Temas Selectos I

U1 Juegos Competitivos

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Section 1

Introduction

Static games

A **static game** a set of players **independently** choose once-for-all actions, after which outcomes are realized. Thus a static game can be ghought of as having two distinct steps:

- *Step 1*: Each player **simultaneously** and **independently** chooses an action. It means that the players must take their actions **without observing** what actions their counterparts take and **without interacting** with other players to coordinate their actions.
- *Step 2*: Conditional on the players' choices of actions, payoffs are distributed to each player. That is, once the players have all made their choices, these choices will result in a **particular outcome**, or **probabilistic distribution over outcomes**. The players have **preferences** over the outcomes of the game given by some payoff function over outcomes.

Games of complete information

Definition

A game of **complete information** requires that the following four components be **common knowledge among all** the players of the game:

- All the **possible actions** of all the players.
- All the **possible outcomes**.
- How each combination of actions of all players **affects** which **outcome** will materialize
- The **preferences** of each and every player over outcomes.

Common knowledge

Definition

An event E is **common knowledge** if

1. everyone knows E,
2. everyone kowns that everyone knows E,
3. and so on *ad infinitum*.

Thus requiring **common knowledge** is not as innocuous as it may seem, but without this assumption it is quite impossible to analyze games within a structured framework. This difficulty arises because we are seeking to depict a situation in which players can engage in **strategic reasoning**. That is, I want to **predict** how you will make your choice, given *my belief that you understand the game*. Your understanding incorporates *your belief about my understanding*, and so on.

Section 2

Normal-Form Games

Normal-form game

Definition

A **Normal-form game** includes three components as follows:

- A **finite set of players**
$$N = \{1, 2, \ldots, n\}$$
- A **collection of sets of pure strategies**
$$\{S_1, S_2, \ldots, S_n\}$$
- A **set of playoff functions** $\{v_1, v_2, \ldots, v_n\}$, each assigning a payoff value to each combination of chosen strategies, that is, a set of functions
$$v_i : \; S_1 \times S_2 \times \ldots \times S_n \to \mathbb{R}, \quad \forall i \in N$$

Thus, the normal-form game is represented as a triple of sets:

$$(N, \{S_i\}_{i=1}^n, \{v_i(\cdot)_{i=1}^n\})$$

Normal-form game

The last representation is very general, and it will capture many situations in which each of the players $i \in N$ must **simultaneously choose** a **possible strategy** $s_i \in S_i$.

In this context, by **simultaneous** we mean the more general construct in which each player is choosing a strategy **without knowing** the choices of the other players.

After strategies are selected, each player will realize his payoff, given by $v_i(s_1, s_2, \ldots, s_n) \in \mathbb{R}$, where $(s_1, s_2, \ldots, s_n) \in S_1 \times S_2 \times \ldots \times S_n$ is the **strategy profile** that was selected by the agents.

Strategy

Definition

A **strategy** can be defined as a **plan of action** intended to accomplish a specific goal.

Definition

A **pure strategy** for player i is a **deterministic** plan of action. The set of all pure strategies for player i is denoted S_i.

Definition

A **profile of pure strategies**

$$s = (s_1, s2, \ldots, s_n), \quad s_i \in S_i, \ \forall i = 1, \ldots, n$$

describes a **particular combination** of pure strategies chosen by all n players in the game.

Example: The Prisioner's Dilemma

The normal form of Prisioner's Dilemma is as follows:

- **Players**:

$$N = \{1, 2\}$$

- **Strategy sets**:

$$S_i = \{C, D\}, \quad \text{for} i \in \{1, 2\}$$

- **Payoffs**:

$$v_1(C, C) = v_2(C, C) = -1$$

$$v_1(D, D) = v_2(D, D) = -9$$

$$v_1(D, C) = v_2(C, D) = 0$$

$$v_1(C, D) = v_2(D, C) = -10$$

Finite games

Definition

A **finite** game is a game with a finite number of players, in which the number of strategies in S_i is finite for all players $i \in N$.

Matrix representation

Definition

Any **two-player finite** game can be represented by a **matrix** that will capture all the relevant information of the **normal-form game**. This is done as follows:

- **Rows**: Each row represents one of player 1's strategies. If there are k strategies in S_1 then the matrix will have k rows.
- **Columns**: Each column represents one of player 2's strategies. If there are m strategies in S_2 then the matrix will have m columns.
- **Matrix entries**: Each entry in this matrix contains a two-element vector $a_{ij} = (v_1(s_i), v_i(s_j))$, where v_l is player l's payoff when the actions of both players correspond to the row i and column j.

Section 3

References

References

- Tadelis, S. (2013). **Game Theory. An Introduction**. Princeton University Press.