



IMPERIAL COLLEGE LONDON

DEPARTMENT OF AERONAUTICS

---

## Semi-Empirical Optimisation of the Shape of a Surface Reducing Turbulent Skin Friction

---

*Author:*

Herman (Hon Man) Mak

*Supervisor:*

Prof. Sergei Chernyshenko

Submitted in partial fulfilment of the requirements for the degree of

*MSc Advanced Aeronautical Engineering*

September, 2021

## **Abstract**

Your abstract goes here. The abstract is a very brief summary of the dissertation's contents. It should be about half a page long. Somebody unfamiliar with your project should have a good idea of what it's about having read the abstract alone and will know whether it will be of interest to them.

### **Acknowledgements**

It is usual to thank those individuals who have provided particularly useful assistance, technical or otherwise, during your project.

# Contents

<b>Acronyms</b>	<b>5</b>
<b>Notation</b>	<b>6</b>
<b>1 Introduction</b>	<b>7</b>
1.1 Motivation . . . . .	7
1.2 Review . . . . .	9
1.2.1 The Spatial Stokes Layer . . . . .	9
<b>2 Background</b>	<b>11</b>
<b>3 Main sections of the project</b>	<b>12</b>
<b>4 Evaluation</b>	<b>13</b>
<b>5 Conclusion</b>	<b>14</b>
<b>A First Appendix</b>	<b>15</b>
<b>Bibliography</b>	<b>19</b>

# List of Figures

1.1	A schematic of triangular riblets the most commonly researched riblets. Figure modified from [27]. . . . .	9
-----	--	---

# List of Tables

# Acronyms

**BL** boundary layer. 7, 8

**CFD** computational fluid dynamics. 9

**DNS** direct numerical simulation. 9

**DR** drag reduction. 7, 9

**TBL** turbulent boundary layer. 7

**TRL** technology readiness level. 8

**TSL** temporal Stokes layer. 10

# Notation

$\rho$  fluid density. 9

$\nu$  kinematic viscosity, defined as the ratio between dynamic viscosity and density  $\frac{\mu}{\rho}$ . 9

$\mu$  dynamic viscosity. 9

$\tau_w$  wall shear stress. 9

$A_g^+$  riblet groove area in wall units. 8

$t$  time. 9

$T$  period. 9

$u_\tau$  friction wall velocity; it is equal to  $\sqrt{\frac{\tau_w}{\rho}}$ . 9

$W_w$  spanwise wall velocity. 9, 10

**$Re$**  Reynolds number, and is equal to  $\frac{UL}{\nu}$ , a characteristic velocity  $U$  multiplied by a characteristic length  $L$  divided by kinematic viscosity  $\nu$ . 7



# Chapter 1

## Introduction

### 1.1 Motivation

Whether it be water in a pipeline, or an aircraft soaring through the skies, every fluid passing by a solid and every solid passing through a fluid will experience drag. The ever pressing need to reduce our impact on the environment requires us to reduce our energy used to combat unwanted drag, which also has the added benefit of reducing costs via increased efficiency. This is especially true in the transportation sector, which accounts for 24% of total global emissions in 2019 according to the IEA, although growth has been limited to only 0.5% per year compared to an average increase of 1.9% annually since 2000 owing to efficiency improvements [1].

The search for these efficiency improvements includes research towards drag reduction (DR) via flow control – that is manipulating the flow characteristics in such a way that somehow produces less overall drag. In fact, Ludwig Prandtl, who revolutionised the study of fluid mechanics with the introduction of the turbulent boundary layer (TBL), pioneered modern flow control as early as 1904, where he demonstrated that suction at the surface of a cylinder delays boundary layer (BL) separation and therefore decreases drag [2, 3]. Indeed, DR is a major focus of research in commercial aviation. In the context of aviation, a 1% reduction in drag corresponds to a 0.75% reduction in fuel and as a result CO<sub>2</sub> emissions [4]. In fact, [4] states that based on estimates on travel demand in 2030, a 1% reduction will constitute a 9 million tonnes reduction in CO<sub>2</sub> emissions.

In transport applications, and in particular aviation, the flows are at high Reynolds numbers  $Re$ , this means the regimes we are dealing with are often turbulent. Moreover, especially in aviation (with the exception of cases where supersonic effects dominate), viscous drag generated in the near-wall BL region constitutes a major component of total drag [5]. These two factors combined mean that “flow control methodology targeting the TBL is the most obvious option to achieve a significant skin-friction-drag reduction and ultimately to reduce emissions” [5].

Flow control is separated into two distinct groups, active and passive control. Active flow control requires an input in energy to affect the flow via the use of actuators, whereas passive

flow control does not. Examples of active control include opposition control [6, 7], spanwise-wall oscillation [8–10], and the aforementioned BL separation control [3]; the former is closed-loop and reacts to sensor inputs from the environment, whereas the latter two can be either open-loop with predetermined control patterns or reactive (feedback/feed-forward systems). The actuators used to perform active flow control can range from zero-net-mass-flux jets [11], to dielectric-barrier-discharge plasma actuators [12], to fluid injection (blowing) and sucking [13], to the ingenious moving surface using “pneumatically actuated compliant structure based on the kagome lattice geometry” [14]. Whereas, examples of passive control include vortex generators [15], discontinuities/notches/fences in the leading/rear edges of a wing [15], compliant surfaces [16], porous coatings [17], superhydrophobic surfaces [18], and a very well studied control technique known as riblets [19–21].

As aforementioned, active flow control allows for reactive responses which can increase the effectiveness of control techniques. Moreover, even open-loop flow control can achieve higher viscous drag reduction than passive control techniques without the need for sensors required for reactive flow control. However, this comes at a cost of the extra energy expended to modify the flow and the difficulty and innovation needed to design actuators. This can clearly be seen in the case of spanwise-wall oscillation where the wall moves as prescribed by a streamwise travelling wave, which, after accounting for the power spent to oscillate the fluid, has a net power saving of around 26% despite a drag reduction of  $> 35\%$  for those conditions [22]. Moreover, in order to emulate a in-plane wall motion in real life, the aforementioned compliant structure from [14] had to be created and trialled in laboratory conditions, and then made at scale and maintained if it were to be used on real-world flows.

On the other hand, passive flow control is necessarily open-loop, and may have decreased performance in comparison to active flow control. However, it does not require actuators and the maintenance thereof. Riblets, for example, “are small surface protrusions aligned with the direction of the flow, which confer an anisotropic roughness to a surface” [21] and can be seen in Figure 1.1. Experiments show that under moderate adverse pressure gradient (i.e. where the pressure increases along the direction of the flow) a 13% skin friction reduction is achievable, compared to 6% reduction in a zero-pressure-gradient BL [23]. Although less efficient compared to active control, due to its relatively simple design, its technology readiness level (TRL) is higher than most other flow control techniques. In fact it has been trialled in scale model aircraft tests in transonic Mach numbers [24], real aircraft tests, and even in commercial service for several years by Cathay Pacific on an Airbus A340 where 30% of the wetted surface was covered with riblets [25]. Based on a flight test on an Airbus A320, in transonic Mach number ranges, an A320 with 70% of the wetted surface covered by riblets could have a drag reduction of about 2% [26]. However, the optimal groove cross section was found to have an optimum at  $A_g^{+1/2} \approx 11$ , where the  $+$  superscript denotes non-dimensionalisation by wall units (see 1.2.1) and spacing of approximately 15 wall units [21]. This is equivalent to approximately 30–70  $\mu\text{m}$  in realistic aerofoil and aircraft flows [21]. Moreover, the sharper the riblets, the more efficient they are at reducing drag [21]. All of these

factors make riblets quite hard to manufacture whilst requiring maintenance/replacements due to the erosion from air moving past.

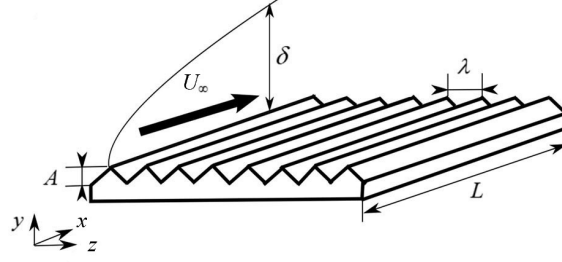


Figure 1.1: A schematic of triangular riblets the most commonly researched riblets. Figure modified from [27].

Therefore, researchers have begun to explore other ways to use passive flow control for turbulent DR. The oblique wavy wall was first proposed by Chernyshenko [28] in 2013 to emulate the motions of in-plane spanwise wall oscillations in hopes that there will be a net energy decrease. We will devote the rest of this report discussing the merits of this curious passive flow control method.

## 1.2 Review

### 1.2.1 The Spatial Stokes Layer

The Stokes layer is one of a few exact solutions to the Navier-Stokes equation describing the motion of a viscous fluid as a function of the wall normal coordinate  $y$ , whereby the infinitely long wall is located at the bottom at  $y = 0$  and oscillating harmonically in its own plane [29]. It turns out that the resulting oscillation in the fluid is only of significant magnitude very close to the wall in a so-called “Stokes layer” and is significantly damped outside of the said-layer.

Jung et al. [8] were the first to suggest using a wall oscillating in the spanwise direction to reduce skin friction in 1992, exploiting the above phenomenon to obtain a maximum drag reduction of 40% at a non-dimensional period of  $T^+ = 100$  using direct numerical simulation (DNS), a computational fluid dynamics (CFD) method [30]. The  $+$  superscript denotes non-dimensionalisation by wall units, which is based upon the wall friction velocity  $u_\tau = \sqrt{\frac{\tau_w}{\rho}}$ , along with the kinematic viscosity  $\nu = \frac{\mu}{\rho}$ , where  $\tau_w$  is the wall shear stress of the fluid flow,  $\rho$  is the density of the fluid, and  $\mu$  is the dynamic viscosity of the fluid flow. The spanwise velocity of the wall is given by

$$W_w = \hat{W}_w \sin\left(\frac{2\pi}{T}t\right), \quad (1.1)$$

where  $\hat{W}_w$  and  $T$  denotes the oscillation amplitude and period, and  $t$  denotes time. Moreover, when only one of the channel walls were oscillating, “the reduction in turbulence activity was observed only near the oscillating wall, while the flow at the other wall remained fully turbulent” [8]. When phase averaged this coincides with the Stokes layer with temporal forcing [10], we will therefore

name it temporal Stokes layer (TSL). Dhanak and Si [31] observed that the duration of sweep events were reduced by 47% and their strength reduced by 23%, suggesting that the skin-friction reduction is a result of the “attenuation in the formation of streamwise streaks [30].

As this is a form of active flow control, despite significant drag reductions, significant energy must also be expended to overcome the extra shear stress to create the spanwise motion of the fluid [10]. Baron and Quadrio [32] was the first to consider the net energy savings from spanwise wall oscillation, and it is now accepted that the net energy savings is 10% [10, 30]. However, this technique requires moving parts and therefore requires actuators, which is hard to implement in practical applications especially in transport applications.

Viotti, Quadrio and Luchini [10] sought to extend the TSL from a time-dependent forcing to a stationary, spatial forcing, which potentially allows an extension into passive solutions which can emulate the oscillatory spatial forcing. (

---

kim and hussain

---

???)

The spatial forcing law is given by

$$W_w = \hat{W}_w \sin\left(\frac{2\pi}{\lambda_x}x\right). \quad (1.2)$$

[28] provides the following context of

Intro: Motivation, Riblets, and others in general, Lit Review – Stokes Layer Results; SSL+Chernyshenko. Refer to Ghebbali DNS. End:Formulation of problem: want to be able to predict drag reduction by WW as f’n of  $k_x, k_z$ , and What

## Chapter 2

# Background

The background section of the dissertation should set the project into context by relating it to existing published work which you read at the start of the project when your approach and methods were being considered. There are usually many ways of solving a given problem, and you shouldn't just pick one at random. Describe and evaluate as many alternative approaches as possible. The background section is often included as part of the introduction but can be a separate chapter if the project involved an extensive amount of research.

The published work may be in the form of research papers, articles, text books, technical manuals, or even existing software or hardware of which you have had hands-on experience. Don't be afraid to acknowledge the sources of your inspiration; you are expected to have seen and thought about other people's ideas; your contribution will be putting them into practice in some other context. However, you must avoid plagiarism: if you take another person's work as your own and do not cite your sources of information/inspiration you are being dishonest; in other words you are cheating.

## Chapter 3

### Main sections of the project

## Chapter 4

# Evaluation

## Chapter 5

## Conclusion

Code is broken

$\tau_w/\tau_0$  problem Ycross is enough to find Pnet Subject to (1) the closed system has been obtained

Wave height versus pressure



# Appendix A

## First Appendix

The appendices contain information which is peripheral to the main body of the dissertation. Information typically included are things like program listings, complex circuit diagrams, tables, proofs, graphs or any other material which would break up the theme of the text if it appeared in situ.

# Bibliography

- [1] IEA. *Transport - Improving the Sustainability of Passenger and Freight Transport*. IEA. 2021. URL: <https://www.iea.org/topics/transport> (visited on 14/09/2021).
- [2] Mohamed Gad-el-Hak. ‘Flow Control: The Future’. In: *Journal of Aircraft* 38.3 (2001), pp. 402–418. DOI: 10.2514/2.2796. URL: <https://doi.org/10.2514/2.2796> (visited on 15/09/2021).
- [3] Ludwig Prandtl. ‘Über Flüssigkeitsbewegung bei sehr kleiner Reibung’. In: *Proceedings of the Third International Mathematical Congress*. Proceedings of the Third International Mathematical Congress. Heidelberg, 8th–13th Aug. 1904, pp. 484–491.
- [4] Michael A. Leschziner, Haecheon Choi and Kwing-So Choi. ‘Flow-Control Approaches to Drag Reduction in Aerodynamics: Progress and Prospects’. In: *Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences* 369.1940 (13th Apr. 2011), pp. 1349–1351. DOI: 10.1098/rsta.2010.0375. URL: <https://royalsocietypublishing.org/doi/10.1098/rsta.2010.0375> (visited on 14/09/2021).
- [5] Adel Abbas et al. ‘Drag Reduction via Turbulent Boundary Layer Flow Control’. In: *Science China Technological Sciences* 60.9 (1st Sept. 2017), pp. 1281–1290. ISSN: 1869-1900. DOI: 10.1007/s11431-016-9013-6. URL: <https://doi.org/10.1007/s11431-016-9013-6> (visited on 14/09/2021).
- [6] Haecheon Choi, Parviz Moin and John Kim. ‘Active Turbulence Control for Drag Reduction in Wall-Bounded Flows’. In: *Journal of Fluid Mechanics* 262 (1994), pp. 75–110. ISSN: 0022-1120. DOI: 10.1017/S0022112094000431. URL: <https://www.cambridge.org/core/article/active-turbulence-control-for-drag-reduction-in-wallbounded-flows/3075211F21E692996F66BF17D63CA649> (visited on 15/09/2021).
- [7] M. Luhar, A. S. Sharma and B. J. McKeon. ‘Opposition Control within the Resolvent Analysis Framework’. In: *Journal of Fluid Mechanics* 749 (June 2014), pp. 597–626. ISSN: 0022-1120, 1469-7645. DOI: 10.1017/jfm.2014.209. URL: <https://www.cambridge.org/core/journals/journal-of-fluid-mechanics/article/opposition-control-within-the-resolvent-analysis-framework/710BD53A61478AFA2DB02355FF7C4FB8> (visited on 15/09/2021).

- [8] W. J. Jung, N. Mangiavacchi and R. Akhavan. ‘Suppression of Turbulence in Wall-bounded Flows by High-frequency Spanwise Oscillations’. In: *Physics of Fluids A: Fluid Dynamics* 4.8 (1st Aug. 1992), pp. 1605–1607. ISSN: 0899-8213. DOI: 10.1063/1.858381. URL: <https://aip.scitation.org/doi/abs/10.1063/1.858381> (visited on 15/09/2021).
- [9] Kwing-So Choi, Jean-Robert DeBisschop and Brian R. Clayton. ‘Turbulent Boundary-Layer Control by Means of Spanwise-Wall Oscillation’. In: *AIAA Journal* 36.7 (1st July 1998), pp. 1157–1163. ISSN: 0001-1452. DOI: 10.2514/2.526. URL: <https://arc.aiaa.org/doi/10.2514/2.526> (visited on 15/09/2021).
- [10] Claudio Viotti, Maurizio Quadrio and Paolo Luchini. ‘Streamwise Oscillation of Spanwise Velocity at the Wall of a Channel for Turbulent Drag Reduction’. In: *Physics of Fluids* 21.11 (Nov. 2009), p. 115109. ISSN: 1070-6631, 1089-7666. DOI: 10.1063/1.3266945. URL: <http://aip.scitation.org/doi/10.1063/1.3266945> (visited on 15/09/2021).
- [11] PanFeng Zhang, JinJun Wang and LiHao Feng. ‘Review of Zero-Net-Mass-Flux Jet and Its Application in Separation Flow Control’. In: *Science in China Series E: Technological Sciences* 51.9 (8th Aug. 2008), p. 1315. ISSN: 1862-281X. DOI: 10.1007/s11431-008-0174-x. URL: <https://doi.org/10.1007/s11431-008-0174-x> (visited on 15/09/2021).
- [12] Jin-Jun Wang et al. ‘Recent Developments in DBD Plasma Flow Control’. In: *Progress in Aerospace Sciences* 62 (1st Oct. 2013), pp. 52–78. ISSN: 0376-0421. DOI: 10.1016/j.paerosci.2013.05.003. URL: <https://www.sciencedirect.com/science/article/pii/S0376042113000535> (visited on 15/09/2021).
- [13] T. L. Chng et al. ‘Flow Control of an Airfoil via Injection and Suction’. In: *Journal of Aircraft* 46.1 (1st Jan. 2009), pp. 291–300. DOI: 10.2514/1.38394. URL: <https://arc.aiaa.org/doi/10.2514/1.38394> (visited on 15/09/2021).
- [14] James Bird, Matthew Santer and Jonathan F. Morrison. ‘Experimental Control of Turbulent Boundary Layers with In-Plane Travelling Waves’. In: *Flow, Turbulence and Combustion* 100.4 (1st June 2018), pp. 1015–1035. ISSN: 1573-1987. DOI: 10.1007/s10494-018-9926-2. URL: <https://doi.org/10.1007/s10494-018-9926-2> (visited on 15/09/2021).
- [15] Paul K. Chang. ‘CHAPTER XII - Control of Separation of Flow’. In: *Separation of Flow*. Ed. by Paul K. Chang. Pergamon, 1st Jan. 1970, pp. 716–752. ISBN: 978-0-08-013441-3. DOI: 10.1016/B978-0-08-013441-3.50016-2. URL: <https://www.sciencedirect.com/science/article/pii/B9780080134413500162> (visited on 15/09/2021).
- [16] K.-S. Choi et al. ‘Turbulent Drag Reduction Using Compliant Surfaces’. In: *Proceedings of the Royal Society of London. Series A: Mathematical, Physical and Engineering Sciences* 453.1965 (8th Oct. 1997), pp. 2229–2240. DOI: 10.1098/rspa.1997.0119. URL: <https://royalsocietypublishing.org/doi/abs/10.1098/rspa.1997.0119> (visited on 15/09/2021).

- [17] Katharina Klausmann and Bodo Ruck. ‘Drag Reduction of Circular Cylinders by Porous Coating on the Leeward Side’. In: *Journal of Fluid Mechanics* 813 (25th Feb. 2017), pp. 382–411. ISSN: 0022-1120, 1469-7645. DOI: 10.1017/jfm.2016.757. URL: [https://www.cambridge.org/core/product/identifier/S0022112016007576/type/journal\\_article](https://www.cambridge.org/core/product/identifier/S0022112016007576/type/journal_article) (visited on 15/09/2021).
- [18] Richard Truesdell et al. ‘Drag Reduction on a Patterned Superhydrophobic Surface’. In: *Physical Review Letters* 97.4 (26th July 2006), p. 044504. DOI: 10.1103/PhysRevLett.97.044504. URL: <https://link.aps.org/doi/10.1103/PhysRevLett.97.044504> (visited on 15/09/2021).
- [19] Michael J. Walsh. ‘Riblets as a Viscous Drag Reduction Technique’. In: *AIAA Journal* 21.4 (1983), pp. 485–486. ISSN: 0001-1452. DOI: 10.2514/3.60126. URL: <https://doi.org/10.2514/3.60126> (visited on 15/09/2021).
- [20] Haechon Choi, Parviz Moin and John Kim. ‘Direct Numerical Simulation of Turbulent Flow over Riblets’. In: *Journal of Fluid Mechanics* 255 (Oct. 1993), pp. 503–539. ISSN: 1469-7645, 0022-1120. DOI: 10.1017/S0022112093002575. URL: <https://www.cambridge.org/core/journals/journal-of-fluid-mechanics/article/direct-numerical-simulation-of-turbulent-flow-over-riblets/8A0DAF9111A41A42EF3F401B10C0594C> (visited on 15/09/2021).
- [21] Ricardo García-Mayoral and Javier Jiménez. ‘Drag Reduction by Riblets’. In: *Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences* 369.1940 (13th Apr. 2011), pp. 1412–1427. DOI: 10.1098/rsta.2010.0359. URL: <https://royalsocietypublishing.org/doi/full/10.1098/rsta.2010.0359> (visited on 15/09/2021).
- [22] Maurizio Quadrio, Pierre Ricco and Claudio Viotti. ‘Streamwise-Travelling Waves of Spanwise Wall Velocity for Turbulent Drag Reduction’. In: *Journal of Fluid Mechanics* 627 (May 2009), pp. 161–178. ISSN: 1469-7645, 0022-1120. DOI: 10.1017/S0022112009006077. URL: <https://www.cambridge.org/core/journals/journal-of-fluid-mechanics/article/abs/streamwisetravelling-waves-of-spanwise-wall-velocity-for-turbulent-drag-reduction/17D9C12129254028993F89BD4451C335> (visited on 10/09/2021).
- [23] J. R. Debiusschop and F. T. M. Nieuwstadt. ‘Turbulent Boundary Layer in an Adverse Pressure Gradient - Effectiveness of Riblets’. In: *AIAA Journal* 34.5 (1st May 1996), pp. 932–937. ISSN: 0001-1452. DOI: 10.2514/3.13170. URL: <https://arc.aiaa.org/doi/10.2514/3.13170> (visited on 15/09/2021).
- [24] E. Coustols and V. Schmitt. ‘Synthesis of Experimental Riblet Studies in Transonic Conditions’. In: *Turbulence Control by Passive Means*. Ed. by E. Coustols. Fluid Mechanics and Its Applications. Dordrecht: Springer Netherlands, 1990, pp. 123–140. ISBN: 978-94-009-2159-7. DOI: 10.1007/978-94-009-2159-7\_8.

- [25] D.W. Bechert and W. Hage. ‘Drag Reduction with Riblets in Nature and Engineering’. In: *WIT Transactions on State of the Art in Science and Engineering*. Ed. by R. Liebe. 1st ed. Vol. 2. WIT Press, 10th Nov. 2006, pp. 457–504. ISBN: 978-1-84564-095-8. DOI: 10.2495/1-84564-095-0/5g. URL: <http://library.witpress.com/viewpaper.asp?pcode=1845640950-507-1> (visited on 15/09/2021).
- [26] J. Szodruch. ‘Viscous Drag Reduction on Transport Aircraft’. In: *29th Aerospace Sciences Meeting*. Aerospace Sciences Meetings. American Institute of Aeronautics and Astronautics, 7th Jan. 1991. DOI: 10.2514/6.1991-685. URL: <https://arc.aiaa.org/doi/10.2514/6.1991-685> (visited on 15/09/2021).
- [27] Shabnam Raayai-Ardakani and Gareth H. McKinley. ‘Geometric Optimization of Riblet-Textured Surfaces for Drag Reduction in Laminar Boundary Layer Flows’. In: *Physics of Fluids* 31.5 (May 2019), p. 053601. ISSN: 1070-6631, 1089-7666. DOI: 10.1063/1.5090881. URL: <http://aip.scitation.org/doi/10.1063/1.5090881> (visited on 15/09/2021).
- [28] Sergei Chernyshenko. *Drag Reduction by a Solid Wall Emulating Spanwise Oscillations. Part 1*. 16th Apr. 2013. arXiv: 1304.4638 [physics.flu-dyn]. URL: <http://arxiv.org/abs/1304.4638> (visited on 10/09/2021).
- [29] Hermann Schlichting et al. *Boundary-Layer Theory*. Ninth edition. Boundary Layer Theory. Berlin ; Springer, 2017. ISBN: 978-3-662-52917-1.
- [30] G.E. Karniadakis and Kwing-So Choi. ‘Mechanisms on Transverse Motions in Turbulent Wall Flows’. In: *Annual Review of Fluid Mechanics* 35.1 (1st Jan. 2003), pp. 45–62. ISSN: 0066-4189. DOI: 10.1146/annurev.fluid.35.101101.161213. URL: <https://doi.org/10.1146/annurev.fluid.35.101101.161213> (visited on 15/09/2021).
- [31] M. R. Dhanak and C. Si. ‘On Reduction of Turbulent Wall Friction through Spanwise Wall Oscillations’. In: *Journal of Fluid Mechanics* 383 (1999), pp. 175–195. ISSN: 0022-1120. DOI: 10.1017/S0022112098003784. URL: <https://www.cambridge.org/core/article/on-reduction-of-turbulent-wall-friction-through-spanwise-wall-oscillations/F524851737C573EFAD0AA19B4AE06CF5> (visited on 16/09/2021).
- [32] Arturo Baron and Maurizio Quadrio. ‘Turbulent Drag Reduction by Spanwise Wall Oscillations’. In: *Applied Scientific Research* 55.4 (1996), pp. 311–326. ISSN: 0003-6994, 1573-1987. DOI: 10.1007/BF00856638. URL: <http://link.springer.com/10.1007/BF00856638> (visited on 15/09/2021).