Semiconductor Materials

2024/06/26

材料工学科 Department of Materials Science

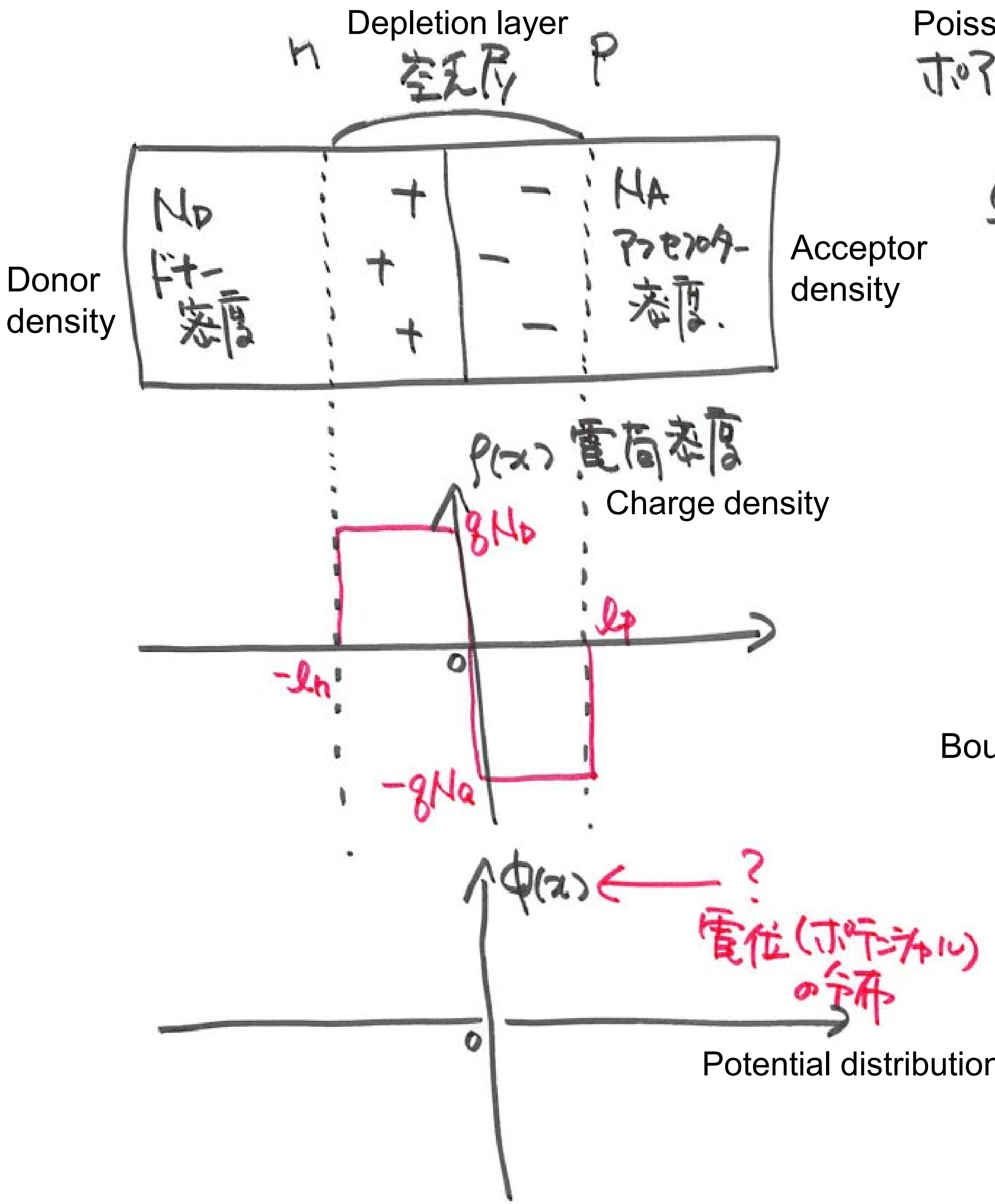
弓野健太郎 Kentaro Kyuno

演習
Exercise

$N_A = 1\times10^{19}/cm^3$, $N_D = 1\times10^{16}/cm^3$ のとき、
空乏層の巾と $x=0$ での電場 $E(0)$ を求めよ。

Derive the depletion layer width and electric field at the junction, E(0), for the pn junction where the acceptor density and donor density in Si are N_A=1 x 10^{19} cm^{-3} and N_D=1 x 10^{16} cm^{-3}, respectively.

$q = 1.6\times10^{-19}$ (C), $\varepsilon_s = 11.7\times8.854\times10^{-12}$ (F/m)



Poisson's equation
ポアソン方程式

$$\frac{d^2\phi(x)}{dx^2} = -\frac{\rho(x)}{\varepsilon_s} \leftarrow \text{Siの誘電率}$$

Permittivity of Si

$$11.7 \times 8.854 \times 10^{-12}\ \mathrm{F/m}$$

Siの比誘電率
relative permittivity of Si

真空の誘電率
Permittivity of vacuum

$\rho(x) \rightarrow \phi(x)$を求めるための方程式
Differential equation

境界条件
Boundary condition

$$E(-l_n) = E(l_p) = 0$$

$$\phi(0) = 0$$

中性条件
Neutral condition

$$N_D l_n = N_A l_p$$

(i) $0 \leq x \leq l_p$, $\rho(x) = -qN_A$

$$\frac{d^2\phi(x)}{dx^2} = \frac{qN_A}{\varepsilon_s}$$

$E(l_p)=0$

$$-E(x) = \frac{d\phi(x)}{dx} = \frac{qN_A}{\varepsilon_s}x + A = \frac{qN_A}{\varepsilon_s}(x - l_p)$$

Electric field
電場

$$\phi(x) = \frac{qN_A}{\varepsilon_s}\frac{(x-l_p)^2}{2} + B = \frac{qN_A}{2\varepsilon_s}\left\{(x-l_p)^2 - l_p^2\right\}$$

$\phi(0)=0$

(ii) $-l_n \leq x \leq 0$, $\rho(x) = qN_D$

$$\phi(x) = -\frac{qN_D}{2\varepsilon_s}\left\{(x+l_n)^2 - l_n^2\right\}$$

$-l_n \le x \le 0$ の区I. $\rho(x) = qN_D$

$$\frac{d^2\phi(x)}{dx^2} = -\frac{qN_D}{\varepsilon_s}$$

$\downarrow$

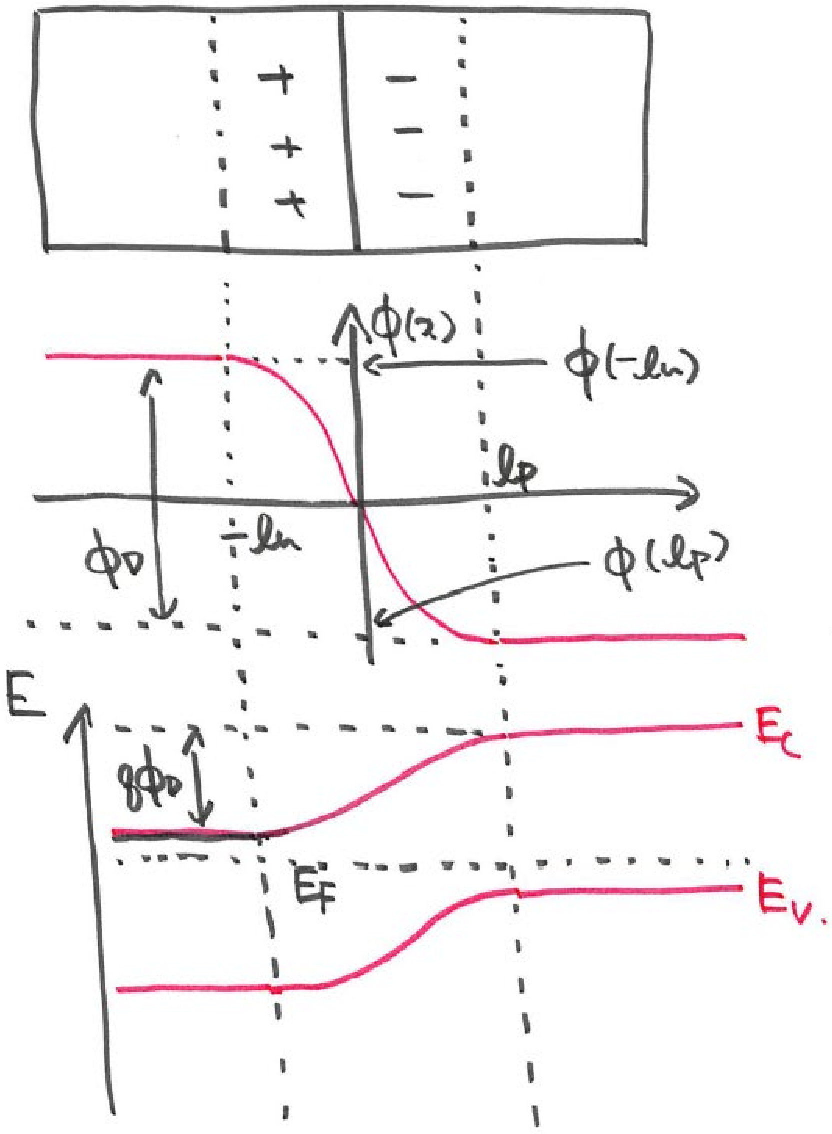
$E(-l_n) = 0.$

$$-E(x) = \frac{d\phi(x)}{dx} = -\frac{qN_D}{\varepsilon_s}x + A = -\frac{qN_D}{\varepsilon_s}(x + l_n)$$

$\downarrow$

$$\phi(x) = -\frac{qN_D}{\varepsilon_s}\frac{(x+l_n)^2}{2} + B = -\frac{qN_D}{2\varepsilon_s}\left\{(x+l_n)^2 - l_n^2\right\}.$$

$\phi(0) = 0$



Built-in potential

内蔵電位

$$\phi_D = \phi(-l_n) - \phi(l_p)$$

$$= \frac{qN_D}{2\varepsilon_s} l_n^2 + \frac{qN_A}{2\varepsilon_s} l_p^2$$

$$\phi_D = \frac{q}{2\varepsilon_s}\left(N_D l_n^2 + N_A l_p^2\right)$$

$N_D l_n$.

$$= \frac{q}{2\varepsilon_s}\left(N_D l_n^2 + (N_A l_p) l_p\right).$$

$$= \frac{q}{2\varepsilon_s} N_D l_n \underbrace{(l_n + l_p)}_{=\, W\ (\text{空乏層の巾})}.$$

$$\begin{cases} l_n + l_p = W. \\ N_D l_n = N_A l_p. \end{cases}$$

$\downarrow$

$$l_n = \frac{N_A}{N_A + N_D} W$$

$\therefore$

$$W = l_n + l_p$$

$$= \sqrt{\frac{2\varepsilon_s}{q} \cdot \frac{N_A + N_D}{N_A N_D} \phi_D}$$

Solution

$$\underset{(l_n+l_p)}{W} = \sqrt{\frac{2\times 11.7\times 8.854\times 10^{-12}}{1.6\times 10^{-19}} \times \frac{10^{19}+10^{16}}{10^{19}\times 10^{16}} \times \frac{1}{10^{6}} \times \underbrace{0.86764}_{\phi_D(V)}}$$

$$= 3.35\times 10^{-7}\,m = 0.335\,\mu m \fallingdotseq l_n.$$

$$\overset{1\times 10^{16}}{N_D}\, l_n = \overset{1\times 10^{19}}{N_A}\, l_p.$$

$$\therefore l_n : l_p = 1000 : 1$$

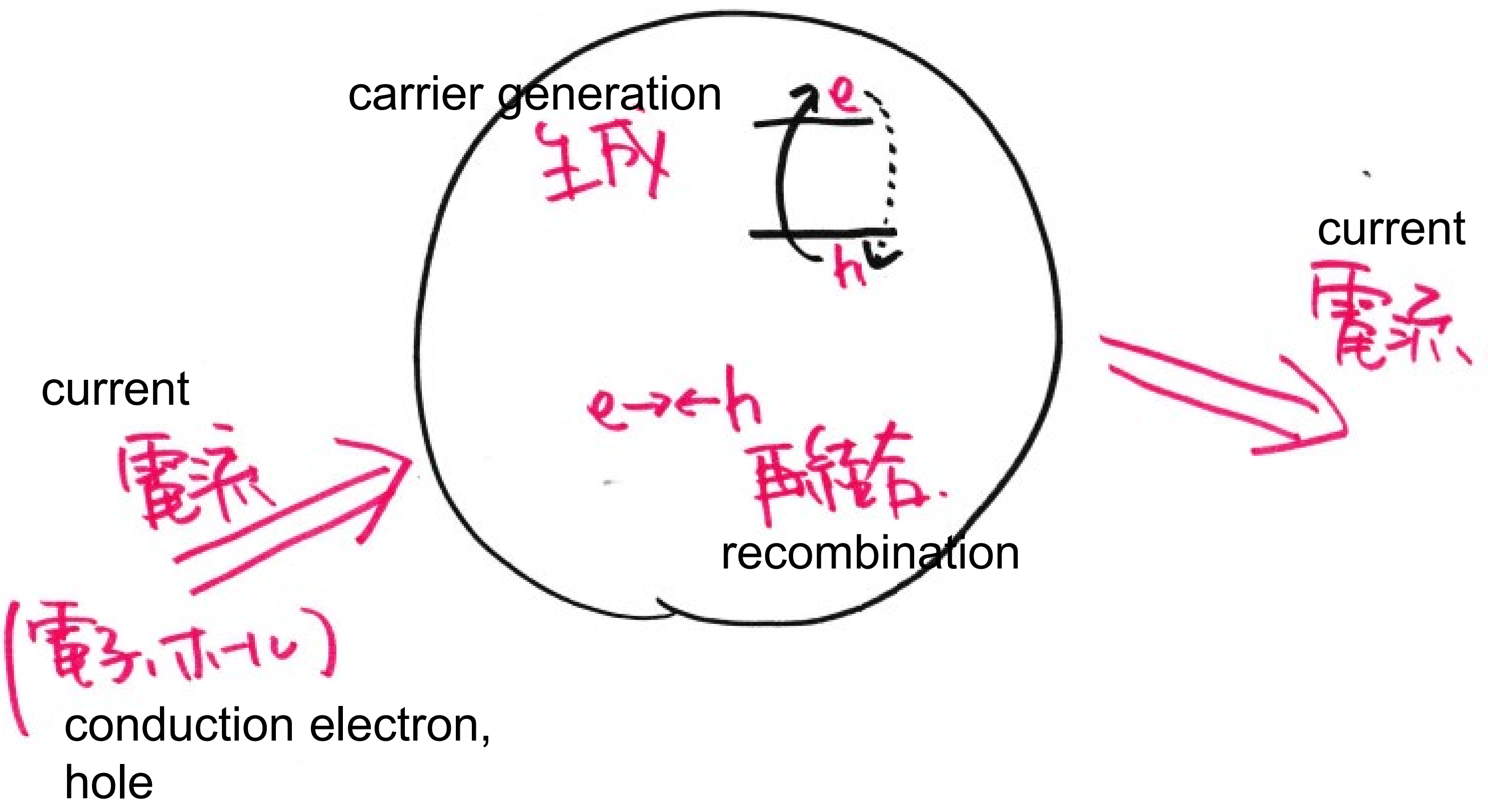
$$E(0) = \frac{8 N_D l_n}{\varepsilon_s}\left(= \frac{8 N_A l_p}{\varepsilon_s}\right)$$

$$= \frac{1.6\times 10^{-19}\times 1\times 10^{16}\times 10^{6}}{11.7\times 8.854\times 10^{-12}} \times 3.35\times 10^{-7}$$

$$= 5.17\times 10^{6}\,V/m = 5.17\times 10^{4}\,V/cm$$

Continuity equation

（電流）連続の式



Rate of carrier density change by generation and recombination

生成・再結合によるキャリア密度の変化速度

Conduction electron density

電子密度 $n_0 \longrightarrow n_0+\Delta n$

ホール密度 $p_0 \longrightarrow p_0+\Delta p$

Hole density

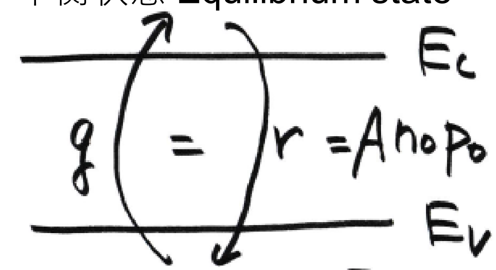
Assume neutral condition

(中性を仮定 $\Delta n=\Delta p$)

平衡状態 非平衡状態

Equilibrium state Non-equilibrium state

平衡状態 Equilibrium state



キャリア密度の変化率

Rate of carrier density change

$$= g - r = An_0p_0 - A(n_0+\Delta n)(p_0+\Delta p)$$

$$= -A\Delta n(n_0+p_0) = -A\Delta p(n_0+p_0)$$

$$= -\frac{\Delta n}{\tau} = -\frac{\Delta p}{\tau} \quad \left(\tau \equiv \frac{1}{A(n_0+p_0)}\right)$$

キャリアの寿命

Carrier lifetime

Generation rate g = 生成速度

Recombination rate r = 再結合速度

$\Delta n, \Delta p > 0$ のとき $g - r < 0$

$\Delta n, \Delta p < 0$ のとき $g - r > 0$

$e \rightarrow$

$\nwarrow h$

Continuity equation for holes

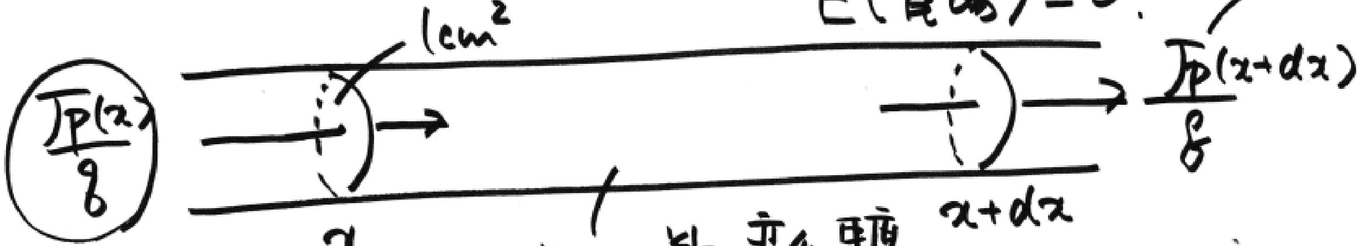
連続の式（ホール）

Electric field

E（電場）$= 0$.

Current density of holes

ホールの電流密度.



ホールの粒子の流れ.

Flow of holes

ホールの数の変化速度.

Change rate of holes

$$\frac{\partial p(x,t)}{\partial t} \times dx$$

$$-\frac{P-P_0}{\tau}$$

$$\frac{\partial p}{\partial t} dx = \frac{J_p(x)}{q} - \frac{J_p(x+dx)}{q} + (g-r)dx$$

$$\frac{\partial p}{\partial t} = -\frac{1}{q}\frac{J_p(x+dx) - J_p(x)}{dx} - \frac{P-P_0}{\tau}$$

$$= -\frac{1}{q}\frac{\partial J_p(x)}{\partial x} - \frac{P-P_0}{\tau}$$

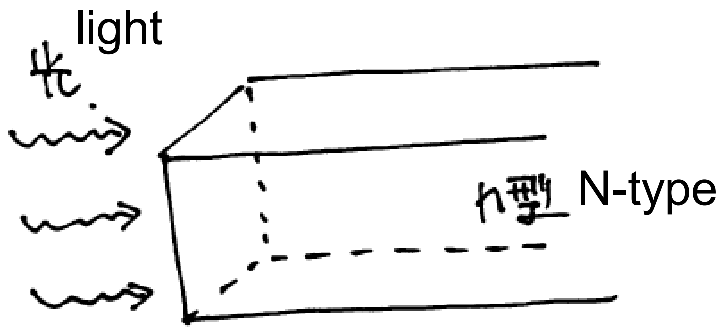
$$J_p(x) = -q D_p \frac{\partial p}{\partial x}$$

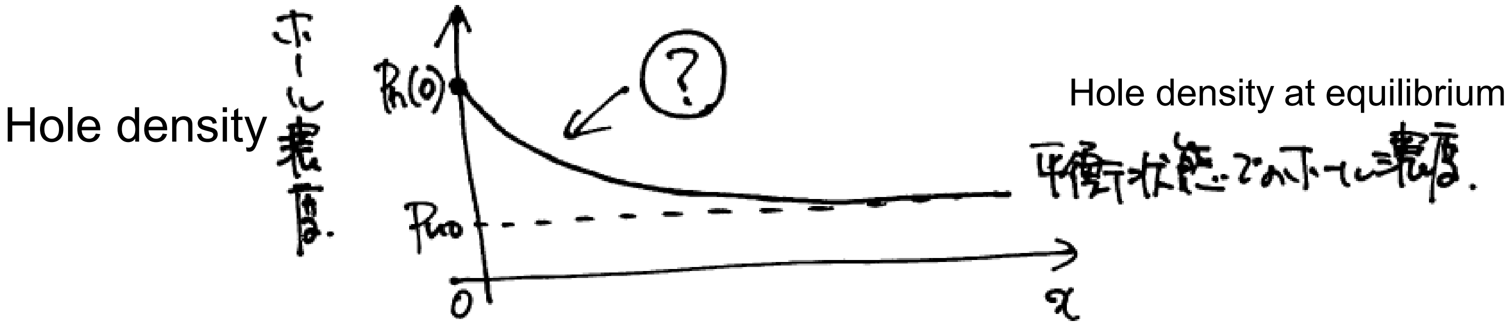
$$= D_p \frac{\partial^2 p}{\partial x^2} - \frac{P-P_0}{\tau}$$

（電子）$\frac{\partial n}{\partial t} = D_n \frac{\partial^2 n}{\partial x^2} - \frac{n-n_0}{\tau}$

Continuity equation for conduction electrons

Example
例題





表面におけるホール濃度を$p_n(0)$とする
(光は半導体内に侵入できないとする)

定常状態における連続の方程式

$$\frac{\partial p_n(x)}{\partial t} = 0 = D_p \frac{\partial^2 p_n}{\partial x^2} - \frac{p_n(x) - p_{n0}}{\tau_p}$$

を解き、$p_n(x)$を求めよ。

Derive $p_n(x)$ by solving the continuity equation.
Assume that the hole density at the surface is $p_n(0)$.

Assume also the steady state condition.

(solution)

$$\frac{d^2(p_n(x)-p_{no})}{dx^2} = \frac{p_n(x)-p_{no}}{D_p\tau_p}$$

$$p_n(x)-p_{no} = A\exp\left(-\frac{x}{\sqrt{D_p\tau_p}}\right) + B\exp\left(\frac{x}{\sqrt{D_p\tau_p}}\right) \times$$

at $x=0$ で $p_n(x)=p_n(0)$ なので $A = p_n(0) - p_{no}$

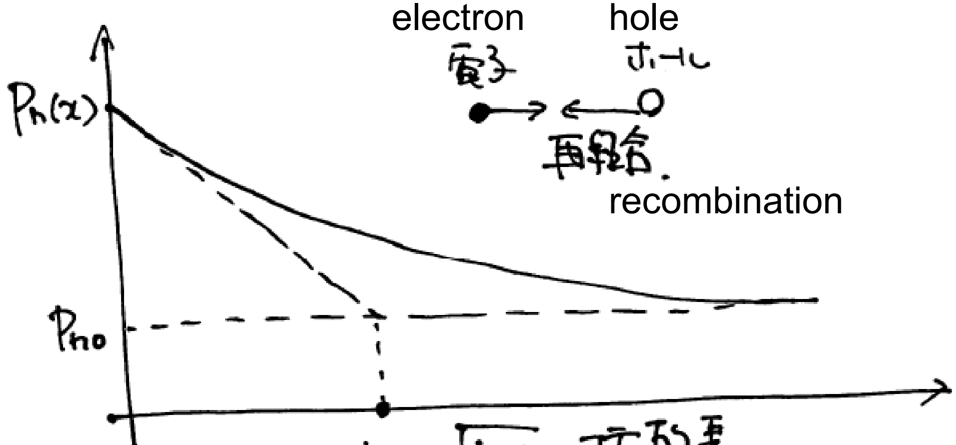
$$\therefore\ p_n(x) = p_{no} + (p_n(0)-p_{no})\exp\left(-\frac{x}{\sqrt{D_p\tau_p}}\right)$$

electron 電子

hole ホール

再結合.

recombination

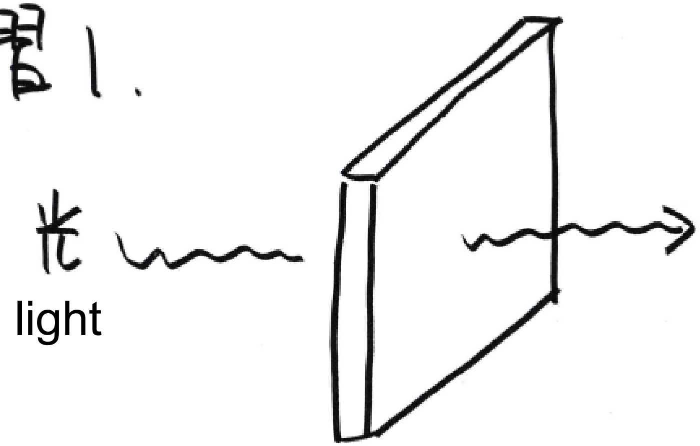


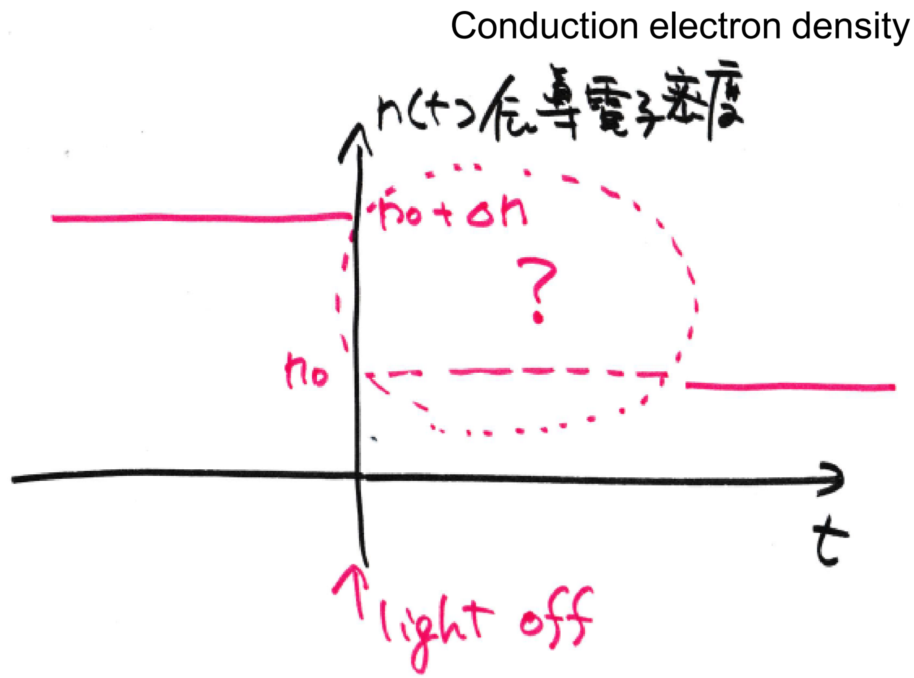
$L_p = \sqrt{D_p\tau_p}$ 拡散長.

Diffusion length

Exercise 1

演習1.





$t>0$での$n(t)$を求めよ。

($n(x,t)$のx依存性はないものとする)

Derive n(t) for t > 0.

The x dependence of n can be neglected.

$$\frac{\partial n}{\partial t} = D_n \cancel{\frac{\partial^2 n}{\partial x^2}} - \frac{n - n_0}{\tau}$$

$$\frac{dn(t)}{dt} = -\frac{n(t) - n_0}{\tau}$$