EE475 HW-4

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<u>Question-1</u>

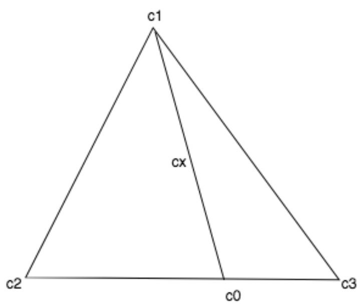

Fig. 1: Initial Positions

c$_1(x_1, y_1)$, c$_2(x_2, y_2)$ and c$_3(x_3, y_3)$ are the points that determine a triangle in a space. c$_x(x_c, y_c)$ is represents any point in the triangle. c$_0(x_0, y_o)$ is the intersection of the line c$_2$-c$_3$ and the line c$_1$-c$_x$. So we should calculate the coordinates of c$_0$ in terms of c$_1$, c$_2$, c$_3$, c$_x$.

The line equation passing the points c$_1$ and c$_x$ is:

$$y = \frac{y_c - y_1}{x_c - x_1} x + y_1 - \frac{y_c - y_1}{x_c - x_1} x_1 \tag{1}$$

$$y = Ax + y_1 - Ax_1 \text{ where } A = \frac{y_c - y_1}{x_c - x_1}$$

The line equation passing the points c$_2$ and c$_3$ is:

$$y = \frac{y_3 - y_2}{x_3 - x_2} x + y_2 - \frac{y_3 - y_2}{x_3 - x_2} x_2 \tag{2}$$

$$y = Bx + y_1 - Bx_1 \text{ where } B = \frac{y_3 - y_2}{x_3 - x_2}$$

The intersection of these two lines gives us the cordinate of c$_0$. From (1) and (2):

$$x(A - B) = y_2 - Bx_2 - y_1 + Ax_1$$

So, we find coordinates of c$_0$:

$$x_0 = \frac{y_2 - y_1 - Bx_2 + Ax_1}{A - B}$$

$$y_0 = Bx_0 y_2 - Bx_2$$

If we put A and B values, the generel expression for c$_0$ is:

$$x_0 = \frac{y_2 - y_1 - \frac{y_3 - y_2}{x_3 - x_2} x_2 + \frac{y_c - y_1}{x_c - x_1} x_1}{\frac{y_c - y_1}{x_c - x_1} - \frac{y_3 - y_2}{x_3 - x_2}}$$

$$y_0 = \frac{y_3 - y_2}{x_3 - x_2} \frac{y_2 - y_1 - \frac{y_3 - y_2}{x_3 - x_2} x_2 + \frac{y_c - y_1}{x_c - x_1} x_1}{\frac{y_c - y_1}{x_c - x_1} - \frac{y_3 - y_2}{x_3 - x_2}} y_2 - \frac{y_3 - y_2}{x_3 - x_2} x_2$$

As a result, when we know the coordinates of the triangle c$_1$, c$_2$, c$_3$ and another point that inside the triangle c$_x$, then we can calculate the coordinates of c$_0$ on Fig. 1 by using the equation found above.

To determine which percentage any coordinate will contribute to the coordinate c$_x$, we should define some probabilities for each coordinates of triangle. By using the the formulation:

$$P_1 = \frac{d(c_1, c_x)}{d(c_1, c_0)} \cdot 100$$

$$P_2 = \frac{d(c_2, c_o)}{d(c_2, c_3)} \cdot (100 - P_1)$$

$$P_3 = \frac{d(c_3, c_o)}{d(c_2, c_3)} \cdot (100 - P_1)$$

or because of $P_1 + P_2 + P_3 = 100$

$$P_3 = 100 - P_2 - P_1$$

Not that: d(a,b) represents distance between point a and point b.

<u>QUESTION-2</u>

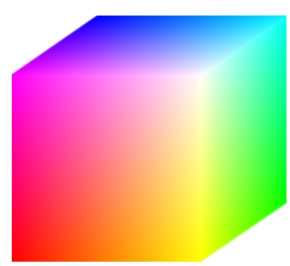

Fig. 2: Initial Positions

The front of the color cube, it also can be expressed mathematically by the plane that R=255. Therefore, when we move on the front part of the image, R value does not change. When we go from left to right again in front part, B value does not change and G value increases. When we go from down to up again on front part, G value does not change and B value increases. To explain this issue with another expression, when we go from left-down part to right-up part, R value does not change but G and B values increase.

Part-b

$$\begin{bmatrix} C \\ M \\ Y \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + \begin{bmatrix} R \\ G \\ B \end{bmatrix} \tag{3}$$

Equation 3 is a transformation from RGB to CMY.

Color	R	G	B	C	M	Y	Result Color
Black	0	0	0	1	1	1	White
Red	1	0	0	0	1	1	Cyan
Yellow	1	1	0	0	0	1	Blue
Green	0	1	0	1	0	1	Magenta
Cyan	0	1	1	1	0	0	Red
Blue	0	0	1	1	1	0	Yellow
Magenta	1	0	1	0	1	0	Green
White	1	1	1	0	0	0	Black

Part-c

So, we can think about the upper edge of the front part of the image. The starting points has [R G B]=[1 0 1], the end points has [R G B]=[1 1 1].

Saturation can be calculated from the equation:

$$S = 1 - \frac{3}{R+B+C}[min(R,G,B)]$$

min(R,G,B) equals to 0, so it implies that saturation value is 1 at the starting point. For end point, because it is white, min(R,G,B) equals to 1, so it implies that saturation value is 0 at the end. Saturation decreases uniformly on the line.

QUESTION-3

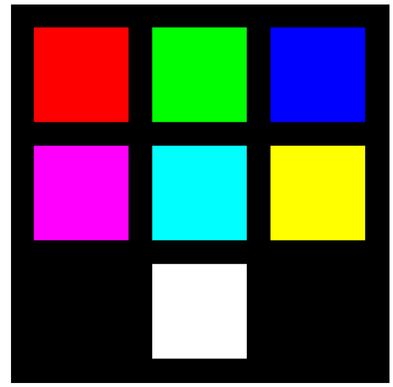

Fig. 3: Original Image

Hue

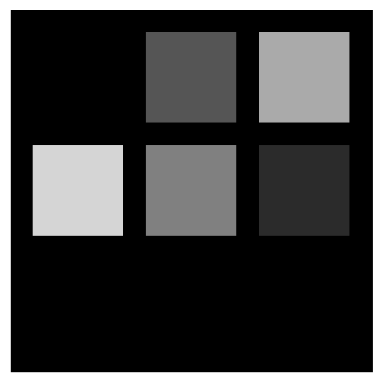

Fig. 4: Hue part of the Image

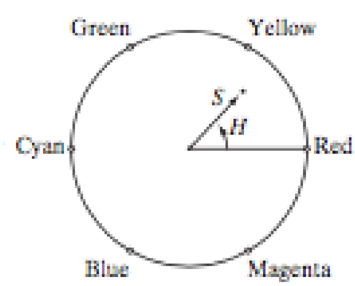

Fig. 5: Circle Representation

From Figure 5 we can understand the hue values, because it is the angle degree from the red position. For example, hue value for red is zero. So, first we should degree of color, then we should map these number to interval [0,1]. Then, we should map them the range [0,7]. If the color has degree θ from the circle representation, then the 8 quanta level should be $\theta \cdot \frac{7}{360}$. Therefore:

$H_R = 0, H_Y = 60\frac{7}{360} = 1, H_G = 120\frac{7}{360} = 2, H_C = 180\frac{7}{360} = 4, H_B = 240\frac{7}{360} = 5,\ H_M = 300\frac{7}{360} = 6$, and $H_W = 0$.

When ploting the figure, we again map them [0,255]:

$H_R = 0, H_Y = 1\frac{255}{7} = 36, H_G = 2\frac{255}{7} = 73, H_C = 4\frac{255}{7} = 146, H_B = 5\frac{255}{7} = 182,\ H_M = 6\frac{255}{7} = 219$, and $H_W = 0$.

Saturation

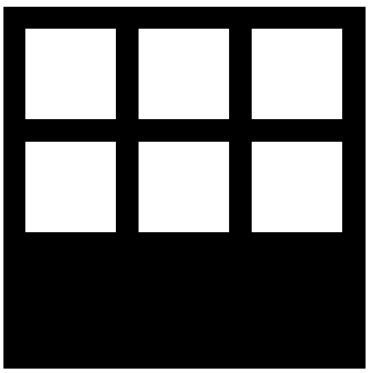

Fig. 6: Saturation part of the Image

From Figure 5, we can understand that the saturation values of the colors red, green, blue, magenta, cyan, yellow should be max. So this parts become white. Because white is at the center of the circle, distance is zero, so it becomes black. Also it can be calculated from the equation below:

$$S = 1 - \frac{3}{R+B+C}[min(R,G,B)]$$

We should scale S to the range [0,7].Therefore:
When we map the saturation values to the range [0,7]:

$S_R = 7, S_G = 7, S_B = 7, S_C = 7, S_M = 7, S_Y = 7$, and $S_W = 0$.

Intensity

Because of white has [R G B]=[255 255 255], intensity part for white equals to 255 and so white. Red, green and blue colors have only one 255 value, R+G+B=255, so intensity value is 85. Magenta, cyan and yellow have R+G+B=255*2, so intensity value is 170. As a result, for image that represents intensity value, the white parts become most white. Magenta, cyan and yellow become darker. Red, green and blue become more dark than than others. Of course black place remains black.

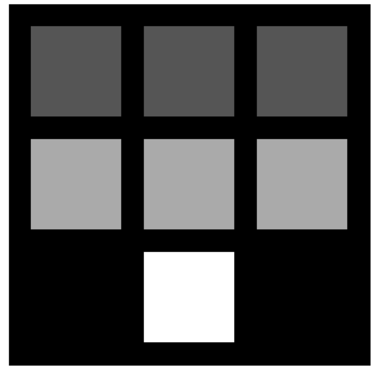

Fig. 7: Intensity part of the Image

$$I = \frac{1}{3}(R+G+B)$$

If we do them for 8 quanta level, then:
$S_R = \frac{7}{3} = 2, S_G = \frac{7}{3} = 2, S_B = \frac{7}{3} = 2, S_C = \frac{14}{3} = 5, S_M = \frac{14}{3} = 5, S_Y = \frac{14}{3} = 5$, and $S_W = \frac{21}{3} = 7$.

When plotting the values that scaled to the range [0-255], so 2 maps to 73, 5 maps 182, and 7 maps to 255.

Question-4

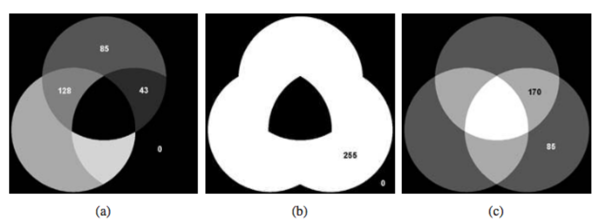

Fig. 8: Question

Part-a

Hue value basically represents the the angle from the red point on Figure 5. $\frac{85}{255} \cdot 360 = 120$ and from that circle we can understand that upper circle circle represents green. $\frac{43}{255} \cdot 360 = 60$ so the place 43 represents yellow. Yellow consist of red and green therefore,right circle is red. $\frac{128}{255} \cdot 360 = 180$ so 128 represents the degree 180 and it is cyan. Because cyan consists of green and blue, left circle is blue. Then, to find the hue values of the remaining parts are easy.

Hue value for blue is $\frac{240}{360} \cdot 255 = 17$
Hue value for magenta is $\frac{300}{360} \cdot 255 = 213$
Hue value for white and black is 0.

Part-b

$$S = 1 - \frac{3}{R+G+B}[min(R,G,B)]$$

Saturation value for red, green, blue, magenta, cyan, yellow is $1 - \frac{3}{...} \cdot 0 = 1$
Saturation value for white is $1 - \frac{3}{255*3} \cdot 255 = 0$

Part-c

$$I = \frac{R+G+B}{3}$$

Intensity value for red, green, blue is 85.
Intensity value for magenta, yellow, cyan is 170.
Intensity value for white is 255.