EE477 HW-3
Phase and Timing Estimation

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Question-1:Phase Estimation

To estimate phase, we implement decision directed method for it, like we have already done formulations at the class.

$$\hat{\phi} = \arg\max_{\phi} \Re[\int r_l(t) \cdot s_l^*(t : \phi)]$$

Starting from this equation, we obtained the decision directed method :

$$\hat{\phi} = \arg\max_{\phi} \Re[e^{j\phi} \sum_{k=0}^{K-1} z_k^* \cdot y_k]$$

where $\quad y_k = \int_{kT}^{(k+1)T} r_l(t) \cdot g(t - kT)dt$

So, we create a sequence z_k, then by using $rcosdesign$ function on matlab we create $g(t)$, we convolve our sequence by $g(t)$. Then we add noise to the our sending signal. For the transmitter side, the receiving signal is $r_l(t)$. By using $r_l(t) and g(t)$ we obtain y_k's first. We have already know the sending sequence z_k's. By usung decision directed method which formulation is below, we found the phase error, for different number of pilot and different snr values.

$$\hat{\phi} = -\arctan\frac{\mathrm{Im}[\sum_{k=0}^{K-1} z_k^* \cdot y_k]}{\Re[\sum_{k=0}^{K-1} z_k^* \cdot y_k]}$$

As we expected, increasing the number of pilot leads to less phase error. On the other hand, increasing the signal to noise ratio implies less phase error. In order to obtain more smooth curve, i repeat this process 200 times and then take the average.

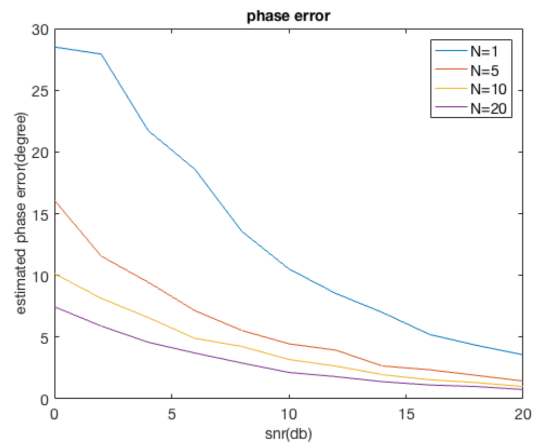

Fig. 1: Phase Estimation Error

Question-2:Time Estimation

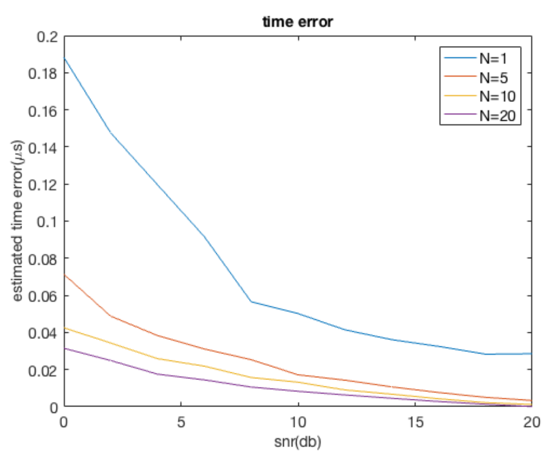

Fig. 2: Time Estimation Error

Similar to the method that been used in question 1, we again find the y_k's and than will find τ that will make correlation higher.

$$\hat{\tau} = \arg\max_{\tau}[\sum_{k=0}^{K-1} z_k^* \cdot y_k(\tau)]$$

where $\quad y_k(\tau) = \int_{kT}^{(k+1)T} r_l(t) \cdot g(t - kT - \tau)dt$

As we expected, increasing the number of pilot implies less time error, because the effect of noise will be eliminated for high N. On the other hand, increasing the snr implies better results. In order to obtain smooth curve, i repeat this process 100 times, however the results were not smooth enough. So i repeat it 500 times and got better graph.

Question-3:Time and Phase Estimation

We will basically use the method that we have already used at the question 2,decision directed method for time estimation. After finding some τ, we can detect optimum $\hat{\phi}$ by using the formulas described belows:

$$\hat{\tau} = \arg\max_{\tau}[\sum_{k=0}^{K-1} z_k^* \cdot y_k(\tau)]$$

$$\sum_{k=0}^{K-1} z_k^* \cdot y_k(\tau) = A(\tau) + j \cdot B(\tau)$$

$$\hat{\tau} = \arg\max_{\tau}[A(\tau) \cdot \cos\phi - B(\tau) \cdot \sin\phi]$$

So, by differentiating it, we can find some τ's that maximize the expression above.

$$-A(\tau) \cdot \sin\phi - B(\tau) \cdot \cos\phi = 0$$

$$\hat{\phi}_{ml} = -\arctan\frac{B(\tau_{ml})}{A(\tau_{ml})}$$

Note for this question that i didnot take $\tau = \frac{T}{30}$. I set it $\tau = \frac{T}{30}$. Because i design my code as taking 40 samples at each time period T. $\frac{T}{30}$ is not appropriate for me, and i didnot change my code for this. Also note that, when we take $\tau = \frac{T}{40}$. $\phi = -2\pi f_c \tau = 50\pi$ and it is equivalent to 0. So, i measure all procedure for this values. The phase error graph is good, but the time estimation error graph is not good enough.

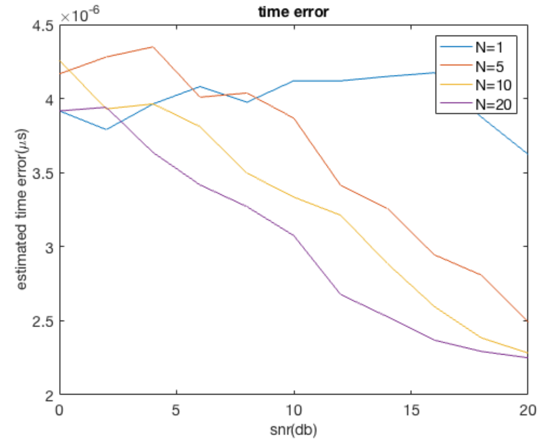

Fig. 3: Time Estimation Error

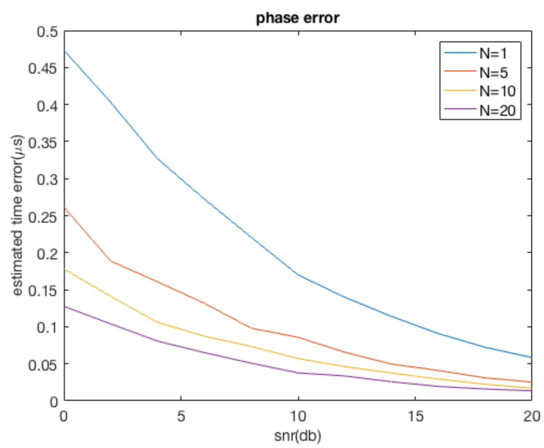

Fig. 4: Phase Estimation Error