual action is something that a person **decides** on his own, **independently** of the other people present in the strategic environment.

Thus, it is accurate to say that noncooperative theories examine **individual decision** making in **strategic settings**.

Non-cooperative games

The **noncooperative** framework treats all of the agents' actions as **individual actions**.

An individual action is something that a person **decides** on his own, **independently** of the other people present in the strategic environment.

Thus, it is accurate to say that noncooperative theories examine **individual decision** making in **strategic settings**.

Non-cooperative games

The framework does not rule out the possibility that one person can limit the options of another; nor is the theory incompatible with the prospect that players can make **decisions in groups**.

In regard to **group decision** making, noncooperative models require the theorist to **specify** the procedure by which decisions get made. The procedure includes a specification of how agents **negotiate** options, which may include offers and counter offers (taken to be individual actions).

Cooperative games

Sometimes, instead of modeling every individual move in negotiation, it is simpler to treat the outcome as a **joint action**.

Cooperative game theory is often preferred for the study of **contractual relations**, in which parties negotiate and jointly agree on the terms of their relationships. Contractual relations form a large fraction of all strategic situations.

Cooperative games

Incorporating **joint actions** into a strategic model is a shorthand device allowing one to represent agents negotiating over certain things, **without explicitly modeling** the **negotiation process** itself.

The objects of negotiation are considered **spot-contractible** in the sense that an **agreement** on a joint action **commits** the agents to take this action.

Synthesis

- In short, **games** are **formal descriptions** of **strategic settings**.
- Thus, **game theory** is a **methodology** of formally studying **situations of interdependence**.
- *Formally* means using a mathematically precise and locally consistent structure.

Game Representation

There are several different ways of **describing games** mathematically, but they all share certain common elements:

- A list of **players**.
- A complete description of what the players can do (their **possible actions**).
- A description of what the players **know** when they act.
- A specification of how the players' actions lead to **outcomes**.
- A specification of the players' **preferences** over outcomes.

Representation of non-cooperative games

There are two common forms in which noncooperative games are represented mathematically:

- The **extensive form**.
- The **normal** or **strategic** form.

Section 2

Examples: Zero-Sum Games

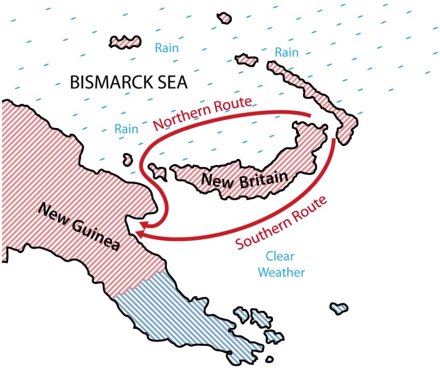
The Battle of the Bismarck Sea

The Battle of the Bismarck Sea

South-Pacific, 1943: The **Japanese admiral Imamura** must move troops across the Bismarck Sea to New Guinea, and the **American admiral Kenney** plans to bomb the transport.

Imamura has two possible **choices**: a short Northern route (2 days) or a long Southern route (3 days). Kenney must choose one to target. If he guesses wrong, planes can be redirected, but bombing time is reduced by one day

We can assume that bombing days are the **payoff**: positive for Kenney, negative for Imamura.



The Battle of the Bismarck Sea

Model:

The Battle of the Bismarck Sea problem can be modelled using the following table:

	North	South
North	2	2
South	1	3

Each player has two possible choices; Kenney (player 1) chooses a row, Imamura (player 2) chooses a column, and these choices are to be made **independently** and **simultaneously**.

This game is an example of a **zero-sum game** because the sum of the payoffs is always equal to zero.

The Battle of the Bismarck Sea

Solution:

By choosing North, Imamura is always **at least as well off as** by choosing South.

So it is safe to assume that Imamura chooses North, and Kenney, being able to perform this same kind of reasoning, will then also choose North, since that is the **best reply** to the choice of North by Imamura.

The game is determined because one of the players (Inamura) has a **dominated choice**, namely South: No matter what the opponent (Kenney) decides to do, North is **at least as good as** South, and sometimes better.

The Battle of the Bismarck Sea

Solution:

Note that the payoff 2 of combination (North, North) is maximal in its column and minimal in its row. Such a position in the matrix is called a **saddle point**. In a saddle point, neither player has an incentive to **deviate unilaterally**.

In this saddle point, the row player maximizes his minimal payoff

$$2 = \min\{2, 2\} \geq 1 = \min\{1, 3\}$$

While the column player minimizes the maximal amount that he has to pay

$$2 = \max\{2, 1\} \leq 3 = \min\{2, 3\}$$

The resulting payoff of 2 from player 2 to player 1 is called the **value** of the game.

Zero-Sum Games

Two-person zero-sum games with finitely many choices, are also called **matrix games** since they can be represented by a single matrix.

Figure: Go (board game)

Section 3

Examples: Nonzero-Sum Games

Prisoner' Dilemma

Prisoner' Dilemma

Two prisoners have committed a crime together and are interrogated **separately**.

Each prisoner has two **possible choices**:

- He may *cooperate* (C) which means *not betray his partner*.
- He may *defect* (D), which means *betray his partner*

The **punishment** for the crime is 10 years of prison. Betrayal yields a reduction of 1 year for the defector (traitor). If a prisoner is not betrayed, he is convicted to 1 year for a minor offense.

Figure: Prisoner's Dilemma - for Magic: The Gathering MKC

Prisoner' Dilemma

Model:

	C	D
C	$-1,\ -1$	$-10,\ 0$
D	$0,\ -10$	$-9,\ -9$

By convention, the first number is the payoff for player 1 (the row player) and the second number is the payoff for player 2 (the column player).

Prisoner' Dilemma

Solution:

For both players option C is a **strictly dominated choice:**

- D is better than C, whatever the other player does.

So it is natural to argue that the outcome of this game will be the pair of choices (D, D), leading to the payoffs -9, -9.

Comments: The payoffs $-9, -9$ are inferior: they are not **Pareto optimal**, the players could obtain the higher payoff of -1 for each by **cooperating**, i.e., both playing C.

There is a large literature on how to establish cooperation, e.g., by **reputation effects** in a **repeated play** of the game. In this case, other (higher) payoffs are possibles.

Battle of the Sexes

Battle of the Sexes

A **man** and a **woman** want to go out together, either to a **activity A** or to a **activity B**.

They forgot to agree where they would go that night, are in different places and have to decide on their own where to go; they have no means to communicate.

Their main concern is to be together. The man has a preference for A and the woman for B.

Battle of the Sexes

Model:

The following table reflects the situation:

	A	B
A	2, 1	0, 0
B	0, 0	1, 2

Battle of the Sexes

Solution

Observe that no player has a dominated choice.

In absence of communication it is hard to give a unique prediction for this game.

The combinations (A, A) and (B, B) are special in the sense that the players' choices are **best replies** to each other: If the man chooses A(B), then it is optimal for the woman to choose A(B) as well, and vice versa.

In the literature, such choice combinations are called **Nash equilibria**.

A Cournot Oligopoly

A Cournot Oligopoly

Two firms produce a similar (*homogenous*) product.

The market price of this product is equal to $p = 1 - Q$ or zero (whichever is larger), where Q is the **total quantity produced**. There are no production costs.

Figure: Duopoly example

A Cournot Oligopoly

Model:

The two firms are the players, 1 and 2.

Each player $i = 1, 2$ chooses a quantity $q_i \geq 0$, and makes a **profit** of

$$K_i(q_1, q_2) = q_i p = q_i(1 - q_1 - q_2)$$

or zero if

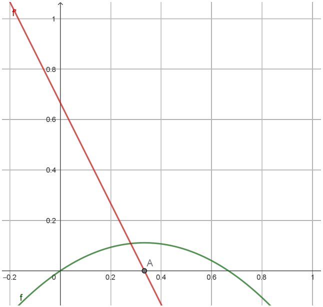
$$q_1 + q_2 \geq 1$$

A Cournot Oligopoly

Solution:

Suppose player 2 produces $q_2 = \frac{1}{3}$. The player 1 maximizes his own profit $q_1(1 - q_1 - \frac{1}{3})$ by choosing $q_1 = \frac{1}{3}$.

$$\max_{q_i \in (0, 2/3)} \; Z = q_1(1 - q_1 - 1/3) = 2/3 q_1 - q_1^2$$



Also the converse holds: if player 1 chooses $q_1 = \frac{1}{3}$ then $q_2 = \frac{1}{3}$ maximizes profit for player 2.

A Cournot Oligopoly

This combination of strategies consists of mutual best replies and is therefore again called a **Nash equilibrium**.

This particular Nash equilibrium is often called **Cournot equilibrium**.

The Cournot equilibrium in this example is again **not Pareto optimal**: if the firms each would produce $1/4$, then they would both be better off.

Section 4

Extensive Form Games

Sequential battle of the Sexes

We assume now that in the battle of the sexes, the man chooses first and the woman can observe the choice of the man.

Model: This situation can be represented by the following **decision tree**:

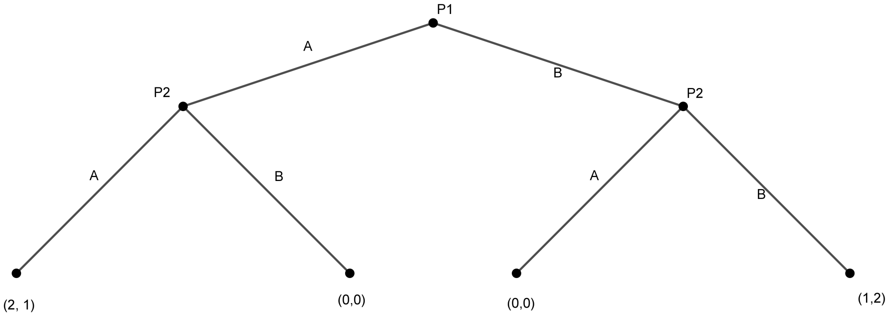


Figure: The decision tree of sequential battle of the sexes

.

Sequential battle of the Sexes

Player 1 (the man) chooses first, player 2 (the woman) observes player 1's choice and then makes her own choice.

The first number in each pair is the payoff to player 1, and the second number is the payoff to player 2.

Filled circles denote **decision nodes** (of a player) or **end nodes** (followed by payoffs).

Solution: If player 1 chooses A, then it is optimal for player 2 to choose A as well, and if player 1 chooses B, then it is optimal for player 2 to choose B as well. Given this choice behavior of player 2 and assuming that player 1 performs this line of reasoning about the choices of player 2, player 1 should choose A.

Imperfect information

The collection of the two nodes of the incumbent, connected by the dashed line, is called an **information set**. In general, information sets are used to model **imperfect information**.

Imperfect information arises when some player does not know the outcome of the chance move or does not observe other's move (for example, when the move is simultaneous).

In the example, player 2, when he moves, does not know what player 1 has chosen.

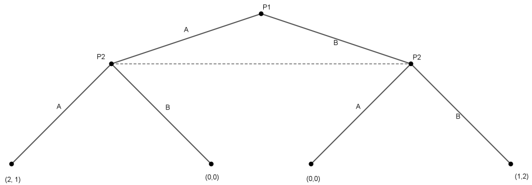


Figure: Extensive form of Battle of the Sexes with imperfect information

Section 5

References

References

- Peters, Hans (2015) **Game Theory. A Multi-Leveled Approach**, 2nd ed., Springer.
- Watson (2013) **Strategy. An Introduction to Game Theory**, 3rd ed., Norton & Company.