

ADA mini homework 3

(1)

Construct a DP table with size of $DP[2][S_a.len() + 1]$, and define each element $DP[i \% 2][j]$ in array stand for the minimum operation cost it needs to transform from $S_b.substr(0, i)$ to $S_a.substr(0, j)$.

Before we start the loop, we need to consider the base case. First, $DP[0][j] = j * d$, as it adds j characters to become S_a if S_b is empty. Second, every time we enter the second loop, we set $DP[i \% 2][0]$ to $i * e$, as it delete all its characters to become an empty string as S_a . For all other general cases, we consider two conditions. First, if $S_b[i] == S_a[j]$, we do nothing and set the cost of $DP[i \% 2][j]$ to $DP[(i - 1) \% 2][j - 1]$. Else, we compare three different steps we can do. Add, remove, and replace, which equals to the cost of $DP[i \% 2][j - 1] + d$, $DP[(i - 1) \% 2][j] + e$, and $DP[(i - 1) \% 2][j - 1] + f$, respectively. We choose the smallest one and assign $DP[i][j]$ to that cost.

We can see that construct the table of $i == S_b.len()$ only required the data of $i == S_b.len() - 1$, therefore, two rows are sufficient for us to calculate the answer.

Example DP table:

			Replace	Remove		
			↙	↖		
			Add ←	(i % 2, j)		

(2)

Transition function:

$DP[i \% 2][j] =$ If $j == 0, i * e$.

Else If $i == 0, j * d$

Else if $S_a[j - 1] == S_b[i - 1], DP[(i - 1) \% 2][j - 1]$

Else $\min(\{DP[i \% 2][j - 1] + d, DP[(i - 1) \% 2][j] + e, \text{ and } DP[(i - 1) \% 2][j - 1] + f\})$

The C++ implementation of this function

```
1. for (int i = 0; i <= Sb.len(); i++) {
2.     for (int j = 0; j <= Sa.len(); j++) {
3.         if(i == 0)
4.             DP[0][j] = j * d
5.         else if (j == 0)
6.             DP[i % 2][j] = i * e;
7.         else if (Sa[j - 1] == Sb[i - 1])
8.             DP[i % 2][j] = DP[(i - 1) % 2][j - 1];
9.         else
10.            DP[i % 2][j] = min({DP[(i - 1) % 2][j] + e, DP[i
    % 2][j - 1] + d, DP[(i - 1) % 2][j - 1] + f});
11.     }
12. }
```