

Verifica del Software - Esercizi parte 2

Università degli Studi di Padova

Mirko Bez

18 febbraio 2017

Indice

Esercizio 1	2
Esercizio 2	2
Esercizio 3	2
Esercizio 4	2
Esercizio 5	2
Esercizio 6	3
Esercizio 7	3
Esercizio 8	3
Esercizio 9	3

Esercizio 1

Consegna Let (α, C, A, γ) be a Galois connection. Prove that:

(A) γ is injective, \iff

(B) $\alpha \circ \gamma = id \iff$

(C) α is surjective.

Svolgimento

Esercizio 2

Consegna Let C and A be complete lattices and let (α, C, A, γ) be a Galois connection. Prove the following properties:

1. $\gamma(\alpha(\top_C)) = \top_C$
2. for any $a \in A, \gamma(a) = \bigwedge \{c \in C \mid \alpha(c) \leq_A a\}$
3. for any $c_1, c_2 \in C, \alpha(c_1 \vee_C c_2) = \alpha(c_1) \vee_A \alpha(c_2)$
4. for any $c \in C, \gamma(\alpha(\gamma(\alpha(c)))) = \gamma(\alpha(c))$

Svolgimento

Esercizio 3

Consegna Let C and A be complete lattices, (α, C, A, γ) be a Galois insertion, $op : C^2 \rightarrow C$ be a monotone concrete operation and $op^a : A^2 \rightarrow A$ be a monotone abstract operation. Prove the following equivalence:

$$\begin{aligned} \forall (a_1, a_2) \in A^2 . \alpha(op(\gamma(\alpha_1), \gamma(\alpha_2))) \leq_A op^a(a_1, a_2) \\ \iff \\ \forall (c_1, c_2) \in C^2 . op(c_1, c_2) \leq_C \gamma(op^a(\alpha(c_1), \alpha(c_2))) \end{aligned}$$

Svolgimento

Esercizio 4

Consegna Let $\langle C, \leq_C \rangle$ be a complete lattice and let $S \subseteq C$ be a subset of C which is meet-closed, that is:

$$\forall Y \subseteq S. \bigwedge_C Y \in S$$

Prove that $\langle S, \leq_C \rangle$ can be viewed as an abstract domain of C where the concretization map $\gamma : S \rightarrow C$ is the identity.

Svolgimento

Esercizio 5

Consegna Let C and A be complete lattices, (α, C, A, γ) be a Galois insertion, $f : C \rightarrow C$ be a monotone concrete operation and $f^\sharp : A \rightarrow A$ be a monotone abstract operation such that: $f \circ \gamma = \gamma \circ f^\sharp$. Prove that $\alpha(gfp(f)) = gf p(f^\sharp)$.

Svolgimento

Esercizio 6

Consegna Let C and A be complete lattices, (α, C, A, γ) be a Galois insertion, $f : C \rightarrow C$ be a monotone concrete operation and $f^\sharp : A \rightarrow A$ be a monotone abstract operation such that: $\alpha \circ f = f^\sharp \circ \alpha$.

1. Prove that $\alpha(lfp(f)) = lfp(f^\sharp)$.
2. Give a counterexample to the equality $lfp(f) = \gamma(lfp(f^\sharp))$.

Svolgimento

Esercizio 7

Consegna Let $(\alpha, \langle A, \leq_A \rangle, \langle \wp(Z), \subseteq \rangle, \gamma)$ be a Galois connection. Let $\mathbb{S}^A, Var \rightarrow A$ and consider the standard pointwise order \sqsubseteq between functions: $s_1^\sharp \sqsubseteq s_2^\sharp$ when for any $x \in Var$, $s_1^\sharp(x) \leq_A s_2^\sharp(x)$. Prove that $(\alpha_s, wp(State), \mathbb{S}^A, \gamma_s)$ is a Galois connection, where:

- $\alpha_s(T) \triangleq \lambda x. \alpha(\{s(x) \mid s \in T\})$
- $\gamma_s(s^\sharp) \triangleq \{s \in State \mid \forall x \in Var. \alpha(\{s(x)\}) \leq_A s^\sharp(x)\}$

Svolgimento

Esercizio 8

Consegna Consider the following abstract domain of $\langle \wp(\mathbb{Z}), \subseteq \rangle$

Svolgimento

Esercizio 9

Svolgimento