EM-HW2

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Problem

First, rewrite the Fourier series as general form

$$f(x) = \frac{1}{2}a_0 + \sum_{k=1}^{n} a_k coskx + b_k sinkx, \quad k = [1, 5]$$

Then we calculate a_0, a_k, b_k respectively.

$$\begin{aligned} a_0 &= \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)dx \\ &= \frac{1}{\pi}\int_{-\pi}^{\pi}(x^2 + x)dx \\ &= \frac{1}{\pi}[\frac{1}{3}x^3 + \frac{1}{2}x^2]_{-\pi}^{\pi} \\ &= \frac{1}{\pi}[(\frac{1}{3}\pi^3 + \frac{1}{2}\pi^2) - (-\frac{1}{3}\pi^3 + \frac{1}{2}\pi^2)] \\ &= \frac{2}{3}\pi^2 \end{aligned}$$

Next, calculate a_k

$$
\begin{aligned}
a_k &= \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)coskxdx \\
&= \frac{1}{\pi}\int_{-\pi}^{\pi}(x^2+x)coskxdx \\
&= \frac{1}{\pi}(\int_{-\pi}^{\pi} x^2coskxdx + \int_{-\pi}^{\pi} xcoskxdx) \\
&= \frac{1}{\pi}\{[x^2\frac{sinkx}{k}]_{-\pi}^{\pi} - \int_{-\pi}^{\pi} 2x\frac{sinkx}{k}dx + [x\frac{sinkx}{k}]_{-\pi}^{\pi} - \\
&\int_{-\pi}^{\pi}\frac{sinkx}{k}dx\} \\
&= -\frac{1}{\pi k}\{2\int_{-\pi}^{\pi} xsinkxdx + \int_{-\pi}^{\pi} sinkxdx\} \\
&= -\frac{1}{\pi k}\{2([x\frac{-coskx}{k}]_{-\pi}^{\pi} - \int_{-\pi}^{\pi}\frac{-coskx}{k}dx) + [\frac{-coskx}{k}]_{-\pi}^{\pi}\} \\
&= \frac{1}{\pi k^2}\{2[xcoskx]_{-\pi}^{\pi} + [coskx]_{-\pi}^{\pi}\} \\
&= \frac{1}{\pi k^2}\{4\pi cosk\pi + 2cosk\pi\} \\
&= (\frac{4}{k^2} + \frac{2}{k^2\pi})(-1)^k
\end{aligned}
$$

Then calculate b_k

$$
\begin{aligned}
b_k &= \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)sinkxdx \\
&= \frac{1}{\pi}\int_{-\pi}^{\pi}(x^2+x)sinkxdx \\
&= \frac{1}{\pi}(\int_{-\pi}^{\pi} x^2sinkxdx + \int_{-\pi}^{\pi} xsinkxdx) \\
&= \frac{1}{\pi}\int_{-\pi}^{\pi} xsinkxdx \\
&= -\frac{1}{k\pi}\{[xcoskx]_{-\pi}^{\pi} - \int_{-\pi}^{\pi} coskxdx\} \\
&= -\frac{2}{k\pi}\{\pi cosk\pi - \frac{sink\pi}{k}\} \\
&= -\frac{2}{k}(-1)^k
\end{aligned}
$$

Finally, we get the Fourier form of $f(x) = x^2 + x$

$$f(x) = \frac{2}{3}\pi^2 + \sum_{k=1}^{n} \frac{2}{k}(-1)^k[(\frac{2}{k} + \frac{1}{k\pi})coskx - sinkx]$$

Then we can sum up to $cos5x$ and $sin5x$

$$f(x) = \frac{2}{3}\pi^2 - (4 + \frac{2}{\pi})cosx + 2sinx + (1 + \frac{1}{2\pi})cos2x - sin2x - (\frac{4}{9} + \frac{2}{9\pi})cos3x + \frac{2}{3}sin3x + (\frac{1}{4} + \frac{1}{8\pi})cos4x - \frac{1}{2}sin4x - (\frac{4}{25} + \frac{2}{25\pi})cos5x + \frac{2}{5}sin5x$$

Graph