Cosmology Assignment 3

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(Code included for exercises 1 and 2 at https://github.com/hermda02/CosmologyAssignment3)

Exercise 1. Numerical calculations were done with code included at the above github address.
i) To find the mean mass per particle, we assume that the medium is highly ionized. Therefore, for each hydrogen atom, the mass is split between the nucleus and the electron. We treat the helium similarly. So we have

$$\mu = \frac{< m >}{m_p} = \frac{1}{2X + 3/4Y}$$

Where X is the amount of hydrogen and Y is the amount of helium.

$$\frac{1}{\mu} = 2X + 3/4Y$$

Using $X = 0.76, Y = 0.24$, we arrrive at $\mu = 0.588$.
ii) The Jeans' mass is derived from the virial theorem:

$$2K = |U| \Rightarrow 3kTN = \frac{3GM^2}{5R}, N = \frac{M}{\mu * m_p}$$
$$\frac{3kTM}{\mu m_p} = \frac{3GM^2}{5R} \Rightarrow \frac{3kT}{\mu m_p} = \frac{4\pi G R^3 \rho}{5R}$$
$$R_J = \sqrt{\frac{15kT}{4\pi G \mu m_p \rho}}$$

Using $\rho = \rho_{crit,0}\Omega_{b,0}(1+z)^3$,

$$R_J = \sqrt{\frac{15kT}{4\pi G \mu m_p \rho_{crit,0}\Omega_{b,0}}}(1+z)^{-3/2}$$
$$= 7.787 * 10^{19}\text{km}(1+z)^{-3/2}$$

So the wave number k is $k = 2\pi/R_J = 8.07 * 10^{-20}\text{km}^{-1}(1+z)^{3/2}$
iii) The velocity width depends on redshift: $v = HR_J = H_0 R_J \sqrt{\Omega_\Lambda + \Omega_M(1+z)^3}$

Redshift (z)	velocity width
0	171.089
1	75.986
2	57.055
3	48.302
4	42.843
5	38.965

iv) The finite width of the Lyα extinction profile causes absorption features to broaden over redshift as a result of the velocity widths shown in part iii).
v) $m = \mu m_p = 0.588\text{amu} = 9.8196 * 10^{-25}\text{g}$

$$v_{TH} = \sqrt{\frac{2kT}{m}} = 16.765\text{km/s}$$

Thus the thermal broadening is far smaller than the Jeans length of the IGM.

Exercise 2.

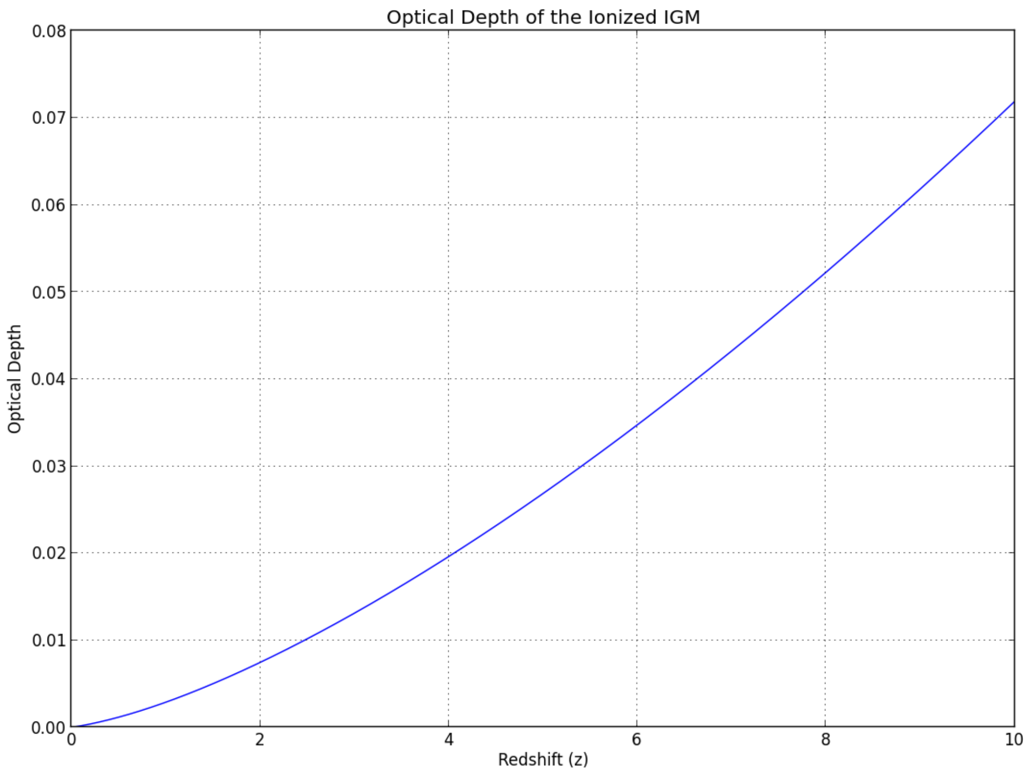

Figure 1: Optical depth of the ionized IGM as a function of redshift.

Exercise 3.

i) $\rho(r) = \frac{A}{r^2}$, $A = \frac{k_B T}{2\pi G M_{DM}}$

$$\begin{aligned}
-\frac{k_B T}{M_{DM} r^2} \frac{d}{dr} r^2 \frac{d}{dr} \ln \rho =& 4\pi G \rho(r) \\
-\frac{k_B T}{M_{DM} r^2} \frac{d}{dr} r^2 \Big[\frac{1}{\rho} \frac{d\rho}{dr}\Big] =& \frac{4\pi G k_B T}{2\pi G M_{DM} r^2} \\
-\frac{k_B T}{M_{DM} r^2} \frac{d}{dr} (-2r) =& \frac{2k_B T}{M_{DM} r^2} \\
\frac{2k_B T}{M_{DM} r^2} =& \frac{2k_B T}{M_{DM} r^2}
\end{aligned}$$

ii) We assume spherical symmetry:

$$\frac{\nabla p}{\rho} = -\nabla \phi \Rightarrow \frac{dp}{dr}\frac{1}{\rho} = -\frac{GM(< r)}{r^2}$$

Where $M = \int_0^r 4\pi r'^2 \rho(r') dr' = 4\pi A \int_0^r \frac{r'^2}{r'^2} dr' = 4\pi A r = \frac{2k_B T}{GM} r$.

So the R.H.S. becomes $-\frac{2k_B T}{M_{DM} r}$

For the L.H.S:

$$p = \frac{\rho k_B T}{M_{DM}} = \frac{k_B T}{M_{DM}}(\frac{A}{r^2}), \frac{dp}{dr} = -\frac{2Ak_B T}{M_{DM} r^3}$$

Multiplying both sides by ρ,

$$-\frac{2k_B T}{M_{DM}}\frac{A}{r^3} = -\frac{2k_B T}{M_{DM}}\frac{\rho}{r}$$
$$-\frac{2k_B T}{M_{DM}}\frac{A}{r^3} = -\frac{2k_B T}{M_{DM}}\frac{A}{r^3}$$