CS 130 SOFTWARE ENGINEERING

HOARE LOGIC:
WEAKEST PRECONDITION

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Based on Materials from Miryung Kim

REVIEW

- {P} skip {P}
- {P[w:=E]} w:=E {P}
  - wp(w := E, R) ≡ R[w := E]
- {P ∧ B} assert B {P}
  - wp(assert P, R) ≡ P ∧ R

REVIEW

- if {P} S {Q} and {Q} T {R}, then {P} S ; T {R}
  - wp(S;T, R) ≡ wp(S, wp(T, R))
- if {P∧B} S {R} and {P∧¬B} T {R}, then {P} if B then S else T end {R}
  - wp(if B then S else T end, R) = (B ∧ wp(S, R)) ∨ (¬B ∧ wp(T, R))

THINK PAIR SHARE

During peer code reviews, under what basis should programmers approve code changes?

- A failure is reproducible. Presence of a test case
- Good test coverage, good code quality
- Does not break existing tests. => show that you run the regression test suites, and no additional test failures.
- Write a weakest pre-condition.

THINK PAIR SHARE

During peer code reviews, under what basis should programmers approve code changes?

- Pre-condition / post-conditions
- Comments
- **Presence of tests**
- Style check – indentation check
- Variable names reasonable in semantics length
- **Run existing tests**

THINK PAIR SHARE

- We talked about the following sequential and composition WP rule. What is the implication of this rule in practice when doing code review?
  - wp(S;T, R) ≡ wp(S, wp(T, R))

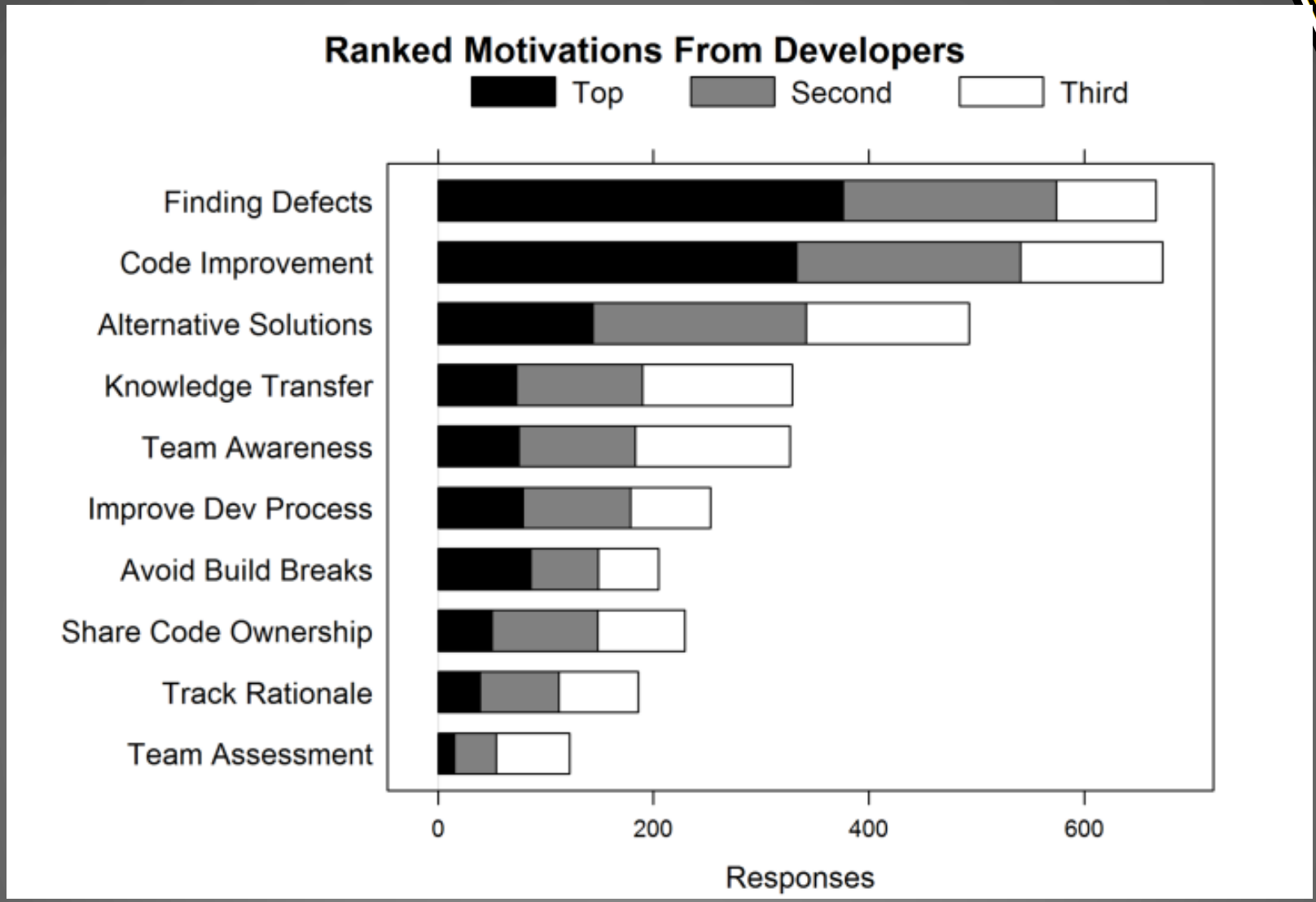
THINK PAIR SHARE

- wp(S;T, R) ≡ wp(S, wp(T, R))
  - If you authored the code, you should write comments or annotations to make the post-condition R explicit.
  - If you are debugging code, print out the failing program state, and apply backward WP reasoning to figure out when the code fails.

MODERN CODE REVIEW PRACTICES

WHY CODE REVIEW DO NOT FIND BUGS. HOW CURRENT CODE REVIEW PRACTICES SLOWS US DOWN.

- *ICSE 2015, Jacek Czerwonka et al. Microsoft*
  - They mined code review tool usage data
  - CodeFlow is open 5 or 6 hours per developer per day.
  - 30 minutes of interaction time per developer per day.
  - **Primary goals:** Find defects, improve maintainability, share knowledge, broadcast progress.



EXPECTATIONS, OUTCOMES, AND CHALLENGES OF MODERN CODE REVIEW, ICSE 2013, BACCHELLI AND BIRD

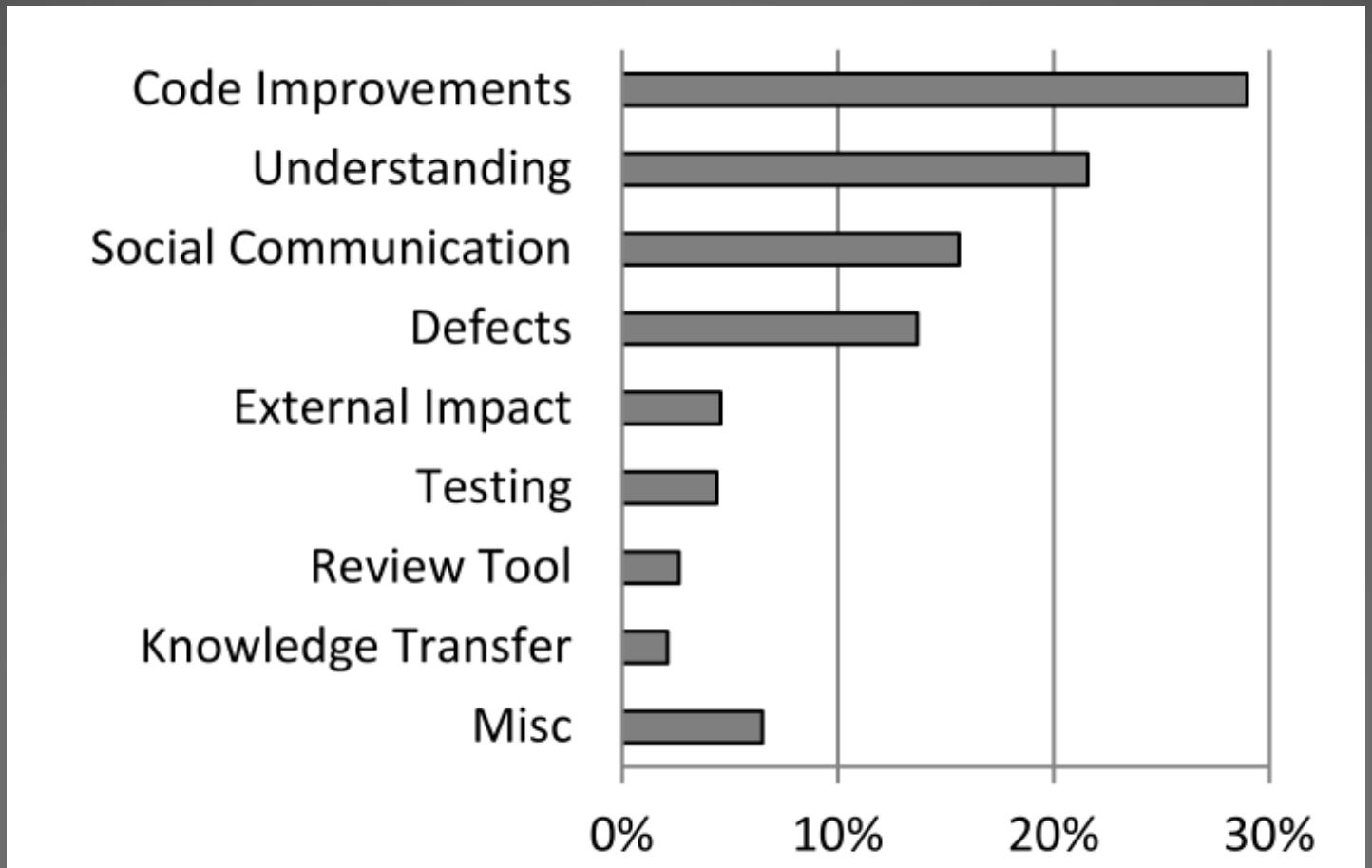


Figure 4. Frequency of comments by card sort category.

EXPECTATIONS, OUTCOMES, AND CHALLENGES OF MODERN CODE REVIEW, ICSE 2013, BACCHELLI AND BIRD

WHY CODE REVIEW DO NOT FIND BUGS. HOW CURRENT CODE REVIEW PRACTICES SLOWS US DOWN.

- Top 3 categories are long term maintenance issues
  - Comments, naming, and styles (22%)
  - Organization of code (16%)
  - Alternative solutions for long term maintenance (9%)
- Only 15% of comments provided by reviewers indicate a possible defect.
- Long term maintainability issues are a much larger portion of comments (50%)
- Only 33% of comments are deemed useful by the author.

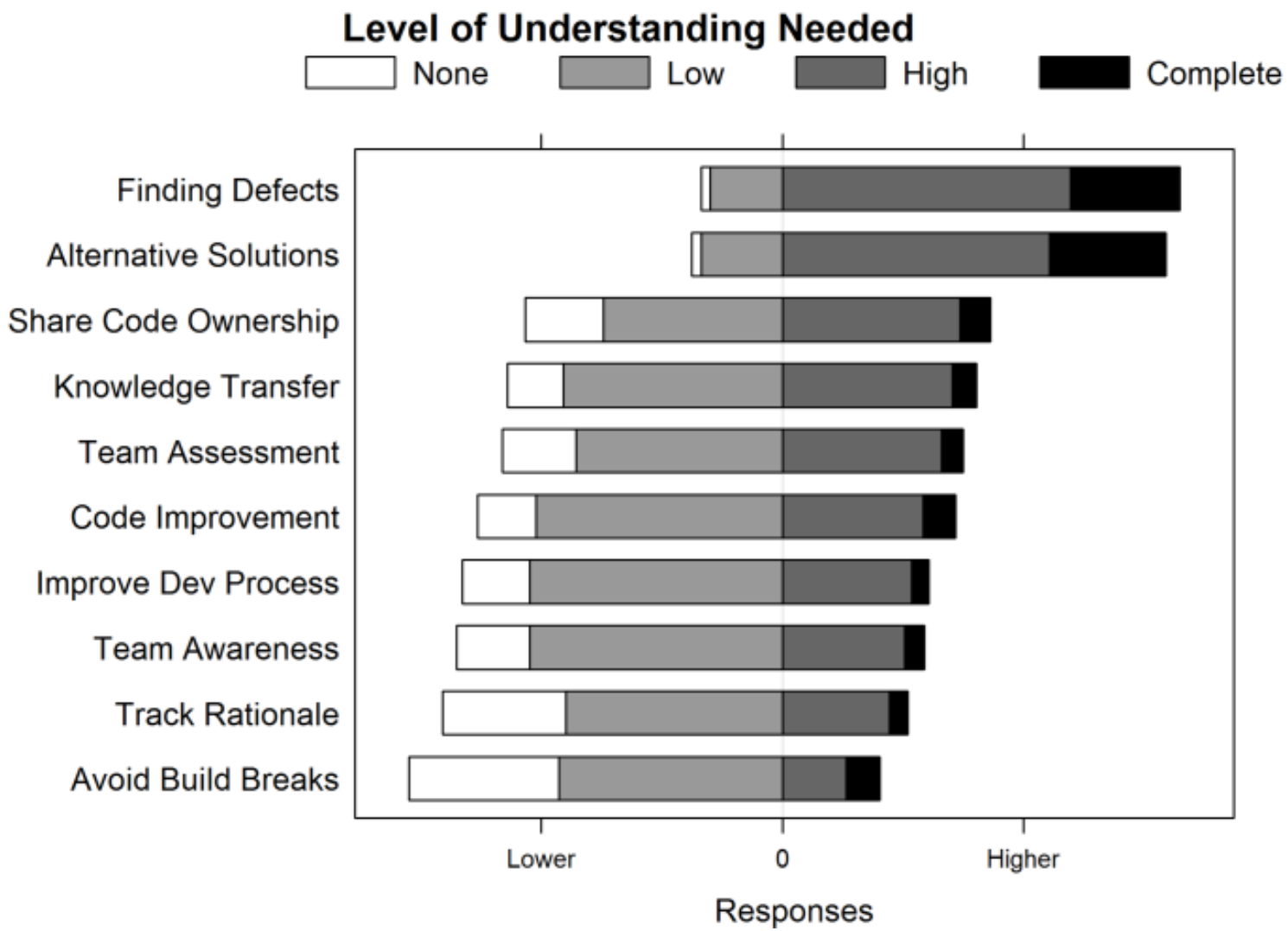


Figure 5. Developers' responses in surveys of the amount of code understanding for code review outcomes.

MODERN CODE REVIEW PRACTICES

- *New reviewers learn fast but need at least 6-12 months to be productive as the rest of the team*
- *People do not actually find bugs – mostly shallow feedback on syntax and style conformance.*
- *We need to have "rigorous criteria" for code reviews to make them effective.*
- Developers need to have "critical eyes" for reviewing code changes, thinking about corner cases.

TAKE AWAY MESSAGE

- Code reviews are time consuming but developers provide shallow style feedback and are not effective at finding bugs.
- Developers need to have "critical eyes" for reviewing code changes, thinking about corner cases.
- I believe this education exercise of "Hoare Logic" is useful to help my students develop as "effective code reviewers for defect finding."

EXERCISE 1.

```
public char[] foo(Object x, int z) {
  if (x != null) {
     n = x.f;
  } else {
     n = z-1;
     z++;
  }
  a = new char[n];
 return a;
}
```

Suppose Alice wrote `foo`. **Which arguments need to be passed to `foo`** so that it returns a non null value without throwing any `NullPointerException` and `ArrayOutOfBoundException`?

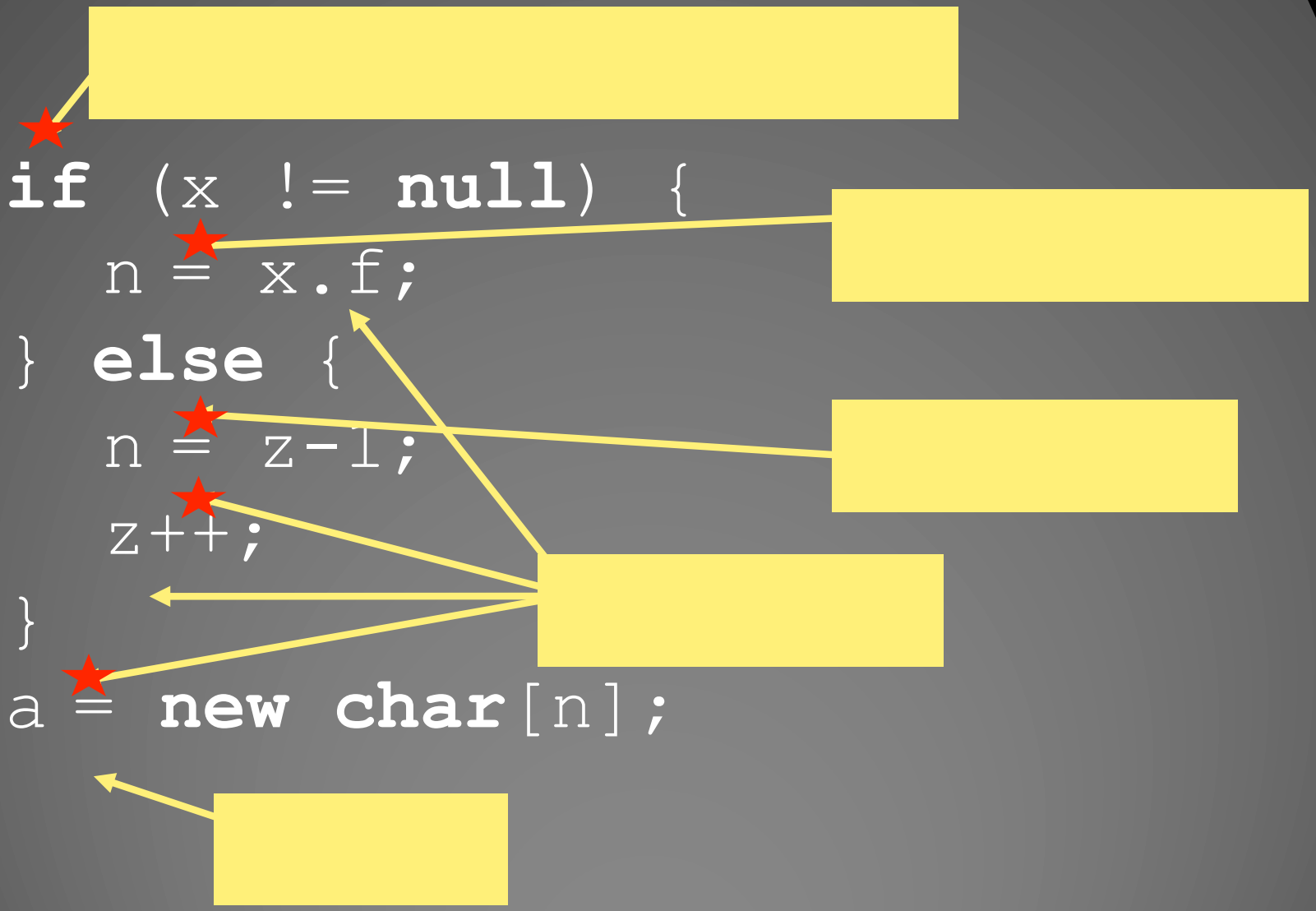
EXERCISE 1.

which pre-condition should hold here?

```
if (x != null) {
   n = x.f;
} else {
   n = z-1;
   z++;
}
a = new char[n];
```

true

EXERCISE 1.



EXERCISE 1.

(x != null AND x.f >= 0) OR
(x == null AND z-1 >= 0)

```
if (x != null) {
   n = x.f;
} else {
   n = z-1;
   z++;
}
a = new char[n];
```

x.f >= 0

z-1 >= 0

n >= 0

true

THINK PAIR SHARE

- Suppose the weakest precondition is
- (x != null AND x.f >= 0) OR
  (x == null AND z-1 >= 0).
- If you are a developer, what should you do?
- If you are a tester, which inputs should you use?

THINK PAIR SHARE

- Suppose the weakest precondition is
- (x != null AND x.f >= 0) OR
  (x == null AND z-1 >= 0).
- If you are a developer, what should you do?
  - Write assertions in the beginning of the code.
  - Update JAVADOC to write the WP as a precondition contract

THINK PAIR SHARE: TESTING

- Suppose the weakest precondition is
- (x != null AND x.f >= 0) OR
  (x == null AND z-1 >= 0).
- If you are a tester, which inputs should you create?
  - T1. X=new Obj(); x.setF(1); => PASS
  - T2. X=new Obj(); x.setF(-1); => FAIL
  - T3. X == null; z=1 => PASS
  - T4. X == null; z=0 => FAIL ...

HOW DO WE APPLY WHAT WE LEARNED JUST NOW?

- Add assertions at a right point during debugging.
- You can later remove them if performance is an issue.
- Create test cases that satisfy weakest preconditions and expect to pass.
- Also create test cases that violate weakest preconditions and expect to fail.

MORE EXERCISES

EXERCISE 2.

```
if (x!= null) {
 n =x.f;
} else {
 n = z+1;
 z = 2*z+1;
   }
a = new char[n-2];
c = a[z];
```

Suppose Tom wrote this code. What is the weakest pre-condition such that this code terminates **without throwing any exceptions**?

```
if (x!= null) {
 n =x.f;
} else {
 n = z+1;
 z = 2*z+1;
   }
 a = new char[n-2]; //
 c = a[z];
```

{x!=null AND wp(n:=x.f, n>=2, n>z+2, z>=0} OR {x==null AND wp (n:=z+1,z:=2z+1, n>=2, n>z+2, z>=0)}

===={x!=null AND x.f>=2, x.f>z+2, z>=0} OR {x==null AND wp (n:=z+1, n>=2, n>2z+1+2, 2z+1>=0)

===={x!=null AND x.f>=2, x.f>z+2, z>=0} OR {x==null AND z+1>=2, z+1>2z+3, 2z+1>=0)}

===={x!=null AND x.f>=2, x.f>z+2, z>=0} OR {x==null AND z>=1, 0>z+2, 2z+1>=0)

===={x!=null AND x.f>=2, x.f>z+2, z>=0} OR {x==null AND FALSE, 2z+1>=0)}

```
if (x!= null) {
  n =x.f;
} else {
  n = z+1;
  z = 2*z+1;
    }
```

wp(a:=new char[n-2], z<a.length, z>=0)

=== {n-2>=0, z<n-2, z>=0}

=== {n>=2, n>z+2, z>=0}

```
a = new char[n-2]; //
```

z>=0 AND z<a.length

```
c = a[z];
```

TRUE

REASONING ABOUT LOOPS

{P} while B do S end {Q}

invariant J and variant function vf such that:

- (1) invariant initially: $P \Rightarrow J$
- (2) invariant maintained: $\{J \wedge B\}\ S\ \{J\}$
- (3) invariant sufficient: $J \wedge \neg B \Rightarrow Q$
- (4) vf bounded: $J \wedge B \Rightarrow 0 \leqq vf$
- (5) vf decreases: $\{J \wedge B \wedge vf=VF\}\ S\ \{vf<VF\}$

EXERCISE 3. ARRAY SUM

```
k := 0; s := 0;
while k ≠ N do
    s:=s+a[k] ; k:=k+1
end
```

Suppose that Tom wrote the above code to compute the sum of integer elements in the array. **Is this code correct?**

THINK-PAIR-SHARE

- Why do we care about loop invariants and what are the implications?

APPLICATION OF HOARE LOGIC/ LOOP INVARIANT

- It's good to actively think about the semantics of a loop.
- To check correctness about a loop, you can first check whether an invariant holds in the beginning after the initialization.
- You can check whether the invariant is maintained in the iteration.
- You can check whether the invariant and the exit condition implies a post condition.

RECAP – LOOP INVARIANT REASONING

- {P} while (B) do {S} {Q}
- J is a loop invariant. vf is a rank function / variant function.
- P=> J // in the beginning, J holds
- {J AND B} S {J} // while a loop is execution, J still holds
- {J AND NOT B} => Q
- vf>=0
- { J AND B AND vf = VF} S {vf<VF}

PEER REVIEW SCENARIO

```
k := 0; s := 0;
while k ≠ N do
    s:=s+a[k] ; k:=k+1
end
```

Q1. Suppose that Tom wrote the above code to compute the sum of integer elements in the array. For Tom to expect that the following post condition holds, given the precondition of {0<=N}, in other words, s is the sum of all elements of array a[], {s = (Σi | 0≦i<N ・ a[i]), **which invariant J must hold during the execution of the loop**?

PEER REVIEW SCENARIO

```
k := 0; s := 0;
while k ≠ N do
    s:=s+a[k] ; k:=k+1
end
```

Q2. **Tom's team lead wants a proof that the above loop behaves correctly.** In other words, provide three proofs that (1) the loop invariant holds initially P=>J, (2) the loop invariant is maintained {J^B} S {J}, and (3) the invariant is sufficient, J^not(B) =>Q where P is the precondition and Q is the post-condition.

EXERCISE. ARRAY SUM

P{0 ≦ N}

k := 0; s := 0;

while k ≠ N do

s:=s+a[k] ; k:=k+1

end

Q: {s = (Σi | 0 ≦ i<N ・ a[i])}

J is to be guessed and 3 verification conditions hold

J: 0<=k<=N AND s = (Σi | o ≦ i<k ・ a[i])}

EXERCISE. ARRAY SUM

Guess Invariant J and a bounded function vf.

- J: $s = (\Sigma i \mid 0 \leqq i < k \cdot a[i]) \wedge 0 \leqq k \leqq N$
- vf: N-k

- We then show how the five conditions hold for J and vf.

THINK-PAIR-SHARE

- How do you guess a loop invariant?
- How do you guess a variant function?
- Can we automatically find loop invariants?
- Can we infer pre and post conditions?

TIPS FOR LOOP REASONING

- How do you guess a loop invariant?
  - Generally by modifying a post condition and modifying a guard.
- How do you guess a variant function?
  - Generally by coming up with a function N-k if k increases by 1.
- Can we automatically find loop invariants?
  - Yes, it's an active research area to find a loop invariant. Mostly they generate candidates and verify.
- Can we infer pre and post conditions?
  - Yes, There is also work that infers pre/post conditions from tests

- Caveats:
  - There are more than one loop invariants.
  - In the absence of a human-provided meaningful post-condition, the loop invariant subsequently is often not meaningful as well.

EXERCISE. ARRAY SUM

—(1) INVARIANT INITIALLY: {P} CODE ⇒ {J}

P:{0 ≦ N}

{0 ≦ N}

k := 0;

{0 = (Σi | 0 ≦ i<k · a[i]) ∧ 0 ≦ k ≦ N}

s := 0;

J:{s = (Σi | 0 ≦ i<k · a[i]) ∧ 0 ≦ k ≦ N}

Proving P=>J

{P} code => {J} if and only if {P} => wp(code, J)

EXERCISE. ARRAY SUM

—(1) INVARIANT INITIALLY: $P \Rightarrow J$

$\{0 \leqq N\}$

$\{0 = (\Sigma i \mid 0 \leqq i < 0 \cdot a[i]) \wedge 0 \leqq 0 \leqq N\}$

$k := 0;$

$\{0 = (\Sigma i \mid 0 \leqq i < k \cdot a[i]) \wedge 0 \leqq k \leqq N\}$

$s := 0;$

$\{s = (\Sigma i \mid 0 \leqq i < k \cdot a[i]) \wedge 0 \leqq k \leqq N\}$

EXERCISE. ARRAY SUM

—(2) INVARIANT MAINTAINED: {J ∧ B} S {J}

J AND B===={s = (Σi | 0≦i<k ・ a[i]) ∧ 0≦k<N}

{s= (Σi | 0≦i<k ・ a[i]) ∧ 0≦k+1≦N}

s := s + a[k];

{s = (Σi | 0≦i<k ・ a[i])+a[k] ∧ 0≦k+1≦N}
{s = (Σi | 0≦i<k+1 ・ a[i]) ∧ 0≦k+1≦N}

k := k+1;

{s = (Σi | 0≦i<k ・ a[i]) ∧ 0≦k≦N}

J: {s = (Σi | 0≦i<k ・ a[i]) ∧ 0≦k≦N}

To show {J ∧ B} S {J} is to show {J ∧ B} => wp (S,J) since {P}S{Q} === P=>wp(S,J)

{s = (Σi | o≦i<k ・ a[i]) ∧ o≦k<N} => {s = (Σi | o≦i<k・ a[i]) ∧ o≦k+1≦N}

EXERCISE. ARRAY SUM

—(2) INVARIANT MAINTAINED: {J ∧ B} S {J}

{s = (Σi | 0≦i<k · a[i]) ∧ 0≦k≦N ∧ k≠N}

{s+a[k] = (Σi | 0≦i<k · a[i])+a[k] ∧ 0≦k<N}

s := s + a[k];

{s = (Σi | 0≦i<k · a[i])+a[k] ∧ 0≦k<N}

{**s = (Σi | 0≦i<k+1 · a[i])** ∧ 0≦k+1≦N}

k := k+1;

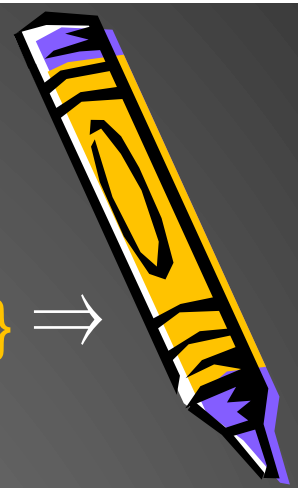
{s = (Σi | 0≦i<k · a[i]) ∧ 0≦k≦N}

s = (Σi | 0≦i<k+1 · a[i])

= a[0] + a[1] … a[k-1] +a[k]

= **sum 0<=i<k a[i]**+a[k]

EXERCISE. ARRAY SUM



—(3) INVARIANT SUFFICIENT: J ∧ ¬B ⇒ Q

JAND NOT B: {s = (Σi | 0≦i<N ・ a[i]) ∧ 0≦N ∧ (k=N)} ⇒

Q: {s = (Σi | 0≦i<N ・ a[i])}

JAND NOT B is simplified as

{s = (Σi | o≦i<k ・ a[i]) ∧ o≦k≦N ∧ k=N }

== {s = (Σi | o≦i<k ・ a[i]) ∧ k=N ∧ 0≦N }

== {s = (Σi | o≦i<N ・ a[i]) ∧ k=N ∧ 0≦N }

{s = (Σi | o≦i<N ・ a[i]) ∧ k=N ∧ 0≦N } => {s = (Σi | o≦i<N ・ a[i])}
(true!) why? A AND B => A

- Requirements you have a program S.
- You have a pre-condition P
- You have a post-condition Q
- {P} S {Q} iff {P} => wp(S,Q)

EXERCISE. ARRAY SUM

—(3) INVARIANT SUFFICIENT: J ∧ ¬B ⇒ Q

J AND NOT (B)

===$\{s = (\Sigma i \mid 0 \leqq i < k \cdot a[i]) \wedge 0 \leqq k \leqq N \wedge \neg(k \neq N)\} \Rightarrow$

===$\{s = (\Sigma i \mid o \leqq i < k \cdot a[i]) \wedge k == N\}$

===$\{s = (\Sigma i \mid o \leqq i < N \cdot a[i]) \wedge k == N\}$

Q: $\{s = (\Sigma i \mid 0 \leqq i < N \cdot a[i])\}$

J AND NOT B => Q

A AND B=> A

A AND B => B

Q is the post condition that you desire.

J ^ NOT B == the condition that holds when you exit the loop

LOOP TERMINATION VERIFICATION CONDITIONS

- vf bounded: J ^ B => (0 <= vf)
- vf decreases: {J ^ B ^ vf = VF} S {vf < VF}

- First, Just like a loop invariant, vf is a piece of puzzle that you need to complete an inductive proof. So its role is similar to an inductive case where you have to guess first and show that your choice of vf satisfies the above two conditions.

LOOP TERMINATION VERIFICATION CONDITIONS

- vf is a monotonically decreasing function that converges towards 0. When vf becomes 0, that's the time that a loop terminates.
- Second, vf bounded: J ^ B => (0 <= vf). This means that while the loop is running (meaning B is true and J is also true), the variant function that you picked has a positive value.

LOOP TERMINATION VERIFICATION CONDITIONS

- Third, vf decreases: {J ^ B ^ vf = VF} S {vf < VF} // This means that while a loop is iterating, vf is monotonically decreasing. Think about is vf's value is equal to VF before the execution of loop's body, but it now becomes less than VF, meaning that it is monotonically decreasing.

PEER REVIEW SCENARIO

```
k := 0; s := 0;
while k ≠ N do
    s:=s+a[k] ; k:=k+1
end
```

Q3. During a peer code review meeting, **Tom's manager asks Tom, whether the following code will always terminate without throwing any exceptions.** Find a variant function vf, and show that vf is bounded and vf decreases.

- We are going to first guess vf as N-k. (N-k is a monotonically decreasing function as k is incremented during loop's execution and it becomes 0 when k becomes N.) Right?

EXERCISE. ARRAY SUM

—(4) VF BOUNDED: J $\wedge$ B $\Rightarrow$ 0 $\leqq$ vf

vf is N-k

{s = (Σi | 0$\leqq$i<k · a[i]) $\wedge$ 0$\leqq$k<N} $\Rightarrow$

{k$\leqq$ N}

The above statement is true.

EXERCISE. ARRAY SUM

—(5) VF DECREASES: {J ∧ B ∧ vf=VF} S {vf<VF}

Left hand side {s = (Σi | 0≦i<k · a[i]) ∧ 0≦k≦N ∧ k≠N ∧ N-k=VF}

{(N-k-1)<VF} (right hand side)

s := s + a[k];

{(N-k-1)<VF}

{N-(k+1)<VF}

k := k+1;

{vf<VF}

{N-k<VF}

Left hand side {s = (Σi | 0≦i<k · a[i]) ∧ 0≦k<N ∧ N-k=VF}

{N-k =VF} => {N-k-1<VF} because VF-1 <VF

EXERCISE. ARRAY SUM

—(5) VF DECREASES: {J ∧ B ∧ vf=VF} S {vf<VF}

{s = (Σi | 0≦i<k ・ a[i]) ∧ 0≦k≦N ∧ k≠N
∧ N-k=VF}

{N-k-1<VF}

s := s + a[k];

{N-k-1<VF}

{N-(k+1)<VF}

k := k+1;

{N-k<VF}

THINK PAIR SHARE

- It's a good practice to think about what holds during a loop execution, and check whether that invariant is satisfied in the beginning and the end.
- If you guess a loop invariant. It's good habit to write assertions during development.

RECAP (1)

- We learned about how to reason about the semantics of assertions, assignment, sequential statements, conditional statements.
- These techniques can help you to find subtle errors during peer code reviews.

RECAP (2)

- Though experienced developers may not use the term, "Hoare Logic or Weakest Preconditions", that's how carefully they analyze the corner cases during peer code reviews.
- We have more exercise questions (Slides 47 to 81).

QUESTIONS?

MORE EXERCISES

CLASS EXERCISE: POWER

```
public static int power(int x, int n) {
  int p = 1, i = 0;
  while (i < n) {
      p = p * x;
      i = i + 1;
  }
  return p;
}
```

- Suppose that Sheryl's manager wants a proof that after the execution of the loop, the return value p is equal to x^n.
- What is a loop invariant that must hold given a precondition n>=0?

The loop invariant J is {p=x^i and 0<=i<=n}

CLASS EXERCISE: POWER

```
P: {n>=0} (given)
public static int power(int x, int n) {
  int p = 1, i = 0;
  while (i < n) {
      p = p * x;
      i = i + 1;
  }
  return p;
}
Q: {p=x^n} (given)
J: {p=x^i, 0<=i<=n} (my guess)
```

The loop invariant J is {p=x^i and 0<=i<=n}

P: {n>=0} (given)

```
public static int power(int x, int n) {
   int p = 1, i = 0;
   while (i < n) {
       p = p * x;
       i = i + 1;
   }
   return p;
}
```

Let's prove this: {n>=0} p:=1; i:=0 {p=x^i, 0<=i<=n}

{n>=0} => wp(p:=1; i:=0 {p=x^I AND 0<=i<=n})

== wp(p:=1, {p=x^0 AND 0<=0<=n})

== {1=x^0 AND 0<=0<=n}

== {TRUE AND 0<=n}

== {0<=n}

The loop invariant J is {p=x^i and 0<=i<=n}

```
public static int power(int x, int n) {
  int p = 1, i = 0;
  while (i < n) {
       p = p * x;
       i = i + 1;
  }
  return p;
}
```

- Suppose that Sheryl's manager wants a proof that after the execution of the loop, the return value p is equal to x^n.
  - Guess J={ p=x^i AND 0=<i<=n } based on the post condition

```
public static int power(int x,
 int n) {
  int p = 1, i = 0;
  while (i < n) {
       p = p * x;
       i = i + 1;
  }
  return p;
}
```

- Guess J={ p=x^i AND 0=<i<=n } based on the post condition

(I) P=>J

```
{n>=0}
p = 1, i = 0;
J={ p=x^i AND  0=<i<=n }
{n=>0} implies wp(p:=1; i:=0; p=x^i
AND  0=<i<=n )
==wp (p:=1; p=x^0 AND  0=<0<=n)
=={1=x^0 AND  0=<0<=n}
=={0<=n}
{n>=0} implies {0<=n}? YES
  while (i < n) {
        p = p * x;
        i = i + 1;
  }
```

(2) {J AND B} S{ J}

```
p = 1, i = 0;
  while (i < n) {
        p = p * x;
        i = i + 1;
  }
```

To show {J AND B} S{ J},
Is to prove {J AND B} =>wp(S,J)

J^B={ p=x^i AND 0=<i<n}

Wp (p = p * x;i = i + 1;{p=x^i AND 0=<i<=n})

==Wp (p = p * x;{p=x^(i+1) AND 0=<i+1<=n})

=={p=x^I AND 0=<i+1<=n}

=={p=x^i AND 0=<i+1<=n}

(2) {J AND B} S{ J}

```
p = 1, i = 0;
  while (i < n) {
        p = p * x;
        i = i + 1;
  }
```

To show {J AND B} S{ J}, {J AND B} =>wp(S,J)

J^B={ p=x^i AND 0=<i<n}

Wp (p = p * x;i = i + 1;{p=x^i AND 0=<i<=n})

==Wp (p = p * x;{p=x^(i+1) AND 0=<i+1<=n})

=={p*x=x^(i+1) AND 0=<i+1<=n}

=={p=x^i AND 0=<i+1<=n}

(3) J AND NOT B =>Q

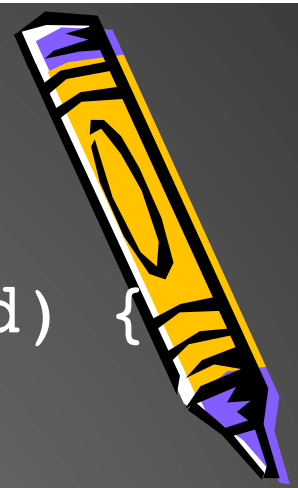
```
p = 1, i = 0;
  while (i < n) {
        p = p * x;
        i = i + 1;
  }
```

J AND(NOT B)= {p=x^i AND 0=<i<=n} AND {i>=n}

=={p=x^i AND i=n}

=={p=x^n AND i=n} implies {p=x^n}

EXERCISE AT HOME: QUOTIENT

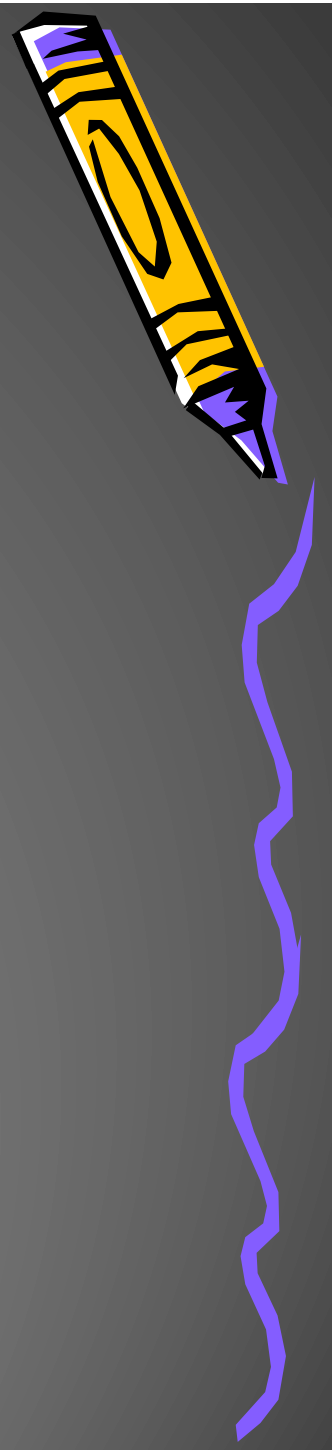


```
public static int quotient(int n, int d) {
  int q = 0, r = n;
  while (r >= d) {
      r = r - d;
      q = q + 1;
  }
  return q;
}
```

This code performs integer division by repeated subtraction. It divides numerator n by divisor d, returning quotient q and also remainder r. What is a loop invariant that satisfies the expected post-condition, `r<d  AND n = q*d +r`?

The loop invariant J is {n=qd+r}

- Guess a loop invariant
- n= qd+r



(I) P=>J

```
int q = 0, r = n;
J={n=qd+r}
  while (r >= d) {
        r = r - d;
        q = q + 1;
  }
```

(2) {J AND B} S{ J}

```
int q = 0, r = n;
  while (r >= d) {
        r = r - d;
        q = q + 1;
  }
```

J={n=qd+r}

(3) J AND NOT B=>Q

```
int q = 0, r = n;
  while (r >= d) {
        r = r - d;
        q = q + 1;
  }
```

J={n=qd+r}