EM-HW1

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Problem 1

Define

$$f(x) = ax + b$$

$$J = \frac{1}{2}\sum_{i=1}^{4}(f(x_i) - y_i)^2 = \frac{1}{2}\sum_{i=1}^{4}(ax_i + b - y_i)^2$$

To find the $f(x)$ with the minimum distance to the points $(x_1, y_1) = (-1, 2), \quad (x_2, y_2) = (0, 1), \quad (x_3, y_3) = (1, -1), \quad (x_4, y_4) = (2, -2)$, we need to solve the following partial derivatives,

$$\frac{\delta J}{\delta a} = 0, \quad \frac{\delta J}{\delta b} = 0$$

$$
\begin{aligned}
\frac{\delta}{\delta a}J &= \frac{\delta}{\delta a}\frac{1}{2}\sum_{i=1}^{4}(ax_i+b-y_i)^2 \\
&= \frac{1}{2}\frac{\delta}{\delta a}\sum_{i=1}^{4}(ax_i+b-y_i)^2 \\
&= \frac{1}{2}\sum_{i=1}^{4}2(ax_i+b-y_i)\frac{\delta}{\delta a}(ax_i+b-y_i) \\
&= \frac{1}{2}\times 2\sum_{i=1}^{4}(ax_i+b-y_i)x_i \\
&= \sum_{i=1}^{4}ax_i^2+bx_i-x_iy_i \\
&= a\sum_{i=1}^{4}x_i^2+b\sum_{i=1}^{4}x_i-\sum_{i=1}^{4}x_iy_i
\end{aligned}
$$

$$
\begin{aligned}
\frac{\delta}{\delta b}J &= \frac{\delta}{\delta b}\frac{1}{2}\sum_{i=1}^{4}(ax_i+b-y_i)^2 \\
&= \frac{1}{2}\frac{\delta}{\delta b}\sum_{i=1}^{4}(ax_i+b-y_i)^2 \\
&= \frac{1}{2}\sum_{i=1}^{4}2(ax_i+b-y_i)\frac{\delta}{\delta b}(ax_i+b-y_i) \\
&= \frac{1}{2}\times 2\sum_{i=1}^{4}(ax_i+b-y_i)\times 1 \\
&= \sum_{i=1}^{4}ax_i+b-y_i \\
&= a\sum_{i=1}^{4}x_i+b\sum_{i=1}^{4}1-\sum_{i=1}^{4}y_i
\end{aligned}
$$

and we have

$$
\sum_{i=1}^{4}x_i^2=6,\quad \sum_{i=1}^{4}x_i=2,\quad \sum_{i=1}^{4}x_iy_i=-7,\quad \sum_{i=1}^{4}y_i=0,\quad \sum_{i=1}^{4}1=4
$$

then

$$6a + 2b = -7$$
$$2a + 4b = 0$$
$$\Rightarrow a = -\frac{7}{5}, \quad b = \frac{7}{10}$$

So, $f(x) = -\frac{7}{5}x + \frac{7}{10}$

Graph

https://www.desmos.com/calculator/9thwlurlid?embed

Problem 2

Define

$$f(x) = ax^2 + bx + c$$

$$J = \frac{1}{2}\sum_{i=1}^{4}(f(x_i) - y_i)^2 = \frac{1}{2}\sum_{i=1}^{4}(ax_i^2 + bx_i + c - y_i)^2$$

To find the $f(x)$ with the minimum distance to the points $(x_1, y_1) = (-1, 0), \quad (x_2, y_2) = (0, -1), \quad (x_3, y_3) = (1, 0), \quad (x_4, y_4) = (2, 4)$, we need to solve the following partial derivatives,

$$\frac{\delta J}{\delta a} = 0, \quad \frac{\delta J}{\delta b} = 0, \quad \frac{\delta J}{\delta c} = 0$$

$$\frac{\delta}{\delta a}J = \frac{\delta}{\delta a}\frac{1}{2}\sum_{i=1}^{4}(ax_i^2 + bx_i + c - y_i)^2$$

$$= \frac{1}{2}\frac{\delta}{\delta a}\sum_{i=1}^{4}(ax_i^2 + bx_i + c - y_i)^2$$

$$= \frac{1}{2}\sum_{i=1}^{4}2(ax_i^2 + bx_i + c - y_i)\frac{\delta}{\delta a}(ax_i^2 + bx_i + c - y_i)$$

$$= \frac{1}{2} \times 2\sum_{i=1}^{4}(ax_i^2 + bx_i + c - y_i)x_i^2$$

$$= \sum_{i=1}^{4} ax_i^4 + bx_i^3 + cx_i^2 - x_i^2 y_i$$

$$= a\sum_{i=1}^{4}x_i^4 + b\sum_{i=1}^{4}x_i^3 + c\sum_{i=1}^{4}x_i^2 - \sum_{i=1}^{4}x_i^2 y_i$$

$$\frac{\delta}{\delta b}J = \frac{\delta}{\delta b}\frac{1}{2}\sum_{i=1}^{4}(ax_i^2 + bx_i + c - y_i)^2$$

$$= \frac{1}{2}\frac{\delta}{\delta b}\sum_{i=1}^{4}(ax_i^2 + bx_i + c - y_i)^2$$

$$= \frac{1}{2}\sum_{i=1}^{4}2(ax_i^2 + bx_i + c - y_i)\frac{\delta}{\delta b}(ax_i^2 + bx_i + c - y_i)$$

$$= \frac{1}{2} \times 2\sum_{i=1}^{4}(ax_i^2 + bx_i + c - y_i)x_i$$

$$= \sum_{i=1}^{4} ax_i^3 + bx_i^2 + cx_i - x_i y_i$$

$$= a\sum_{i=1}^{4}x_i^3 + b\sum_{i=1}^{4}x_i^2 + c\sum_{i=1}^{4}x_i - \sum_{i=1}^{4}x_i y_i$$

$$
\begin{aligned}
\frac{\delta}{\delta c}J &= \frac{\delta}{\delta c}\frac{1}{2}\sum_{i=1}^{4}(ax_i^2+bx_i+c-y_i)^2 \\
&= \frac{1}{2}\frac{\delta}{\delta c}\sum_{i=1}^{4}(ax_i^2+bx_i+c-y_i)^2 \\
&= \frac{1}{2}\sum_{i=1}^{4}2(ax_i^2+bx_i+c-y_i)\frac{\delta}{\delta c}(ax_i^2+bx_i+c-y_i) \\
&= \frac{1}{2}\times 2\sum_{i=1}^{4}(ax_i^2+bx_i+c-y_i)\times 1 \\
&= \sum_{i=1}^{4}ax_i^2+bx_i+c-y_i \\
&= a\sum_{i=1}^{4}x_i^2+b\sum_{i=1}^{4}x_i+c\sum_{i=1}^{4}1-\sum_{i=1}^{4}y_i
\end{aligned}
$$

and we have

$$
\sum_{i=1}^{4}x_i^4=18,\quad \sum_{i=1}^{4}x_i^3=8,\quad \sum_{i=1}^{4}x_i^2=6,\quad \sum_{i=1}^{4}x_i=2,
$$
$$
\sum_{i=1}^{4}x_i^2y_i=16,\quad \sum_{i=1}^{4}x_iy_i=8,\quad \sum_{i=1}^{4}y_i=3,\quad \sum_{i=1}^{4}1=4
$$

then

$$
\begin{gathered}
18a+8b+6c=16 \\
8a+6b+2c=8 \\
6a+2b+4c=3 \\
\Rightarrow a=\frac{5}{4},\quad b=\frac{1}{20},\quad c=-\frac{23}{20}
\end{gathered}
$$

So, $f(x)=-\frac{5}{4}x^2+\frac{1}{20}x-\frac{23}{20}$

Graph

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https://www.desmos.com/calculator/t9irv2w81d
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