GIT Tutorial Project

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Hello World! (First Commit)

Gauss's Lemma: If $f(x) \in \mathbb{Z}[x]$ is reducible over $\mathbb{Q}$, then $f(x)$ is reducible over $\mathbb{Z}$.

Theorem: If R is a UFD, then $R[x]$ is a UFD.

Eisenstein's Criterion: Let P be a prime ideal of the integral domain R and let $f(x) = x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ be a polynomial in $R[x]$ (here $n \geq 1$). Suppose $a_{n-1}, \ldots, a_1, a_0$ are all elements of P and suppose a_0 is not an element of P^2. Then $f(x)$ is irreducible in $R[x]$.

Theorem: Let F be a field. Then $f(x)$ is irreducible if and only if $F[x]/(f(x))$ is a field.

Definition: Let F be a field. A **field extension** of F is any field K containing F.

Proposition: Let K be a field extension of the field F. Then K is a vector space over F, with vector addition the same as addition in K as a field and F-scalar multiplication the same as multiplication in K as a field.

Definition: Let F be a field and $f(x)$ an irreducible polynomial over $F[x]$. The *degree* of $F[x]/(f(x))$ over F is dimension of $F[x]/(f(x))$ as a vector space over F.