HOMEWORK #8 (Submit solutions through blackboard in PDF format)

Problem 1

Given the Z-transform

$$X(z) = \frac{10-4z^{-1}}{1-\frac{3}{4}z^{-1}+\frac{1}{8}z^{-2}}$$

Using partial fraction expansion (assuming causal ROC), invert X(z) to obtain the time domain signal x(n)

Problem 2

Using unilateral Z-transform, find the total solution of the difference equation for the input signal $x(n) = 2\,u(n)$ (work in Z+ domain and then invert the $Y^{+}(z)$ using any method you prefer).

$$y(n) + y(n-1) - 2y(n-2) = x(n) - 4x(n-1)$$

$$y(-1) = 2$$

$$y(-2) = -2$$

Problem 3

For the continuous periodic signal:

$$x_p(t) = A\sin(2\pi F_0 t + \varphi)$$

a) Calculate Fourier Series expansion coefficients c_k for the signal $x_p(t)$
b) Which coefficients c_k are non-zero? How does the phase angle φ affect the magnitude of c_k?
c) Calculate the power of the signal using Parseval's Theorem.

Problem 4

For the continuous aperiodic signal (that consists of one period of the continuous sine-wave):

$$x_a(t) = \begin{cases} A\sin(2\pi F_0 t), & -T_0/2 < t < T_0/2 \\ 0, & elsewhere \end{cases}$$

where $F_0 = 1/T_0$

a) Obtain the (continuous aperiodic) Fourier Transform $X_a(F)$ of the signal $x_a(t)$
b) Sketch the magnitude of $X_a(F)$. Hint: the FT consists of a specific waveform shifted by ± F_0
c) Compare the Fourier series coefficients from problem 3a) to the $X_a(F)$sampled in frequency domain with "sampling period" F_0 (i.e. $X_a(F)$ evaluated at discrete frequency points $X_a(kF_0)$ where k=0,±1, ±2, …).