Verifica del Software - Esercizi parte 2

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Indice

Esercizio 1

Consegna *Let (α, C, A, γ) be a Galois connection. Prove that:*

(A) γ *is injective,* $\iff$

(B) $\alpha \circ \gamma = id$ $\iff$

(C) α *is surjective.*

Svolgimento

Esercizio 2

Consegna *Let C and A be complete lattices and let (α, C, A, γ) be a Galois connection. Prove the following properties:*

1. $\gamma(\alpha(\top_C)) = \top_C$
2. *for any* $a \in A, \gamma(a) = \wedge\{c \in C \mid \alpha(c) \leq_A a\}$
3. *for any* $c_1, c_2 \in C, \alpha(c_1 \vee_C c_2) = \alpha(c_1) \vee_A \alpha(c_2)$
4. *for any* $c \in C, \gamma(\alpha(\gamma(\alpha(c)))) = \gamma(\alpha(c))$

Svolgimento

Esercizio 3

Consegna *Let C and A be complete lattices, (α, C, A, γ) be a Galois insertion, $op : C^2 \to C$ be a monotone concrete operation and $op^a : A^2 \to A$ be a monotone abstract operation. Prove the following equivalence:*

$$\forall (a_1, a_2) \in A^2 \ . \ \alpha(op(\gamma(a_1), \gamma(a_2))) \leq_A op^a(a_1, a_2)$$
$$\iff$$
$$\forall (c_1, c_2) \in C^2 \ . \ op(c_1, c_2) \leq_C \gamma(op^a(\alpha(c_1), \alpha(c_2)))$$

Svolgimento

Definizione 1. *(Galois Insertion) Galois connection + $\forall a \in A \alpha(\gamma(a)) = a$*

Noto che C^2 e A^2 sono anch'essi dei reticoli completi:

- $(C \times C, \leq_{C^2}, \sqcup_{C^2}, \perp_{C^2}, \top_{C^2})$
- $(A \times A, \leq_{A^2}, \sqcup_{A^2}, \perp_{A^2}, \top_{A^2})$

Dimostrazione. Dimostro le due implicazioni separatamente.

- $\boxed{\implies}$ Assumo che $\forall (a_1, a_2) \in A^2 \ .\alpha(op(\gamma(a_1), \gamma(a_2))) \leq_A op^a(a_1, a_2)$.

  $op(c_1, c_2) \leq_C \gamma(\alpha(op(c_1, c_2)))$ Perchè $op(c_1, c_2) \in C$ e abbiamo una Galois connection

  Inoltre valgono le seguenti relazioni

  $$c_1 \leq_C \gamma(\alpha(c_1))$$
  $$c_2 \leq_C \gamma(\alpha(c_2))$$

Siccome op, α, γ sono monotone

$$
\begin{array}{ll}
op(c_1, c_2) \leq_C & \text{Perchè } op(c_1, c_2) \in C \text{ e abbiamo una Galois connection} \\
\gamma(\alpha(op(c_1, c_2))) \leq_C & \text{Per le relazioni precedenti+} op, \alpha, \gamma \text{ sono monotone} \\
\gamma(\alpha(op(\gamma(\alpha(c_1)), \gamma(\alpha(c_2))))) & \text{Per l'assunzione e la monotonicità di } \gamma \\
\gamma(op^A(\alpha(c_1), \alpha(c_2))) &
\end{array}
$$

- $\boxed{\Longleftarrow}$ Assumo che $\forall (c_1, c_2) \in C^2 \ . \ op(c_1, c_2) \leq_C \gamma(op^a(\alpha(c_1), \alpha(c_2)))$.

$$
\begin{array}{lcr}
\alpha(op(\gamma(a_1), \gamma(a_2))) & \leq_A & \text{Assunzione + monotonia di } \alpha \\
\alpha(\gamma(op^a(\alpha(\gamma(a_1)),\ \alpha(\gamma(a_2))))) & = & \text{GI, i.e. } \alpha \circ \gamma = id \\
op^a(\alpha(\gamma(a_1)),\ \alpha(\gamma(a_2))) & = & \alpha \circ \gamma = id \\
op^a(a_1, a_2) & &
\end{array}
$$

□

Esercizio 4

Consegna *Let $\langle C, \leq_C \rangle$ be a complete lattice and let $S \subseteq C$ be a subset of C which is meet-closed, that is:*

$$\forall Y \subseteq S. \wedge_C Y \in S$$

Prove that $\langle S, \leq_C \rangle$ can be viewed as an abstract domain of C where the concretization map $\gamma : S \to C$ is the identity.

Svolgimento

Esercizio 5

Consegna *Let C and A be complete lattices, (α, C, A, γ) be a Galois insertion, $f : C \to C$ be a monotone concrete operation and $f^\sharp : A \to A$ be a monotone abstract operation such that: $f \circ \gamma = \gamma \circ f^\sharp$. Prove that $\alpha(gfp(f)) = gfp(f^\sharp)$.*

Svolgimento

Esercizio 6

Consegna *Let C and A be complete lattices, (α, C, A, γ) be a Galois insertion, $f : C \to C$ be a monotone concrete operation and $f^\sharp : A \to A$ be a monotone abstract operation such that: $\alpha \circ f = f^\sharp \circ \alpha$.*

1. *Prove that $\alpha(lfp(f)) = lfp(f^\sharp)$.*
2. *Give a counterexample to the equality $lfp(f) = \gamma(lfp(f^\sharp))$.*

Svolgimento

Esercizio 7

Consegna *Let* $(\alpha, \langle A, \leq_A \rangle, \langle \wp(Z), \subseteq \rangle, \gamma)$ *be a Galois connection. Let Let* $\mathbb{S}^A$, $Var \to A$ *and consider the standard pointwise order* $\sqsupseteq$ *between functions:* $s_1^\sharp \sqsubseteq s_2^\sharp$ *when for any* $x \in Var$, $s_1^\sharp(x) \leq_A s_2^\sharp(x)$. *Prove that* $(\alpha_s, wp(State), \mathbb{S}^A, \gamma_s)$ *is a Galois connection, where:*

- $\alpha_s(T) \triangleq \lambda x.\alpha(\{s(x) | s \in T\})$
- $\gamma_s(s^\sharp) \triangleq \{s \in State | \forall x \in Var.\alpha(\{s(x)\}) \leq_A s^\sharp(x)\}$

Svolgimento

Esercizio 8

Consegna *Consider the following abstract domain of* $\langle \wp(\mathbb{Z}, \subseteq) \rangle$

Svolgimento

Esercizio 9

Svolgimento