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EM-HW2

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Problem

First, rewrite the Fourier series as general form

$$f(x)=rac{1}{2}a_0+\sum_{k=1}^n a_k coskx+b_k sinkx,\quad k=[1,5]$$

Then we calculate a_0 , a_k , b_k respectively.

$$egin{align} a_0 &= rac{1}{\pi} \int_{-\pi}^{\pi} f(x) dx \ &= rac{1}{\pi} \int_{-\pi}^{\pi} (x^2 + x) dx \ &= rac{1}{\pi} [rac{1}{3} x^3 + rac{1}{2} x^2]_{-\pi}^{\pi} \ &= rac{1}{\pi} [(rac{1}{3} \pi^3 + rac{1}{2} \pi^2) - (-rac{1}{3} \pi^3 + rac{1}{2} \pi^2)] \ &= rac{2}{3} \pi^2 \end{array}$$

Next, calculate a_k

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$$a_k = rac{1}{\pi} \int_{-\pi}^{\pi} f(x) coskx dx \ = rac{1}{\pi} \int_{-\pi}^{\pi} f(x) coskx dx \ = rac{1}{\pi} \int_{-\pi}^{\pi} (x^2 + x) coskx dx \ = rac{1}{\pi} (\int_{-\pi}^{\pi} x^2 coskx dx + \int_{-\pi}^{\pi} x coskx dx) \ = rac{1}{\pi} \{ [x^2 rac{sinkx}{k}]_{-\pi}^{\pi} - \int_{-\pi}^{\pi} 2x rac{sinkx}{k} dx + [x rac{sinkx}{k}]_{-\pi}^{\pi} - \int_{-\pi}^{\pi} rac{sinkx}{k} dx \} \ = -rac{1}{\pi k} \{ 2 \int_{-\pi}^{\pi} x sinkx dx + \int_{-\pi}^{\pi} sinkx dx \} \ = -rac{1}{\pi k} \{ 2 ([x rac{-coskx}{k}]_{-\pi}^{\pi} - \int_{-\pi}^{\pi} rac{-coskx}{k} dx) + [rac{-coskx}{k}]_{-\pi}^{\pi} \} \ = rac{1}{\pi k^2} \{ 2 [x coskx]_{-\pi}^{\pi} + [coskx]_{-\pi}^{\pi} \} \ = rac{1}{\pi k^2} \{ 4\pi cosk\pi + 2 cosk\pi \} \ = (rac{4}{k^2} + rac{2}{k^2\pi}) (-1)^k$$

Then calculate b_k

$$egin{aligned} b_k &= rac{1}{\pi} \int_{-\pi}^{\pi} f(x) sinkx dx \ &= rac{1}{\pi} \int_{-\pi}^{\pi} (x^2 + x) sinkx dx \ &= rac{1}{\pi} (\int_{-\pi}^{\pi} x^2 sinkx dx + \int_{-\pi}^{\pi} x sinkx dx) \ &= rac{1}{\pi} \int_{-\pi}^{\pi} x sinkx dx \ &= -rac{1}{k\pi} \{ [x coskx]_{-\pi}^{\pi} - \int_{-\pi}^{\pi} coskx dx \} \ &= -rac{2}{k\pi} \{ \pi cosk\pi - rac{sink\pi}{k} \} \ &= -rac{2}{k} (-1)^k \end{aligned}$$

Finally, we get the Fourier form of $f(x)=x^2+x$

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$$f(x) = rac{2}{3}\pi^2 + \sum_{k=1}^{n}rac{2}{k}(-1)^k[(rac{2}{k} + rac{1}{k\pi})coskx - sinkx]$$

Then we can sum up to cos5x and sin5x

$$f(x) = rac{2}{3}\pi^2 - (4 + rac{2}{\pi})cosx + 2sinx + \ (1 + rac{1}{2\pi})cos2x - sin2x - \ (rac{4}{9} + rac{2}{9\pi})cos3x + rac{2}{3}sin3x + \ (rac{1}{4} + rac{1}{8\pi})cos4x - rac{1}{2}sin4x - (rac{4}{25} + rac{2}{25\pi})cos5x + rac{2}{5}sin5x$$

