Basic Circuit for sensing

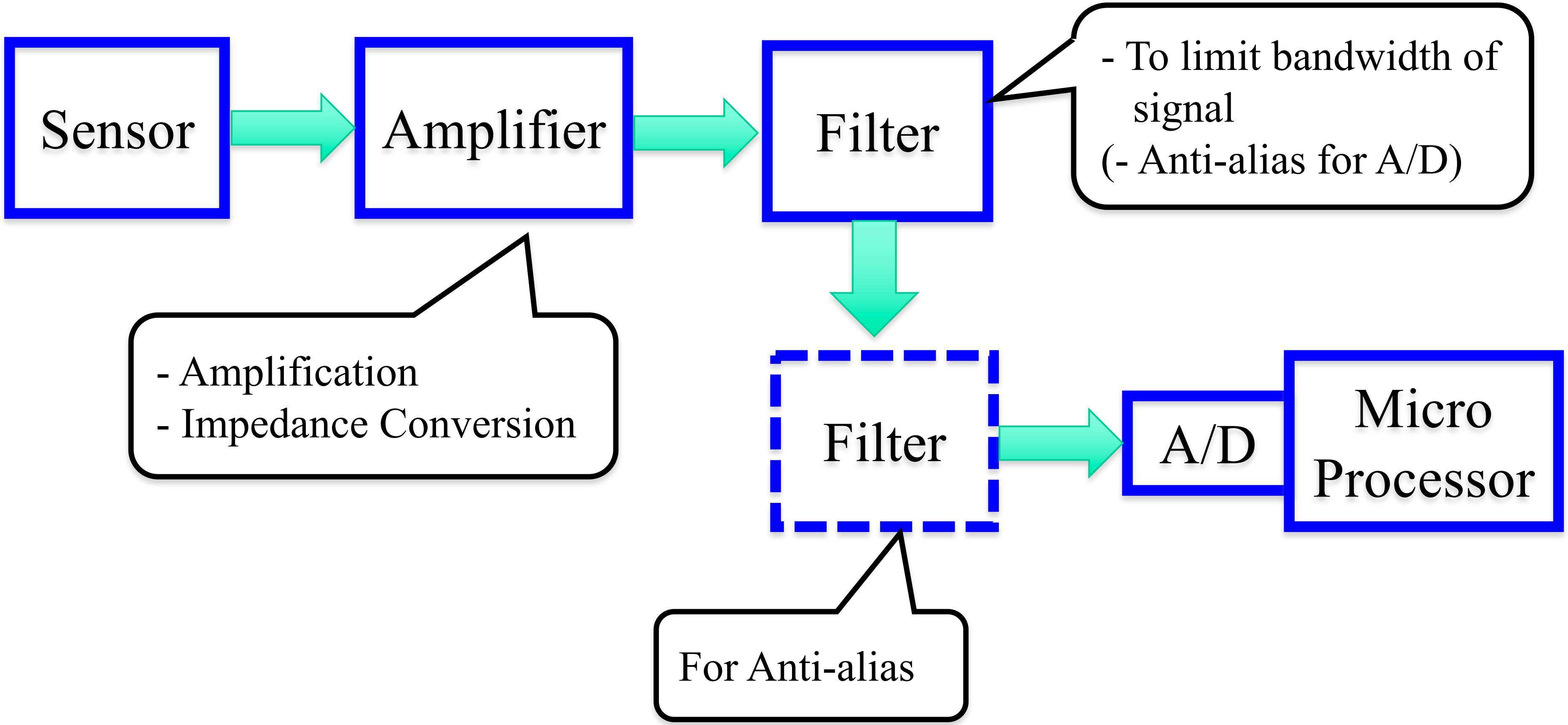
May. 15th, 2024

Importance of circuits around sensor

- To bring out the performance of sensor
  - High S/N (Signal to Noise ratio)
  - Signal with sufficient amplitude
  - Strong signal which has low output impedance

- To convert from analog signal to digital data easily, and correctly
  - Voltage output and conditioning its amplitude for ADC (A/D converter)
  - Limitation of output signal's band width
    - * For reduction noise
    - * For conversion under sampling theory

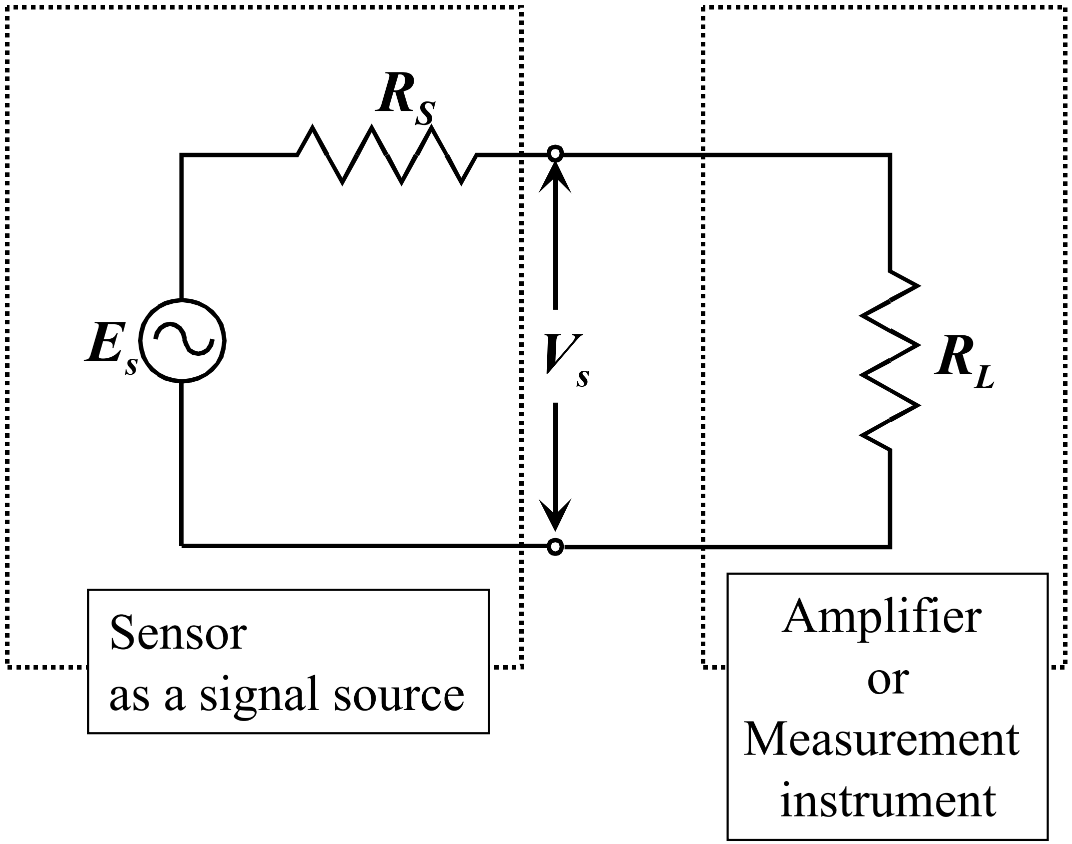
Common composition of Sensor system



Circuit component for sensing and its purpose

- From “Weak signal” to “Strong signal”
  → Impedance converter
- From “small signal” to “large signal”
  → Amplifier
- Limitation of band width of signal
  → Filter
  (Passive Filter, Active Filter)
- Signal converter
  → Current amplifier, Charge amplifier,
  f-V converter, etc…

Impedance of signal source and load



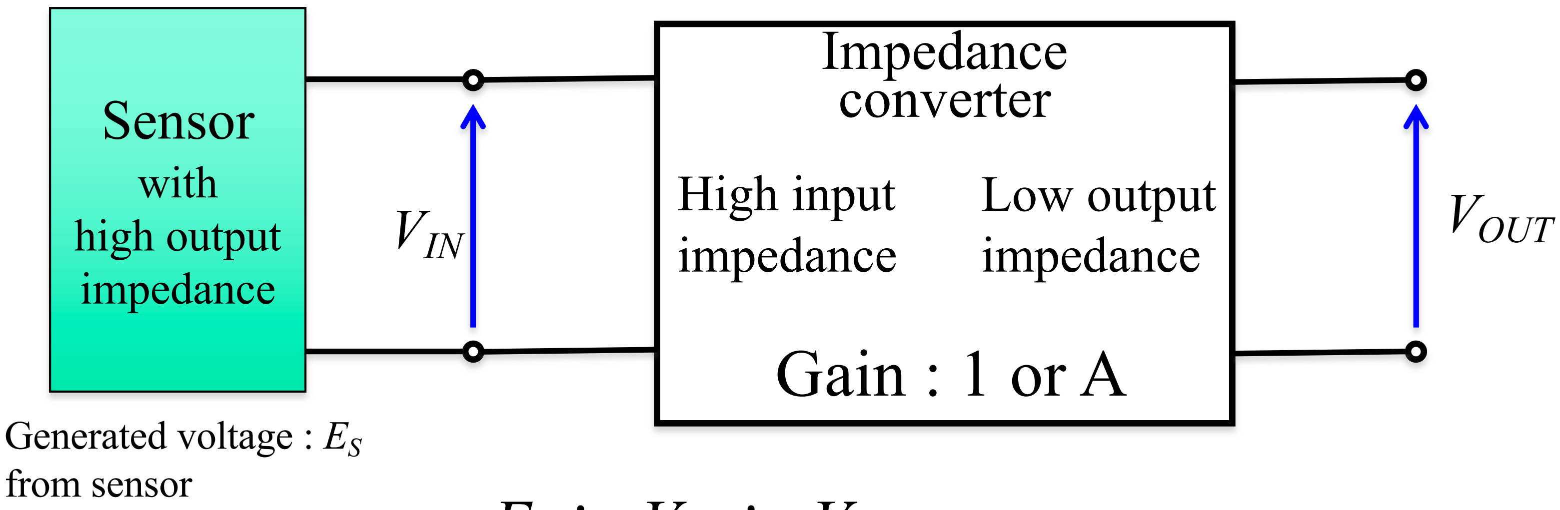
R_S : Output (inert) impedance of sensor
E_S : Signal source in sensor
V_S : Measured sensor output
R_L : Input impedance of amplifier
or measurement instrument

- Signal source is expressed by an ideal voltage source E_S and an output impedance R_S.
- R_L is an input impedance of a load connected with signal source.
  * Load is an amplifier, filter or measurement instrument…
- E_S is generated signal depending on state of sensing target.
  But, V_S is amplified or measured.

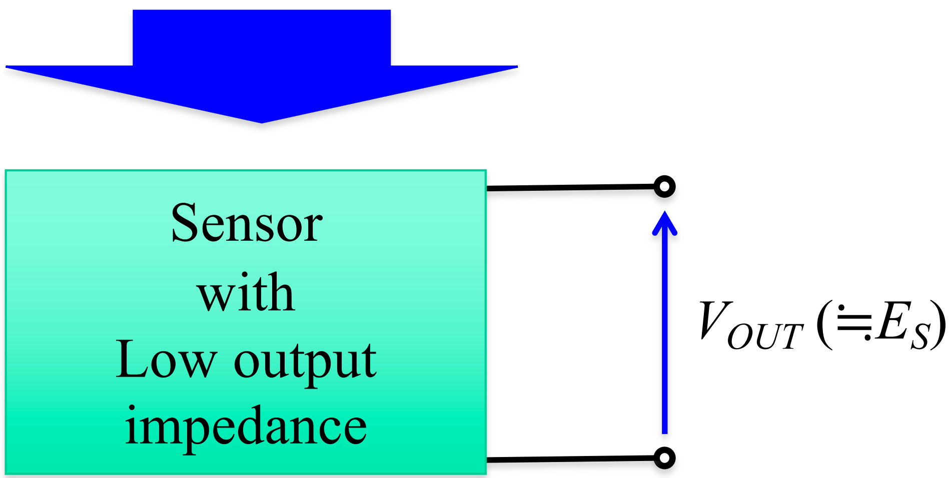
E_S should be measured.
To realize that, always…

The relation between R_S and R_L is important!!

Impedance conversion for measurement

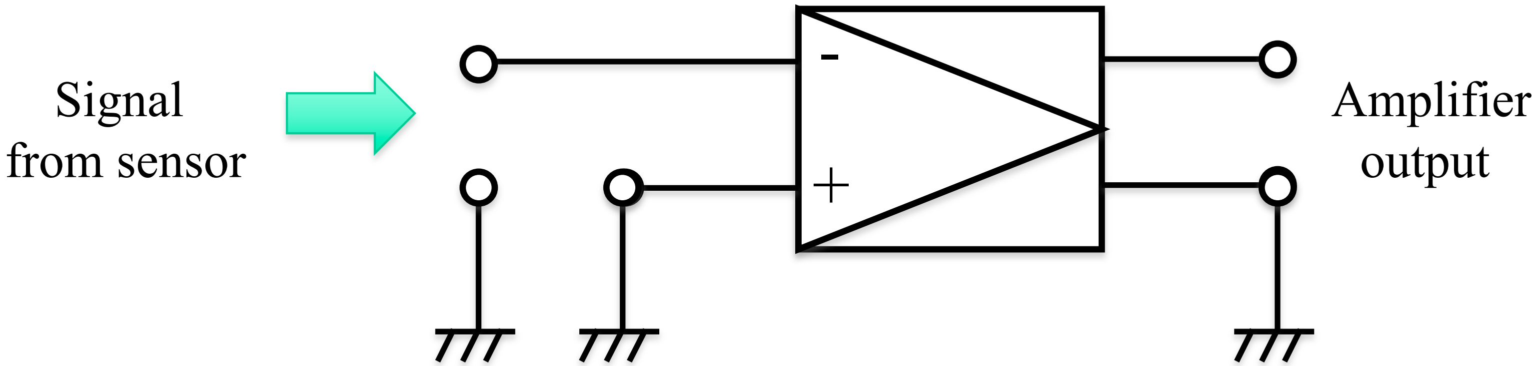


$$E_S \fallingdotseq V_{IN} \fallingdotseq V_{OUT}$$

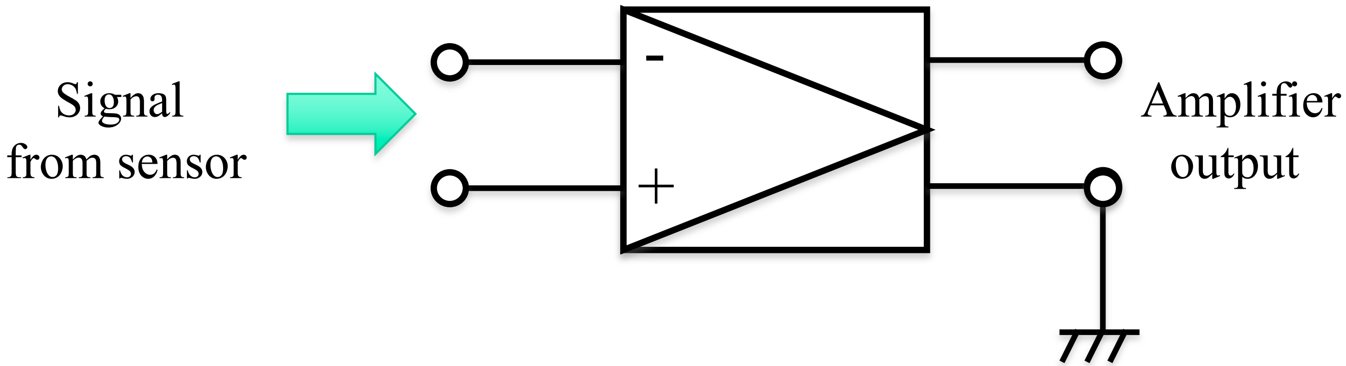


Amplifire

Types of Amplifier



(a) Single-End input amplifier



(b) Differential input amplifier

Operational Amplifier

Operational amplifier (OP-amp.) is an almost ideal amplifier.

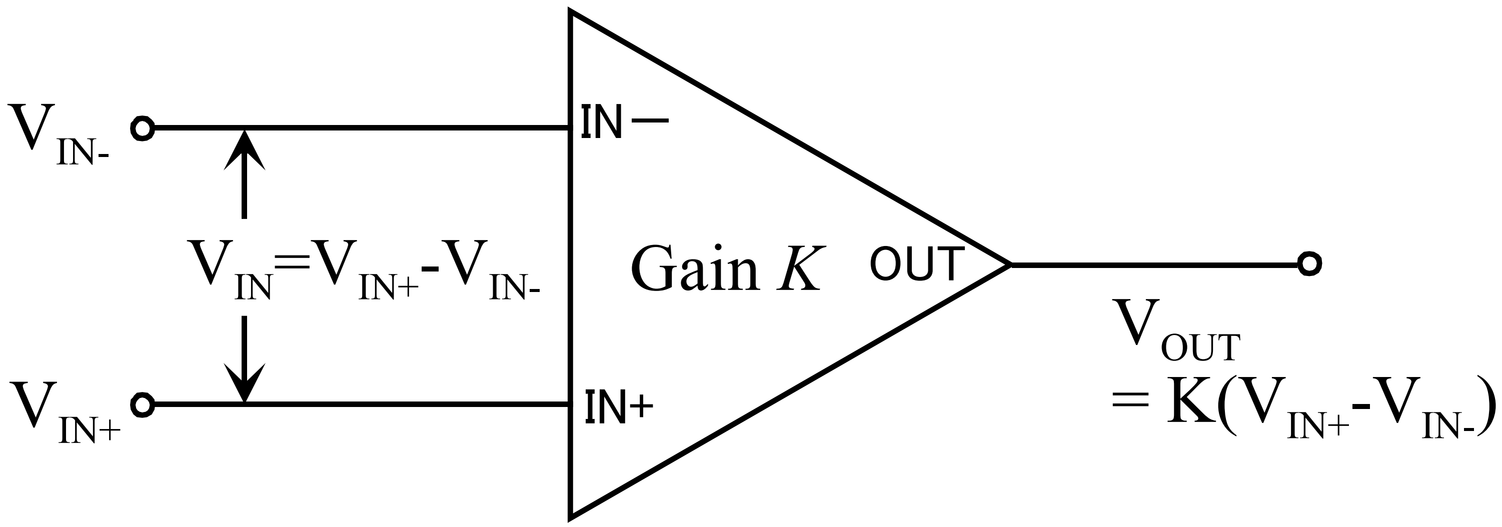
It is used for measurement circuit.

Table : Features of Operational amplifier

Feature	Ideal Value	Practical Value
Differential voltage gain	∞	$10^5 \sim 10^7$
Common mode voltage gain	0	10^{-5}
Input impedance	∞	Order of $10^6 \sim 10^9$ [ohm]
Output impedance	0	Order of $\sim 10^2$[ohm]
Frequency band width	∞	Order of $10^6 \sim 10^7$[Hz]
Input bias current	0	Order of $\sim 10^2$[nA]
Offset voltage on inputs	0	Order of $\sim 10^1$[mV]

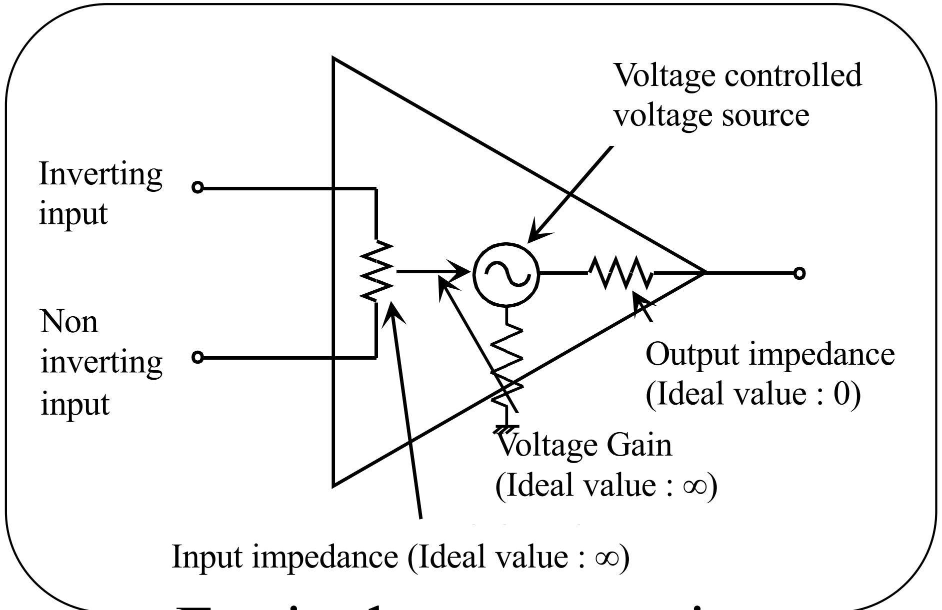
Basic operation of OP-amp.

- OP-amp. has 2 input terminals, “Inverting input”, and “Non-inverting input”.
- OP-amp. amplifies the voltage difference between those terminals.

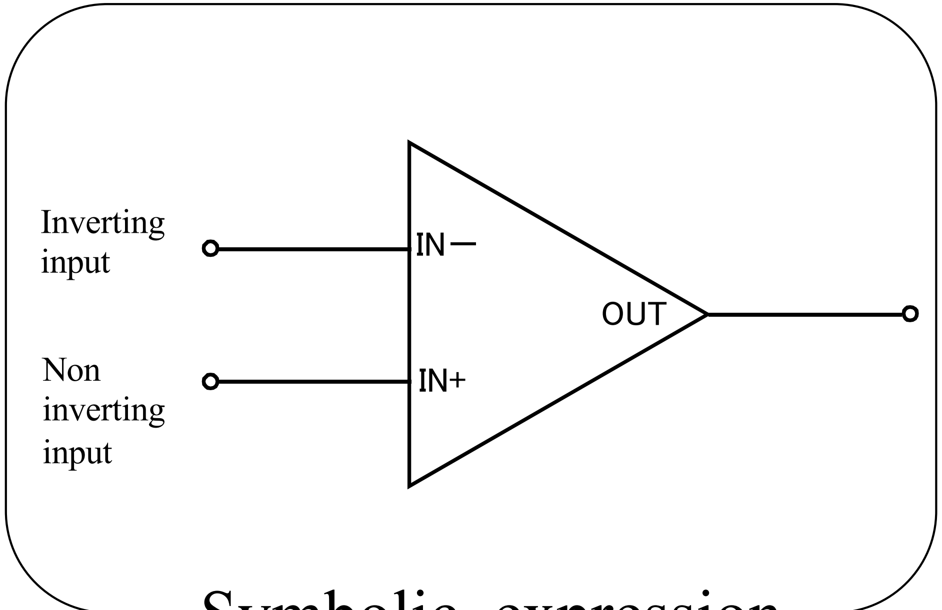


K : Open loop gain of OP-amp. itself.

Equivalent expression and Symbol of OP amp.

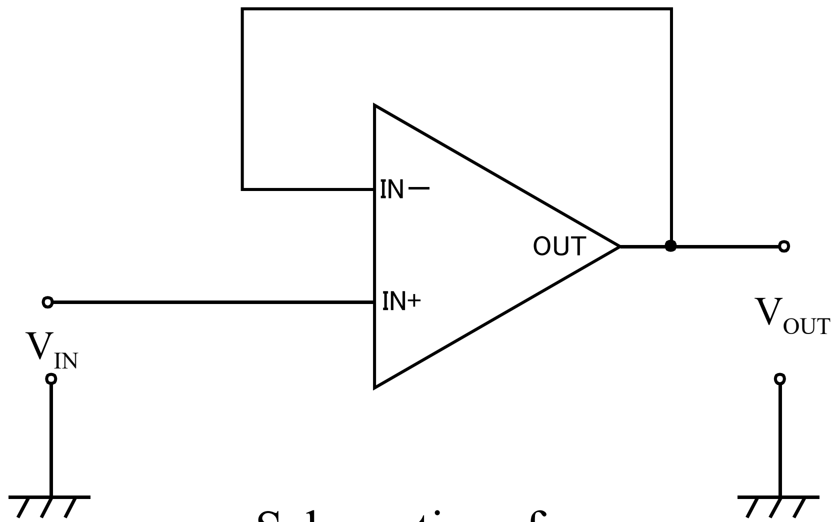


Equivalent expression



Symbolic expression

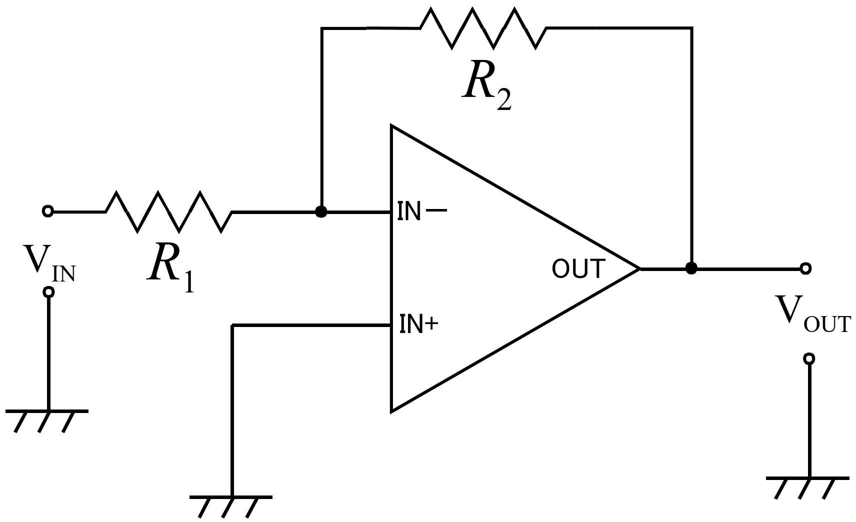
Voltage-Follower as an impedance converter



Schematics of
Voltage Follower Circuit

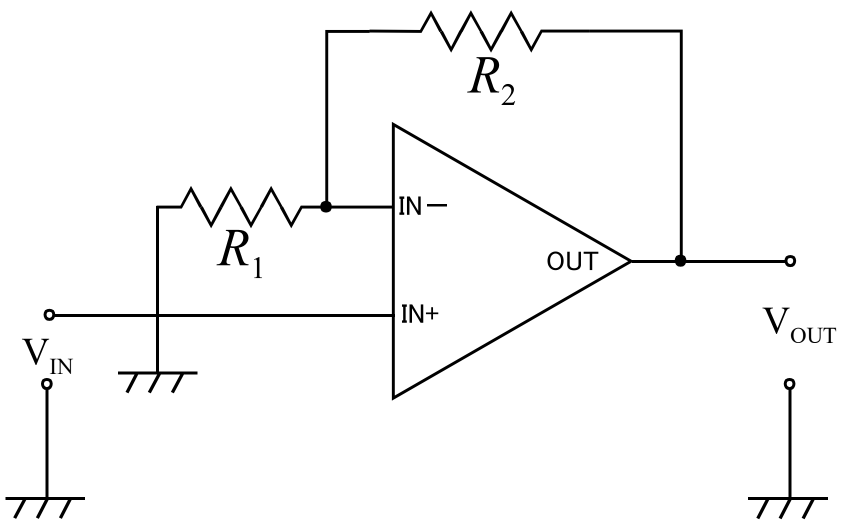
- High input impedance
- Low output impedance
- Voltage Gain : 1 => $V_{IN} = V_{OUT}$

Basic amplifier using OP-Amp.



Inverting amplifier

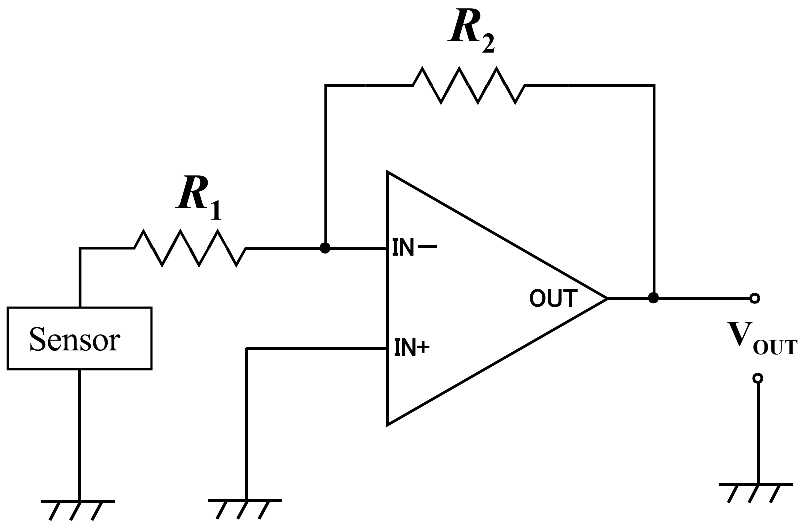
- Inverting amplifier
  - Signal inputs to Inverting input terminal "IN-"
  - Output signal has 180° phase difference to input signal.
  - Input impedance is not high (almost R_1).



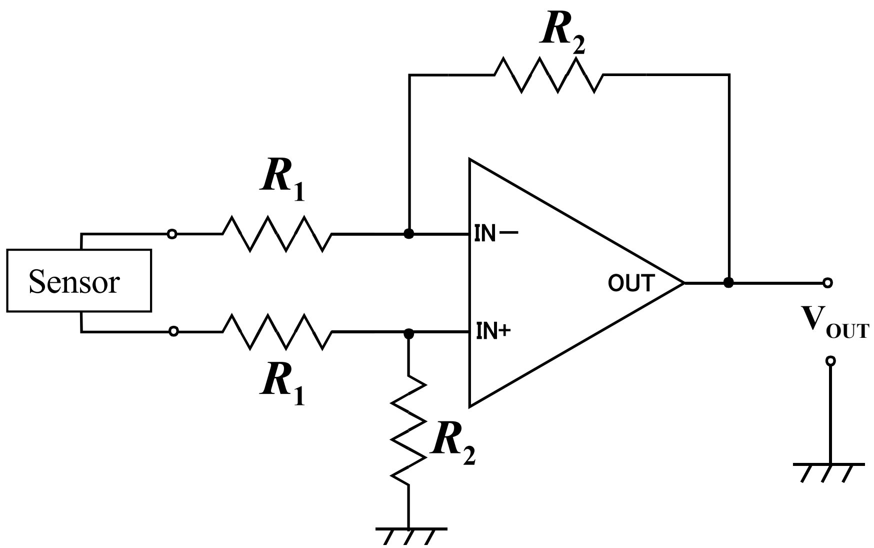
Non-inverting amplifier

- Non-inverting amplifier
  - Signal inputs non-inverting terminal "IN+"
  - Output signal is co-phase with input signal
  - Input impedance is high. (Important!!!)

Single end input amp. and Differential input amp. by OP amp.



Single-End input amplifier



Differential input amplifier

- Single-End input amplifier
  - A port of sensor and of OP amp. is grounded, directly.
  * Non-inverting amp. is also a single end input amp..

- Differential input amplifier
  - Both inputs are not grounded, Those are connected to OP amp. directly.

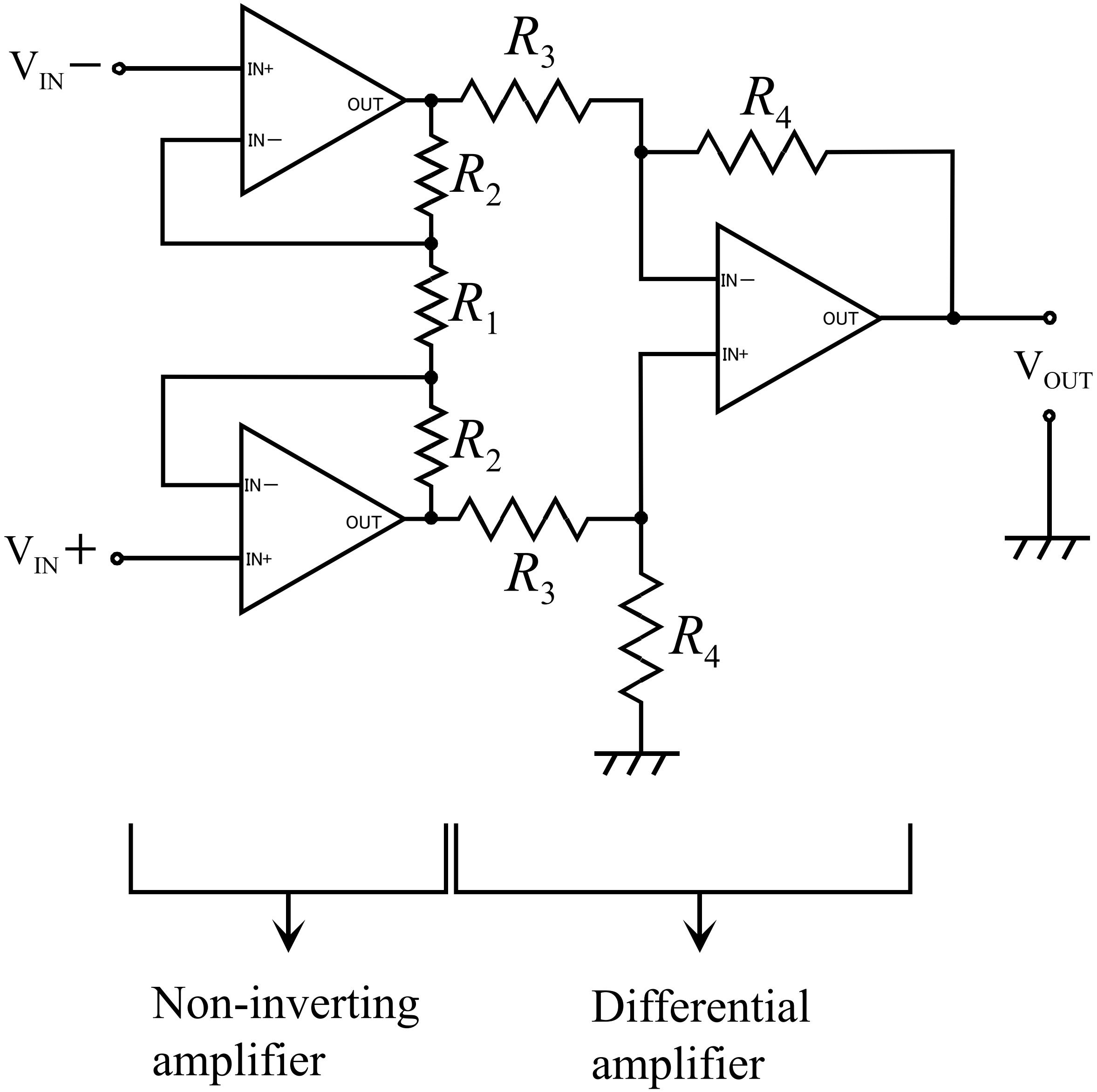
Instrumentation amplifier

★ Instrumentation amplifier is used for sensor with high input Impedance, and for improving S/N in noisy measurement.

Features of instrumentation amplifier

- High input impedance
- Low output impedance
- Differential amplifier

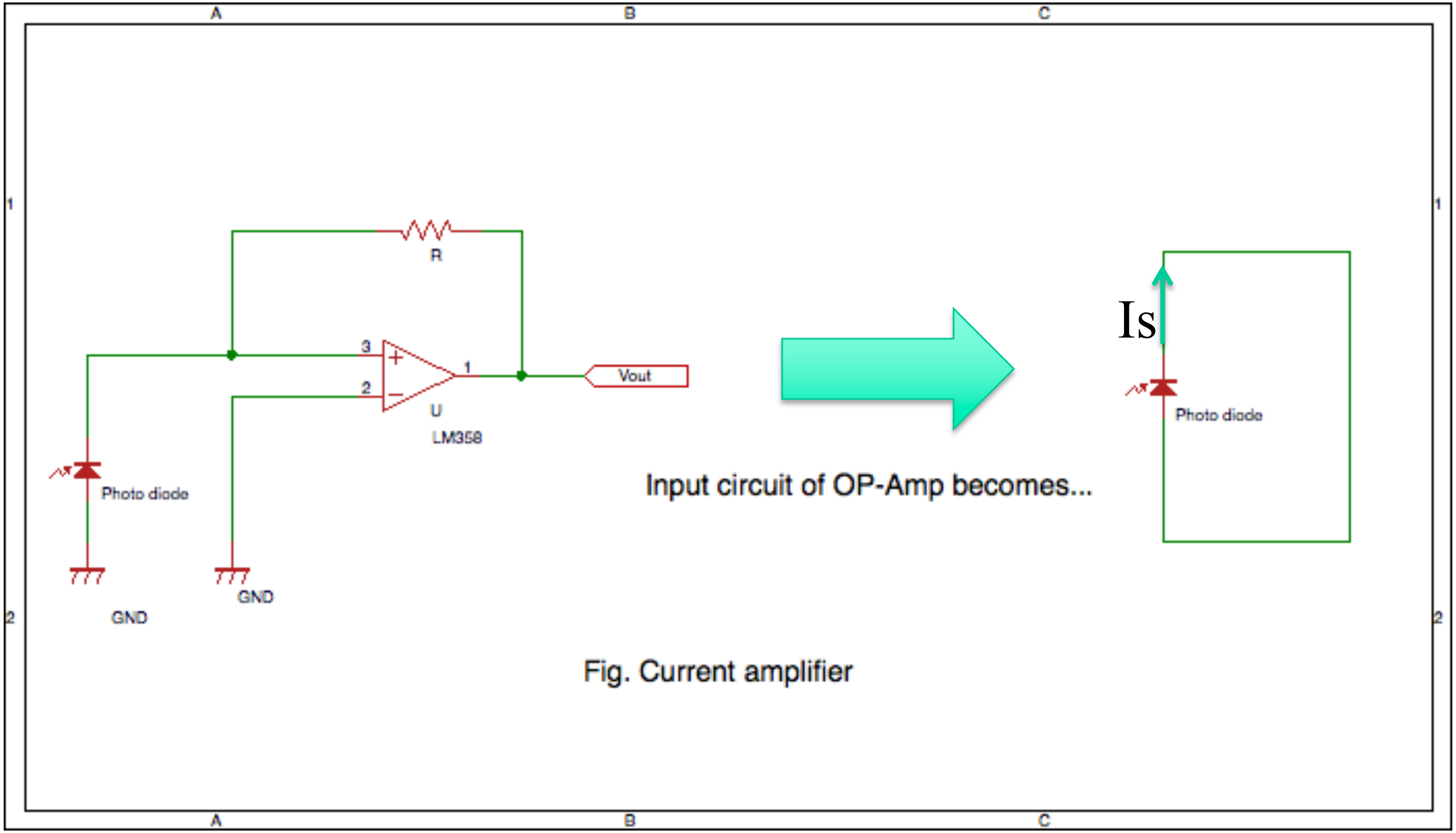
Instrumentation Amplifier



Other amplifier for measurement

- Current amplifier
- Charge amplifier

Current amplifier



Is : Short current of photo diode.
This current is generated depending on inputted photo energy to pn junction.

Charge amplifier

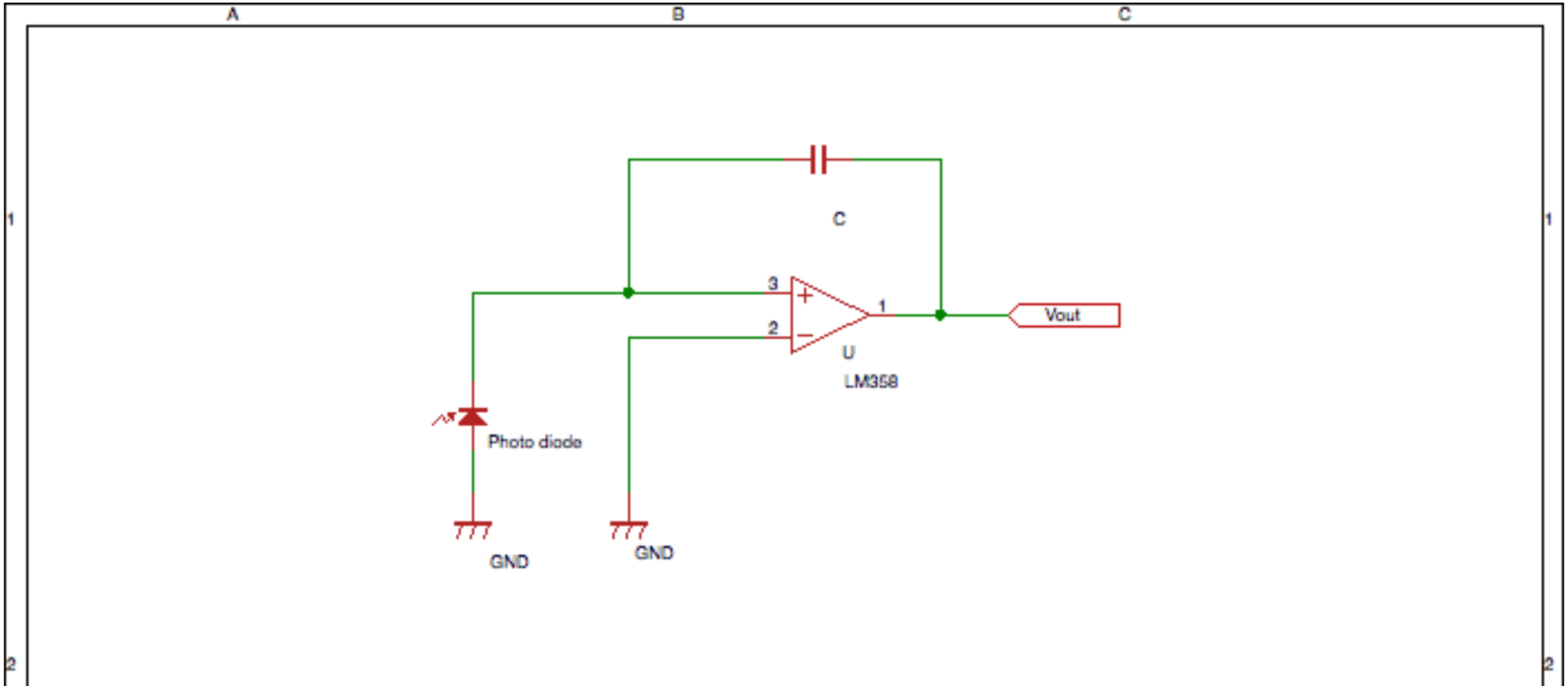


Fig. Charge amplifier

Practical amplifier circuit using OP-amp.

- OP-amp. is almost ideal. But, OP-amp. is not ideal.
- Important point should be considered.
  - off-set voltage
  - bias current
  - off-set current
  - input and output impedance
  - frequency response etc…

It's necessary to select a suitable OP amp.
for measurement aim.

Offset voltage and bias current of commercially available OP-Amp

	IC type number	Offset voltage		Bias current	
		typ	max	typ	max
Bi-polar (general purpose)	NJM4558	0.5mV	6mV	25nA	500nA
Bi-polar (general purpose)	LM358	2mV	7mV	45nA	250nA
Bi-polar (high precision)	AD8599	0.015mV	0.12mV	40nA	210nA
FET (general purpose)	LF411	0.8mV	2mV	50pA	200pA
FET (general purpose)	LF412	1mV	3mV	50pA	200pA
FET (high precision)	AD8610A	0.085mV	0.2mV	2pA	10pA
CMOS (general purpose)	LMC6482AI	0.11mV	1.35mV	0.02pA	4pA
CMOS (general purpose)	LMC6442AI	0.75mV	4mV	0.005pA	4pA
CMOS (high precision)	LMC6462AI	0.25mV	1.2mV	0.15pA	10pA

Active Filter

Importance of Filter in measurement

- Signal from sensor always includes noise and unwanted signal.
  Those should be reduced before A/D conversion.
- In the viewpoint of A/D conversion, inputted analog signal to ADC has to be based on Shannon's sampling theorem

↓

Filtering by electrical circuit

Features of active filter

- Active filters are composed by active components and passive components

- Active filters can filter and amplify signal at a time.

- High order filter can be realized by 1st order filter and 2nd order filters.

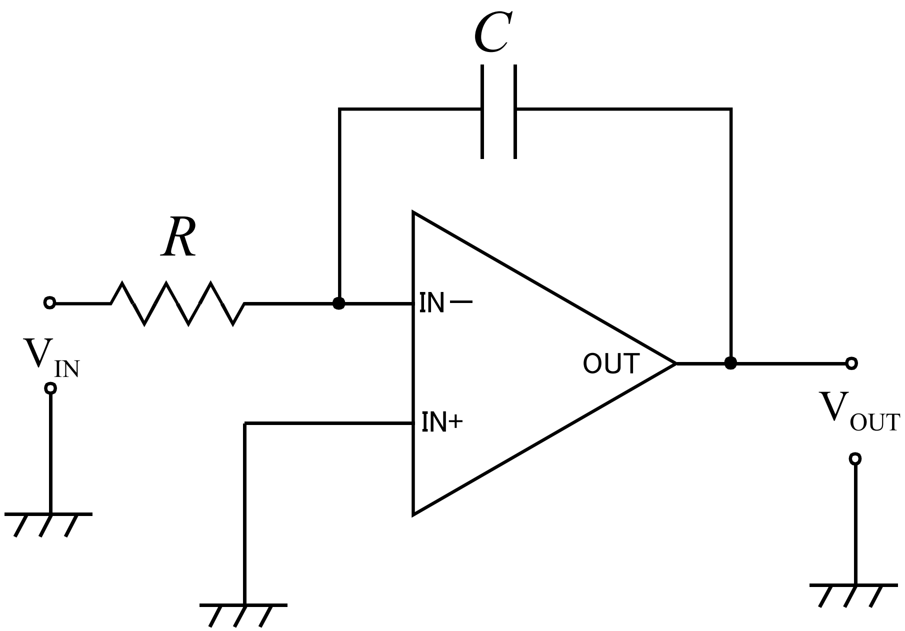
- Filter for low frequency is realized without inductor.

etc. . . . .

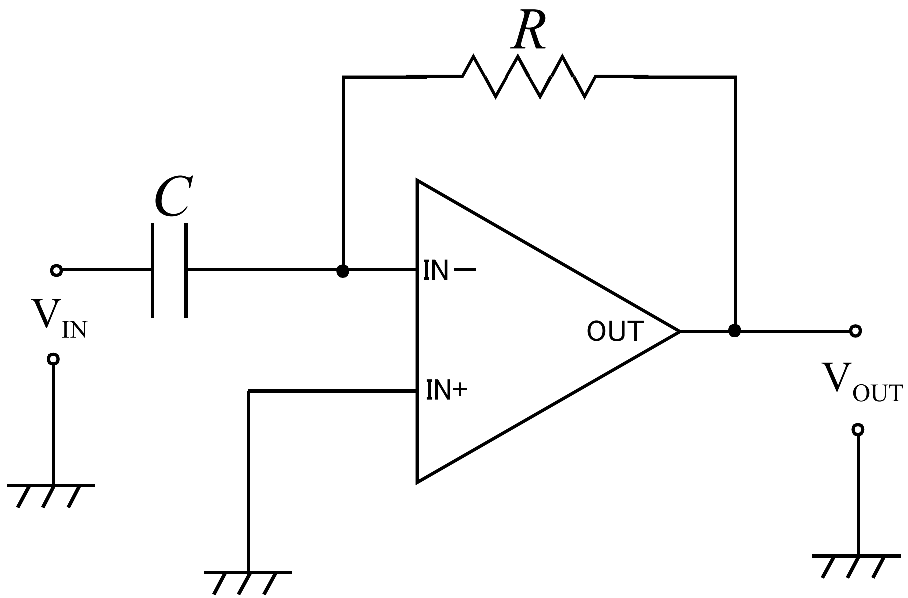
Active filter using OP-amp.

- Active filter by using OP-amp. is based on following circuits.
  - Integrator circuit (material for LPF)
  - Differentiating circuit (material for HPF)

Integrator circuit and Differentiating circuit by OP-amp.



Integrator circuit



Differentiating circuit

Transfer function and Amplitude characteristic of integrator circuit

● I/O function of integrator circuit

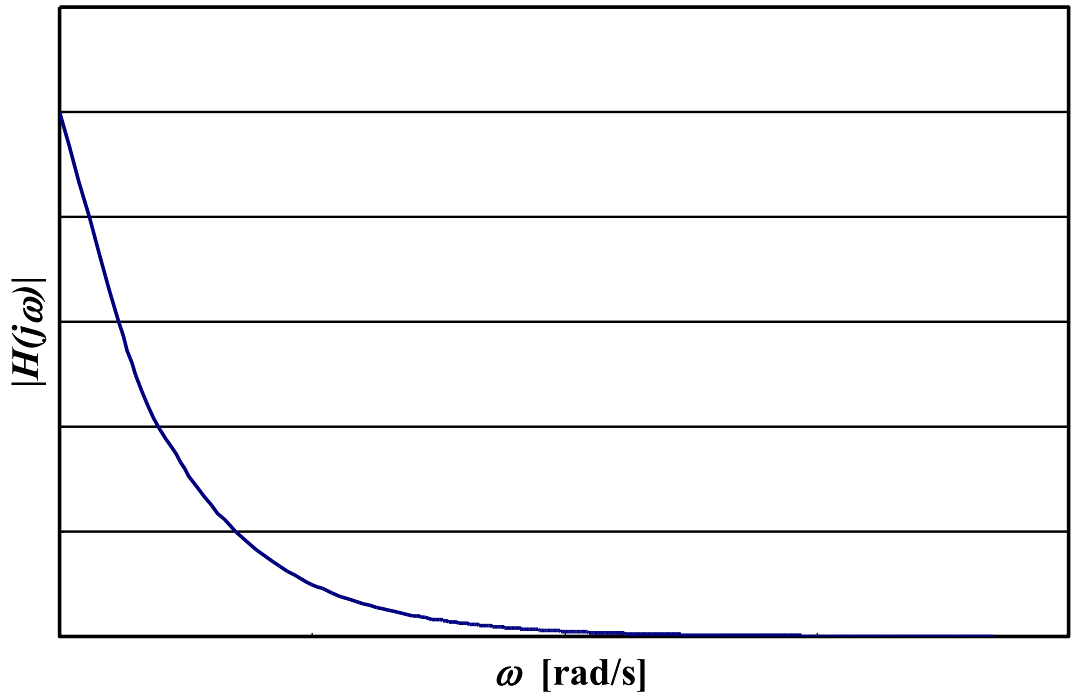
$$V_{out}(t) = -\frac{1}{CR}\int V_{in}(t)dt \quad (1)$$

● Transfer function

$$H(j\omega) = -\frac{1}{j\omega CR} \quad (2)$$

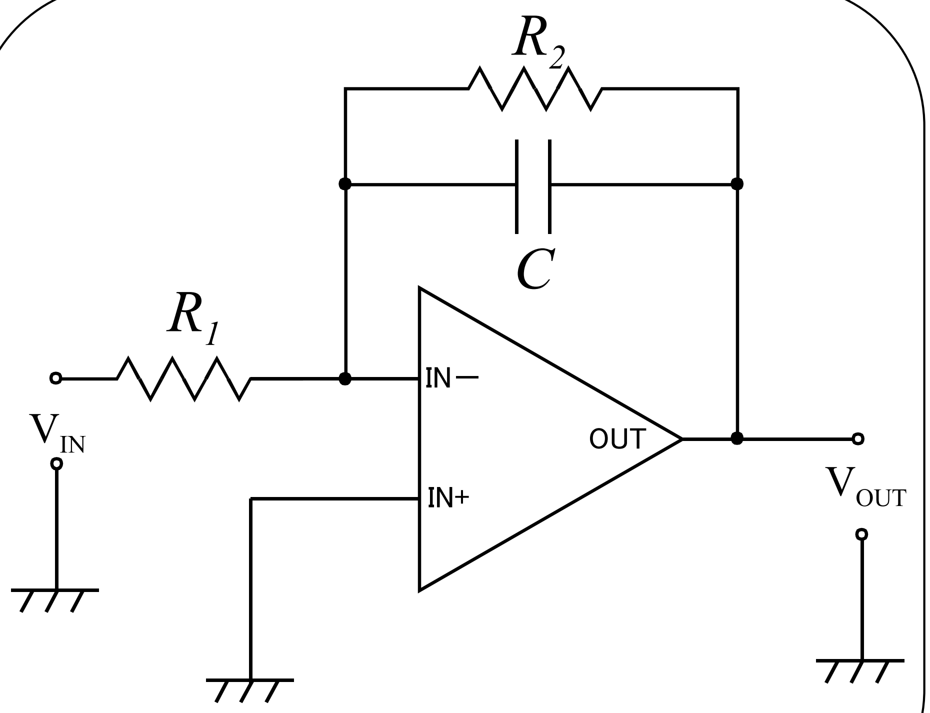
● Amplitude characteristic

$$\left|H(j\omega)\right| = \frac{1}{\omega CR} \quad (3)$$

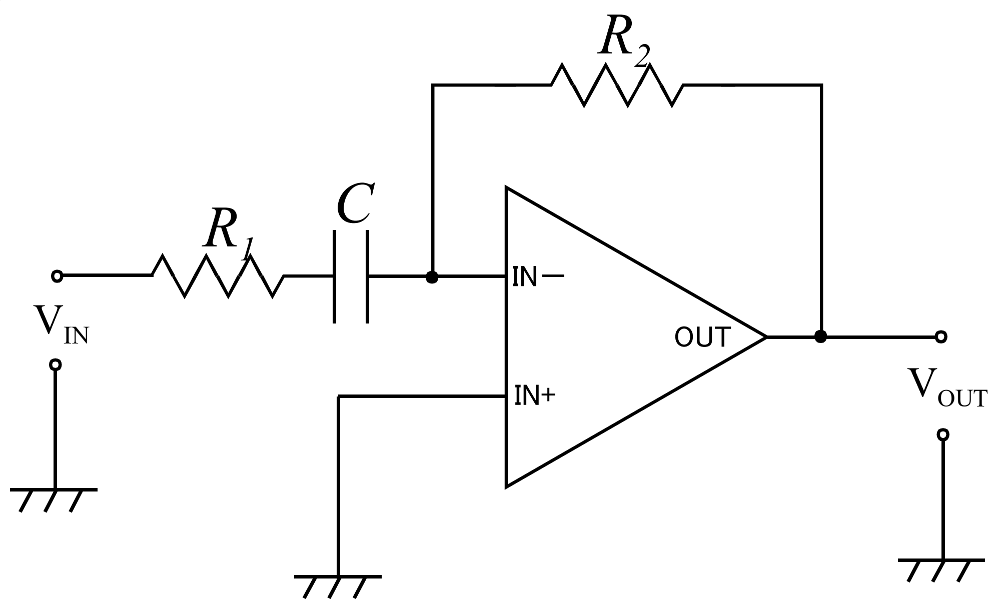


Amplitude characteristic

Filter circuits based on integrator and differentiation circuit



Low pass filter
based on integrator circuit



High pass filter
based on integrating circuit

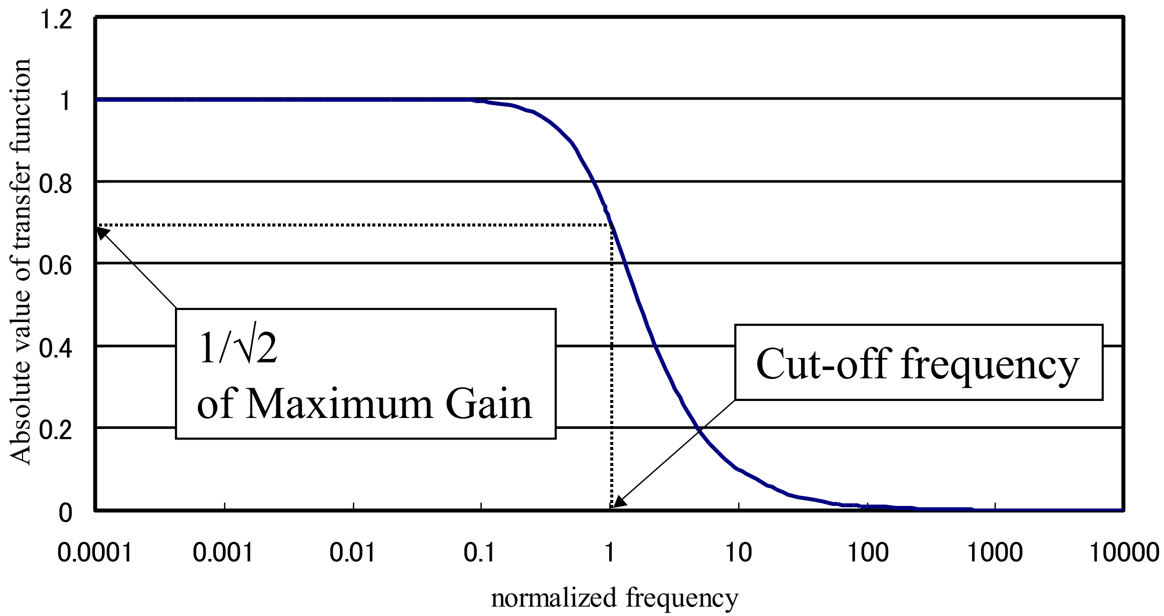
Basic 1st order active filter

● Transfer function

$$H(j\omega) = -\frac{R_2}{R_1}\frac{1}{1 + j\omega CR_2} \quad (1)$$

● Amplitude characteristic

$$|H(j\omega)| = \frac{R_2}{R_1}\frac{1}{\sqrt{1 + (\omega CR_2)^2}} = \frac{R_2}{R_1}\frac{1}{\sqrt{1 + \{\omega/(1/CR_2)\}^2}} \quad (2)$$



- Horizontal axis expresses normalized frequency by the cut-off frequency.
- Vertical axis expresses normalized value by maximum gain.

Attenuation gradient (slope)

☆What is "attenuation gradient"?

It means attenuation property of filter over the cut-off frequency.
This property is determined by the property of measuring signal.

Usually, following units are used for attenuation gradient.

dB/dec : "dec" means "decade". Attenuation ratio defined as follows;

Attenuation gradient [dB/dec] = 20 log $(|H(j\omega_2)|/|H(j\omega_1)|)$

* $\omega_2 = 10 \cdot \omega_1$

dB/oct : "oct" means "octave".

Attenuation gradient [dB/oct] = 20 log $(|H(j\omega_2)|/|H(j\omega_1)|)$

* $\omega_2 = 2 \cdot \omega_1$

Design of high order filter

- High order filter
  → Transfer function is expressed
  by the equation of high order "$j\omega$" .

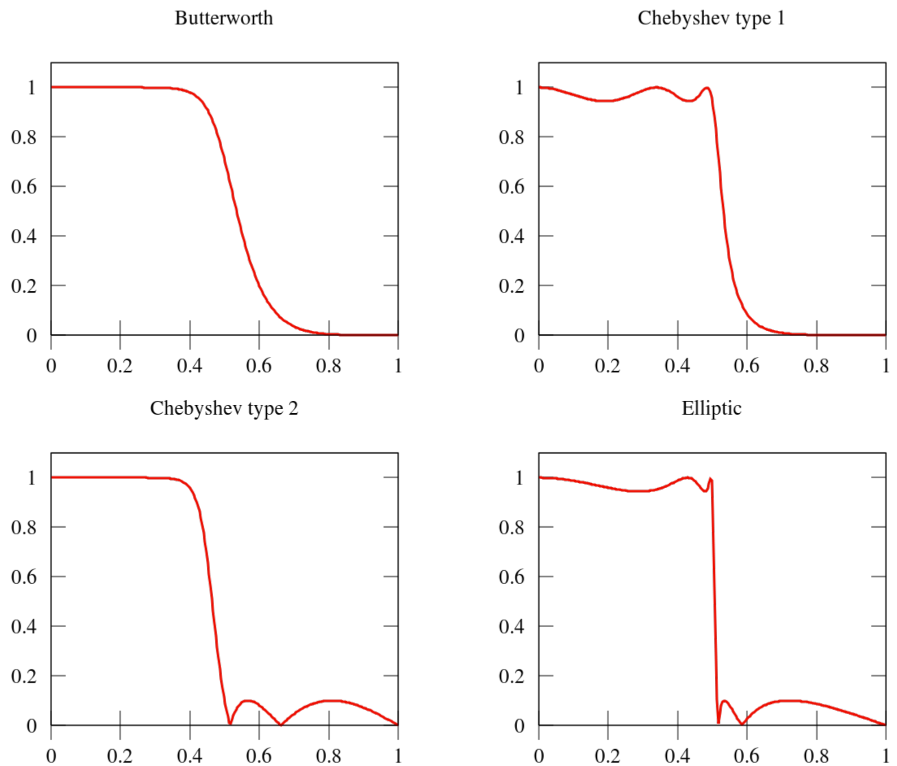
To get high order transfer function

- Amplitude characteristics is defined.
- Transfer function from the characteristics.

The amplitude characteristics

- Butterworth filter
  → Pass band and stop band have flat frequency response.
- Chebyshev filter
  → Pass band or stop band have ripple feature.
- Elliptic filter
  → Pass band and stop band have ripple feature.

…etc.



* From Wikipedia

Design of Butterworth filter

- Amplitude characteristic of Butterworth filter

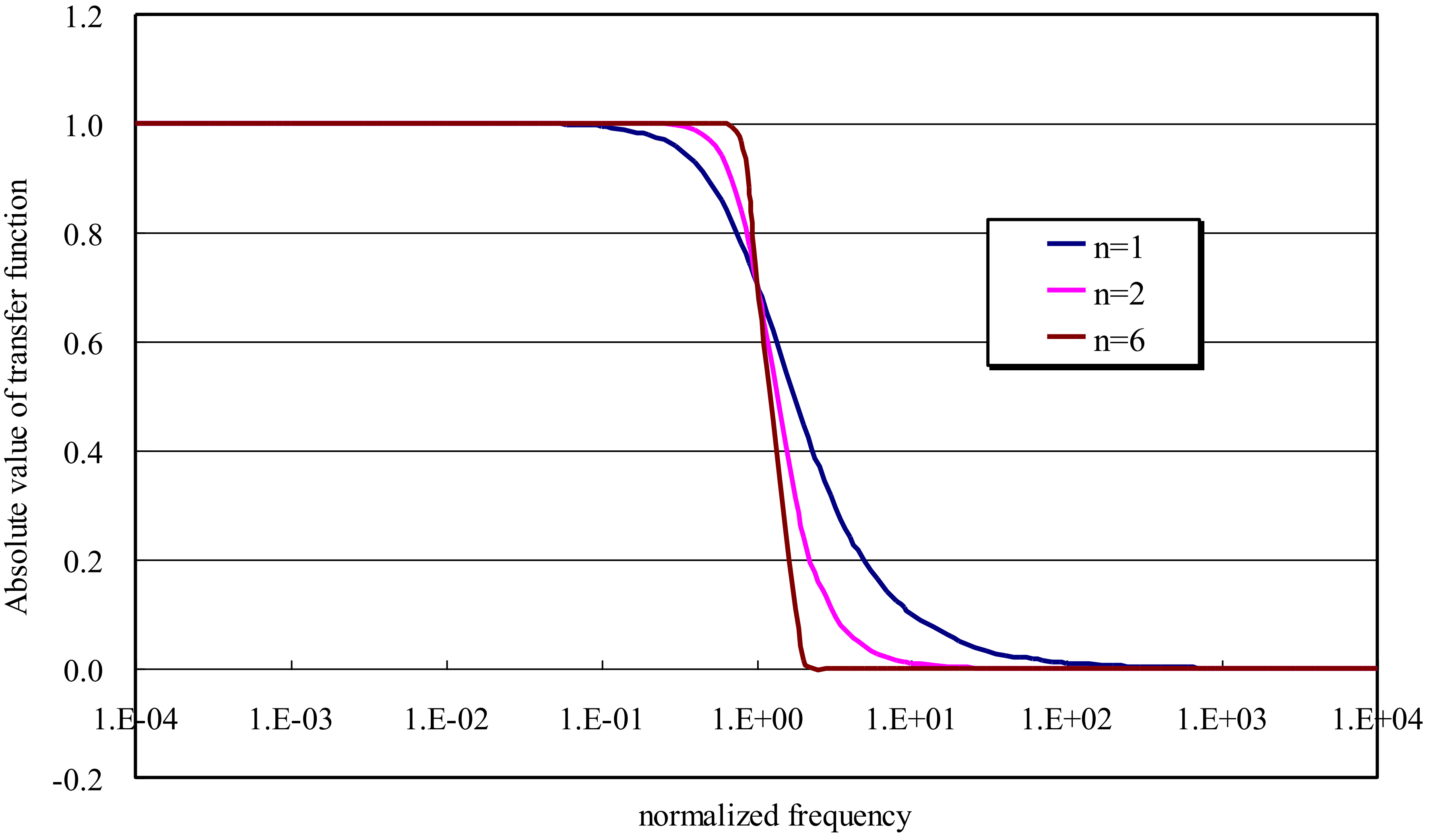
$$|G(jx)| = \frac{1}{\sqrt{1 + x^{2n}}}$$

※n : The order of transfer function
$x = \omega/\omega_c$: Normalized frequency by cut-off frequency.

Transfer function of prototype filter $G(jx)$

※ Prototype filter:
Filter designed under using normalized frequency.

Amplitude characteristics of Butterworth filter



Butterworth filter's transfer function

Transfer function $G(jx)$
from $|G(jx)|$ of Butterworth filter.

$$G(jx) = \frac{1}{(jx - jx_0)(jx - jx_1)\cdots(jx - jx_{n-1})}$$

$$x_m = e^{j\frac{2m+1}{2n}\pi} \qquad \{m = 0,\ 1,\ \cdots,\ n\text{-}1\}$$

n : The order of transfer function

$$|G(jx)| = \frac{1}{\sqrt{1+x^{2n}}}$$

$$|G(jx)|^2 = G(jx)\cdot G(-jx)$$

$$= \frac{1}{1+x^{2n}}$$

To get $G(jx)$, the denominator is factorized.

$$1 + x^{2n} = 0$$

$$x^{2n} = -1$$

$$x^n = \pm j = e^{\pm j\frac{(2m+1)}{2}\pi}$$

$$x_m = e^{\pm j\frac{(2m+1)}{2n}\pi}$$

m = 0, 1, 2,･･･, n-1

Denominator of transfer function of Butterworth filter

Order n	Denominator of transfer function
1	jx+1
2	(jx)^2+1.4142(jx)+1
3	(jx+1) {(jx)^2+(jx)+1}
4	{(jx)^2+0.7654(jx)+1} {(jx)^2+1.8478(jx)+1}
5	(jx+1) {(jx)^2+0.6180(jx)+1} {(jx)^2+1.6180(jx)+1}
6	{(jx)^2+0.5176(jx)+1} {(jx)^2+1.4142(jx)+1} {(jx)^2+1.9319(jx)+1}
7	(jx+1) {(jx)^2+0.4450(jx)+1} {(jx)^2+1.2470(jx)+1} {(jx)^2+1.8019(jx)+1}
8	{(jx)^2+0.3902(jx)+1} {(jx)^2+1.1111(jx)+1} {(jx)^2+1.6629(jx)+1} {(jx)^2+1.9616(jx)+1}

Design example

★ 1st order filter:

n=1, and m=0

$$x_0 = e^{j\frac{\pi}{2}} = j$$

1st order transfer function

$$G(jx) = \frac{1}{1 + jx}$$

★ 2nd order filter:

n=2, and m=0, 1

$$x_0 = e^{j\frac{\pi}{4}} = \frac{\sqrt{2}}{2} + j\frac{\sqrt{2}}{2}, x_1 = e^{j\frac{3\pi}{4}} = -\frac{\sqrt{2}}{2} + j\frac{\sqrt{2}}{2}$$

2nd order transfer function

$$G(jx) = \frac{1}{\left(jx + \sqrt{2}/_2 - j\sqrt{2}/_2\right)\left(jx + \sqrt{2}/_2 + j\sqrt{2}/_2\right)} = \frac{1}{(jx)^2 + \sqrt{2}(jx) + 1}$$

Design example (2)

★ 3rd order filter:

n=3, and m=0, 1 , 2

$$x_0 = \frac{\sqrt{3}}{2} + j\frac{1}{2}, x_1 = j, x_2 = -\frac{\sqrt{3}}{2} + j\frac{1}{2}$$

3rd order transfer function

$$G(jx) = \frac{1}{(jx+1)\{(jx)^2 + (jx) + 1\}} = \underbrace{\frac{1}{(jx+1)}}_{1^{\text{st}}\text{ order}} \cdot \underbrace{\frac{1}{(jx)^2 + (jx) + 1}}_{2^{\text{nd}}\text{ order}}$$

* 3rd order filter is composed by 1st order and 2nd order filter.

Design rule …

Design rule of Butterworth filter

① Pairs of conjugate complex exist in the transfer function above 2^{nd} order.

n : odd number ⇒ (n-1)/2 pairs

even number ⇒ n/2 pairs

From these,
2^{nd} order filter is composed.

② In the case of odd number's order, 1^{st} order filter exists in the transfer function.
→ Odd number's filter is composed by 1^{st} order filter and some 2^{nd} order filter,

and cascade connection of those.

③ In the case of even number's order, the filter is composed
by some 2^{nd} order filter, and cascade connection of those.

1^{st} order filter circuit, and 2^{nd} order filter circuit

1st order filter circuit

☆1st order filter's transfer function (Prototype filter)

$$G(jx) = \frac{1}{1 + jx}$$

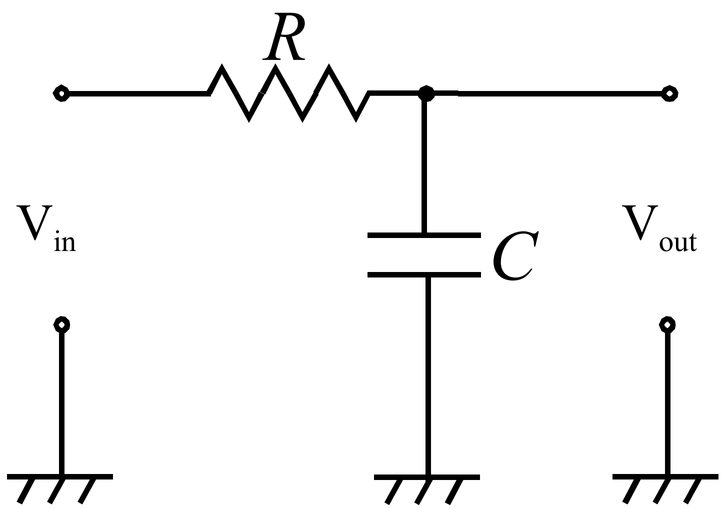
1st order filter is always realized by passive filter.

Passive RC circuit's transfer function

$$H(j\omega) = \frac{1}{1 + j\omega CR}$$

$$\boxed{\omega_c = 1/CR}$$

$$G(jx) = \frac{1}{1 + jx},\ x = {}^{\omega}/_{\omega_c}$$



Passive RC filter circuit

About 2nd order filter

☆ General expression of 2nd order filter's transfer function

$$G(jx) = \frac{1}{(jx)^2 + {}^{1}/_{a} \cdot (jx) + 1}$$

* "*a*" is determined by pair of conjugate complex number.

@ cut-off frequency (x=1)

$$|G(j1)| = a$$

⇒ "*a*" means the gain at cut-off frequency.

⇓

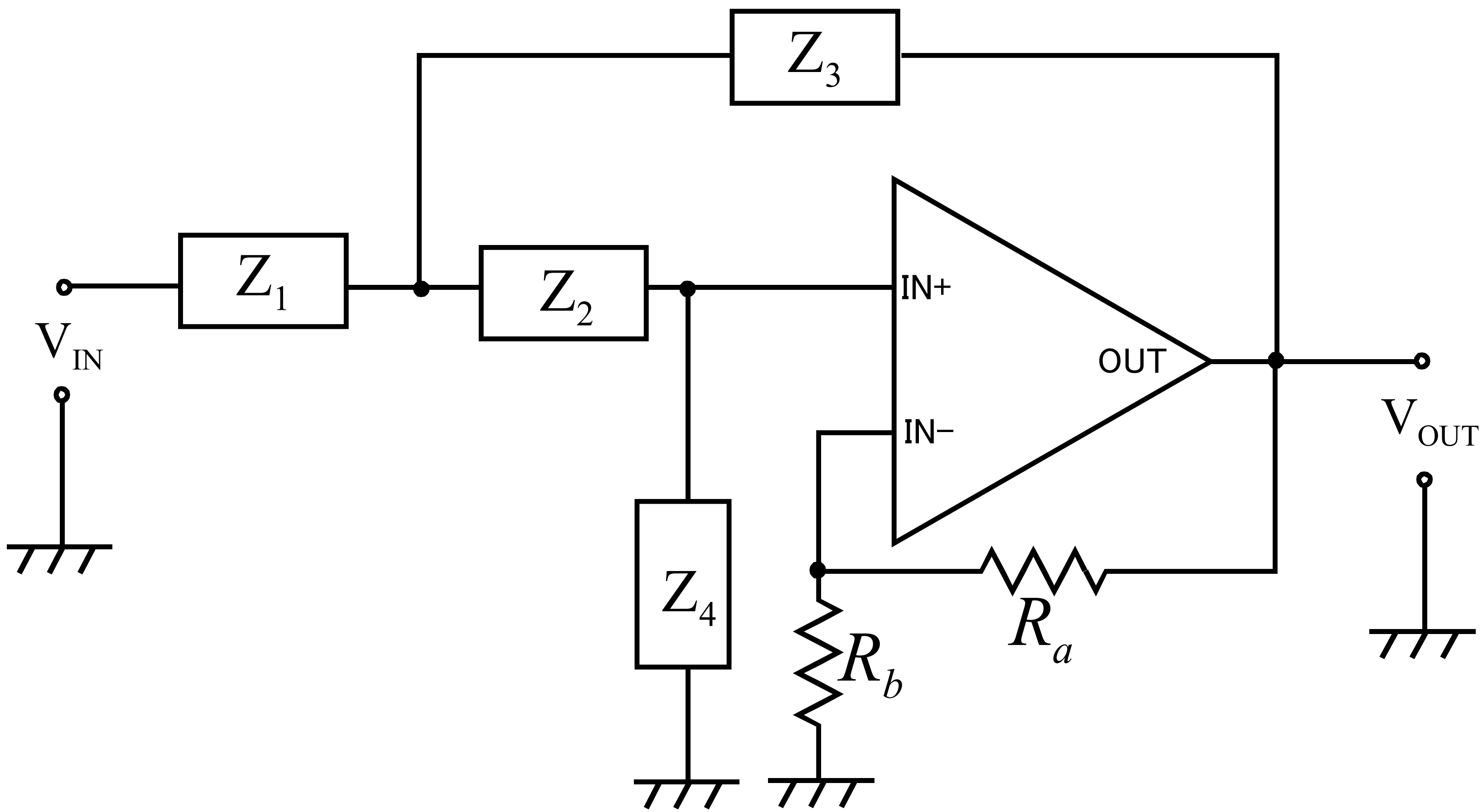
2nd order filter circuit which have same transfer function.

Circuit for 2nd order filter

- VCVS(Voltage-Controlled Voltage Source) circuit (Sallen-Key circuit)
- MFB(Multi Feed Back) circuit

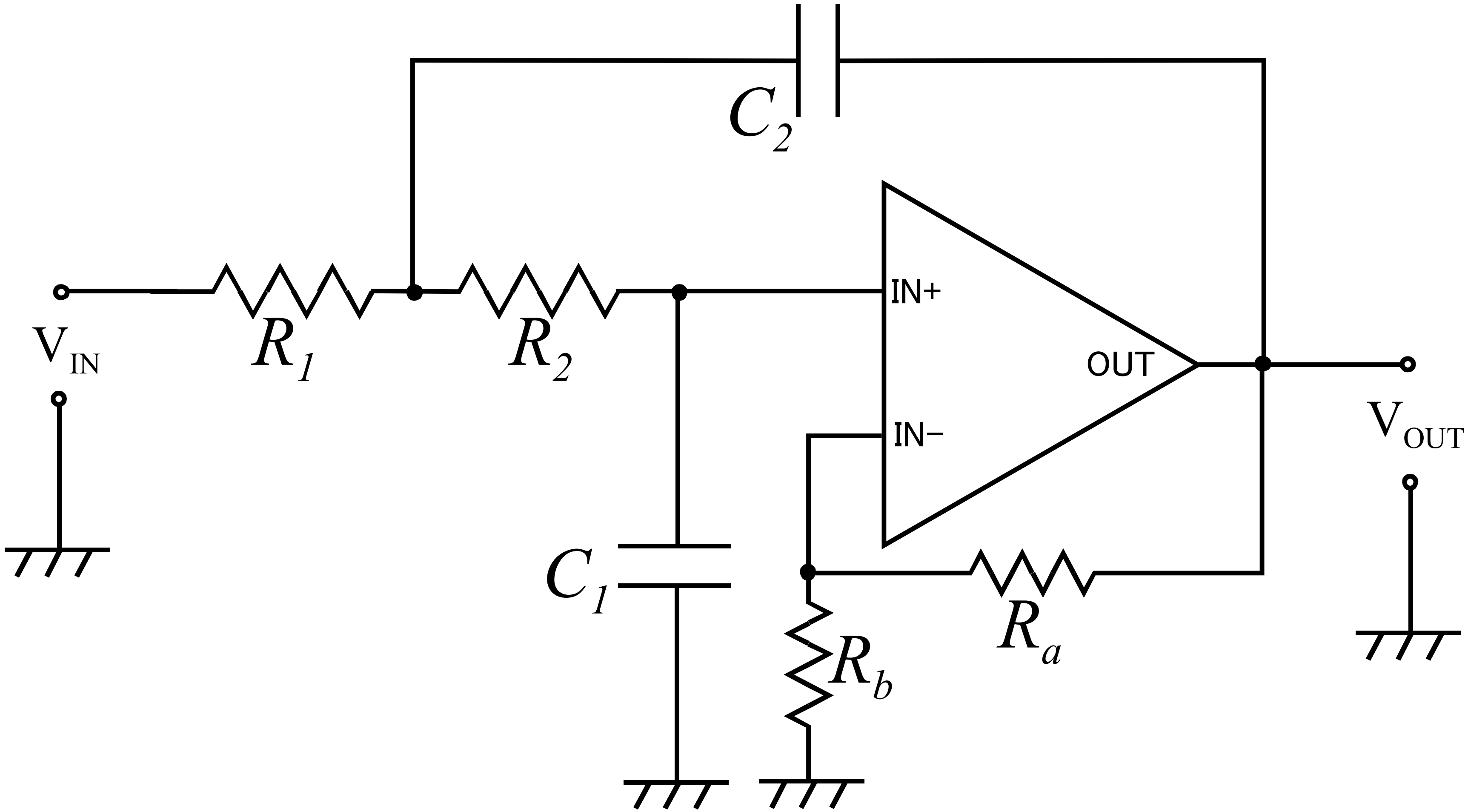
…etc.

VCVS circuit



VCVS : Voltage Controlled Voltage Source

2^{nd} order filter using VCVS circuit



2^{nd} order LPF using VCVS circuit

Transfer function of VCVS circuit

$$H(j\omega) = \frac{A}{(j\omega)^2 R_1 C_1 R_2 C_2 + (j\omega)\{(1-A)R_1 C_2 + R_2 C_1 + R_1 C_1\} + 1}$$

$* \; A = 1 + R_a/R_b$

$$\Downarrow \quad R_1 = R_2 = R, \quad C_1 = C_2 = C$$

$$H(j\omega) = \frac{A}{(j\omega)^2 (RC)^2 + (j\omega)(3-A)RC + 1}$$

$$\Downarrow \quad \omega_c = 1/RC$$

$$\boxed{H(jx) = \frac{A}{(jx)^2 + (jx)(3-A) + 1}}$$

Comparison between transfer functions

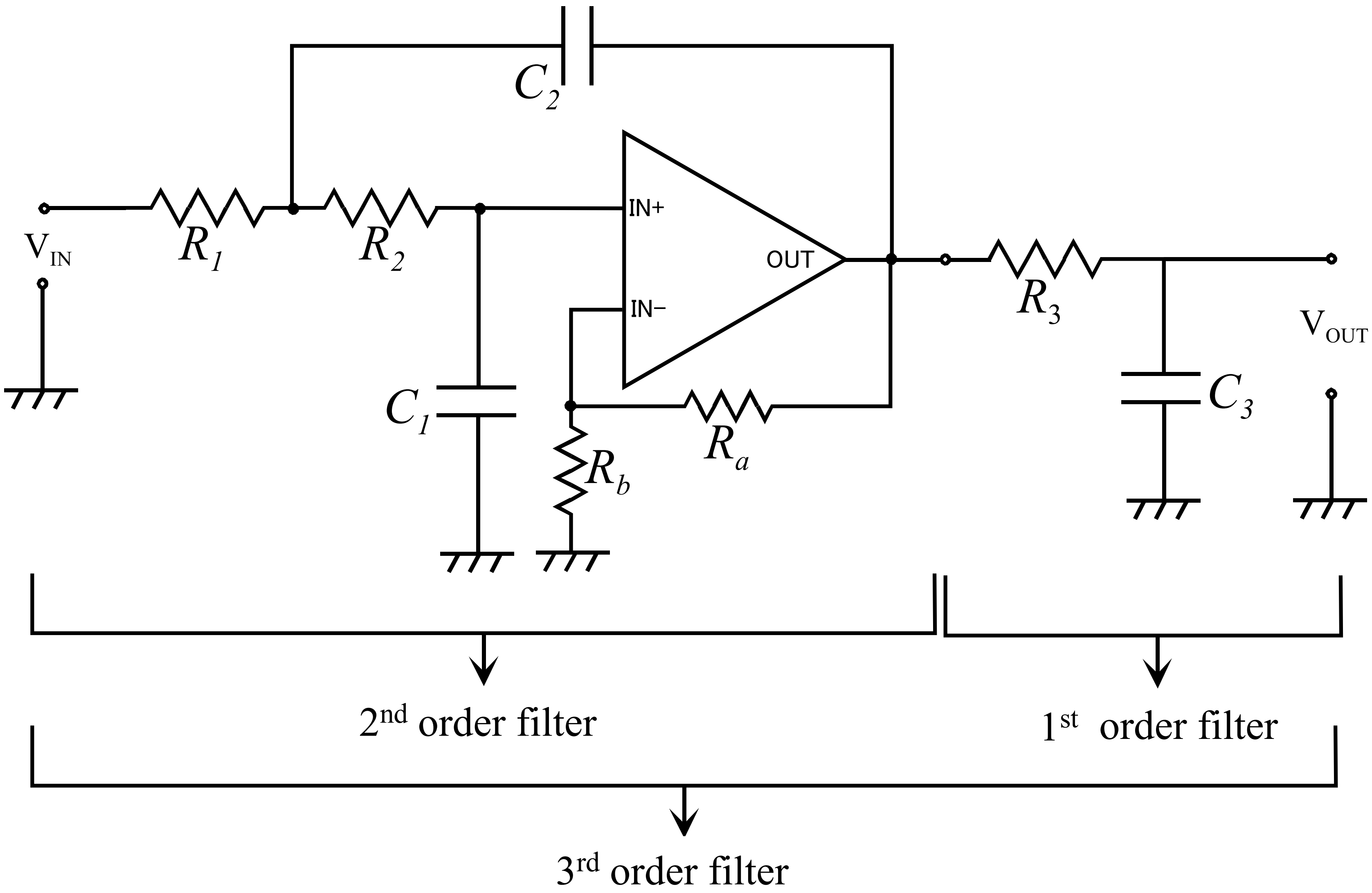
Prototype Butterworth filter's : transfer function (2nd order)

$$G(jx) = \frac{1}{(jx)^2 + {}^{1}/_{a} \cdot (jx) + 1}$$

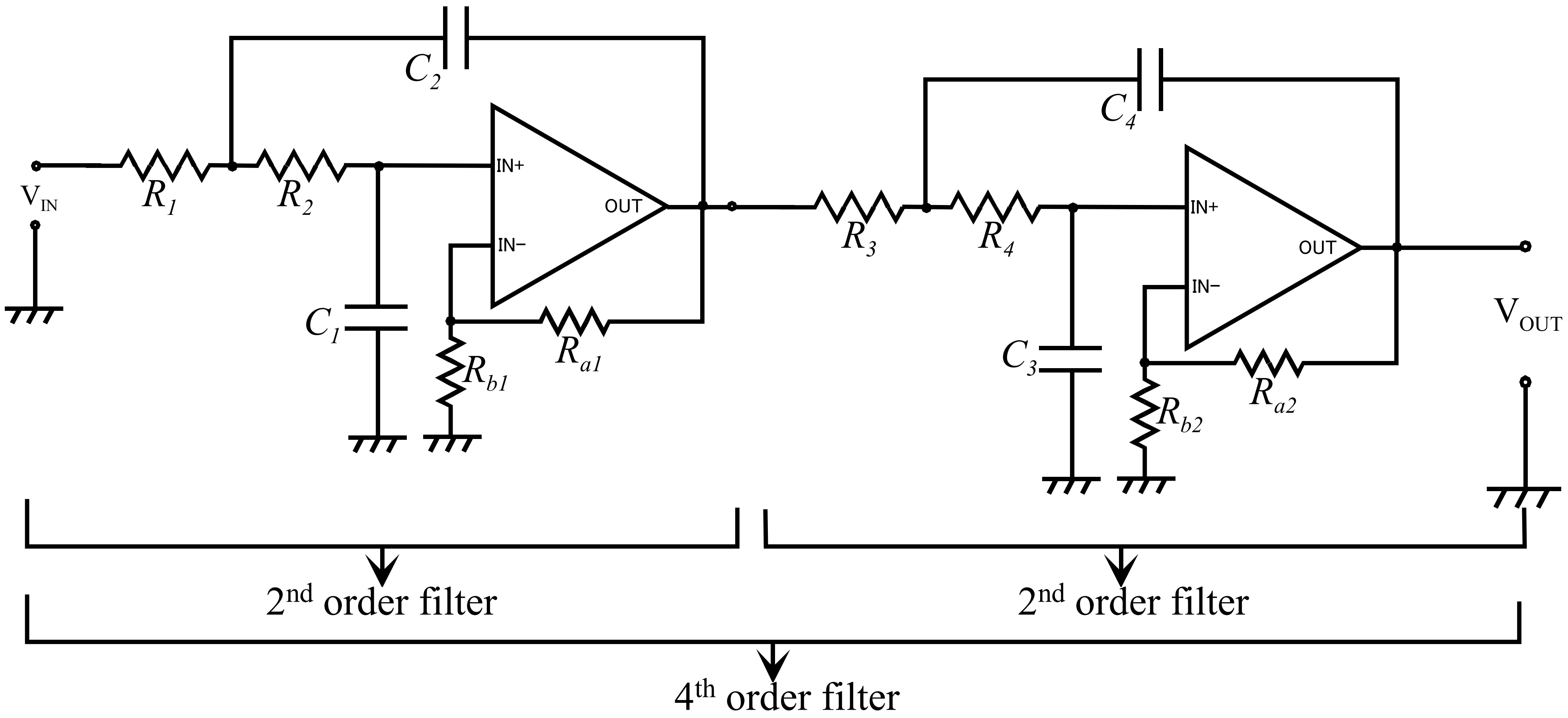
VCVS circuit's transfer function :

$$H(jx) = A \cdot \frac{1}{(jx)^2 + (jx)(3 - A) + 1}$$

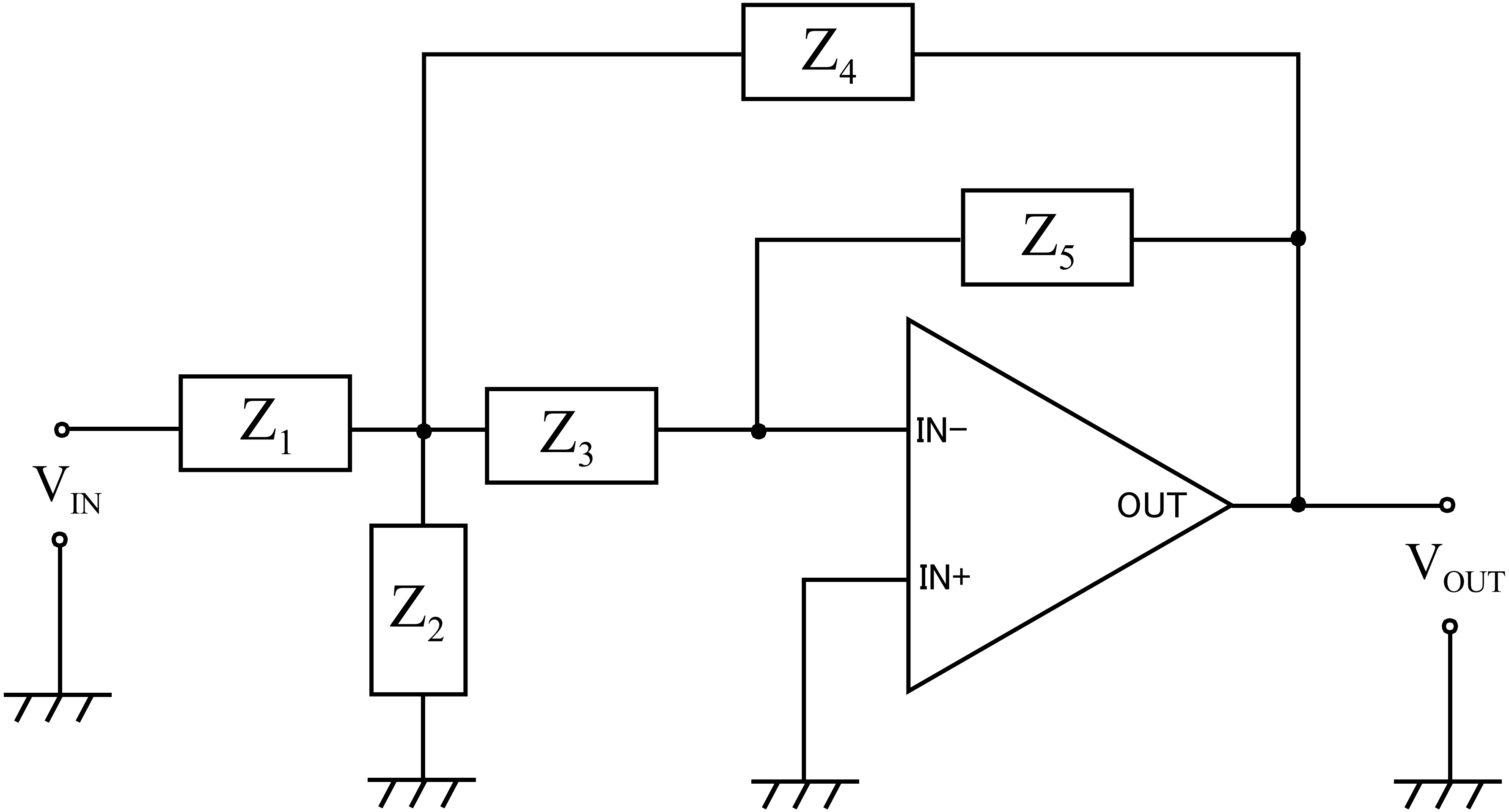
3rd order LPF



4^{th} order LPF



MFB circuit



MFB : Multiple Feed Back