Verifica del Software - Esercizi parte 2

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Indice

Esercizio 1

Consegna   Let (α, C, A, γ) be a Galois connection. Prove that:

(A) γ is injective, $\Longleftrightarrow$

(B) $\alpha \circ \gamma = id$ $\Longleftrightarrow$

(C) α is surjective.

Svolgimento

Esercizio 2

Consegna   Let C and A be complete lattices and let (α, C, A, γ) be a Galois connection. Prove the following properties:

1. $\gamma(\alpha(\top_C)) = \top_C$

2. for any $a \in A, \gamma(a) = \wedge\{c \in C \mid \alpha(c) \leq_A a\}$

3. for any $c_1, c_2 \in C, \alpha(c_1 \vee_C c_2) = \alpha(c_1) \vee_A \alpha(c_2)$

4. for any $c \in C, \gamma(\alpha(\gamma(\alpha(c)))) = \gamma(\alpha(c))$

Svolgimento

Esercizio 3

Consegna   Let C and A be complete lattices, (α, C, A, γ) be a Galois insertion, $op : C^2 \to C$ be a monotone concrete operation and $op^a : A_2 \to A$ be a monotone abstract operation. Prove the following equivalence:

$$\forall(a_1, a_2) \in A^2 \ . \ \alpha(op(\gamma(\alpha_1), \gamma(\alpha_2))) \leq_A op^a(a_1, a_2)$$
$$\Longleftrightarrow$$
$$\forall(c_1, c_2) \in C^2 \ . \ op(c_1, c_2) \leq_C \gamma(op^a(\alpha(c_1), \alpha(c_2)))$$

Svolgimento

Esercizio 4

Consegna   Let $\langle C, \leq_C \rangle$ be a complete lattice and let $S \subseteq C$ be a subset of C which is meet-closed, that is:

$$\forall Y \subseteq S. \wedge_C Y \in S$$

Prove that $\langle S, \leq_C \rangle$ can be viewed as an abstract domain of C where the concretization map $\gamma : S \to C$ is the identity.

Svolgimento

Esercizio 5

Consegna *Let C and A be complete lattices, (α, C, A, γ) be a Galois insertion, $f : C \to C$ be a monotone concrete operation and $f^\sharp : A \to A$ be a monotone abstract operation such that: $f \circ \gamma = \gamma \circ f^\sharp$. Prove that $\alpha(gfp(f)) = gfp(f^\sharp)$.*

Svolgimento

Esercizio 6

Consegna *Let C and A be complete lattices, (α, C, A, γ) be a Galois insertion, $f : C \to C$ be a monotone concrete operation and $f^\sharp : A \to A$ be a monotone abstract operation such that: $\alpha \circ f = f^\sharp \circ \alpha$.*

1. *Prove that $\alpha(lfp(f)) = lfp(f^\sharp)$.*

2. *Give a counterexample to the equality $lfp(f) = \gamma(lfp(f^\sharp))$.*

Svolgimento

Esercizio 7

Consegna *Let $(\alpha, \langle A, \leq_A \rangle, \langle \wp(Z), \subseteq \rangle, \gamma)$ be a Galois connection. Let Let $\mathbb{S}^A$, $Var \to A$ and consider the standard pointwise order $\sqsupseteq$ between functions: $s_1^\sharp \sqsubseteq s_2^\sharp$ when for any $x \in Var$, $s_1^\sharp(x) \leq_A s_2^\sharp(x)$. Prove that $(\alpha_s, wp(State), \mathbb{S}^A, \gamma_s)$ is a Galois connection, where:*

- $\alpha_s(T) \triangleq \lambda x.\alpha(\{s(x) | s \in T\})$
- $\gamma_s(s^\sharp) \triangleq \{s \in State | \forall x \in Var.\alpha(\{s(x)\}) \leq_A s^\sharp(x)\}$

Svolgimento

Esercizio 8

Consegna *Consider the following abstract domain of $\langle \wp(\mathbb{Z}), \subseteq \rangle$*

Svolgimento

Esercizio 9

Svolgimento