



# SISTEM ROBOT OTONOM

Section 7:

*A brief introduction to fuzzy inference system part 2*

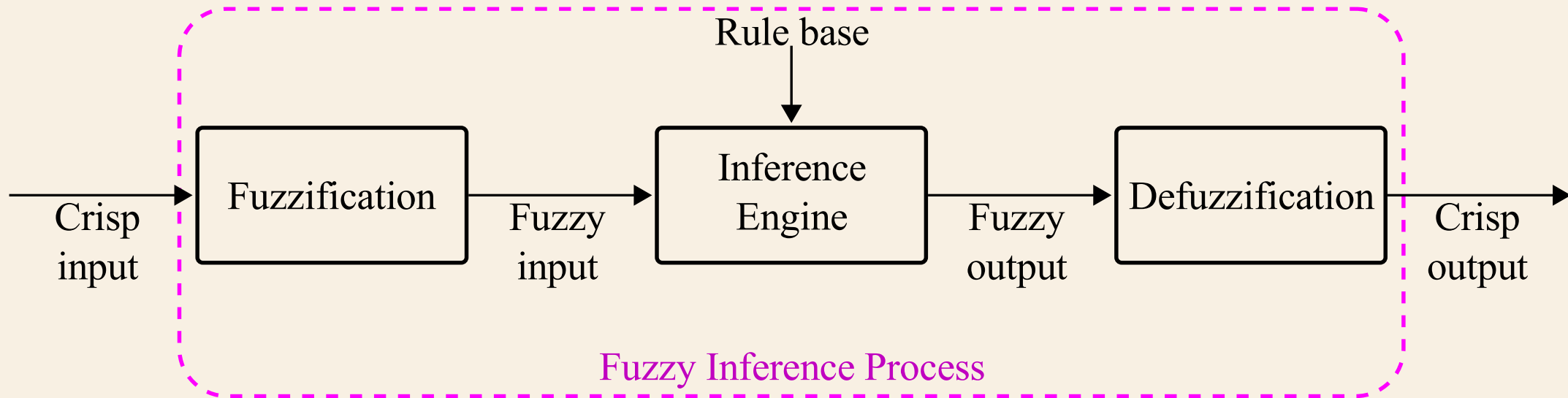
Djoko Purwanto; M. Q. Zaman

[djoko@its.ac.id](mailto:djoko@its.ac.id); [muhammad.zaman@its.ac.id](mailto:muhammad.zaman@its.ac.id)



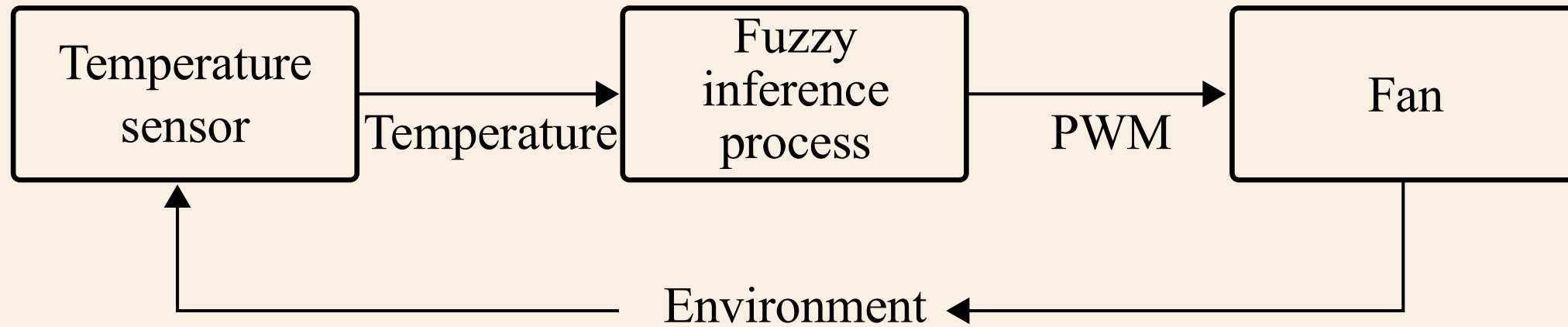
# FUZZY INFERENCE SYSTEM

# THE OVERALL INFERENCE PROCESS



What we learned in previous section is called Fuzzification process

# THE OVERALL FEEDBACK PROCESS

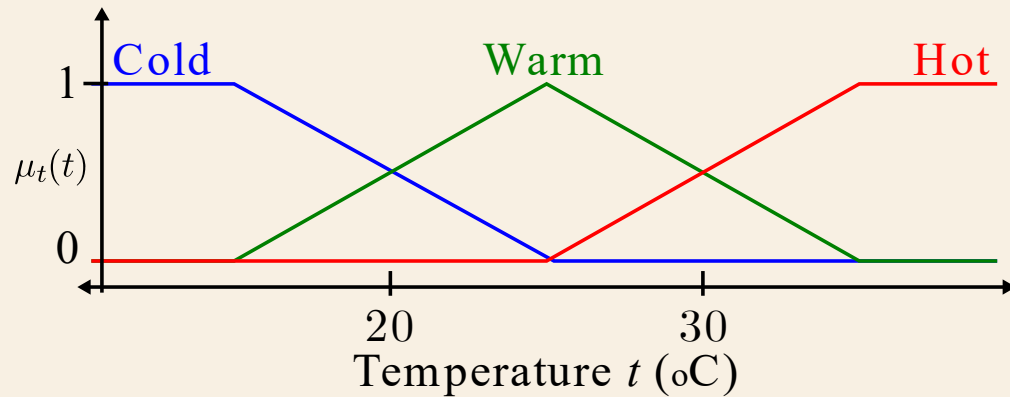


Please note that, for now, there is no temperature set point

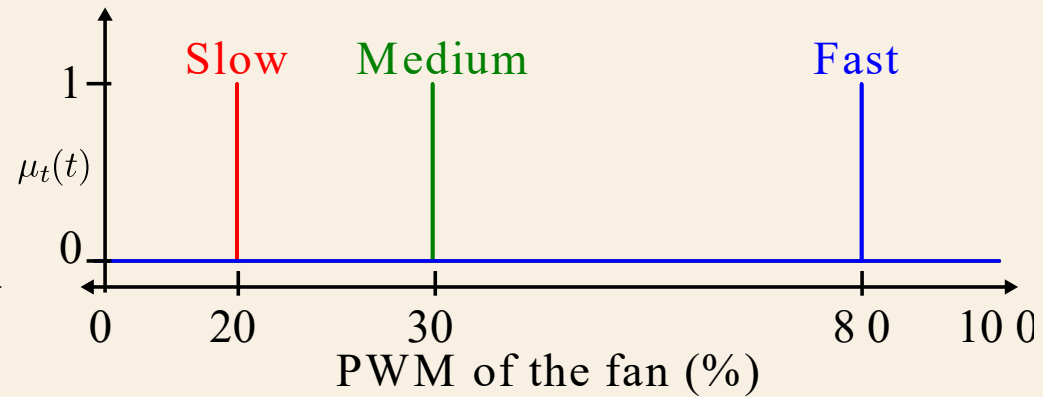


# FUZZY OUTPUT

# SINGLETON FUZZY OUTPUT



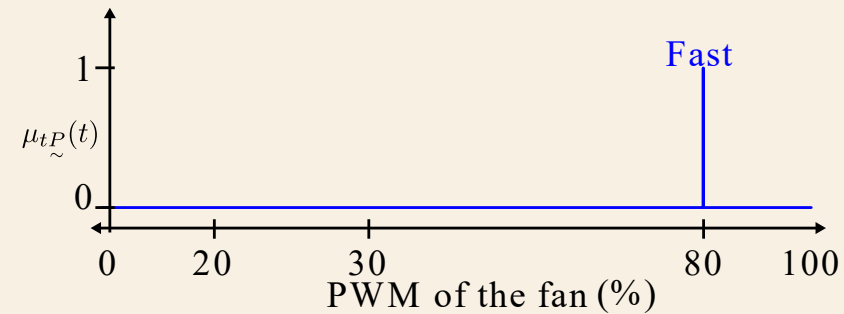
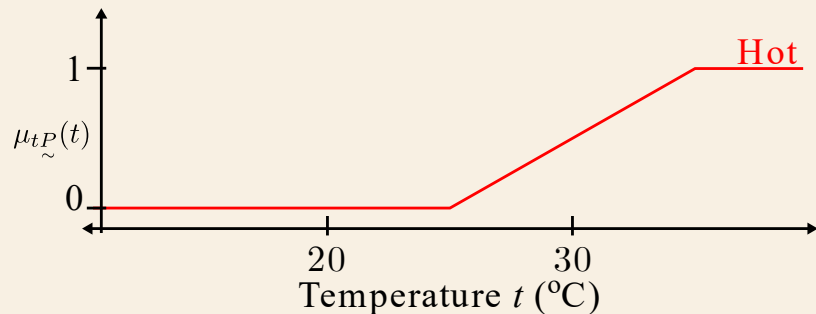
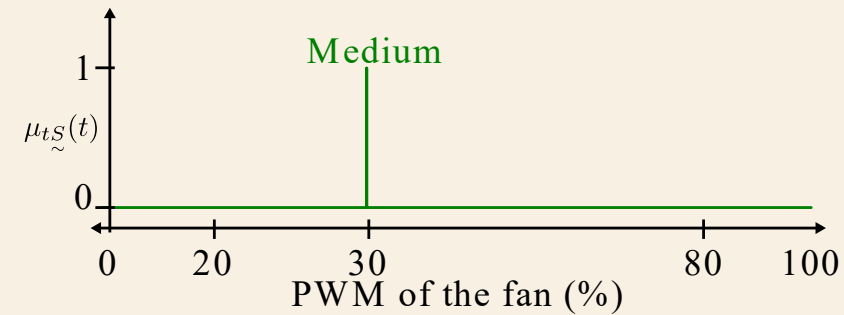
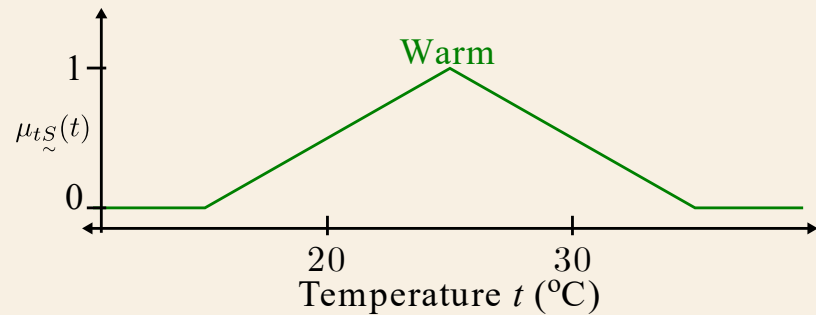
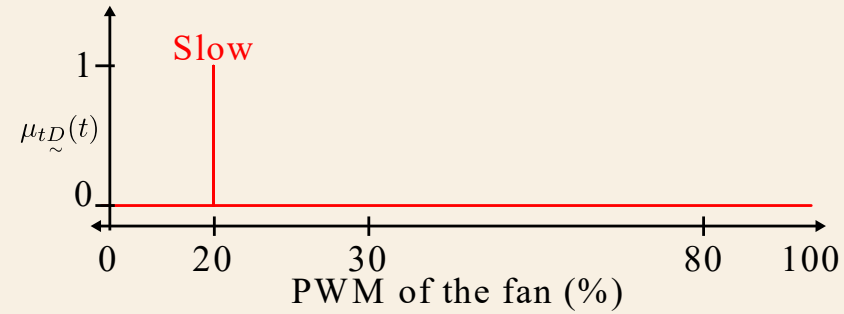
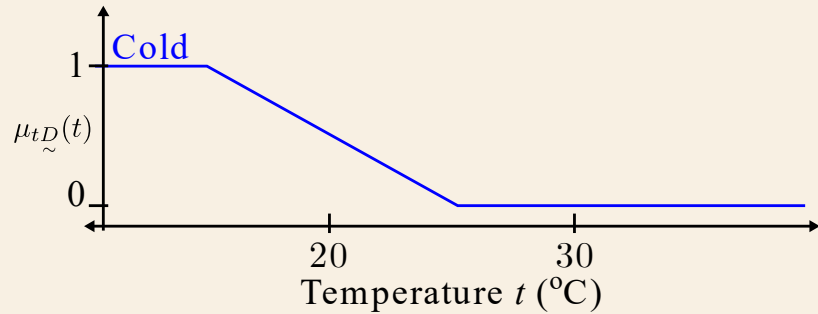
Fuzzy membership functions of input



Singleton fuzzy membership output

Input membership function for comparison

# SINGLETON FUZZY OUTPUT



Fuzzy membership functions of input

Singleton fuzzy membership output

Input membership function for comparison

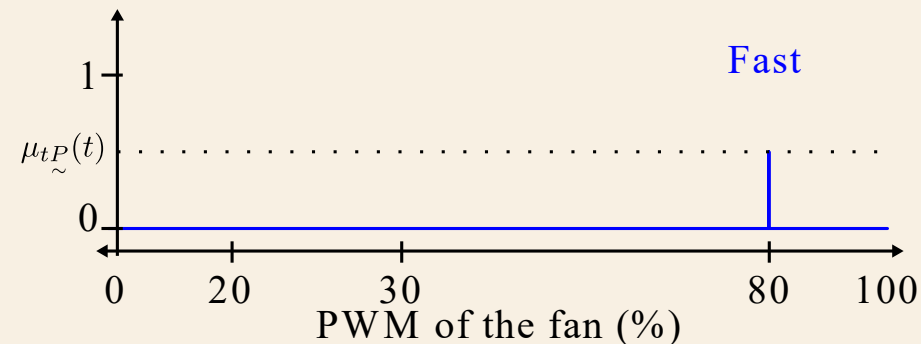
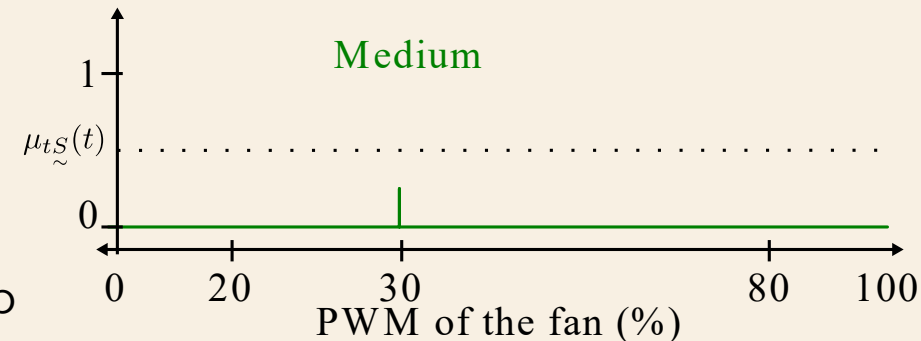
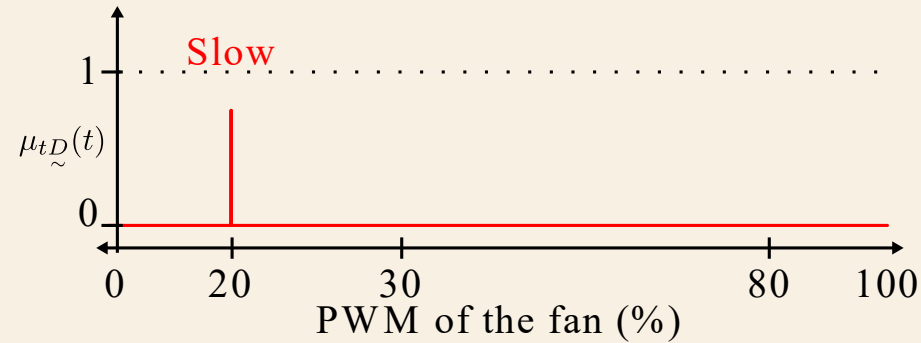
# SINGLETON FUZZY OUTPUT

Singleton fuzzy output also has varying membership degree, for example:

- 0.75 of "slow", or 0.75 of "20"
- 0.3 of "medium", or 0.3 of "30"
- 0.5 of "fast", or 0.5 of "80"

Fuzzy output must be converted to crisp output, such as:

- 25% of PWM output
- 70% of PWM output



Singleton fuzzy membership output





# RULE-BASED INFERENCE ENGINE

1 input and 1 output

# THE RULE

1. **IF** Temp. is *Cold*, **THEN** PWM output is *Slow*
2. **IF** Temp. is *Warm*, **THEN** PWM output is *Medium*
3. **IF** Temp. is *Hot*, **THEN** PWM output is *Fast*

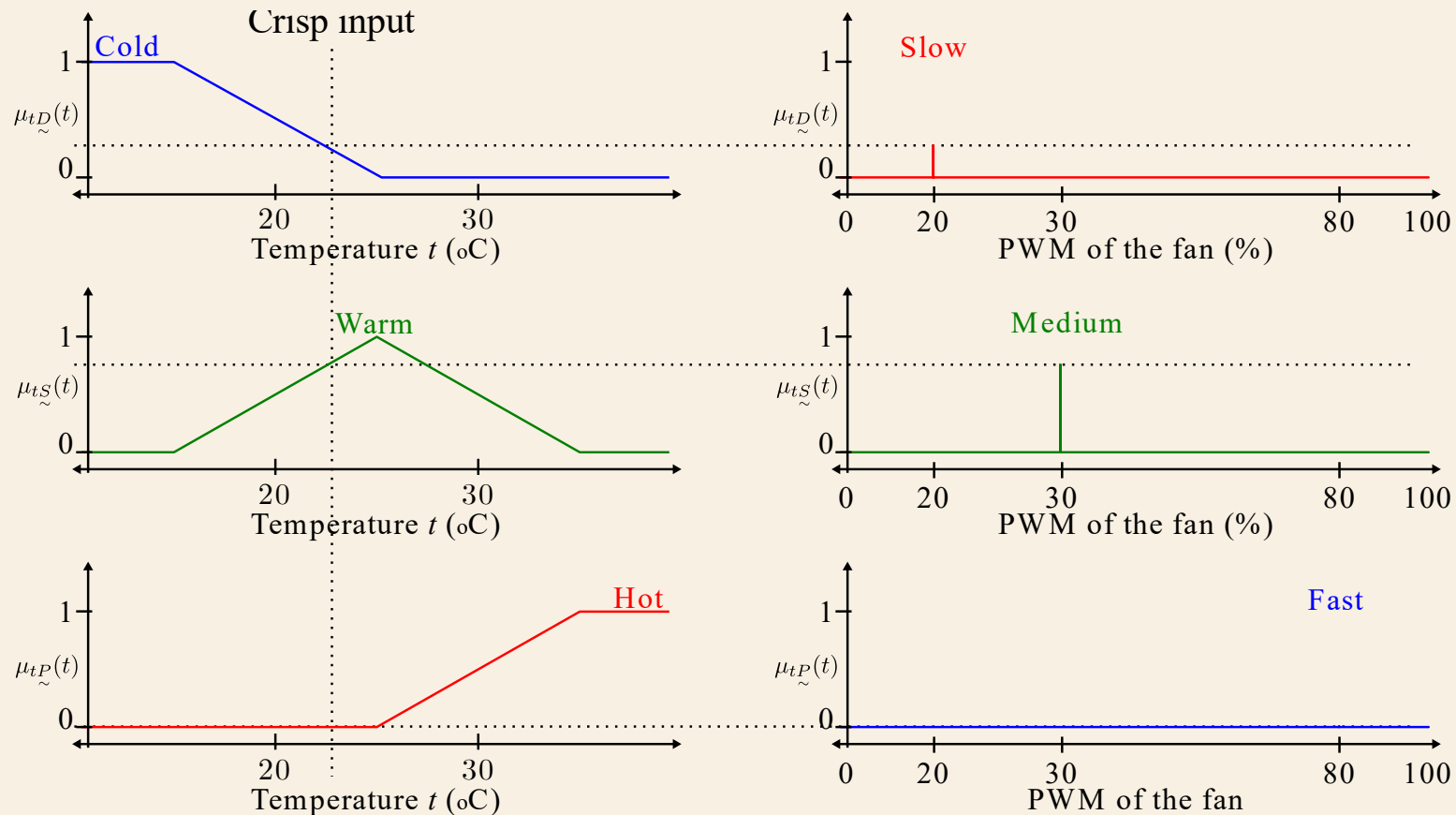
# THE OUTPUT'S FIRING DEGREE

Assigning output's firing degree of 1 input fuzzy inference rule

1. **IF** Temp. is (0.3) *Cold*, **THEN** PWM output is (0.3) *Slow*
2. **IF** Temp. is (0.7) *Warm*, **THEN** PWM output is (0.7) *Medium*
3. **IF** Temp. is (0.0) *Hot*, **THEN** PWM output is (0.0) *Fast*

# THE OUTPUT'S FIRING DEGREE

Assigning output's firing degree of 1 input fuzzy inference rule



Fuzzy membership functions of input

Singleton fuzzy membership output

Visualization of inference process



# RULE-BASED INFERENCE ENGINE

2 input and 1 output

# THE RULE

Example 2 fuzzy inputs and 1 fuzzy output rule:

- Input is fuzzified into 2 memberships.
- Output membership number is the same number with input combinations.

Rules:

1. **IF** left sensor is *near* and right sensor is *near*, **THEN** Left motor is -5
2. **IF** left sensor is *far* and right sensor is *near*, **THEN** Left motor is -5
3. **IF** left sensor is *near* and right sensor is *far*, **THEN** Left motor is 5
4. **IF** left sensor is *far* and right sensor is *far*, **THEN** Left motor is 5

# THE OUTPUT'S FIRING DEGREE

Assigning output's firing degree of 2 inputs fuzzy inference rule.

In this case, memberships of input 1 is  $0.3$  *near* or  $0.7$  *far*

While memberships of input 2 is  $0.1$  *near* or  $0.9$  *far*

1. **IF** left sensor is  $(0.3)$  *near* and right sensor is  $(0.1)$  *near*, **THEN** Left motor is  $(0.1)$  -5
2. **IF** left sensor is  $(0.7)$  *far* and right sensor is  $(0.1)$  *near*, **THEN** Left motor is  $(0.1)$  -5
3. **IF** left sensor is  $(0.3)$  *near* and right sensor is  $(0.9)$  *far*, **THEN** Left motor is  $(0.3)$  5
4. **IF** left sensor is  $(0.7)$  *far* and right sensor is  $(0.9)$  *far*, **THEN** Left motor is  $(0.7)$  5

`min()` function is used to implement *and* logic

# THE OUTPUT'S FIRING DEGREE

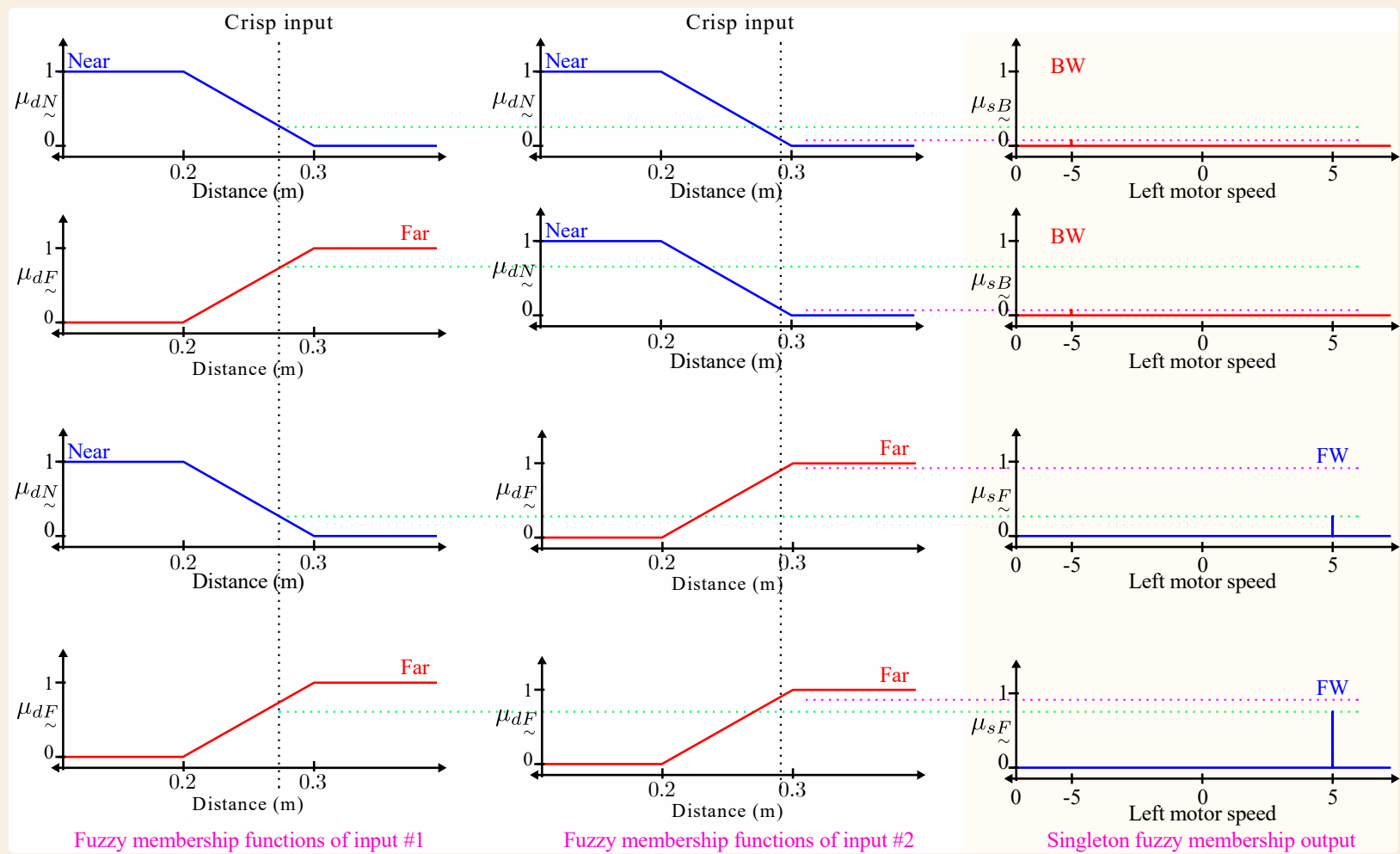
Assigning output's firing degree of 2 inputs fuzzy inference rule.

In this case, memberships of input 1 is **0.3** *near* or **0.7** *far*

While memberships of input 2 is **0.1** *near* or **0.9** *far*

Visualization of inference

Process:





# THE OUTPUT'S FIRING DEGREE

Assigning output's firing degree of 2 inputs fuzzy inference rule.  
In this case, memberships of input 1 is **0.3** *near* or **0.7** *far*  
While memberships of input 2 is **0.1** *near* or **0.9** *far*

Left motor output		Left sensor	
		<b>0.3</b> <i>near</i>	<b>0.7</b> <i>far</i>
Right sensor	<b>0.1</b> <i>near</i>	(0.1) -5	(0.1) -5
	<b>0.9</b> <i>far</i>	(0.3) 5	(0.7) 5

The rule in a table form



# DEFUZZIFICATION

Using singleton fuzzy output

# WEIGHTED AVERAGE

*r-th* singleton output of the rule

$$z^* = \frac{\sum_r (f_r \cdot z_r)}{\sum_r f_r}$$

*r-th* firing degree of the rule

Crisp output

The diagram shows the formula for a weighted average. A blue arrow points from the symbol  $z^*$  to the text 'Crisp output'. A red arrow points from the text '*r-th* singleton output of the rule' to the symbol  $z_r$  in the numerator. A green arrow points from the text '*r-th* firing degree of the rule' to the symbol  $f_r$  in the numerator.

# WEIGHTED AVERAGE

1. **IF** Temp. is (0.3) *Cold*, **THEN** PWM output is (0.3) 20
2. **IF** Temp. is (0.7) *Warm*, **THEN** PWM output is (0.7) 30
3. **IF** Temp. is (0.0) *Hot*, **THEN** PWM output is (0.0) 80

Firing degree of the rule

Singleton output of the rule

$$\begin{aligned} z^* &= \frac{\sum_r (f_r \cdot z_r)}{\sum_r f_r} \\ &= \frac{(0.3 \cdot 20) + (0.7 \cdot 80) + (0.0 \cdot 30)}{0.3 + 0.7 + 0.0} \\ &= 27 \end{aligned}$$

# WEIGHTED AVERAGE

Left motor output		Left sensor	
		<b>0.3</b> <i>near</i>	<b>0.7</b> <i>far</i>
Right sensor	<b>0.1</b> <i>near</i>	(0.1) -5	(0.1) -5
	<b>0.9</b> <i>far</i>	(0.3) 5	(0.7) 5

Firing degree of the rule

Singleton output of the rule

$$\begin{aligned}
 z^* &= \frac{\sum_r (f_r \cdot z_r)}{\sum_r f_r} \\
 &= \frac{(0.1 \cdot -5) + (0.1 \cdot -5) + (0.3 \cdot 5) + (0.7 \cdot 5)}{0.1 + 0.1 + 0.3 + 0.7} \\
 &= 3.33
 \end{aligned}$$



# PYTHON IMPLEMENTATION

# TRIANGULAR MEMBERSHIP FUNCTION

```
import numpy as np
import matplotlib.pyplot as plt

def triangleMF(x, a, b, c, FD0, FD2):
    if x < a:
        FD = FD0
    elif x >= a and x < b:
        FD = (x-a)/(b-a)
    elif x >= b and x < c:
        FD = (c-x)/(c-b)
    elif x >= c:
        FD = FD2
    return FD
```

# FUZZIFICATION FUNCTION

```
# input membership fuzzification
def dis_MF(x):
    near = triangleMF(x, 0.2, 0.2, 0.3, 1, 0)
    far = triangleMF(x, 0.2, 0.3, 0.3, 0, 1)
    y = np.array([[near],
                  [far]],
                 dtype=float)
    return y
lt = ["near", "far"]
```

```
# singleton membership fuzzification of output
singleton_PWM_outputs = np.array([
    [-5],
    [5]
], dtype=float)

# singleton firing degree of output
# initiated with ones
# later, the firing degree will be assigned using min() function
FD_PWM_outputs = np.ones(shape=singleton_PWM_outputs.shape, dtype=float)
```



# FUZZIFICATION PROCESS

```
# Fuzzification of input 1 (column)
x1 = 0.27
y1 = dis_MF(x1);
print(f"Left sensor (input#1) {x1} Firing Degree vector is [{y1[0][0]:.2f},{y1[1][0]:.2f}]")

# Fuzzification of input 2 (row)
x2 = 0.29
y2 = dis_MF(x2);
print(f"Right sensor (input#2) {x2} Firing Degree vector is [{y2[0][0]:.2f},{y2[1][0]:.2f}]")
```

```
Left sensor (input#1) 0.27 Firing Degree vector is [0.30,0.70]
Right sensor (input#2) 0.29 Firing Degree vector is [0.10,0.90]
```

# RULE TABLE

```
# rule table
# List of indices pointing out to the singleton
# membership function output index
# here, index 0 is -5 and index 1 is 5
rule_table = np.array(
    [
        [0, 0],
        [1, 1]
    ],
    dtype=int)
```

# OUTPUT FIRING DEGREE AND DEFUZZIFICATION

```
# r-th rule firing degree calculation
# and defuzzification
num = 0;
den = 0;
for r, rval in enumerate(y2):
    for c, cval in enumerate(y1):
        tab_idx = rule_table[r][c]

        fd1andfd2 = min(cval, rval)

        FD_PWM_outputs[tab_idx] = fd1andfd2

        print(f"IF input 1 {lt[c]} FD = {cval} AND
        input 2 {lt[r]} FD = {rval}, THEN FD of
        {singleton_PWM_outputs[tab_idx][0]} (MF idx
        {tab_idx}) = {fd1andfd2}")

        num = num + (fd1andfd2*singleton_PWM_outputs
        [tab_idx][0])
        den = den + (fd1andfd2)

crisp_out = num/den
```

IF input 1 near FD = [0.3] AND input 2 near FD = [0.1], THEN FD of -5.0 (MF idx 0) = [0.1]  
IF input 1 far FD = [0.7] AND input 2 near FD = [0.1], THEN FD of -5.0 (MF idx 0) = [0.1]  
IF input 1 near FD = [0.3] AND input 2 far FD = [0.9], THEN FD of 5.0 (MF idx 1) = [0.3]  
IF input 1 far FD = [0.7] AND input 2 far FD = [0.9], THEN FD of 5.0 (MF idx 1) = [0.7]

```
print(f"Left motor vel output is {crisp_out[0]:.1f}")
```

Left motor vel output is 3.3

# ASSIGNMENT

- Create a membership function for ultrasound reading in CoppeliaSim.
- The number of used sensors is by your design.
- The ultrasound reading is fuzzified into some number of membership functions by your design.
- Put an obstacle in CoppeliaSim.
- Design left and right wheel velocities based on fuzzy inference system to avoid obstacles.
- All the code is done in Python language using only NumPy and Matplotlib libraries.

In the report (PDF file):

- - Plot your fuzzification design of sensors
- - Show your fuzzy rule design in table style

Files to be submitted:

- CoppeliaSim \*.ttt file
- Python code \*.py
- Report in \*.pdf
- Demo video (youtube link)