Xi'an Jiaotong-Liverpool University

PAPER CODE	EXAMINER	DEPARTMENT	TEL
CPT 107	K.L. Man	Discrete Mathematics and Statistics	1509

2021/22 SEMESTER 1 – Assessment I

BACHELOR DEGREE – Year 2

Discrete Mathematics and Statistics

DEADLINE: 29 October, 2021 at 5 pm

INSTRUCTIONS TO CANDIDATES

1. The Assessment should be done individually.
2. Total marks available are 100, accounting for 10% of the overall module marks.
3. The number in the column on the right indicates the marks for each question.
4. Answer all questions.
5. Answers should be written in English.
6. Relevant and clear steps should be included in your answers.
7. Your solutions should be submitted electronically through the Learning Mall via the submission link.
8. The naming of Report (in pdf) is as follows: CPT107_StudentID_CW1 (e.g., CPT107_12345678_CW1.pdf).
9. Answers can also be handwritten, fully and clearly scanned or photographed for submission as one single PDF document through the Learning Mall via the submission link.

Notes:

- **To obtain full marks for each question, relevant and clear steps must be included in the answers.**
- **Partial marks may be awarded depending on the degree of completeness and clarity.**

QUESTION 1: Proof Techniques (20 marks)

(a) Use proof by *contradiction* to show that there does not exist any integers x and y such that $6x + 18y = 1$.

(3 marks)

(b) Let a be a positive real number, and the Statement S be: "if a is not rational number, then $\sqrt[6]{a}$ is also not a rational number".

1. Prove the Statement S by *contradiction*.
2. If you think the converse of the Statement S is true, prove it. If not, give a counter-example.

(8 marks)

(c) For x, y, z $\in \mathbb{Z}$, use proof by *contradiction* to show the following statement:

if $x^2 + y^2 = z^2$, then at least one of x and y is even.

(5 marks)

(d) For all integer $m \geq 1$, use proof by *induction* to show that:

$$2m(\frac{4m+1}{3})(m+\frac{1}{2}) = (1^2 + 2^2 + 3^2 + \ldots + (2m)^2).$$

(4 marks)

QUESTION 2: Set Theory **(32 marks)**

(a) Let the universal set be $\mathbb{N}$, $\mathrm{A} = \{x \in \mathbb{N} \mid x \text{ is even and } 3 < x < 11\}$, $\mathrm{B} = \{x \in \mathbb{N} \mid x \text{ is odd and } 2 < x < 10\}$, $\mathrm{C} = \{x \in \mathbb{N} \mid x \text{ is odd and } 3 < x < 9\}$ and $\mathrm{D} = \{\mathrm{d} \in \mathbb{N} \mid \mathrm{d}^2 = 5\}$. Find the elements of the following expressions:

1. $\mathrm{B} \cap (\mathrm{A} \cup \mathrm{D})$
2. $\sim((\mathrm{B} - \mathrm{D}) \cap \mathrm{A})$
3. $(\mathrm{C} \cup \mathrm{D}) \cap (\mathrm{B}\, \Delta\, \mathrm{A})$
4. $\big(((\mathrm{D} \cup \mathrm{B}) \cap (\mathrm{A}\, \Delta\, \mathrm{B})\big) \cup (\mathrm{B} - (\mathrm{A} - \mathrm{D}))) \cap \mathrm{C}$

(8 marks)

(b) Let X and Y be sets, then $X \subseteq Y$ if and only if $\sim X \subseteq \sim Y$. If you think that it is true, prove it. Otherwise, give a counterexample to show that it is false.

(6 marks)

(c) Prove that the following sets are equal:

1. A x (B ∪ C) = (A x B) ∪ (A x C)
2. A x (B − C) = (A x B) − (A x C)

(10 marks)

(d) Let X and Y be sets, and X ⊆ Y if and only if (Y − X) ∪ X = Y. If you think that it is true, prove it. Otherwise, give a counterexample to show that it is false.

(8 marks)

QUESTION 3: Relations (48 marks)

(a) Let $A = \{a \in \mathbb{Z} \mid a \neq 0\}$ and R be the relation on the set A defined by $R = \{(a,b) \mid a + b = 2c\}$, where $a,b,c \in A$. Prove or disprove that R over A is equivalence notion.

(6 marks)

(b) Let $A = \{1,2,3,4\}$. What is the transitive closure of the relation $\{(1,2),(2,3),(3,4),(1,3),(1,4),(2,4)\}$ on A? Express your answer using ordered pairs.

(4 marks)

(c) If A and B are both transitive relations, then their intersection is also transitive. If you think that it is true, prove it. If not, give a counter-example.

(8 marks)

(d) Let R and S be two partial orders on a set X, and T is a relation on X such that aTb (i.e. $a,b \in X$) if and only if both aRb and aSb hold. Is T also a partial order on X? Justify your answer with proofs and/or counterexamples

(7 marks)

(e) Given the partial ordering on the following set A = {a, b, c, d, e} as {(a,a), (b,b), (c,c), (d,d), (e,e), (a,b), (a,c), (a,d), (a,e), (d,e)}, draw the corresponding Hasse diagram.

(3 marks)

(f) Let A be the points in the plane ($A = \mathbb{R}\text{x}\mathbb{R}$). We say two points are equivalent if they are equal distance from the origin. So (x, y) ◎ (w, z) if $x^2+y^2 = w^2+z^2$, where $(x,y), (w,z) \in A$. Show this is an equivalence relation on A and find the equivalent classes.

(10 marks)

(g) For all $a, b \in \mathbb{Z}^+$, we define a◎b if $a/b \in \mathbb{Z}^+$. Prove or disprove ◎ is a partial order and/or a total order on $\mathbb{Z}^+$.

(10 marks)

END OF ASSESSMENT PAPER