

$$y \sim \mathcal{NP}(0, \underbrace{C_d + C_e + f^2 \cdot P \cdot C_u \cdot P^T}_{K_y})$$

$$K_y = f^2 \cdot P \cdot C_u \cdot P^T + C_d + C_e \quad K_y$$

$$-\log(p(y)) = \frac{n}{2} \log 2\pi + \frac{1}{2} \log |K_y| + \frac{1}{2} (f \cdot P \cdot \bar{u} - y)^T \times K_y^{-1} (f \cdot P \cdot \bar{u} - y)$$

$$\frac{\partial K_y}{\partial f} = 2 f \cdot P \cdot C_u \cdot P^T$$

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$$\begin{aligned} \frac{\partial K_y}{\partial \sigma_d} &= \frac{\partial C_d}{\partial \sigma_d} ; \quad C_d = \sigma_d^2 \cdot \exp\left(-\frac{1}{2\ell_d^2} (x - x')^2\right) \\ &= 2\sigma_d \cdot \exp\left(-\frac{1}{2\ell_d^2} (x - x')^2\right) \end{aligned}$$


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$$\begin{aligned} \frac{\partial K_y}{\partial \ell_d} &= \frac{\partial C_d}{\partial \ell_d} = \sigma_d^2 \cdot \exp\left(-\frac{1}{2\ell_d^2} (x - x')^2\right) \cdot \\ &\quad \cdot \frac{1}{\ell_d^3} (x - x')^2 \end{aligned}$$


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$$\Rightarrow \text{z.B.} \quad \frac{\partial}{\partial \ell_d} -\log(p(y)) = -\frac{1}{2} \text{tr}\left((a a^T - K_y^{-1}) \cdot \frac{\partial K_y}{\partial \ell_d}\right)$$

$$\text{mit } a = K_y^{-1} y$$