

AST4320 COSMOLOGY AND EXTRAGALACTIC ASTRONOMY ASSIGNMENT 3

Deadline: Friday, November 10

Exercise 1 The Ly α forest (20 points) Use cosmological parameter $\Omega_\Lambda = 0.692$, $\Omega_m = 0.308$, $\Omega_b = 0.048$ and $\Omega_r = 0$ for the following questions.

- Assuming the intergalactic medium (IGM) contains only hydrogen and helium, and the hydrogen and helium mass fractions are $X = 0.76$ and $Y = 0.24$, respectively, compute the mean molecular weight of the IGM, μ .
- Compute the Jeans length of the IGM as a function of redshift. You can assume that the IGM temperature is $T = 10^4$ K. What value for k does this correspond to?
- In redshift-space, the Jeans length would correspond to a velocity width. What is this velocity width?
- What does the finite width of the Ly α extinction profile do to absorption features in the Ly α forest?
- In lectures we discussed that the intrinsic Ly α extinction profile function (i.e. the Lorentz profile) is broadened by gas thermal motions, and the thermal broadening scale is $v_{th} = \sqrt{(2k_bT/m)}$, where T is the temperature of the gas and m is the mass of the particle. Compare the thermal broadening scale to the Jeans length.

Exercise 2 Optical depth of the ionised IGM (10 points)

In the lectures we derived that the total optical depth of the ionised IGM due to electron scattering is given by

$$\tau_e(z) = c \int_0^z \frac{n_e(z) \sigma_T dz}{(1+z)H(z)},$$

where $\sigma_T = \frac{8\pi}{3} r_e^2 = 6.65 \times 10^{-25} \text{ cm}^2$ denotes the cross-section for electron scattering. Assuming the IGM is highly ionised, and the average number density of hydrogen evolves as $\bar{n}_H(z) \sim 1.9 \times 10^{-7} (1+z)^3 \text{ cm}^{-3}$, compute the optical depth as a function of z under cosmological parameter $\Omega_\Lambda = 0.692$, $\Omega_m = 0.308$ and $\Omega_r = 0$). Plot $\tau_e(z)$ as function of z for $z = 0-10$.

Exercise 3 Isothermal density profile (10 points)

In the lectures we obtained a second order differential equation for $\rho(r)$ if assuming the halo is “isothermal” (i.e. dark matter particles have the same velocity dispersion everywhere in the halo):

$$-\frac{k_{\text{b}}T}{m_{\text{DM}}r^2} \frac{d}{dr} r^2 \frac{d}{dr} \ln \rho = 4\pi G \rho(r),$$

- Show that

$$\rho(r) = \frac{A}{r^2}, \quad A = \frac{k_{\text{b}}T}{2\pi G m_{\text{DM}}}.$$

provides a solution to this equation.

- For gas in hydrostatic equilibrium with gravity we have

$$\frac{dp}{dr} = -\frac{GM(< r)\rho}{r^2}.$$

Show that isothermal gas settles into a similar state.