Math 151A Sample Midterm (Questions from previous midterms)

(Winter 2018: Anderson)

1. Consider the data

x	f(x)
0	1
2	-1
3	1
5	2

   (a) Give the polynomial that interpolates the above data.

   (b) Give an approximation to the derivative of f at $x = 1$ that uses all of the data points.

2. On my computer I've determined that the value of ϵ for which $1.0 + \epsilon = 1.0$ is $\epsilon = 1 \times 10^{-16}$.

   (a) Does ϵ indicate the smallest real number which the computer can represent?

   (b) I want to use the formula

$$f'(x) \approx \frac{f(x+h) - f(x)}{h}$$

   to approximate the derivative of $\cos(x)$ at $x = 1$.

   Should I use $h = \epsilon$ to obtain the most accurate approximation? **Explain your answer**.

3. **True or False:**

   (a) Newton's method converges for all initial guesses.

   (b) If f is continuous on an interval [a,b] and if $f(a) * f(b) < 0$ the bisection method will always converge to a root in [a,b].

   (c) If one is given five distinct data points $(x_i, f(x_i))$, $i = 1...5$, then there is a unique polynomial of degree at most 4 that interpolates the data.

   (d) If there is a root between the two initial guesses for the secant method, then the secant method will converge to that root

   (e) Since integer operations are performed without error by a computer, one never runs into difficulty using integers

4. Consider the fixed point iteration $x_{k+1} = \cos(x_k) + x_k$ to compute roots of $\cos(x)$. Does this iteration converge, and if so, what is the expected order of convergence?

5. For k sufficiently large, the error in Newton's method satisfies the relation $e_{k+1} \approx \lambda {e_k}^{\alpha}$ where λ is a constant depending on derivatives of the function and $\alpha = 2$. Derive the formula that allows you to estimate α from the results of a computation in which you compute the e_k's.

6. To check an implementation of Newton's method, I applied the technique to the problem $\sqrt{x} - 2 = 0$. When I used an initial guess of 0, I had to abort the program because it appeared to want to run forever. When I used an initial guess of 3.0 here are the results of the computation:

   (a)

k	x_k	ERROR_k
1	2.32842712474619	1.67157287525381
2	3.77524668665126	0.22475331334874
3	3.99675093213506	0.00324906786494
4	3.99999933995428	0.00000066004572
5	3.99999999999997	0.00000000000003

   Is the program working correctly? And if the program is working correctly, why did I get peculiar behavior with an initial guess of 0?

   Explain your answers