Temas Selectos I

U1 Juegos Competitivos

Héctor Maravillo

Section 1

Introduction

Solution concepts

If we want **to predict** or **prescribe** actions for a game, we need to make **assumptions** about the **behavior** and the **beliefs** of the players.

We therefore need a **solution concept** that will result in predictions or prescriptions.

A solution concept is a method of analyzing games with the objective of **restricting** the **set of all possible outcomes** to those that are more reasonable than others.

Assumptions

- Players are **rational**: A rational player is one who chooses his action, $s_i \in S_i$, **to maximize** his payoff **consistent** with **his beliefs** about what is going on in the game.
- Players are **intelligent**: An intelligent player **knows** everything **about the game**: the actions, the outcomes, and the preferences of all the players.
- **Common knowledge**: The fact that players are rational and intelligent is common knowledge among the players of the game.
- **Self-enforcement**: It is the core of **noncooperative game theory**. Each player is in control own action, and he will stick to an action only if he finds it to be in **his best interest**.

Pareto dominance

Definition

A strategy profile $s \in S$ **Pareto dominates** strategy profile $s' \in S$ if

$$v_i(s) \geq v_i(s') \; \forall \; i \in N$$

and

$$v_i(s) > v_i(s')$$

for at least one $i \in N$ (in which case, we will also say that s' is **Pareto dominated** by s).

A strategy profile is **Pareto optimal** if it is not Pareto dominated by any other strategy profile.

Pareto dominance

	M	F
M	$-2,\ -2$	$-5,\ -1$
F	$-1,\ -5$	$-4,\ -4$

Note that the solution (F, F) is **Pareto dominated** by (M, M), but it is not the only one, the solutions (M, F) and (F, M) are also **Pareto-optimal** outcomes because there is no other profile that dominates any of them.

Pareto dominance

Notes:

- Economists use a particular criterion for evaluating whether an outcome is **socially undesirable**. An outcome is considered to be socially undesirable if there were a different outcome that would make **some** people **better off** without **harming anyone else**.
- Don't confuse **Pareto optimality** with the best symmetric outcome that leaves all players **equally happy**.

Notation

We denoted the **payoff of a player** i from a profile of strategies

$$s = (s_1, s_2, \ldots, s_{i-1}, s_i, s_{i+1}, \ldots, s_n)$$

as $v_i(s)$.

It will soon be very useful to refer specifically to the strategies of a **player's opponents** in a game.

To simplify we use a common shorthand notation as follows: We define

$$S_{-i} \equiv S_1 \times S_2 \times \cdots \times S_{i-1} \times S_{i+1} \times \cdots \times S_n$$

as the set of all the strategy sets of all players **who are not player** i.

Notation

We then define

$$s_{-i} \in S_{-i}$$

as a particular **possible profile** of strategies for all players who are not i.

Hence, we can rewrite the payoff of player i from strategy s as

$$v_i(s_i, s_{-i}),$$

where

$$s = (s_i, s_{-i}).$$

Section 2

Dominance

Strictly dominated

Definition

Let $s_i \in S_i$ and $s_i' \in S_i$ be possible strategies for player i.

We say that s_i' is **strictly dominated** by s_i if for **any possible combination** of the other players' strategies, $s_{-i} \in S_{-i}$, player i's payoff from s_i' is strictly less than that from s_i.

That is, if

$$v_i(s_i, s_{-i}) > v_i(s_i', s_{-i}), \qquad \forall s_{-i} \in S_{-i}$$

We will write $s_i \succ s_i'$ to denote that s_i' is **strictly dominated** by s_i to player i.

Strictly dominated

Affirmation

A **rational player** will never play a strictly dominated strategy.

This affirmation is obvious. If a player plays a dominated strategy then he cannot be playing optimally because, by the definition of a dominated strategy, the player has another strategy that will yield him a **higher payoff** regardless of the strategies of his opponents.

Hence **knowledge** of the game implies that a player should **recognize dominated strategies**, and **rationality** implies that these strategies will be **avoided**.

Strictly dominated

	M	F
M	$-2,\ -2$	$-5,\ -1$
F	$-1,\ -5$	$-4,\ -4$

Playing M is worse than playing F for each player **regardless** of what the player's opponent does. What makes it unappealing is that there is another strategy F, that is better than M regardless of what one's opponent chooses. We say that such a strategy is **dominated**.

When we apply the notion of a dominated strategy to the Prisoner's Dilemma we argue that each of the two players **has one dominated strategy** that he should never use, and hence each player is left with one strategy that is not dominated.

Strictly dominant strategy

Because a **strictly dominated** strategy is one **to avoid** at all costs, there is a counterpart strategy, represented by F in the Prisoner's Dilemma, that would be desirable. This is a strategy that **is always the best** thing you can do, **regardless** of what your opponents choose.

Definition

$s_i \in S_i$ is a **strictly dominant strategy** for player i if **every** other strategy of i is **strictly dominated** by it, that is,

$$v_i(s_i, s_{-i}) > v_i(s_i', s_{-i}) \quad \forall\ s_i' \in S_i,\ s_i' \neq s_i,\ \text{and}\ \forall\ s_{-i} \in S_{-i}$$

Strict dominant strategy equilibrium

Definition

The strategy profile $s^D \in S$ is a strict **dominant strategy equilibrium** if $s_i^D \in S_i$ is a strict dominant strategy for all $i \in N$.

Using this solution concept for any game is not that difficult. It basically requires that we **identify** a strict dominant strategy **for each player** and then use this profile of strategies to predict or prescribe behavior.

Solutions and payoffs

Be careful not to make a common error by referring to the pair of payoffs $(-4, -4)$ as the solution.

The **solution** should always be described as the **strategies** that the players will choose. Strategies are a **set of actions** by the players, and **payoffs** are a **result** of the outcome.

When we talk about **predictions**, or **equilibria**, we will always refer to what players do as the equilibrium, not their payoffs.

Strict dominant strategy equilibrium

Proposition

If the game

$$\Gamma = (N, \{S_i\}_{i=1}^n, \{v_i\}_{i=1}^n)$$

has a **strictly dominant strategy equilibrium** s^D, then s^D is the **unique** dominant strategy equilibrium.

For example, in the Prisoner's Dilemma, (F, F) is a dominant strategy equilibrium.

Weak dominance

Definition

We say that s_i' is **weakly dominated** by s_i if, for any possible combination of the other players' strategies, player i's payoff from s_i' is weakly less than that from s_i.

That is,

$$v_i(s_i, s_{-i}) \geq v_i(s_i', s_{-i}) \quad \forall\ s_{-i} \in S_{-i}$$

This means that for some $s_{-i} \in S_{-i}$ this weak inequality may hold strictly, while for other $s_{-i} \in S_{-i}$ it will hold with equality. We define a strategy to be weakly dominant in a similar way.

An important difference between **weak** and **strict dominance** is that if a **weakly dominant equilibrium** exists, it **need not be unique**.

Section 3

Iterated Elimination

Rational players

The requirement that players be **rational** implied two important conclusions:

- A rational player will **never play** a **dominated strategy**.
- If a rational player **has a dominant strategy** then he will **play it**.

The first conclusion is by itself useful in that it **rules out** what players **will not do**. As a result, we conclude that rationality tells us which strategies will never be played.

Common knowledge

Now turn to another important assumption: the **structure** of the game and the **rationality** of the players are **common knowledge among the players**.

The introduction of common knowledge of rationality **allows us** to do much more than identify strategies that rational players will avoid. If indeed all the players know that each player will never play a strictly dominated strategy, they can **effectively ignore** those **strictly dominated strategies** that **their opponents** will never play, and their opponents can do the same thing.

If the original game has some players with some strictly dominated strategies, then all the players know that effectively they are facing a **smaller restricted game** with fewer total strategies.

Common knowledge

In this smaller restricted game, everyone knows that players will not play strictly dominated strategies.

In fact we may indeed find **additional strategies** that are dominated in the restricted game that **were not dominated in the original game**.

Because it is common knowledge that players will perform this kind of reasoning again, the process can continue **until no more strategies can be eliminated** in this way.

Example

Consider the following two-player finite game:

		Player 2		
		L	C	R
	U	4, 3	5, 1	6, 2
Player 1	M	2, 1	8, 4	3, 6
	D	3, 0	9, 6	2, 8

Note that there is not **strictly dominant strategy**, neither for player 1 nor for player 2.

Also note that there is no **strictly dominated strategy** for player 1.

There is, however, a **strictly dominated strategy** for player 2: the strategy C is strictly dominated by R because $2 > 1$ (row U), $6 > 4$ (row M), and $8 > 6$ (row D).

Example

Thus, because this is **common knowledge**, both players know that we can **effectively eliminate** the strategy C from player 2's strategy set, which results in the following reduced game:

		Player 2: L	Player 2: R
Player 1	U	4, 3	6, 2
	M	2, 1	3, 6
	D	3, 0	2, 8

In this reduced game, both M and D are **strictly dominated** by U for player 1, allowing us to perform a second round of eliminating strategies, this time for player 1.

Example

Eliminating these two strategies yields the following trivial game:

		Player 2	
		L	R
Player 1	U	**4, 3**	6, 2

in which player 2 has **a strictly dominated strategy**, playing R.

Thus for this example the **iterated** process of **eliminating dominated strategies** yields a unique prediction: the strategy profile we expect these players to play is (U, L), giving the players the payoffs of $(4, 3)$.

Example

As the example demonstrates, this process of **iterated elimination of strictly dominated strategies (IESDS)** builds on the assumption of **common knowledge of rationality**.

The first step of iterated elimination is a consequence of player 2's rationality; the second stage follows because players **know that players are rational**; the third stage follows because players know that players know that they are rational, and this ends in a unique prediction.

Iterated elimination of strictly dominated strategies (IESDS)

Let S_i^k denote the strategy set of player i that survives k rounds of IESDS.

- **Step 1**: Define $S_i^0 = S_i$ for each i, the original strategy set of player i in the game, and set $k = 0$.
- **Step 2**: **Are there** players for whom there are strategies $s_i \in S_i^k$ that are **strictly dominated**?
  - If yes, go to **step 3**.
  - If not, go to **step 4**.
- **Step 3**: For all the players $i \in N$, **remove** any strategies $s_i \in S_i^k$ that are **¿strictly dominated**. Set $k = k + 1$, and define a new game with strategy sets S_i^k that **do not include** the strictly dominated strategy that have been removed. Go back to step 2.
- The **remaining strategies** in S_i^k are reasonable predictions for behavior.

Iterated elimination of strictly dominated strategies (IESDS)

Using the process of IESDS we can define a new solution concept

Definition

We will call any strategy profile

$$s^{ES} = (s_1^{ES}, \dots, s_n^{ES})$$

that **survives** the process of **IESDS** an **Iterated-elimination equilibrium**.

Like the concept of a **strictly dominant strategy equilibrium**, the iterated-elimination equilibrium starts with the **premise of rationality**.

However, in addition to rationality, **IESDS** requires a lot more: **common knowledge of rationality**.

IESDS

Proposition

If for a game

$$\Gamma = (N, \{S_i\}_{i=1}^n, \{v_i\}_{i=1}^n)$$

s^* is a **strict dominant strategy equilibrium**, then s^* **uniquely survives** IESDS.

Section 4

References

References

- Tadelis, S. (2013). **Game Theory. An Introduction**. Princeton University Press.