IZVJEŠTAJ ZA 6. DOMAĆU ZADAĆU IZ PREDMETA

"NEIZRAZITO, EVOLUCIJSKO I NEURO RAČUNARSTVO"

SVEUČILIŠTE U ZAGREBU, FAKULTET ELEKTROTEHNIKE I RAČUNARSTVA

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FUNKCIJE PRIPADNOSTI:

$$\mu_A(x_1; a, b) = \frac{1}{1 + e^{a(x_1 - b)}}$$

$$\mu_B(x_2; c, d) = \frac{1}{1 + e^{c(x_2 - d)}}$$

UZORCI ZA UČENJE:

x_1	x_2	y
$x_1^{(1)}$	$x_2^{(1)}$	$y^{(1)}$
$x_1^{(2)}$	$x_2^{(2)}$	$y^{(2)}$
$\vdots$	$\vdots$	$\vdots$
$x_1^{(N)}$	$x_2^{(N)}$	$y^{(N)}$

$$X^{(k)} = \left[x_1^{(k)}, x_2^{(k)} \right]^T$$

KONSEKVENS:

$$z = p \cdot x_1 + q \cdot x_2 + r$$

Dakle, imamo 7 parametara za jedno pravilo: a, b, c, d, p, q i r

Gradijentnim spustom želimo pronaći optimalne parametre.

Definirajmo još jakost paljenja pravila:

$$\alpha = \mu_A(x_1) \cdot \mu_B(x_2) \qquad \text{(T-norma algebarski produkt)}$$

Odnosno za M pravila i N primjera, i neko i-to pravilo i za k-ti primjer:

$$\alpha_i(X^{(k)}) = \mu_{A_i}(x_1^{(k)}) \cdot \mu_{B_i}(x_2^{(k)}),$$ gdje su μ_{A_i} i μ_{B_i} parametrizirani sa: a_i, b_i, c_i i d_i.

Izlaz iz sustava za neki ulaz $X^{(k)}$:

$$o(X^{(k)}) = \frac{\sum_{i=1}^{M} \alpha_i(X^{(k)}) \cdot z_i(X^{(k)})}{\sum_{i=1}^{M} \alpha_i(X^{(k)})}, \quad z_i(X^{(k)}) = p_i \cdot x_1^{(k)} + q_i \cdot x_2^{(k)} + r_i$$

FUNKCIJA POGREŠKE ZA k-ti UZORAK:

$$E\left(x^{(k)}\right) = \frac{1}{2}\left(y^{(k)} - o\left(x^{(k)}\right)\right)^2$$

AŽURIRANJE NEKOG PRAVILA ψ:

$$\psi \leftarrow \psi - \eta \cdot \frac{\partial E\left(x^{(k)}\right)}{\partial \psi}, \qquad \eta \text{ - stopa učenja.}$$

Dakle, trebamo pronaći parcijalne derivacije funkcije pogreške po svim parametrima: a, b, c, d, p, q i r.

Budući da su parametri a i c, i parametri b i d slični, provesti ću postupak samo za: a, b, p, q i r.

Izvod za ažuriranje pravila r_i:

$$r_i \leftarrow r_i - \eta \cdot \frac{\partial E_k}{\partial r},$$

$$\frac{\partial E_k}{\partial r_i} = \frac{\partial E_k}{\partial o_k} \cdot \frac{\partial o_k}{\partial z_i} \cdot \frac{\partial z_i}{\partial r_i}$$

$$\boxed{\frac{\partial E_k}{\partial o_k} = y_k - o_k} \qquad (2.1)$$

$$\boxed{\frac{\partial o_k}{\partial z_i} = \frac{\alpha_i}{\sum_{j=1}^{M} \alpha_j}} \qquad (2.2)$$

$$\boxed{\frac{\partial z_i}{\partial r_i} = 1} \qquad (2.3)$$

Iz (2.1), (2.2) i (2.3) slijedi: $\boxed{\frac{\partial E_k}{\partial r_i} = (y_k - o_k) \cdot \frac{\alpha_i}{\sum_{j=1}^{M} \alpha_j}}$

Izvod za ažuriranje pravila q_i:

- $$q_i \leftarrow q_i - \eta \cdot \frac{\partial E_k}{\partial q_i}$$

$$\frac{\partial E_k}{\partial q_i} = \frac{\partial E_k}{\partial o_k} \cdot \frac{\partial o_k}{\partial z_i} \cdot \frac{\partial z_i}{\partial q_i}$$

$$\boxed{\frac{\partial z_i}{\partial q_i} = x_2} \qquad (3.1)$$

- Iz (2.1), (2.2) i (3.1) slijedi: $$\boxed{\frac{\partial E_k}{\partial q_i} = (y_k - o_k) \cdot \frac{\alpha_i}{\sum_{j=1}^{M} \alpha_j} \cdot x_2}$$

Izvod za ažuriranje pravila p_i:

$$p_i \leftarrow p_i - \eta \cdot \frac{\partial E_k}{\partial p_i}$$

$$\frac{\partial E_k}{\partial p_i} = \frac{\partial E_k}{\partial o_k} \cdot \frac{\partial o_k}{\partial z_i} \cdot \frac{\partial z_i}{\partial p_i}$$

$$\boxed{\frac{\partial z_i}{\partial p_i} = x_1} \qquad (3.2)$$

Iz (2.1), (2.2) i (3.2) slijedi: $$\boxed{\frac{\partial E_k}{\partial p_i} = (y_k - o_k) \cdot \frac{\alpha_i}{\sum_{j=1}^{M} \alpha_j} \cdot x_1}$$

Izvod za ažuriranje pravila a_i:

$$a_i \leftarrow a_i - \eta \cdot \frac{\partial E_k}{\partial a_i}$$

$$\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{g(x)^2}$$

$$\frac{\partial E_k}{\partial a_i} = \frac{\partial E_k}{\partial o_k} \cdot \frac{\partial o_k}{\partial \alpha_i} \cdot \frac{\partial \alpha_i}{\partial a_i}$$

$$\sigma'(x) = \sigma(x) \cdot (1 - \sigma(x))$$

$$\boxed{\frac{\partial o_k}{\partial \alpha_i} = \frac{\sum_{j=1}^{M} \alpha_j (z_i - z_j)}{\left(\sum_{j=1}^{M} \alpha_j\right)^2}} \quad (4.1)$$

$$\boxed{\frac{\partial \alpha_i}{\partial a_i} = -\mu_{B_i}(x_2) \cdot (x_1 - b_i) \cdot \mu_{A_i}(x_1) \cdot (1 - \mu_{A_i}(x_1))} \quad (4.2)$$

Izvod za ažuriranje pravila b_i:

$$b_i \leftarrow b_i - \eta \cdot \frac{\partial E_k}{\partial b_i}$$

$$\frac{\partial E_k}{\partial b_i} = \frac{\partial E_k}{\partial o_k} \cdot \frac{\partial o_k}{\partial \alpha_i} \cdot \frac{\partial \alpha_i}{\partial b_i}$$

$$\boxed{\frac{\partial \alpha_i}{\partial b_i} = a_i \cdot \mu_{B_i}(x_2) \cdot \mu_{A_i}(x_1) \cdot (1 - \mu_{A_i}(x_1))}$$