

ECE 437 Project Report

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Purpose

The purpose of this project was to code an implementation of a Direct I filter by using MATLAB.

Additionally, students were tasked with calculating the values of $y(n)$ using some given MATLAB functions and a few specifications.

Implementation

The different methods for implementing my own functions are described in the following sections:

Writing the iir() function

I started by defining all the arrays required for the filter. These were “a” for the a array, “b” for the b array, x_array for the calculated $b_k * x(n-k)$ values, and the y_array for the $a_k * y(n-k)$ calculations. It should be noted that x_array and y_array were initialized to zero. The next step was looping through the x(n-k) loop to obtain values for the x_array vector. The looping for y_array was next, and since all the initial values for $y(n)$ were zero, I summed over an empty vector during the first iteration. I added these two sums to generate $y(0)$. From here, the loop repeated n times, depending on my value for n.

Calculating $y(n)$ method 1

Unlike the previous function, writing the program that relied solely on the impz function was much more straightforward. Before anything could be written, I needed to do some algebraic manipulation to obtain the $X(z) H(z)$ form. This is shown below in Equation 1.

$$\frac{1}{1 - az^{-1}} \cdot \frac{1 - r \cos(\omega) z^{-1}}{1 - 2r \cos(\omega) z^{-1} + r^2 z^{-2}} = \frac{1 - r \cos(\omega) z^{-1}}{1 - 2r \cos(\omega) z^{-1} + r^2 z^{-2} - az^{-1} + az^{-2} 2r \cos(\omega) - r^2 az^{-3}}$$

Equation 1: $X(z) H(z)$

Once this was complete, the coefficients of the numerator were assigned to a vector called “b” and the denominator was assigned to a vector named “a”. The `impz()` function was then used to obtain $y(n)$.

Calculating $y(n)$ method 2

In this case, $x(n)$ and $h(n)$ were needed before anything could be done. The coefficients for both $X(z)$ and $H(z)$ were recorded into two sets of “a” and “b” variables. The `impz()` function was used on both sets of data to obtain the values in the time domain. Once this was complete, the convolution was taken using the `conv ()` function.

Calculating $y(n)$ method 3

This was the most involved of the three calculation problems. Like the previous iteration, I started by making two sets of “a” and “b” variables that represented the coefficients of $X(z)$ and $H(z)$. Following this, I used the `impz ()` function to obtain approximations of $x(n)$ and $h(n)$. The next step was to use the Discrete Fourier Transform on both of these. Lastly, I multiplied the two results together and took the inverse Discrete Fourier Transform to finally obtain $y(n)$.

Results

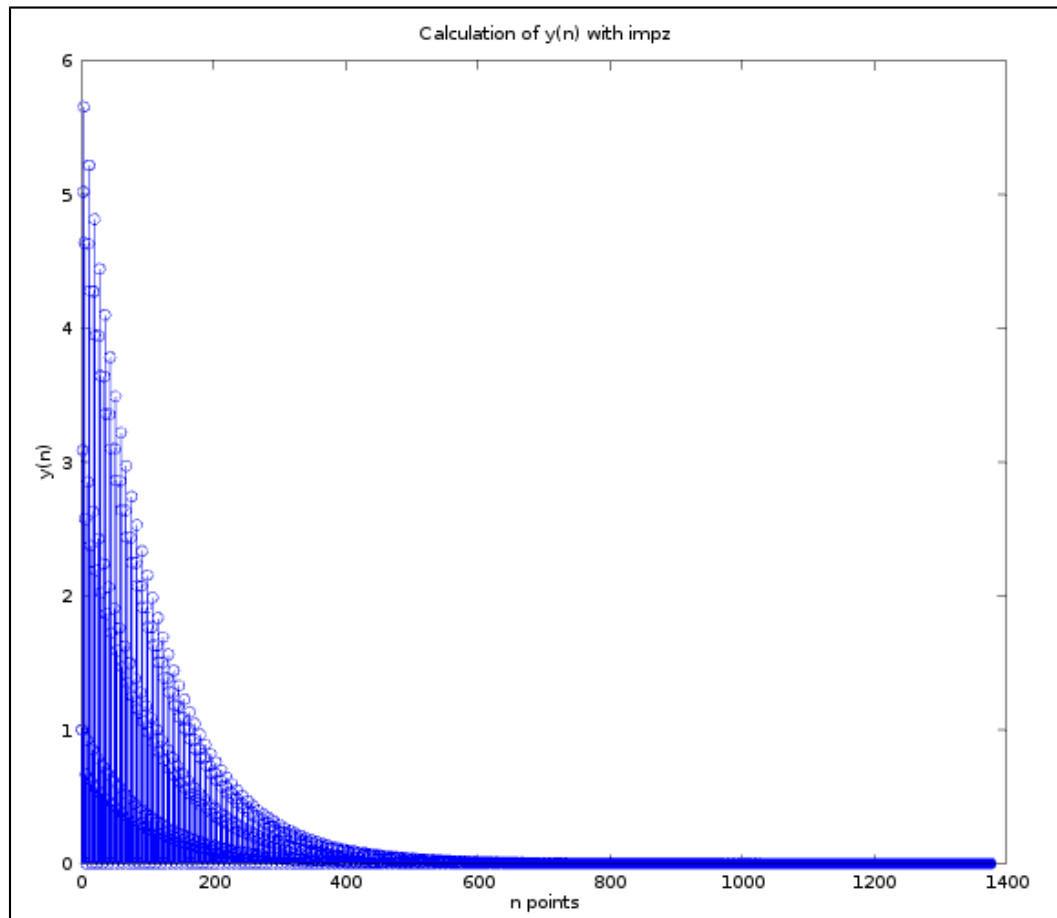


Figure 1: Calculation of $y(n)$ with impz

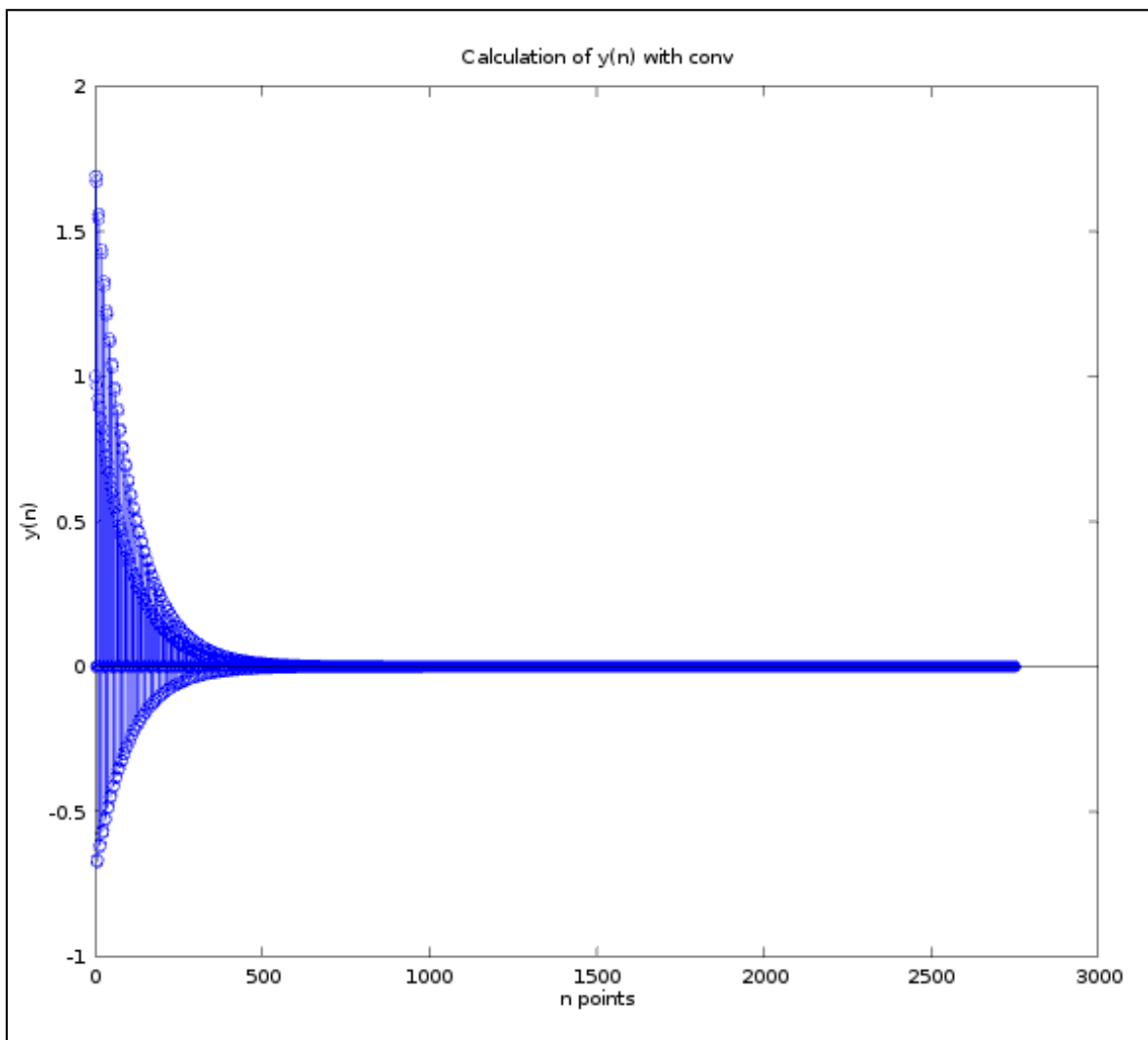


Figure 2: Calculation of $y(n)$ with conv

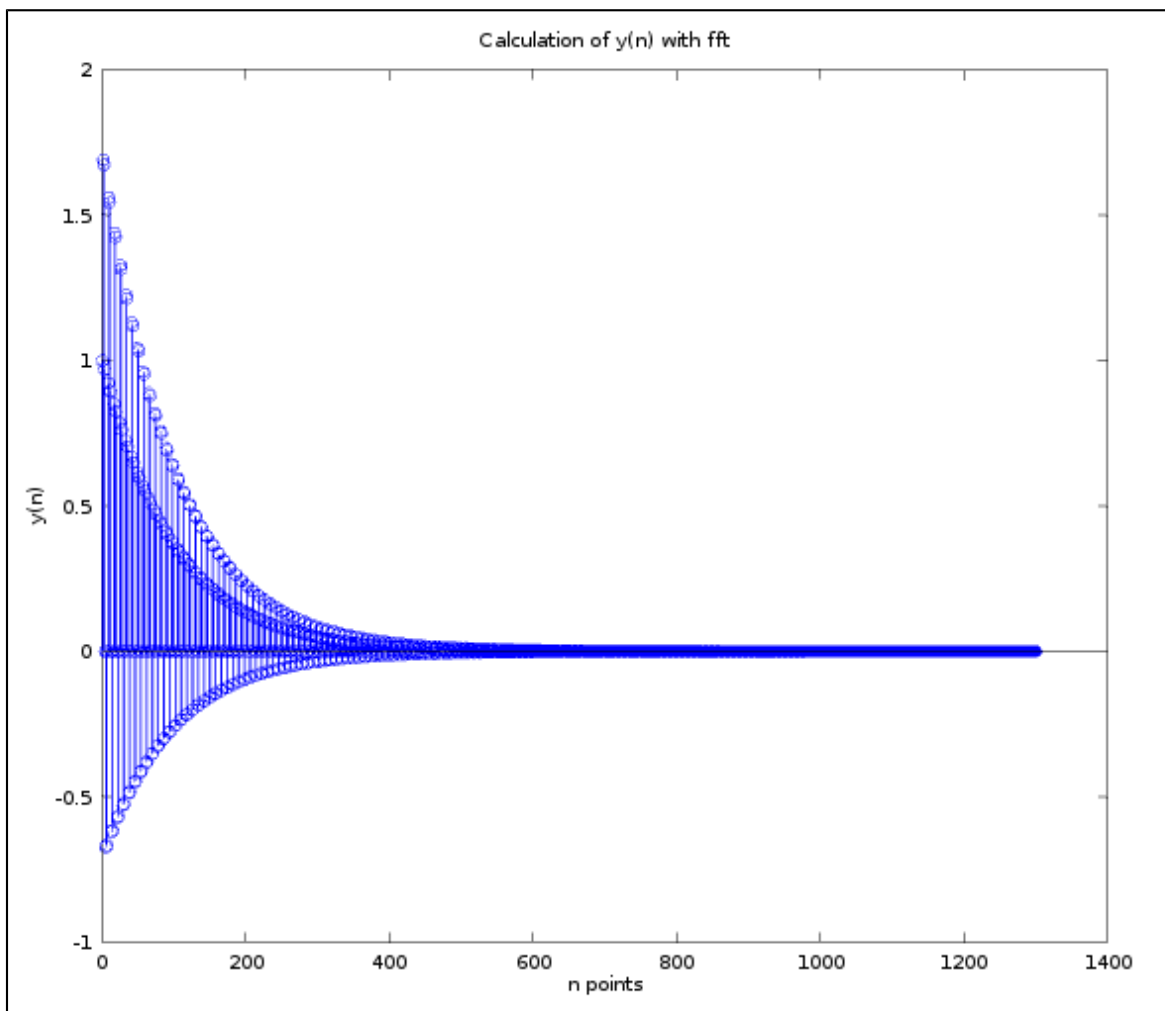


Figure 3: Calculation of $y(n)$ with fft

Discussion

In this computer exercise observe and comment how changes in the length of calculated $x(n)$ and $h(n)$ (e.g. experiment with $N=8, 16, 32, 64, 128$ and 256 samples), changes the calculated output $y(n)$.

By increasing N samples by the listed intervals, the output $y(n)$ followed suit by increasing in resolution. That is, the number of points received as output directly corresponded to the numbers used for N .

Justify difference between results obtained by `impz`, vs. `conv` vs. `fft` vs. your own `iir()` function, i.e.. for which samples in the result $y(n)$ will the discrepancies/errors be the highest and why?

The highest number of discrepancies came from the `fft` approach, as there were complex values appearing in the results. So many approximations and domain changes negatively affected the results. This also happened in the `conv()` method but not as severely. I attribute this to the taking the approximations with `impz()` while also transforming them again.

Which method(s) produce the result closest to the actual $y(n)$?

The methods that produce the results closest to the actual $y(n)$ is the `impz()` (Method 1).

Is there a K where both `conv` and `fft` provide the same result? Why?

The `conv()` and `ff()` functions will provide the same result if K is set to be 2750. This is because that is the number of points that were automatically generated when the `conv()` method was used.

Conclusion

For this project, I wrote several different functions to implement a Direct I filter and various approaches to calculating $y(n)$. My actual `iir()` function did not perform as intended due to difficulties with the indexing, but the other three functions worked as intended.

Appendix

Source Code

```
function [y_n] = iir (x_n)
    M = 2; #Adjusted to be one less than normal because of non-zero indexing of array
    N = 3;
    n = 0;

    #Calculating x(n)
    #a1 = [1, -.99];
    #b1 = 1;
    # [h1, t1] = impz(b1, a1);
    # x = h1;

    #Used for H(z) coefficients
    a = [1, -2*.99*cos(pi/4), .99^2];
    b = [1, -.99*cos(pi/4)];

    # x
    x_array = zeros(1,15);
    # y
    y_array = zeros(1,15);

    for n = 0:9 #Calculate terms from n = 0 to n = 10.

        x_sum = 0;
        y_sum = 0;

        ## Computing b * x(n-k) terms
        for k = 0:M # Adjusted loop iteration value
            x_array(k+1) = b(k+1) * x_n(n - (k-(2k+1))); #Must add 1 to adjust the indices (arrays not zero-indexed)
            x_sum = x_sum + x_array(k);
        end

        ## Computing a * y(n-k) terms
        for k = 1:N
            if n-k >= 0 #Check if next y(n-k) will be initial condition or not (aka: is n-k positive?)
                y_sum = a(k) * y_array(k); #Calculates bi * x(n-M). Required index adjustments
            end
        end

        y_array(n+1) = x_sum - y_sum;
        out = ['y(n) = ', num2str(y_array(n+1)), ', n = ', num2str(n)] ;
```



```
disp(out);  
disp(" ");  
  
end
```

```
endfunction
```

Program Output

The program does not output properly. I could not figure out the indexing after generating $x(n)$ for the function. See below:

```
iir(x)  
error: iir: subscript indices must be either positive integers less than 2^31 or logicals  
error: called from  
    iir at line 33 column 18
```