Semiconductor Materials

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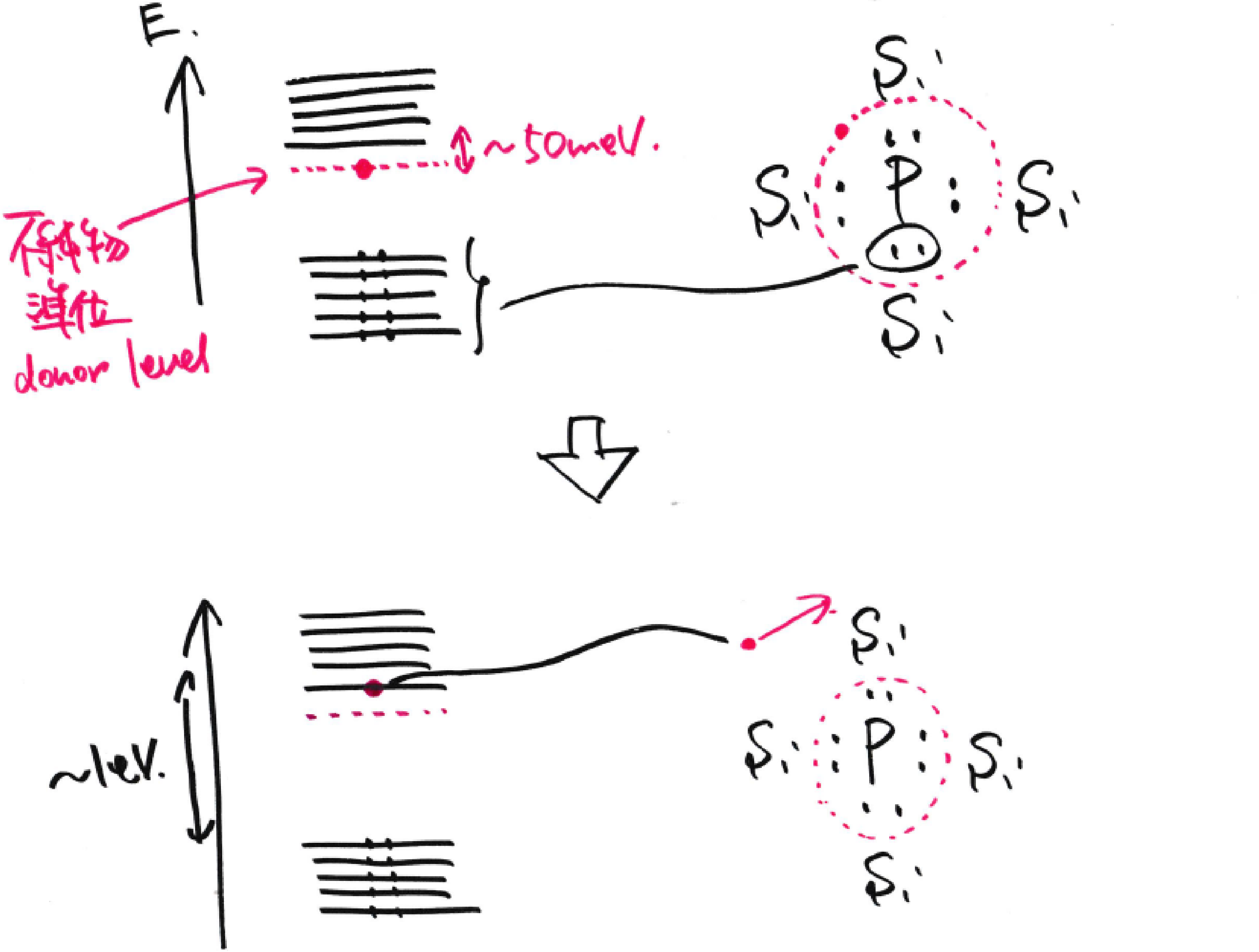
Exercise

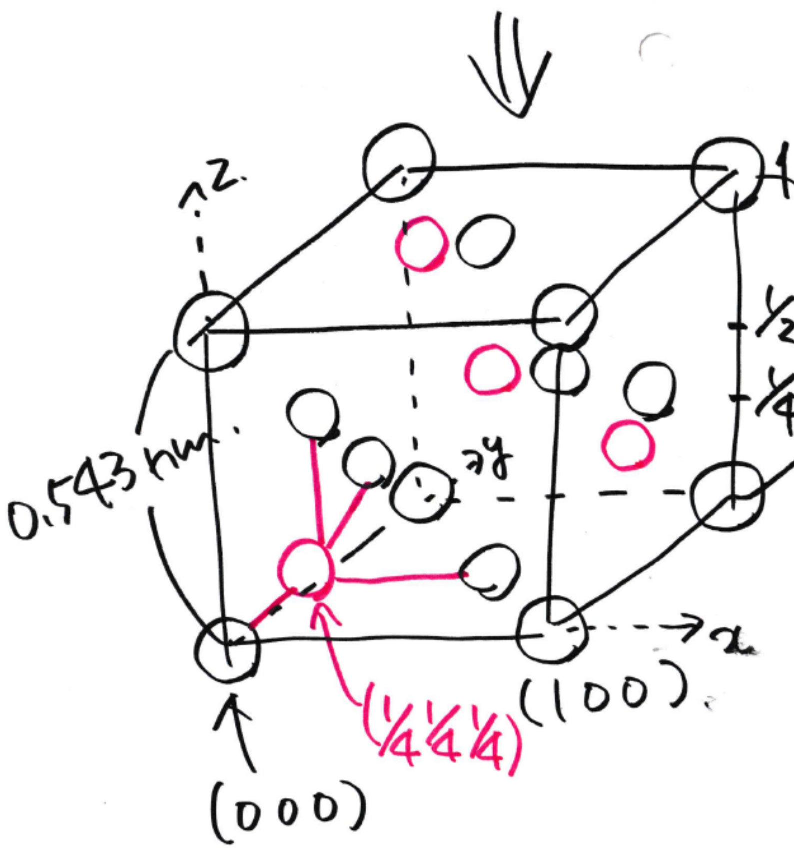
Evaluate the concentration of P atoms (at%), which is necessary to increase the conduction electron density in Si at room temperature to 1 x 10^{16} /cm^3.

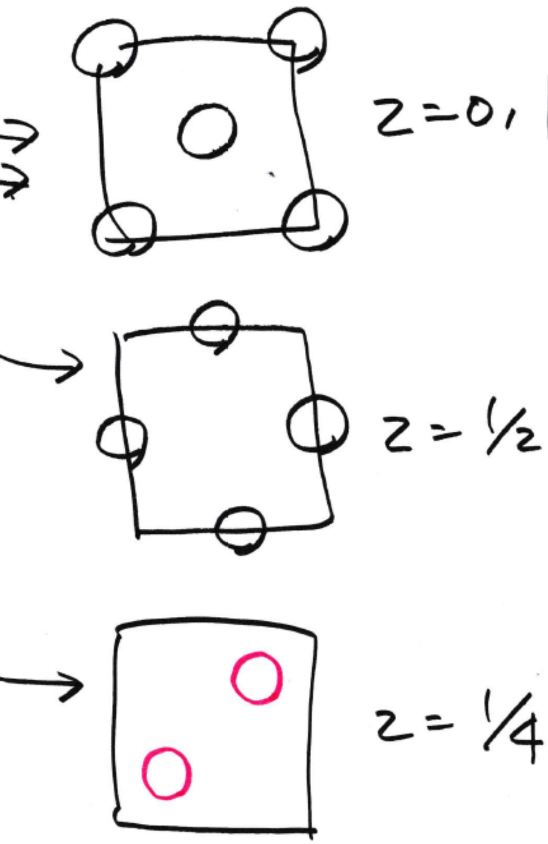
（室温におけるSiの伝導電子密度を 1 x 10^{16} /cm^3 とするために必要なP原子の濃度 (at%) を計算せよ。）

ドナーのドープ doping with donors

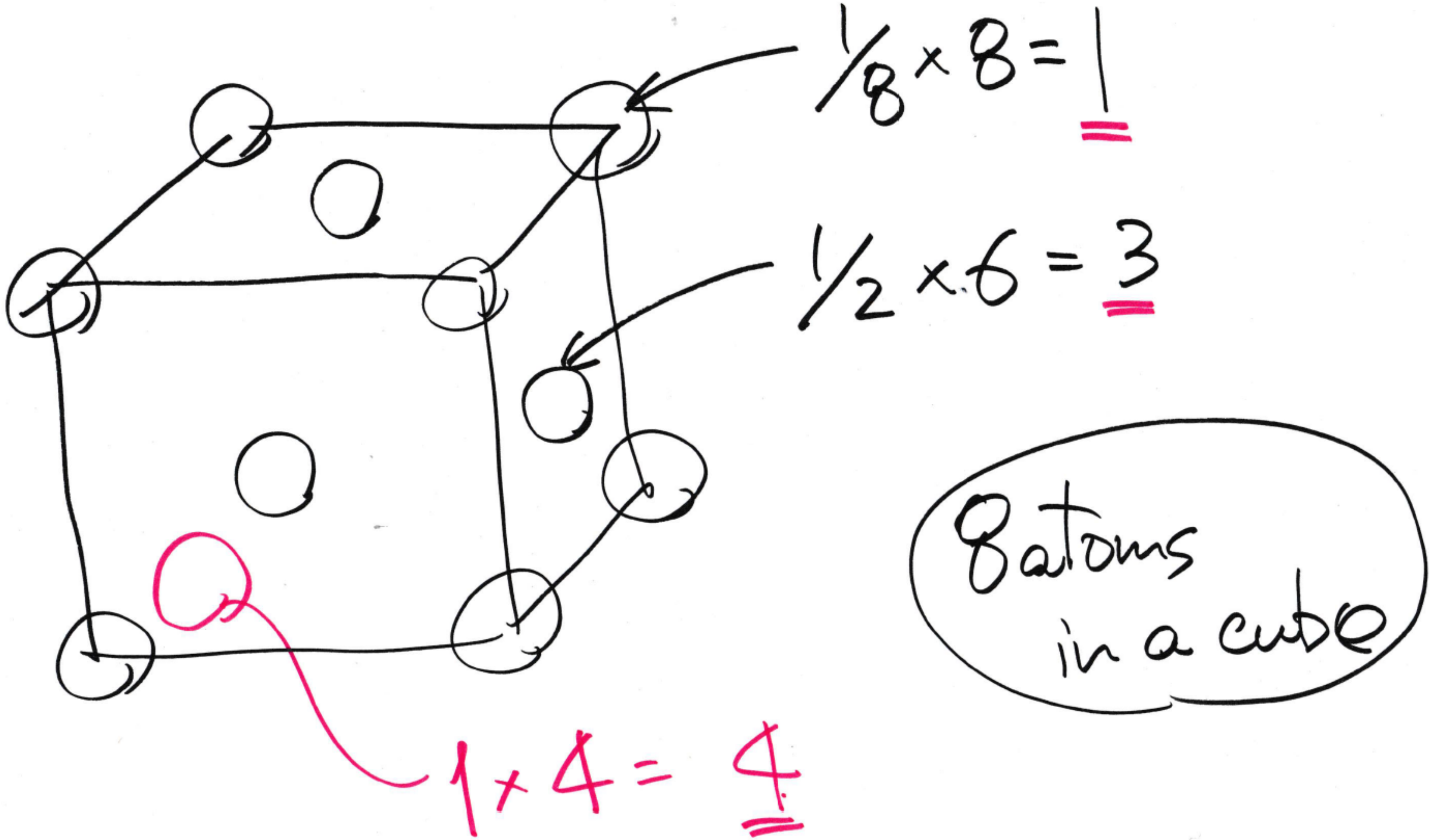


ダイヤモンド構造 diamond structure





正四面体のネットワーク
network of tetrahedron



1/8 × 8 = 1
1/2 × 6 = 3
1 × 4 = 4
8 atoms in a cube

$$\frac{8 \text{ atoms}}{(0.543 \text{ nm})^3} = \frac{8}{(0.543 \times 10^{-7} \text{ cm})^3}$$

$$= 5 \times 10^{22}/\text{cm}^3$$

P density.

$$\frac{1\times10^{16}}{5\times10^{22}} = 2\times10^{-7} = 2\times10^{-5}\%$$

$$\frac{2}{10000000}$$ (2 P atoms in 10000000 atoms).

purity of Si water $\underbrace{99.999999999}_{11\text{. nine}}\%$.

状態密度 Density of states

effective density of states (conduction band)

伝導帯の有効状態密度.

$N_c = 2.86\times10^{19}/cm^3$ (Si)

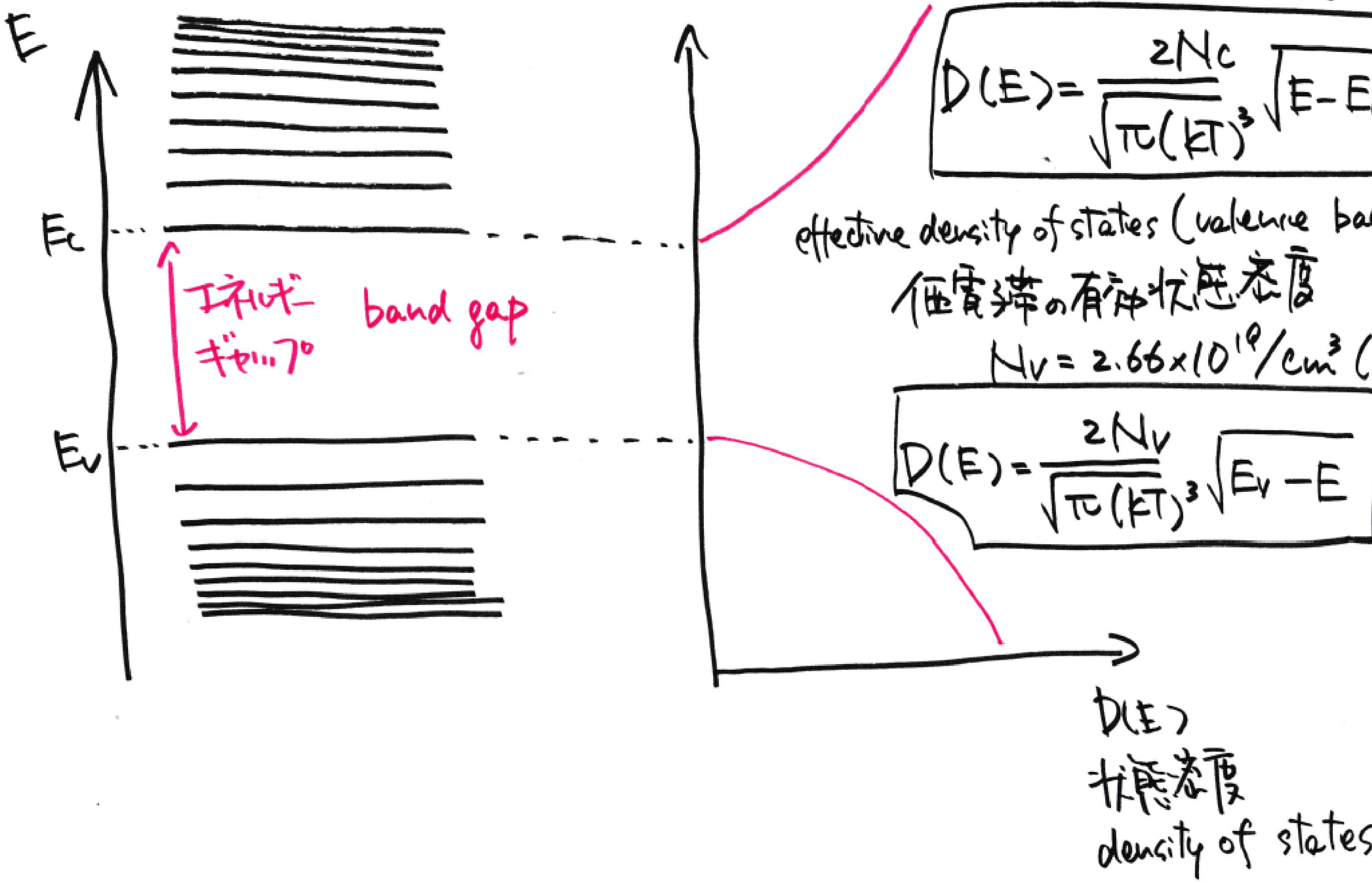
$$\boxed{D(E) = \frac{2N_c}{\sqrt{\pi(kT)^3}}\sqrt{E-E_c}}$$

effective density of states (valence band)

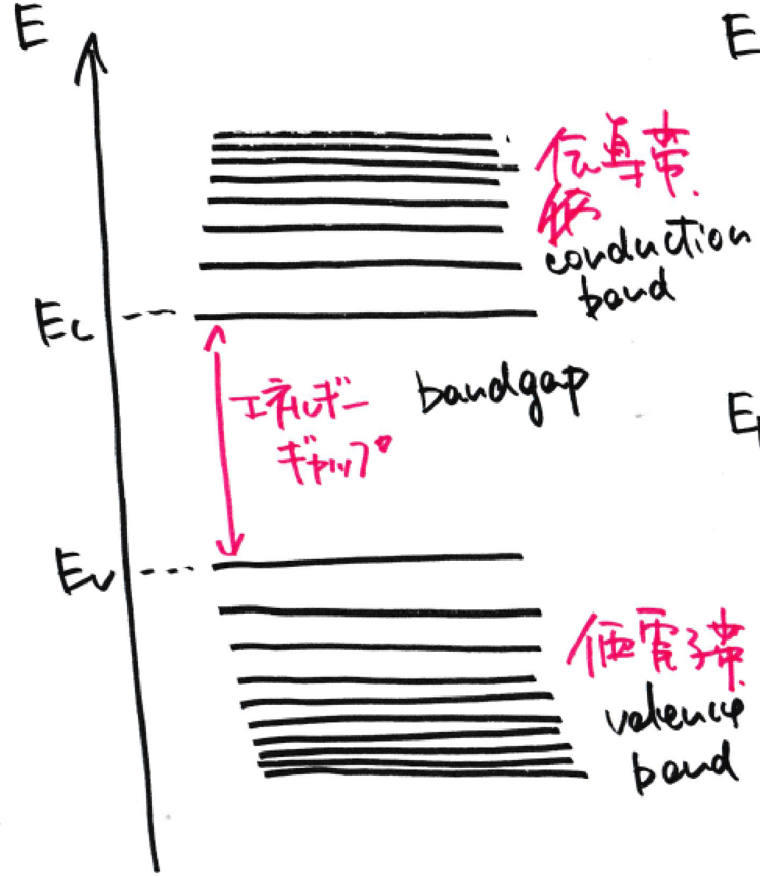
価電子帯の有効状態密度

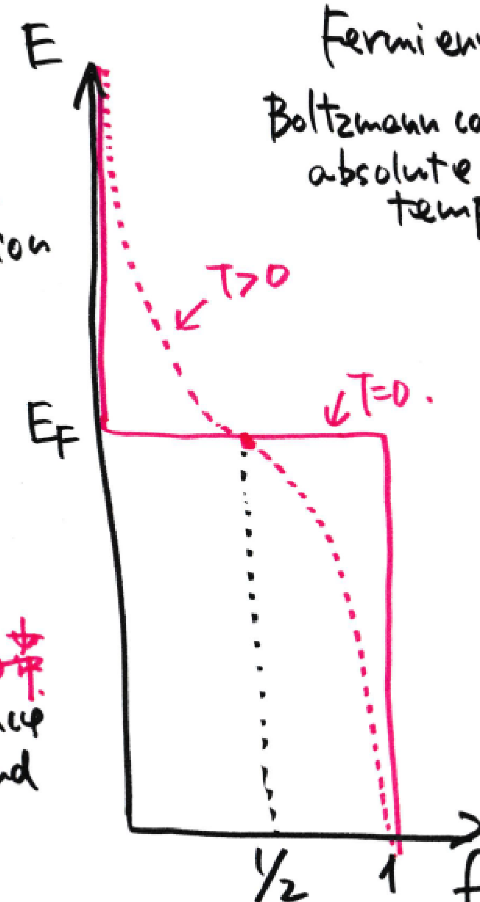
$N_v = 2.66\times10^{19}/cm^3$ (Si)

$$\boxed{D(E) = \frac{2N_v}{\sqrt{\pi(kT)^3}}\sqrt{E_v-E}}$$



フェルミ・ディラック分布
Fermi・Dirac distribution





f(E) 準位の状態の占有率
フェルミ・ディラック分布 occupation probability

$$f(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{kT}\right)}$$

Fermi energy E_F フェルミエネルギー
Boltzmann const. k ボルツマン定数
absolute temp. T 絶対温度

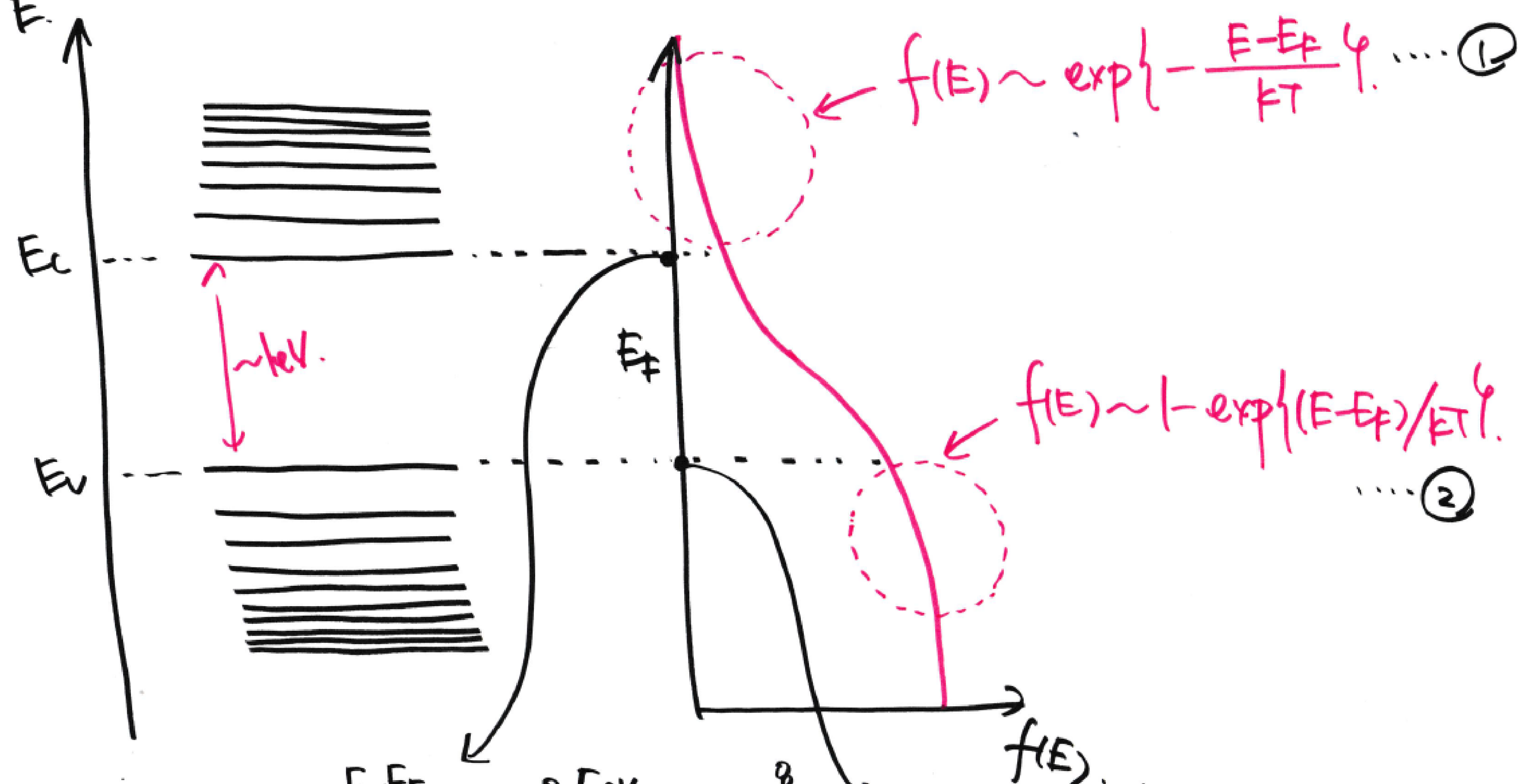
$T \sim 0(K)$

- $E - E_F > 0 \rightarrow \exp\left(\frac{E-E_F}{kT}\right) \rightarrow \infty$
  $\rightarrow f(E) \rightarrow 0$
- $E - E_F < 0 \rightarrow \exp\left(\frac{E-E_F}{kT}\right) \rightarrow 0$
  $\rightarrow f(E) \rightarrow 1.$

$$f(E_F) = 1/2$$

フェルミ分布の近似形

approximation of Fermi distribution



$$\exp\frac{E-E_F}{kT} \sim \exp\frac{0.5\,eV}{0.026\,eV} \sim 2\times10^{8} \gg 1, \quad \exp\frac{E-E_F}{kT} \sim \exp\left(-\frac{0.5}{0.026}\right) \sim 4\times10^{-9} \ll 1$$

フェルミ・ディラック分布　approximation of Fermi distribution.

$$f(E)=\frac{1}{1+\exp\left(\frac{E-E_F}{kT}\right)}$$

- $E>E_c$ のとき.

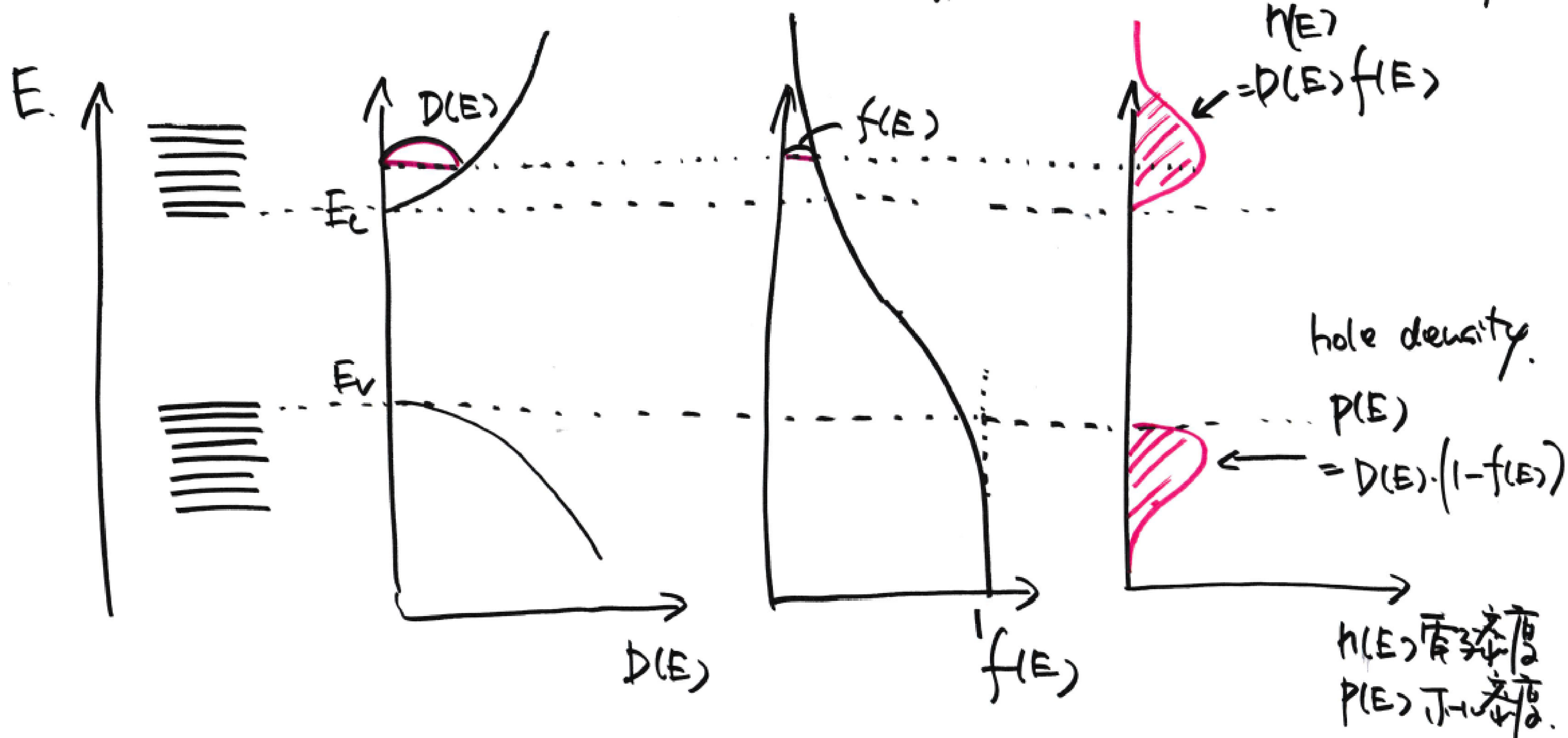
$\exp\left(\frac{E-E_F}{kT}\right)\gg 1$ より.

$$f(E)\sim\exp\left(-\frac{E-E_F}{kT}\right)$$

- $E<E_v$ のとき.　$\exp\left(\frac{E-E_F}{kT}\right)\ll 1$ より.

$$f(E)=\left\{1+\exp\left(\frac{E-E_F}{kT}\right)\right\}^{-1}\sim 1-\exp\left(\frac{E-E_F}{kT}\right)$$

キャリア密度の計算 calculation of carrier density.



伝導電子密度 conduction electron density

$$\rightarrow \exp\left(-\frac{E-E_c+(E_c-E_F)}{kT}\right)$$

$$n=\int_{E_c}^{\infty} D(E)f(E)\,dE=\frac{2N_c}{\sqrt{\pi(kT)^3}}\int\sqrt{E-E_c}\,\exp\left(-\frac{E-E_F}{kT}\right)dE$$

$$=\frac{2N_c}{\sqrt{\pi(kT)^3}}\exp\left(-\frac{E_c-E_F}{kT}\right)\int\sqrt{E-E_c}\,\exp\left(-\frac{E-E_c}{kT}\right)dE$$

$$\left(\frac{E-E_c}{kT}\equiv x,\quad \frac{dE}{kT}=dx,\quad \begin{array}{c|c} E & E_c\to\infty \\ \hline x & 0\to\infty \end{array}\right)$$

$$\rightarrow \frac{\sqrt{\pi}}{2}$$

$$=\frac{2N_c}{\sqrt{\pi(kT)^3}}\exp\left(-\frac{E_c-E_F}{kT}\right)\sqrt{kT}\cdot kT\int_0^{\infty}\sqrt{x}\cdot e^{-x}\,dx$$

$$=N_c\exp\left(-\frac{E_c-E_F}{kT}\right)$$

$T\nearrow$ で指数関数的に $n\nearrow$

increase exponentially

Exercise 1.

Prove $p = N_V \exp\left(-\frac{E_F - E_V}{kT}\right)$.

$p = N_V \exp\left(-\frac{E_F - E_V}{kT}\right)$ となることを示せ。