# Verifica del Software - Esercizi parte 2

**Università degli Studi di Padova**

Mirko Bez

19 febbraio 2017

## Indice

## Esercizio 1

**Consegna** *Let $(\alpha, C, A, \gamma)$ be a Galois connection. Prove that:*

(A) $\gamma$ *is injective,* $\iff$

(B) $\alpha \circ \gamma = id \iff$

(C) $\alpha$ *is surjective.*

**Svolgimento**

## Esercizio 2

**Consegna** *Let $C$ and $A$ be complete lattices and let $(\alpha, C, A, \gamma)$ be a Galois connection. Prove the following properties:*

1. $\gamma(\alpha(\top_C)) = \top_C$
2. *for any* $a \in A, \gamma(a) = \wedge\{c \in C \mid \alpha(c) \leq_A a\}$
3. *for any* $c_1, c_2 \in C, \alpha(c_1 \vee_C c_2) = \alpha(c_1) \vee_A \alpha(c_2)$
4. *for any* $c \in C, \gamma(\alpha(\gamma(\alpha(c)))) = \gamma(\alpha(c))$

**Svolgimento**

## Esercizio 3

**Consegna** *Let $C$ and $A$ be complete lattices, $(\alpha, C, A, \gamma)$ be a Galois insertion, $op : C^2 \to C$ be a monotone concrete operation and $op^a : A_2 \to A$ be a monotone abstract operation. Prove the following equivalence:*

$$\begin{gathered}\forall (a_1, a_2) \in A^2 \ .\ \alpha(op(\gamma(\alpha_1), \gamma(\alpha_2))) \leq_A op^a(a_1, a_2) \\ \iff \\ \forall (c_1, c_2) \in C^2 \ .\ op(c_1, c_2) \leq_C \gamma(op^a(\alpha(c_1), \alpha(c_2)))\end{gathered}$$

**Svolgimento**

## Esercizio 4

**Consegna** *Let $\langle C, \leq_C \rangle$ be a complete lattice and let $S \subseteq C$ be a subset of $C$ which is meet-closed, that is:*

$$\forall Y \subseteq S. \wedge_C Y \in S$$

*Prove that $\langle S, \leq_C \rangle$ can be viewed as an abstract domain of $C$ where the concretization map $\gamma : S \to C$ is the identity.*

**Svolgimento** Osservo che $\gamma$ è montona. Consideriamo $A = \langle S, \leq_C \rangle$ (per un qualsiasi $S$ meet-closed) questo è chiaramente un poset. L'ordinamento parziale deve essere coerente con la funzione di concretizzazione $\gamma$:

$$x \leq_C y \iff \gamma(x) \leq_C \gamma(y)$$

che è valido banalmente in quanto $\gamma$ è la funzione identità. Sia $\alpha : C \to S$ una funzione qualsiasi. Dimostro che valgono le relazioni per soddisfare la Galois Adjunction:

$$\forall c \in C.\ \forall s \in S.\ \alpha(c) \leq_C s \iff c \leq_C \gamma(s)$$

*Dimostrazione.* Dimostro le due inclusioni separatamente:

- $\boxed{\implies}$
- $\boxed{\impliedby}$

□

# Esercizio 5

**Consegna** *Let $C$ and $A$ be complete lattices, $(\alpha, C, A, \gamma)$ be a Galois insertion, $f : C \to C$ be a monotone concrete operation and $f^\sharp : A \to A$ be a monotone abstract operation such that: $f \circ \gamma = \gamma \circ f^\sharp$ . Prove that $\alpha(gfp(f)) = gfp(f^\sharp)$.*

**Svolgimento**

# Esercizio 6

**Consegna** *Let $C$ and $A$ be complete lattices, $(\alpha, C, A, \gamma)$ be a Galois insertion, $f : C \to C$ be a monotone concrete operation and $f^\sharp : A \to A$ be a monotone abstract operation such that: $\alpha \circ f = f^\sharp \circ \alpha$.*

1. *Prove that $\alpha(lfp(f)) = lfp(f^\sharp)$.*
2. *Give a counterexample to the equality $lfp(f) = \gamma(lfp(f^\sharp))$.*

**Svolgimento**

# Esercizio 7

**Consegna** *Let $(\alpha, \langle A, \leq_A \rangle, \langle \wp(Z), \subseteq \rangle, \gamma)$ be a Galois connection. Let Let $\mathbb{S}^A$ , $Var \to A$ and consider the standard pointwise order $\sqsupseteq$ between functions: $s_1^\sharp \sqsubseteq s_2^\sharp$ when for any $x \in Var$ , $s_1^\sharp(x) \leq_A s_2^\sharp(x)$. Prove that $(\alpha_s, wp(State), \mathbb{S}^A, \gamma_s)$ is a Galois connection, where:*

- $\alpha_s(T) \triangleq \lambda x.\alpha(\{s(x) | s \in T\})$
- $\gamma_s(s^\sharp) \triangleq \{s \in State | \forall x \in Var.\alpha(\{s(x)\}) \leq_A s^\sharp(x)\}$

**Svolgimento**

# Esercizio 8

**Consegna** *Consider the following abstract domain of* $\langle \wp(\mathbb{Z}, \subseteq) \rangle$

**Svolgimento**

# Esercizio 9

**Svolgimento**