Using Numerical Analysis to Simulate Fresnel Diffraction for Square and Rectangular Apertures

Samuel Hopkins
(Dated: February 2021)

This report details the use of the Composite Simpson's Rule to compute the Fresnel equation provided, to demonstrate the various diffraction patterns for both near-field(Fresnel) and far-field(Fraunhoffer) effects.

INTRODUCTION

Fresnel and Fraunhoffer Diffraction

The diffraction effects to be simulated in this experiment involve the Fresnel and Fraunhoffer diffraction equations, where the Fresnel diffraction equation is applied to the propagation of waves in the near field. The Fraunhoffer diffraction equation is applied to the far field region. The general Fresnel diffraction equation for a square or rectangular equation is given by;

$$E(x,y,z) = \frac{kE_0}{2\pi z}\int_{x_1'}^{x_2'} exp\left\{\frac{ik}{2z}(x-x')^2\right\}dx' \int_{y_1'}^{y_2'} exp\left\{\frac{ik}{2z}(y-y')^2\right\}dy' \tag{1}$$

Where z is the distance from the aperture to the screen and k is defined by $k = \frac{2\pi}{\lambda}$.The general formula for Fresnel diffraction is impossible to solve by analytical methods, but can be solved numerically. The Fresnel number, F, can be defined as;

$$F = \frac{a^2}{L\lambda}, \tag{2}$$

where a is the radius of the aperture, L is the distance to the screen from the aperture, and λ is the wavelength of light. In the case of square and rectangular apertures, this is only an approximation as we can't define a radius, only the width of the aperture, $D = 2a$. For near field effects to take place, $F < 1$, and for far field effects, $F > 1$.

Simpson's Rule

To numerically evaluate the Fresnel equation, Simpson's rule is used to approximate its value for given coordinates and parameters. Simpson's composite Rule is defined as;

$$\int_a^b f(x)dx \approx \frac{h}{3}\left[f(x_0) + 2\sum_{j=1}^{n/2-1} f(x_{2j}) + 4\sum_{j=1}^{n/2} f(x_{2j-1}) + f(x_n)\right] \tag{3}$$

Here, n is an even number and $x_j = a + jh$ for $j = 0, 1, ..., n-1, n$. h is defined as; $h = (b-a)/n$. This rule can be implemented with relative ease into mathematical programming languages such as python, MATLAB etc. The use of the composite Simpson's rule over the Simpsons rule is due to the fact that for functions with oscillatory behaviour or functions that lack derivatives at certain points, Simpson's rule will not give an accurate approximation. By dividing the interval up and summing the results for each integral, a better approximation can be made.

METHOD

One Dimensional Evaluation

For part (a) of the task, the 1 dimensional integral(Equation 4) had to be evaluated and then a plot of Intensity against screen coordinate was to be made. The 1 dimensional integral is defined as;

$$E(x,z) = \frac{kE_0}{2\pi z}\int_{x_1'}^{x_2'} exp\left\{\frac{ik}{2z}(x-x')^2\right\}dx' \tag{4}$$

This was evaluated by defining a function that implemented the Composite Simpson's rule in python and then squared the result to give a value for the intensity for a given screen coordinate. The plot was then generated by repeating the function evaluation in a 'for loop', and plotting the values against the screen coordinate. Part (b) of the task involved changing the values of the aperture width and distance to the screen, to investigate the near field effects.

Two Dimension Evaluation

In part (c), equation (1) had to be evaluated and this was done by modifying the equation in part (a), duplicating this for the y-component and then simply multiplying these results together. The functions then had to be repeated in a 'for loop' to give a value for every screen coordinate and was then plotted on a colour map using the matplotlib.plyplot.imshow function.

ONE DIMENSIONAL RESULTS

Far Field and Near Field

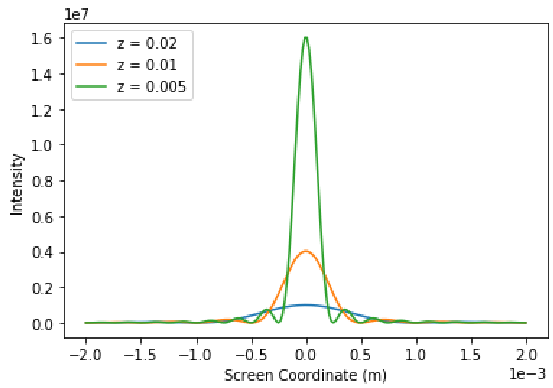
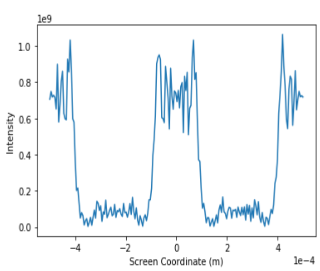
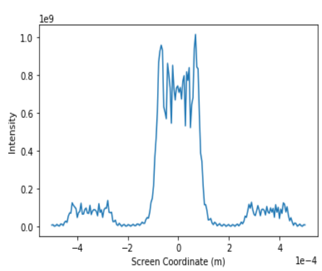


FIG. 1: Plot of Intensity against Screen Coordinate (m), for 3 different values of z in the far field Region. $\lambda = 1\mu m$, $x_2' - x_1' = 20\mu m$. Here the Fresnel numbers are, $F = 0.01$ for $z = 0.01$, $F = 5 \times 10^{-3}$, for $z = 0.02$, and $F = 0.02$ for $z = 0.005$.

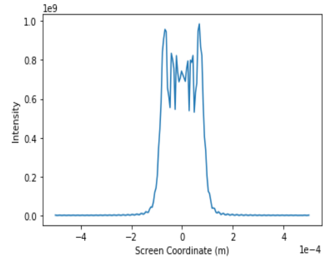
It can be seen, that to observe near field effects by just changing z, the screen distance must be much less than 0.01. Near field effects are best shown by varying both z and the aperture width.



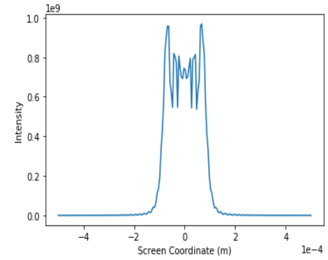
(a) $N = 70$



(b) $N = 100$



(c) $N = 200$

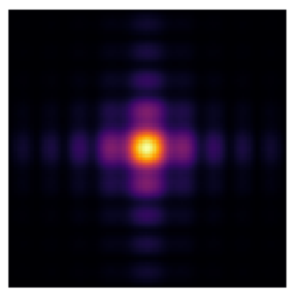


(d) $N = 500$

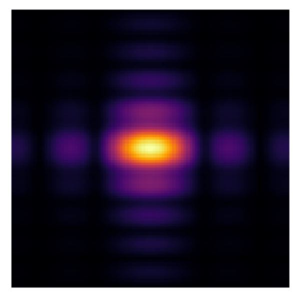
FIG. 3: Intensity plots for 4 different values of N, the number of intervals for integration.Since $z = 1.4mm$ and the aperture width, $200\mu m$, all 4 show nearfield effects. The Fresnel number here is $F = 7.143$

DIFFRACTION PATTERNS

Both square and rectangular aperture shapes were investigated for different screen distances, shown below.



(a) Square Aperture



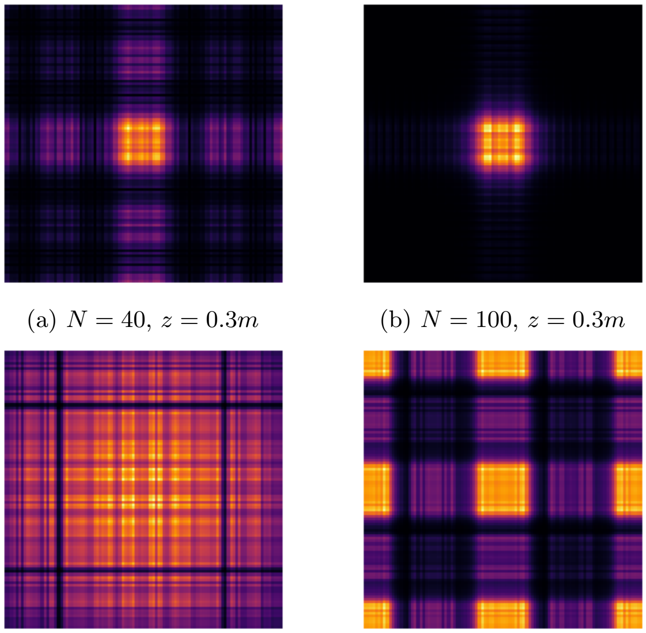
(b) Rectangular Aperture

FIG. 4: Plots of both square and rectangular aperture shapes for the diffraction patterns simulated using the Composite Simpson's rule. Here $z = 2m$, $D_s = 2mm$, $D_{rx} = 2mm$, $D_{ry} = 1mm$ where D_s is the width of the square aperture, D_{rx} is the width of the rectangular aperture, and D_{ry} is the height of the rectangular aperture.

The effects of changing the aperture width and screen distance are the same as before in the 1 dimensional case, this time however it affects both the x and y to produce symmetric images to show both far field and near field cases.

DISCUSSION

It can be seen in the figures, that the diffraction patterns for both far field and near field regions could be simulated using Simpson's rule. Figure 1 demonstrates the far field diffraction pattern for the one dimensional case, and it shows that for larger z, a larger peak intensity will be recorded. Figure 3 shows the near field effects,



(a) $N = 40$, $z = 0.3m$ (b) $N = 100$, $z = 0.3m$

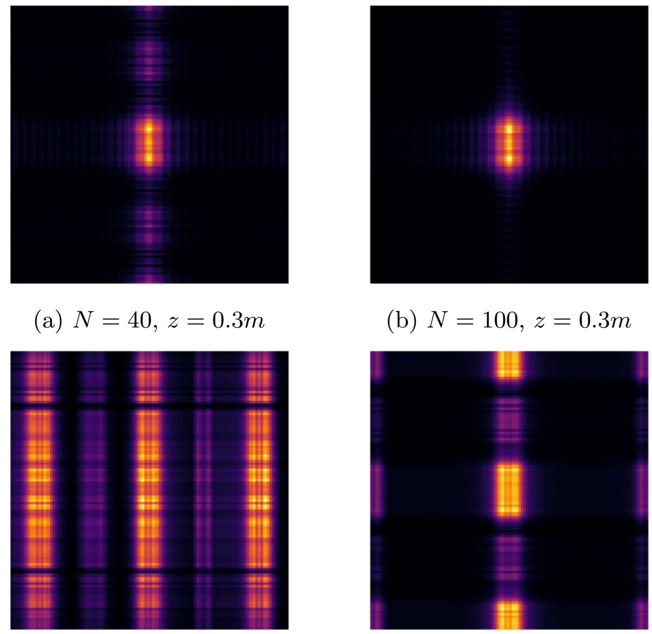
(c) $N = 40$, $z = 0.1m$ (d) $N = 100$, $z = 0.1m$

FIG. 5: Square diffraction patterns in the near field region for various N and z. Intensity is shown through colour, with the brighter regions having higher intensity than the darker regions.

and the effects of varying N, on the appearance of the intensity plots. N generally needs to be larger in order to obtain accurate plots in the near field region as there is a higher variance in the output across the screen coordinate range. Lower N in this region will cause details and values to be misinterpreted, and more 'artifacts' will be present. Higher values of N, are then required for this region to 'smooth' the output, so that a more accurate plot is obtained.

In the two dimensional case, higher values of N are also needed in the near field, as artifacts on the image or much more present and can completely obscure the diffraction pattern shown in figures 5 and 6. It can be seen that for the square pattern, for $N = 40$, numerous artifacts appear on the plot as the near-field region provides more variance for both coordinate ranges.

The method could have been improved by repeating the demonstration for other aperture shapes such as, a circle, triangle etc. The program could also have been improved by adding customizable inputs for values such as N, z, and a or by showing plots for more values of z, N. A 3 dimensional plot of the intensity could also have been possible as shown in [1]. It also would have been possible to trial different methods of integration or methods to demonstrate the diffraction patterns, such as Monte-Carlo integration or the fast Fourier transform method [3]. Overall, the demonstration was a success as



(a) $N = 40$, $z = 0.3m$ (b) $N = 100$, $z = 0.3m$

(c) $N = 40$, $z = 0.1m$ (d) $N = 100$, $z = 0.1m$

FIG. 6: Rectangular diffraction patterns in the near field region for various N and z.

the various diffraction patterns and effects were shown through the use of the Composite Simpson's rule in numerical analysis of the two dimension Fresnel equation.

[1] M. Abedin, M.R. Islam, A.F.M.Y. Haider. Computer simulation of Fresnel diffraction from rectangular apertures and obstacles using the Fresnel integrals approach. Department of Physics, University of Dhaka, Dhaka 1000, Bangladesh (2005).

[2] Dean E. Dauger. Simulation and study of Fresnel diffraction for arbitrary two-dimensional apertures. Computers in Physics 10, 591 (1996)

[3] Heideman, Michael T.; Johnson, Don H.; Burrus, Charles Sidney. Gauss and the history of the fast Fourier transform. (1984)