Introduction to Bayesian Data Analysis
Homework 2

This homework assignment is due by midnight of 3/31. Please submit your homework through eeclass. 50% penalty will be applied if it is submitted on 4/1. No submission will be accepted after 4/1.

1. (50 points) Suppose that if θ = 1, then y has a normal distribution with mean 1 and standard deviation σ, and if θ = 2, then y has a normal distribution with mean 2 and standard deviation σ. Also, suppose P(θ = 1) = 0.5 and P(θ = 2) = 0.5.
   A. For σ = 2, write the formula for the marginal probability density for y and sketch it.
   B. What is P(θ = 1|y = 1), again supposing σ = 2?
   C. Describe how the posterior density of θ changes in shape as σ is increased and as it is decreased.

2. (50 points) Write PPL programs using PyMC3 and ArviZ to solve the following questions. For all these cases, suppose the data show 63 heads in 97 flips.
   A. Suppose the prior is beta(θ|a=0.01,b=0.01), which is (approximately) the so-called Haldane prior. What is mode and 94% HPD of the posterior distribution? Show the result graphically.
   B. Suppose the prior is beta(θ|a=1,b=1), which is uniform. What is mode and 94% HPD of the posterior distribution? Show the result graphically.
   C. Suppose the prior is beta(θ|a=2,b=4), which is gently biased toward tails. What is mode and 94% HPD of the posterior distribution? Show the result graphically.
   D. Is there much difference in the posterior distributions (modes and HPD's) across the different priors? Briefly explain why.

3. (20 bonus points) Prove that the value of $\hat{\theta}$ that minimizes the quadratic loss function is the mean of the posterior $p(\theta|D)$.