TTK4190 - Assignment 3

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1 Part 2: Heading Autopilot

1.1 Problem 1 - Environmental Disturbances

1.1.1 Problem 1.a

With $\beta_{V_c} = 45°$ and $V_c = 1$, and from using the following equations from [1]

$$u_c = V_c cos(\beta_{V_c} - \psi)$$
$$v_c = V_c sin(\beta_{V_c} - \psi)$$

```
for i=1:Ns+1

    t = (i-1) * h;                      % time (s)
    R = Rzyx(0,0,eta(3));

    % current (should be added here)
    nu_c(1) = Vc*cos(betaVc - eta(3));  % surge current
    nu_c(2) = Vc*sin(betaVc - eta(3));  % sway current

    nu_r = nu - nu_c;
```

Listing 1: MATLAB code for 2D irrotational ocean current in surge and sway

and the resulting plots become

1.1.2 Problem 1.b

With zero ocean currents (and wind) affecting the vessel, there will be no crab or sideslip angle. This will result in the vessel following a straight path.

1.1.3 Problem 1.c

The angle of attack between the wind and the boat is given as

$$\gamma_w = \psi - \beta_{V_w} - \pi \tag{1}$$

The wind-coefficients are approximated as in [1] in sway and yaw direction as

$$C_Y(\gamma_w) \approx c_y \sin(\gamma_w) \tag{2}$$

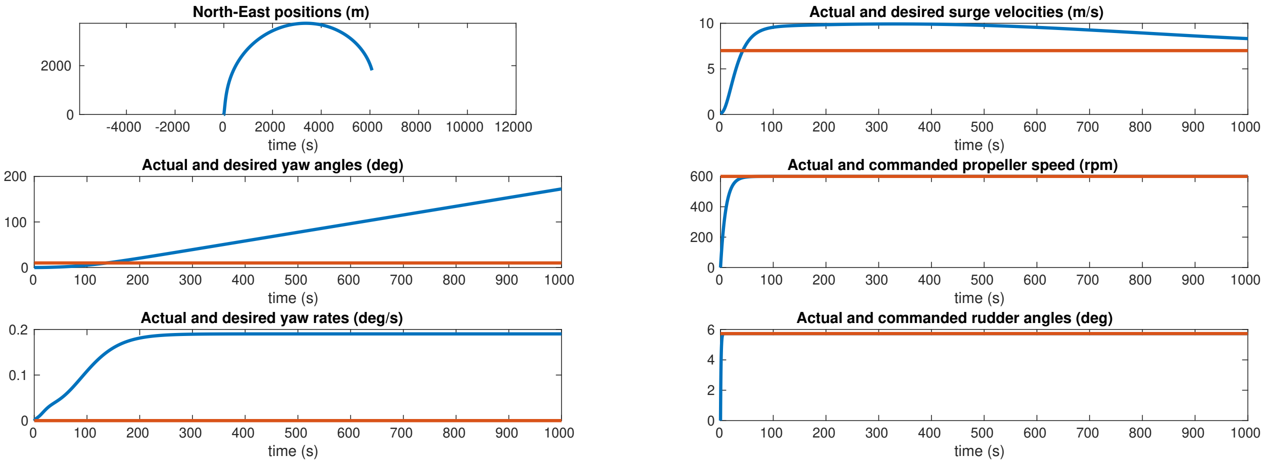
$$C_N(\gamma_w) \approx c_n \sin(2\gamma_w) \tag{3}$$

The moment

$$\tau_{wind} = \begin{bmatrix} \tau_{Y_{wind}} \\ \tau_{N_{wind}} \end{bmatrix} = \begin{bmatrix} \frac{\rho_a {V_{ra}}^2}{2} C_Y(\gamma_{rw}) A_{L_w} \\ \frac{\rho_a {V_{ra}}^2}{2} C_N(\gamma_{rw}) A_{L_w} L_{oa} \end{bmatrix} \tag{4}$$

```
    if t > 200
        % UNSURE ABOUT RELATIVE VS ABSOLUTE WIND SPEED AND ANGLE-OF-ATTACK
        gamma_w = eta(3) - betaVw - pi;
        C_Ywind = cy*sin(gamma_w);                % sway wind coefficient
        C_Nwind = cn*sin(2*gamma_w);              % yaw wind coefficient
        Ywind = 1/2*rho_a*Vw^2*C_Ywind*A_Lw;      % expression for wind moment in ...
            sway should be added.
        Nwind = 1/2*rho_a*Vw^2*C_Nwind*A_Lw*L;    % expression for wind moment in ...
            yaw should be added.
    else
        Ywind = 0;
        Nwind = 0;
    end
    tau_env = [0 Ywind Nwind]';
```

Listing 2: Wind movement in sway and yaw.



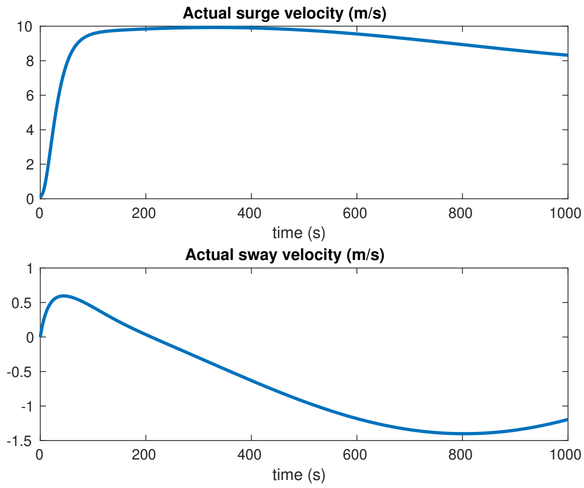


Figure 1: Boat behaviour with current disturbance

1.2 Problem 2 - Heading Autopilot

1.2.1 Problem 2.a

From the given matlab code we get that

$$\mathbf{C}_A = \begin{bmatrix} 0 & 0 & Y_{\dot{v}}v + Y_{\dot{r}}r \\ 0 & 0 & -X_{\dot{u}}u \\ -Y_{\dot{v}}v - Y_{\dot{r}}r & X_{\dot{u}}u & 0 \end{bmatrix} \tag{5}$$

$$C_{RB} = \begin{bmatrix} 0 & -mr & -mx_g r \\ mr & 0 & 0 \\ mx_g r & 0 & 0 \end{bmatrix} \tag{6}$$

and the nonlinear coriolis forces become

$$\mathbf{C}_A(\nu)\nu = \begin{bmatrix} Y_{\dot{v}}vr + Y_{\dot{r}}r^2 \\ -X_{\dot{u}}ur \\ -Y_{\dot{v}}vu - Y_{\dot{r}}ru + X_{\dot{u}}uv \end{bmatrix} \tag{7}$$

$$C_{RB}(\nu)\nu = \begin{bmatrix} -mrv - mx_g r^2 \\ mru \\ mx_g ru \end{bmatrix} \tag{8}$$

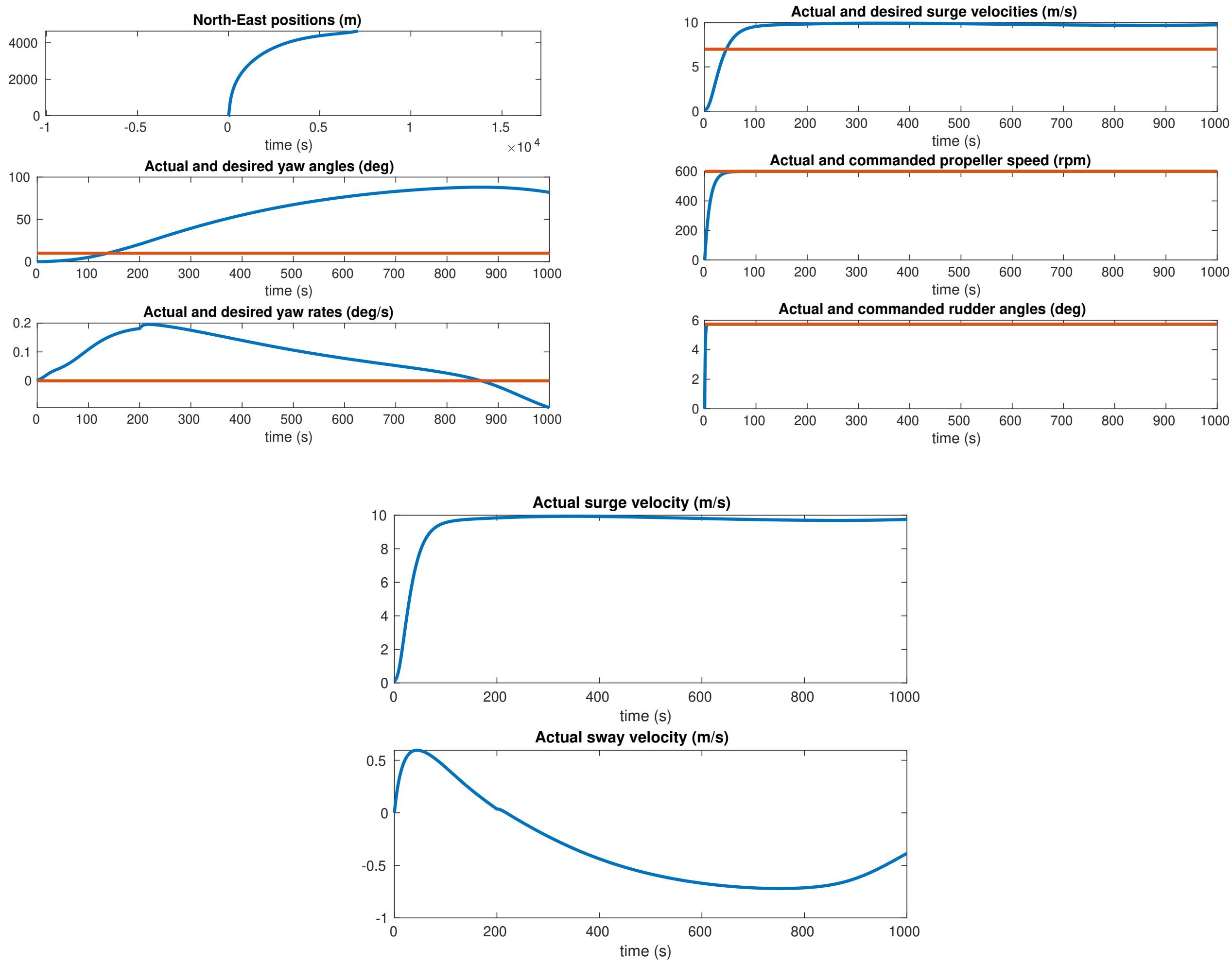


Figure 2: Boat behaviour with current disturbance and wind disturbance after 200 seconds

Linearized at $\nu = \nu^*$ we get

$$\mathbf{C}_A^* = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -X_{\dot{u}}U_d \\ 0 & -Y_{\dot{v}}U_d + X_{\dot{u}}U_d & -Y_{\dot{r}}U_d \end{bmatrix} \tag{9}$$

$$C_{RB}^* = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & mU_d \\ 0 & 0 & mx_gU_d \end{bmatrix} \tag{10}$$

1.2.2 Problem 2.b

The model is given in [1] as

$$M\dot{\nu}_r + N\nu_r = b\delta \tag{11}$$

and by rearranging we get that

$$\dot{\nu}_r = -M^{-1}N\nu_r + M^{-1}b\delta \tag{12}$$

The maneuvering model can be expressed as a state-space model as

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u},$$

$$\mathbf{y} = \mathbf{C}\mathbf{x} + \mathbf{D}\mathbf{u}$$

where

$$\mathbf{A} = -\mathbf{M}^{-1}N$$
$$\mathbf{B} = \mathbf{M}^{-1}b$$
$$\mathbf{C} = \begin{bmatrix} 0 & 1 \end{bmatrix}$$
$$\mathbf{D} = 0$$

The resulting transfer function can be approximated by using ss2tf(.)

$$\frac{r}{\delta}(s) = \frac{8.04 \cdot 10^{-5}s + 6.15 \cdot 10^{-6}}{s^2 + 0.151s + 8.20 \cdot 10^{-4}} = \frac{K(T_3 s + 1)}{(T_1 s + 1)(T_2 + 1)} \tag{13}$$

With the numerical values yielding $K = 7.50 \cdot 10{-}3$, $T_3 = 14.054$, $T_2 = 117.70$ and $T_1 = 6.90$.

1.2.3 Problem 2.c

Going from Equation (13) to the Nomoto model

$$\frac{r}{\delta}(s) = \frac{K}{Ts + 1} \tag{14}$$

where $T := T_1 + T_2 - T_3 = 169.55$ and $K = 7.50 \cdot 10^{-3}$

1.2.4 Problem 2.d

A PID controller for heading autopilot is created using pole placement. We can also see that the actual yaw angle and yaw rate is following the desired yaw angle and yaw rate. From the north east position we can see that the course angle is different from zero, however this is because we have created a heading controller, not a course controller. Because of the slow dynamics, the controller use some time to reach the desired angle. That is what causes the north east angle.

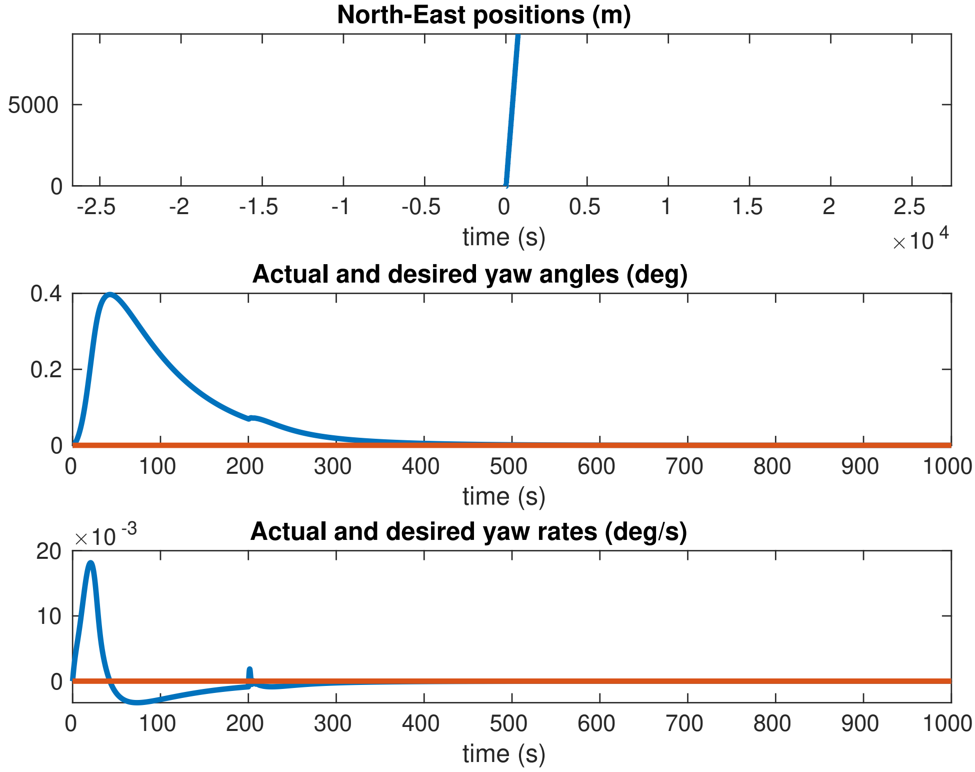


Figure 3: Boat behaviour with PID controller for heading autopilot.

1.2.5 Problem 2.e

The heading autopilot can compensate for the 10 deg to 20 deg maneuver, as can be seen in the North East position plot in fig. 4. Integral windup is not necessary in this case because we don't have saturation in the actuator. However it is good practice to implement.

```
M_lin = MRB + MA;
M_lin = M_lin(2:3,2:3);
M_lin_inv = inv(M_lin);

CRB_lin = [ 0     m*U_d
            0     m*xg*U_d   ];

CA_lin = [   0                        -Xudot*U_d
            -Yvdot*U_d+Xudot*U_d      -Yrdot*U_d   ];

N_lin = CRB_lin + CA_lin + D(2:3,2:3);
b_lin = [-2*U_d*Y_Δ -2*U_d*N_Δ]';

[num, den] = ss2tf(-M_lin_inv*N_lin, M_lin_inv*b_lin, [0 1], 0);

K_lin = num(3)/den(3);            % gain can be found by using the steady-state ...
    value (s=0) of the transfer function

poles_lin = roots(den);
zeros_lin = roots(num);

T3_lin = -1/zeros_lin;
T1_lin = -1/poles_lin(1);
T2_lin = -1/poles_lin(2);

```

```
T_lin = T1_lin+T2_lin-T3_lin;   % nomoto first-order time constant, eq. (7.24)

% controller gains (example 15.7)
Kp = wn^2*T_lin/K_lin;
Kd = (2*zeta*wn*T_lin-1)/K_lin;
Ki = wn/10*Kp;
```

Listing 3: Linearized C_a and C_{RB} and the gains for PID controller using pole placement

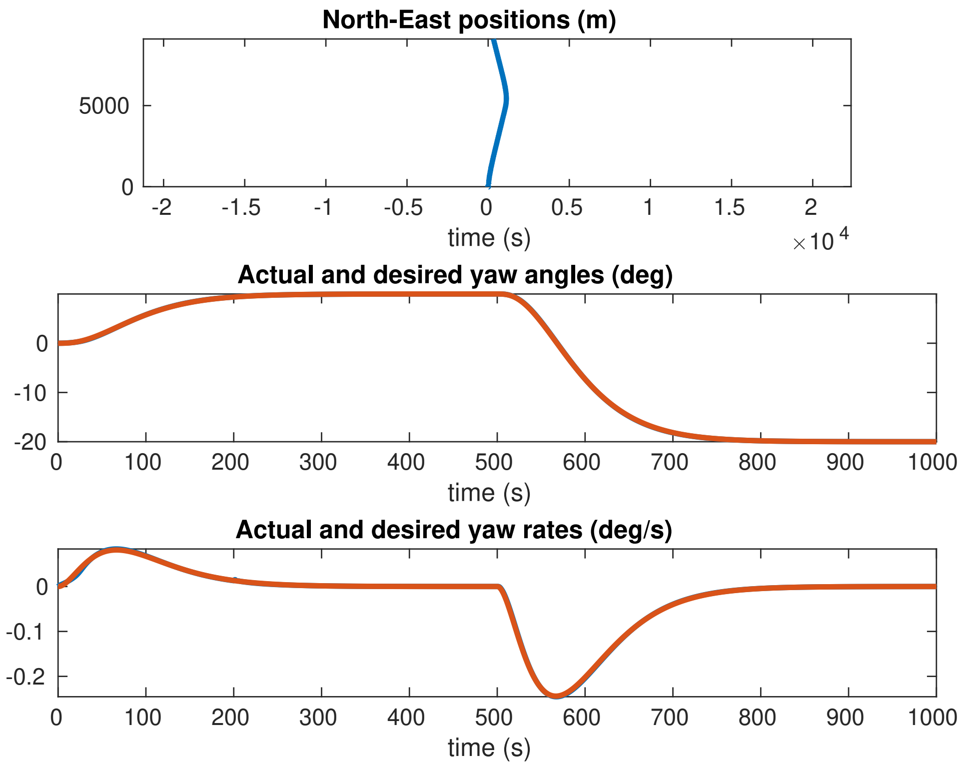


Figure 4: Boat behaviour when doing a 10-20 degree maneuver.

References

[1] T.I. Fossen. *Handbook of Marine Craft Hydrodynamics and Motion Control*. John Wiley & Sons, 2021.