1 Introduction

Balancer is an automated market maker protocol, where there are token pools which allows traders to buy and sell those tokens given they pay a fee, whereas some others can invest their tokens in these pools and collect their fees for providing liquidity.

From the Balancer whitepaper we have:

$$\text{Value function: } V = \prod_i B_i^{W_i}$$

$$\text{Spot price of a token in relation to another: } SP_i^j = \frac{\frac{B_i}{W_i}}{\frac{B_j}{W_j}}$$

Any time someone invests in a given pool they receive a “pool token” which represents their share in the total pool reserves. We need to calculate this pool token’s price in a given commodity. In this case, ETH.

The naive approach is

$$P_{pt} = \frac{\sum_i B_i P_{eth}^i}{T_{pt}}$$

where:

P_{pt} is the pool token price.

T_{pt} is the total amount of pool tokens in existence.

B_i is the balance of the i-th pool component token.

P_{eth}^i is the price of the i-th pool component token in ETH.

Our issue is that these balances can be manipulated by an attacker. Thus, we cannot trust each B_i. We need to find the price P_{pt} without using them, or using them in such a way that is immune to manipulations.

2 A stable formula

For each pool component we consider the quantity $X_i = \frac{B_i P_{eth}^i}{W_i}$. If the pool has not been manipulated, the spot price SP_i^j will match the relative price of the external world: P_{eth}^j / P_{eth}^i.

$$\frac{\frac{B_i}{W_i}}{\frac{B_j}{W_j}} = \frac{P_{eth}^j}{P_{eth}^i} \quad \text{and this is the same as } X_i = X_j \text{ for every } i \text{ and } j$$

As a matter of fact, in this case any X_i gives the total value of the pool in *eth*. This is because, calling $X_i = X$ for every i,

$$\sum_i B_i P_{eth}^i = \sum_i W_i X_i = \sum_i W_i X = X$$

since $\sum_i W_i = 1$. Then we also have $P_{pt} = X/T_{pt}$. Now we want to express X in terms of V, instead of one specific B_i. Using again $\sum_i W_i = 1$, we can write:

$$X = \prod_i X_i^{W_i} = V \prod_i \left(\frac{P_{eth}^i}{W_i}\right)^{W_i}$$

Therefore we have reached the stable formula:

$$P_{pt} = \frac{V}{T_{pt}} \prod_i \left(\frac{P_{eth}^i}{W_i}\right)^{W_i}$$

$$P_{pt} = \frac{1}{T_{pt}} \prod_i \left(\frac{B_i P_{eth}^i}{W_i}\right)^{W_i}$$