DSMA (M2021) EndSem – Group 1 – 20 Marks

1. (a) [3 Marks] Solve the following recurrence relation:

$S_n + 4S_{n-1} + 4S_{n-2} = 8$ for $n \geq 2$ with $S_0=1$ and $S_1=2$.

(b) [2+2 Marks] Let S be given by span{[1,2,2],[-1,0,2]}.

Find orthonormal basis of S and $S^{\perp}$ (orthogonal complement of S). [Span{x,y} indicates the space spanned by x and y]

2. [3 Marks] Let A = {1,2,4,5,10,15,20} and R be the partial ordering on A defined by aRb if a divides b.

(a) Determine the relational matrix(zero one matrix) for R

(b) Draw the Hasse diagram of the poset (A, R)

(c) Find its maximal, minimal elements. Is there a greatest and a least element?

3. [3 Marks] Let R be a relation on A = {1,2,3,4,5,6,7,8,9,10} defined by xRy if and only if $(x - y)$ is a multiple of 5.

(a) Verify whether R is an equivalence relation on A.

(b) Determine the equivalence classes and partition of A induced by R

4.(a)[3 Marks] Find the set of solutions for Ax = b where

$$A = \begin{bmatrix} 1 & 0 & 5 \\ 2 & 1 & 6 \\ 0 & 0 & 0 \end{bmatrix} \text{ and } b = \begin{bmatrix} 6 \\ 9 \\ 0 \end{bmatrix}$$

Is the set of solutions a vector space? Justify your answer.

(b) [4 Marks] Let V be the column space of A. Find projection matrix which projects a vector onto the orthogonal complement of V. {Note, the orthogonal complement of V contains vectors which are perpendicular to every vector in V}.

$$A = \begin{bmatrix} 3 & 1 \\ 2 & 1 \\ 3 & 4 \end{bmatrix}$$

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