Univariate and Multivariate GARCH: t-GARCH, DCC and Stress Testing

2025-02-10

1 Introduction

The chosen stocks are Tesla and Nvidia on the time frames 2011-01-01 to 2023-12-31. The implemented univariate models are GARCH(1,1) and t-GARCH(1,1) and the multivariate model implemented is DCC-GARCH. The models have been compared and analysed, while the DCC-GARCH model has also been subjected to stress testing.

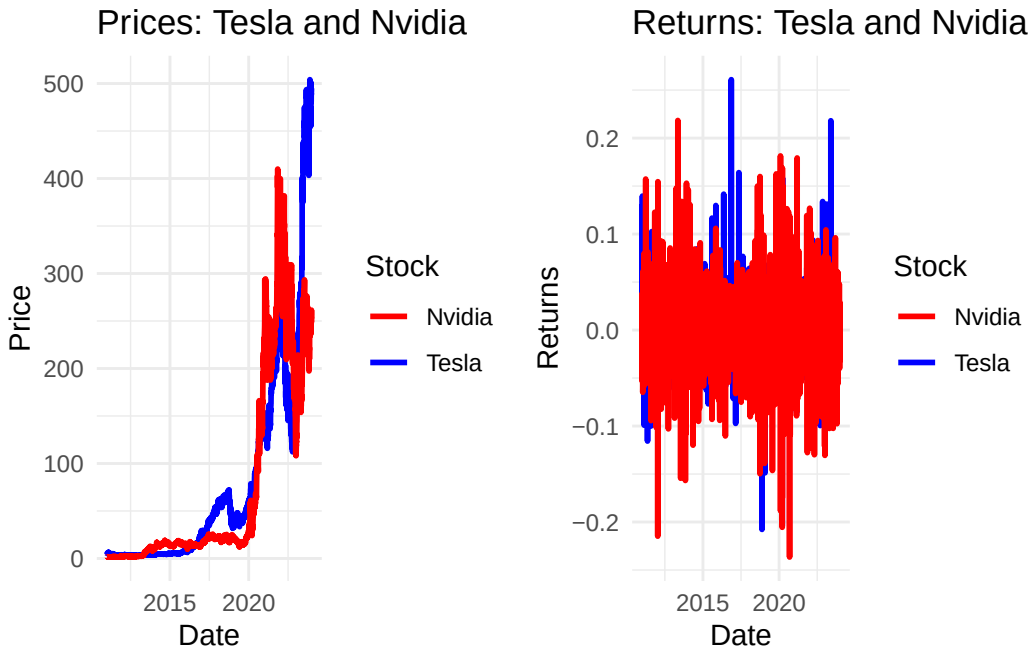


Table 1: Summary Statistics for Tesla and Nvidia Returns

Statistic	Tesla	Nvidia
Mean	0.0015	0.0015
Standard Deviation	0.0353	0.0281
Skewness	-0.0216	0.2859
Kurtosis	7.8848	10.6351

2 Summary Statistics

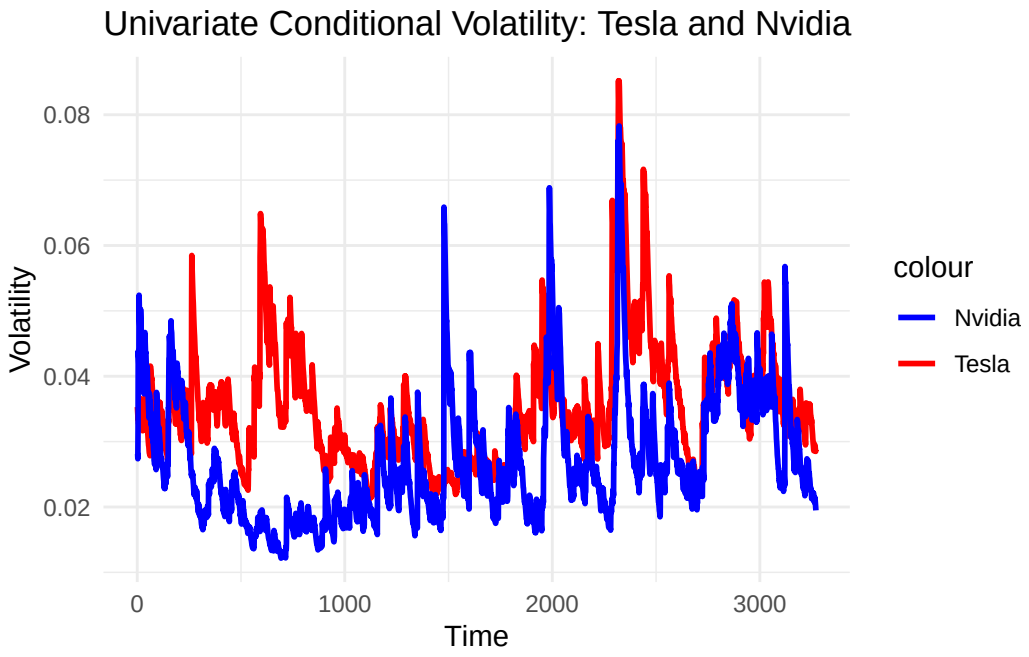
Both Tesla and Nvidia show positive, but small average daily returns, where Tesla shows slightly higher volatility in its day-to-day price movements compared to Nvidia. Tesla's return distribution is nearly symmetric, while Nvidia's has a slight positive skew, which indicates a tendency for more frequent moderate positive returns than large negative ones.

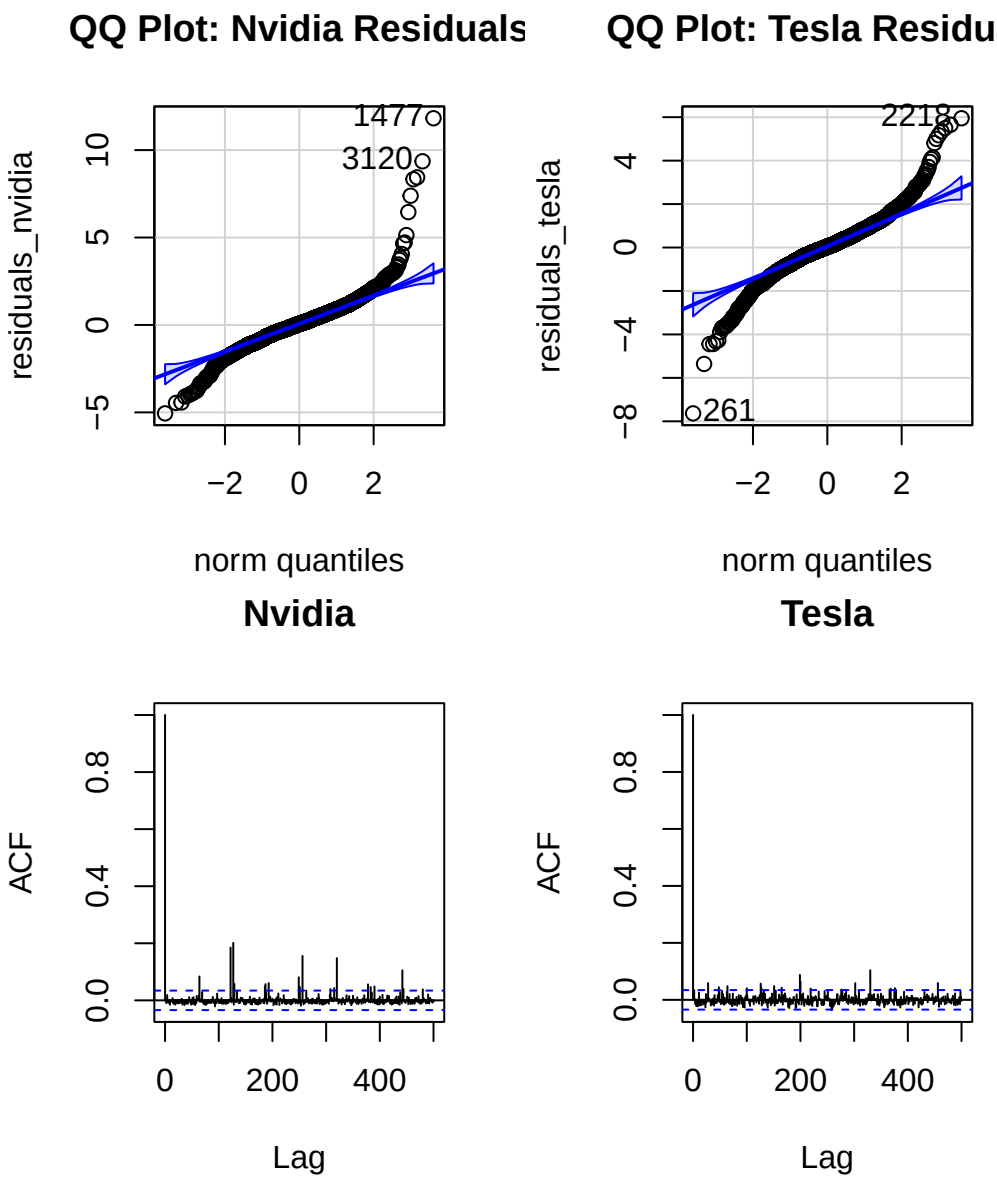
Table 2: Likelihood Ratio Test: Tesla and Nvidia

Stock	Normal Log-Likelihood	t-Log-Likelihood	LR Statistic	P-Value
Tesla	6477.483	6706.354	457.7416	0
Nvidia	7283.234	7557.454	548.4402	0

After running both the Normal Univariate GARCH(1,1) model and the Student-t Univariate GARCH(1,1) model, the Student-t Model was observed to be better by having a large log-likelihood value for both stocks and large likelihood ratio statistic value, where t-GARCH(1,1) was the unrestricted model and GARCH(1,1) was the restricted model. As such, t-GARCH(1,1) was the model that fit the data best and the one that was used for both the Univariate GARCH and DCC-GARCH.

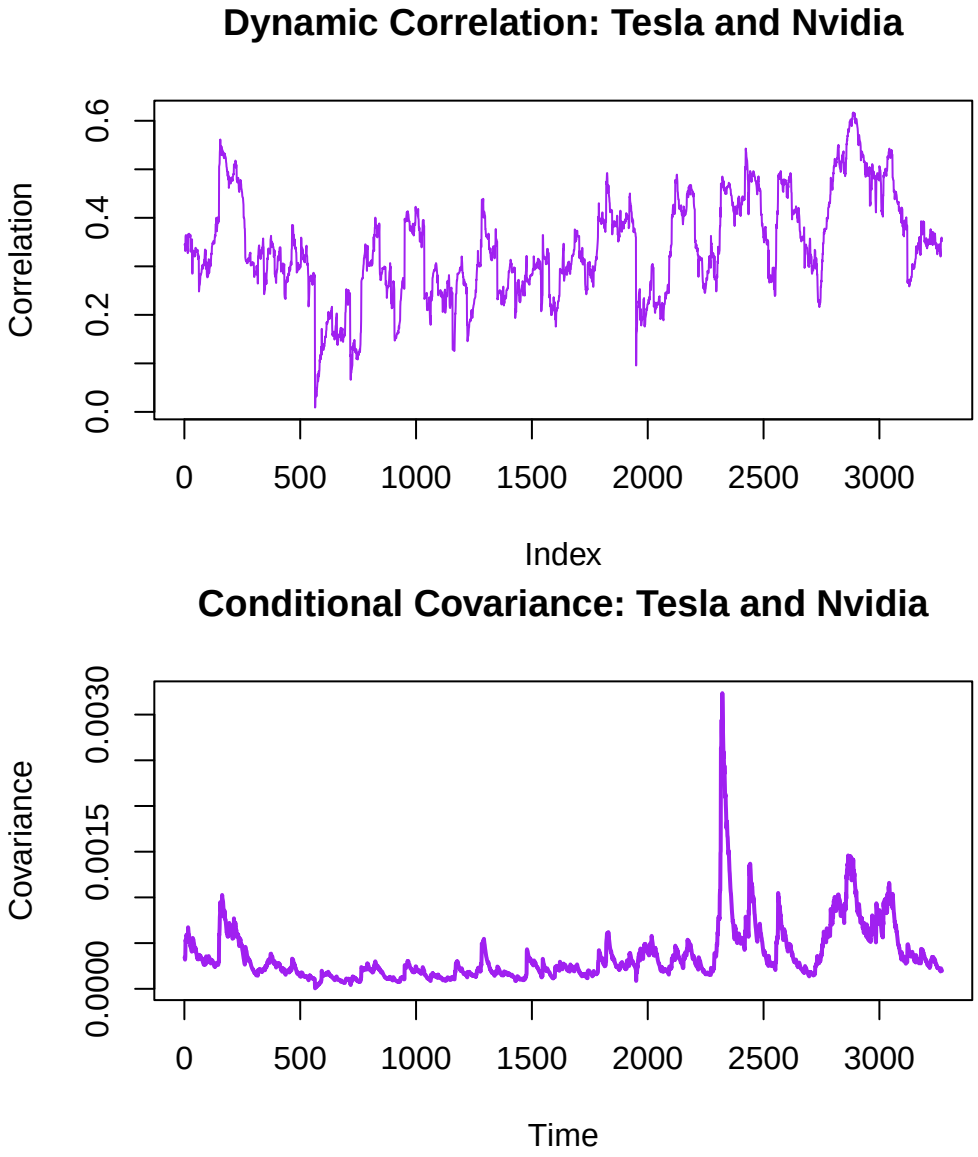
3 Univariate t-GARCH(1,1)





Overall, the conditional volatilities reflect time-varying heteroskedasticity but have the drawback that they ignore the spillover effects between the assets. t-GARCH(1,1) shows that Tesla has higher volatility spikes, while both the QQ plots show fatter tails which suggest that using t-GARCH over GARCH is better, while outliers still exist (such as 1477, 2218). The values of the ACFs of the residuals are consistently very low which shows that the model was succesful in removing serial correlation, while however, the residuals likely retain cross correlation between the stocks, as they are treated independently by the univariate models.

4 Multivariate: DC-GARCH



DCC has the advantage of adding a dynamic correlation component, which adjusts the joint volatility forecasts based on changing correlations over time. The dynamic correlation table shows moderately high fluctuations which reflects the differences in volatility that were also seen in the univariate case. When it comes to conditional covariance, the spikes in the graph illustrate periods of high volatility, when stocks tend to correlate more. Periods of low volatility in the stocks lead to lower covariance, even if correlations remain steady. Conditional covariance is useful practically for determining value at risk and can only be determined using DCC and not univariate GARCH.

Table 3: Log-Likelihood Results for Univariate and Multivariate Models

Model	Log-Likelihood
Tesla (Univariate)	6706.354
Nvidia (Univariate)	7557.454
DCC-GARCH (Multivariate)	14538.205

Table 4: Likelihood Ratio Test Results

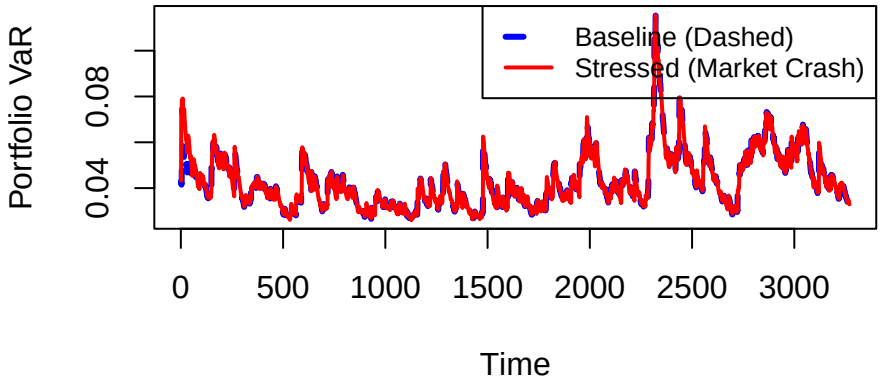
Test	Likelihood Ratio Statistic	P-Value
Likelihood Ratio Test	548.7955	0

5 Overall comparison

The value for the LR Statistics is large, which implies that running DCC jointly for Nvidia and Tesla shows a statistically significant improvement over running t-GARCH separately. Moreover, as there are heavy tails in the QQ plots, Student-t was the better choice but DCC also better accounted for this behaviour. The DCC dynamic correlation and conditional covariance plots revealed significant shifts which are unaccessible by using univariate GARCH.

6 Stress testing of DCC-GARCH

A market crash of -20% for both stocks has been simulated in order to observe the difference in Value at Risk within the framework of the DCC-GARCH model.



The almost perfect overlap between the baseline and stressed values shows that the model is highly resilient at least to a mild shock. This implies that the DCC-GARCH model can dynamically adjust to extreme events through time-varying correlation.