Numerical Study of Electrohydrodynamic Floating Liquid Bridges

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Literature Review

- Laws governing the Electric field system as established by Melcher and Taylor [1]

$$\nabla \times E = 0$$
$$\nabla . D = q$$
$$\nabla . J + \frac{\partial q}{\partial t} = 0$$
$$D = \varepsilon_0 E + P$$

- Boundary Conditions at the interface of a Electric field system as established by Melcher and Taylor [1]

$$n.[D] = Q$$
$$n \times [E] = 0$$

- Hydrodynamic Equations as established by Melcher and Taylor [1]

$$\rho \frac{Dv}{Dt} = \rho g + \nabla . (T^m + T^e)$$
$$T^m_{ij} = \mu \left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right) - \delta_{ij} p$$
$$\nabla . v = 0$$

- Hydrodynamic Boundary conditions as established by Melcher and Taylor [1]

$$n[p] = n.[T^m + T^e]$$
$$n.[v] = 0$$

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Literature Review

- Provided the charge convection at the interface is negligible and the liquid has uniform current density and permittivity, volumetric electrical forces are not present [3]
- Tracer particles introduced in the liquid and ions originating at the electrodes can give free charge in the bulk and can lead to volume forces [2]
- According to Melcher [2,3]

Discontinuous Permittivity and Conductivity across interface

Discontinuity in Normal Electric Field across interface

Surface Charge Density at the interface

$$q_s = \varepsilon^a E_n^a - \varepsilon^b E_n^b = \varepsilon_0 \varepsilon_r^a E_n^a - \varepsilon_0 \varepsilon_r^b E_n^b$$

Coulomb’s force on the Surface Charge due to the tangential electric field

$$T_{xy} = q_s E_t$$

Electric Shear force is balanced by Viscous Shear stress leading to Fluid motion

$$q_s E_t = \mu \frac{\partial u_s}{\partial y}$$

Literature Review

- Yuan Lin [4] used a *leaky dielectric model* in COMSOL Multiphysics built using prebuilt physics interfaces, “electric current” and “two-phase laminar level set” to solve EHD phenomena in a single droplet in an uniform electric field.
- He also noted that, when using the dielectric model, it is the conductivity ratio between the two fluids that influences the solution rather than the absolute values of the conductivities.

- Feng and Scott [5] showed that the actual EHD behaviour becomes insensitive to the actual value of the conductivity ratio when one fluid is more conductive than the other by a factor of 100. Therefore, he concluded that it’s not necessary to measure the actual values of conductivities.

- Burcham and Saville numerically solved the problem of a liquid bridge of a leaky dielectric and they found a heterogenous distribution of charge at the bridge surface. This gave rise to a recirculating flow pattern within the bridge.
- Marin and Lohse [10] also noted that the electrical shear stresses generated at the surface of the bridge is quite unsteady and inhomogeneous. So, it gave rise to spatial and temporal variations in the velocity field. However, the temporal variations happen in the order of seconds.

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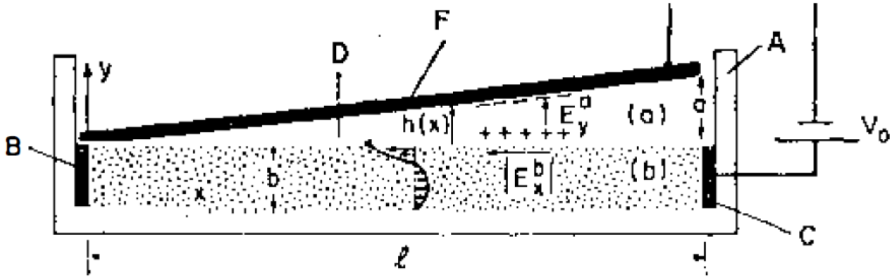
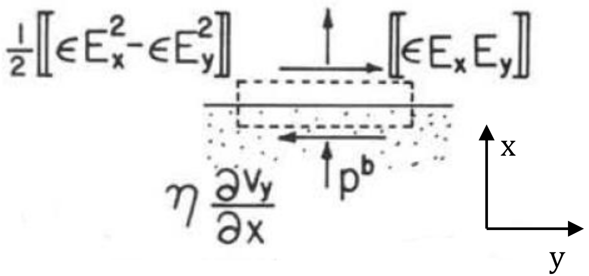
Taylor Cavity – Cellular Convection

Sketch of Taylor's experiment



Force Balance



Leaky Dielectric Model ⟶ Electric Currents Interface (ec) + Laminar Flow Interface (spf)

Interface Boundary Conditions ⟶ Slip Wall + Boundary Stress (Weak Contribution)

A	Insulating Container
B	Cathode
C	Anode
D	Interface
F	Slanted Electrode
(a)	Air
(b)	Corn Oil

Weak Expressions

Slip Wall ⟶ v = 0

Boundary Stress ⟶ F = ec.Dy*ec.Ex*test(u)

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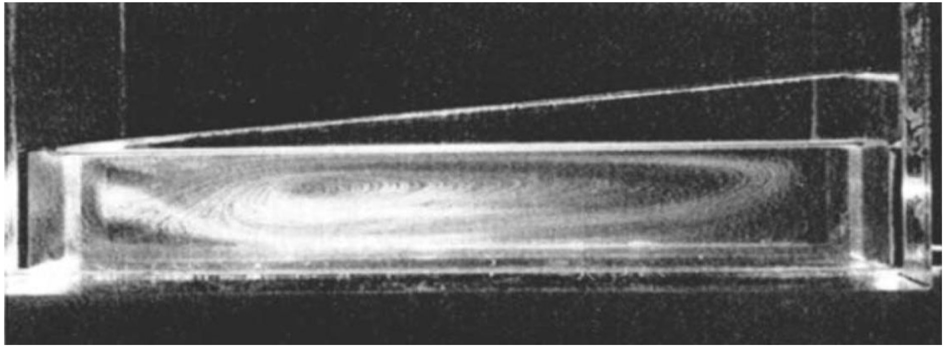
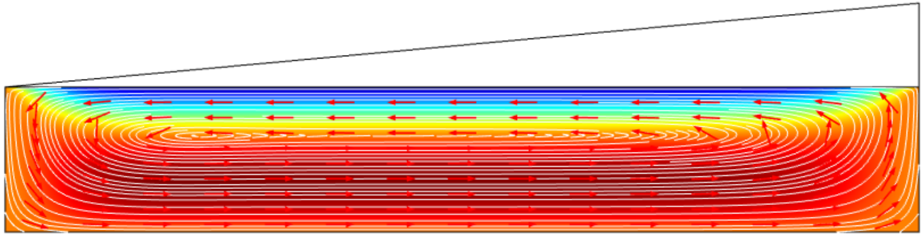
Qualitative Observation



Streak Photograph of Cellular Convection in Taylor Cavity

Unlike for a single droplet [5], the Conductivity ratio for the Taylor cavity should be above 1000 to observe the same EHD behaviour as in experiments



Horizontal Velocity Contour of Taylor Cavity

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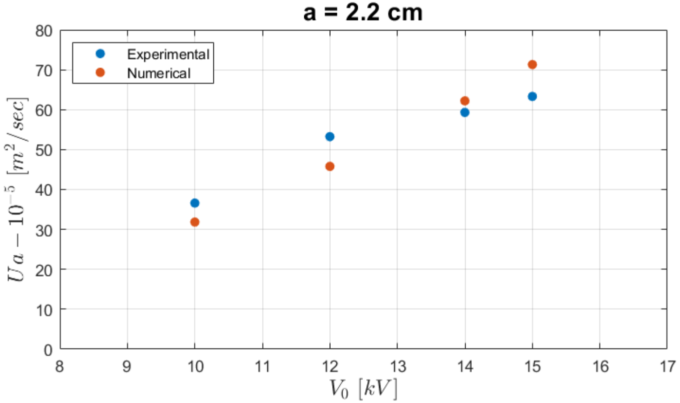
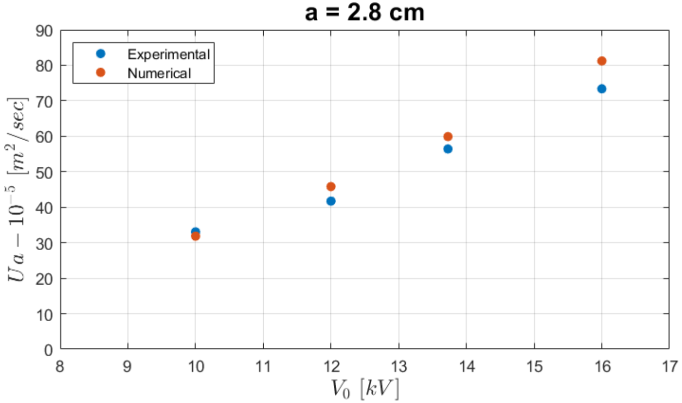
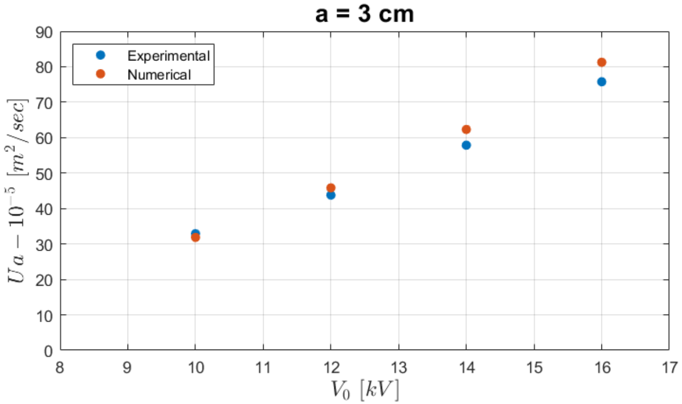
Quantitative Validation

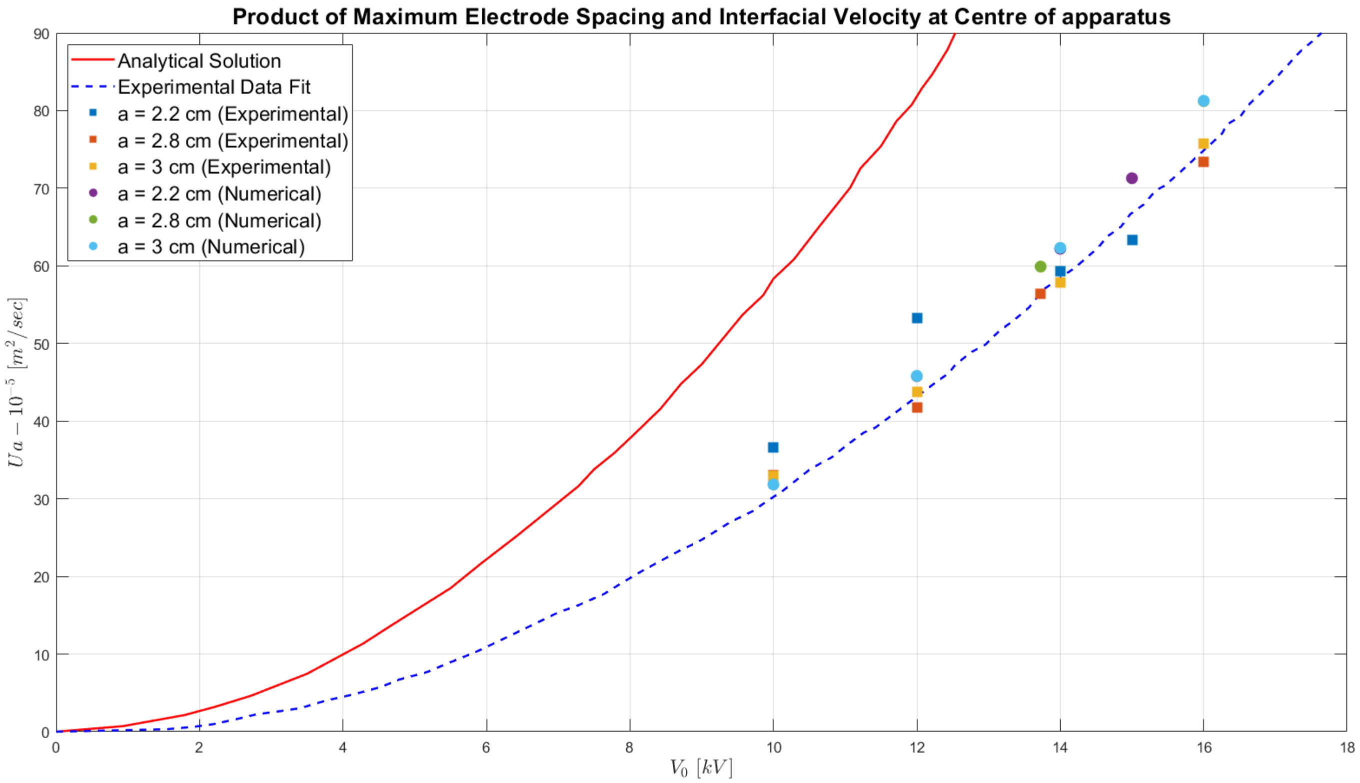






Comparison plots between Numerical and Experimental results of Taylor Cavity for different heights of the Slanted electrode

Quantitative Validation

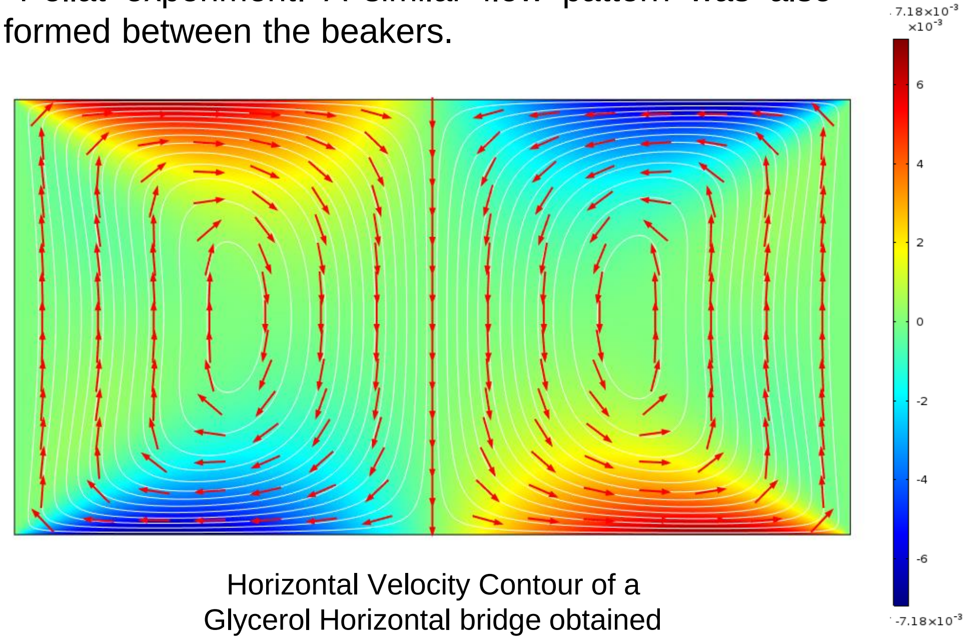


Experimental Data and Experimental Data Fit obtained from Melcher and Taylor [1]

Modified Pellat Experiment

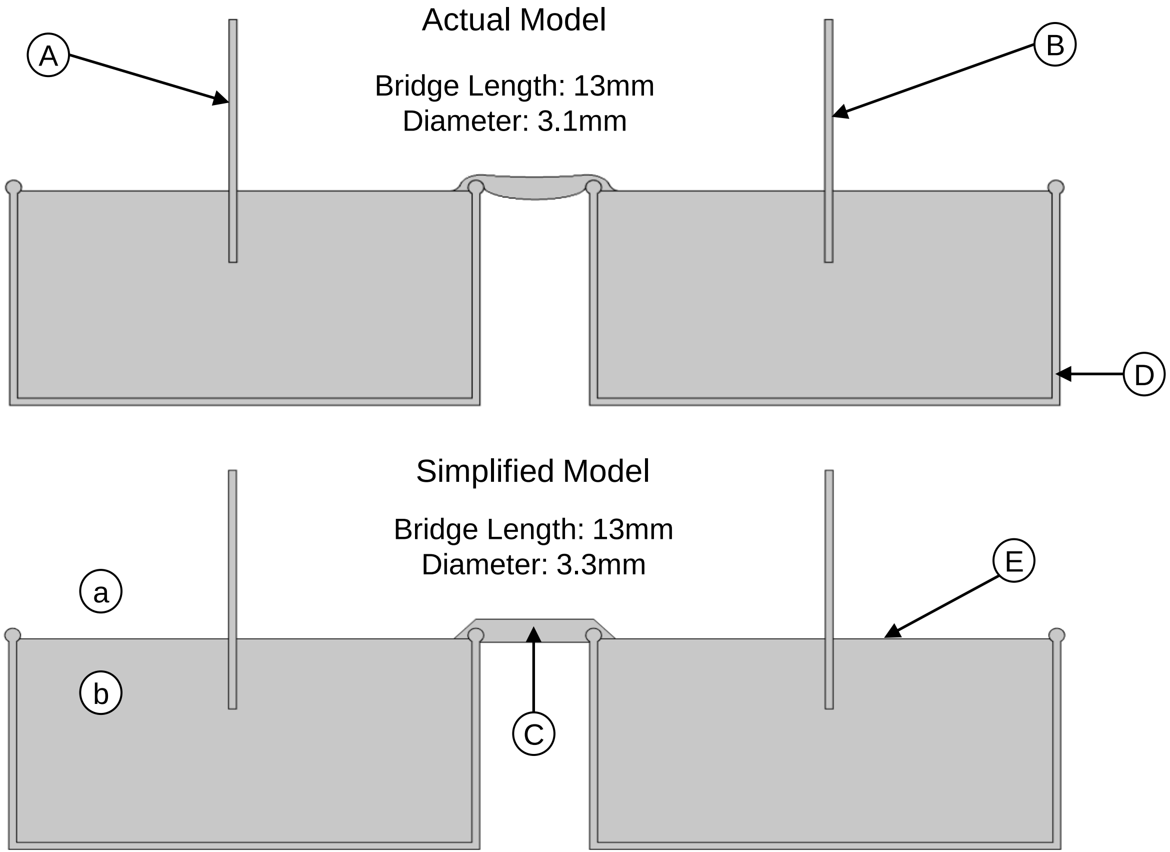
Jakob Woisetschläger et al. [6] observed cellular convection inside the horizontal bridges formed by the modified Pellat experiment. A similar flow pattern was also observed in the horizontal bridge formed between the beakers.

Horizontal Bridge formed by Pellat experiment using Glycerol between two electrodes as observed by Jakob Woisetschläger et al.



Horizontal Velocity Contour of a Glycerol Horizontal bridge obtained using COMSOL with Cathode at the left and Anode at the right for a potential difference of 18 kV DC

Liquid Bridges



Liquid Bridges – Numerical Setup

Leaky Dielectric Model → Electric Currents Interface (ec) + Laminar Flow Interface (spf)

Boundary Conditions

A	Cathode	→ Ground
B	Anode	→ Electric Potential
C	Liquid Bridge	
D	Beaker	→ Electric Insulation
E	Interface	→ Slip Wall + Boundary Stress (Weak Contribution)
a	Air	
b	Liquid	

Weak Expressions

T_{xy} → (ec.Dx*spf.nxmesh + ec.Dy*spf.nymesh + ec.Dz*spf.nzmesh)*(ec.Ey*spf.nxmesh)*test(v)

T_{xz} → (ec.Dx*spf.nxmesh + ec.Dy*spf.nymesh + ec.Dz*spf.nzmesh)*(ec.Ez*spf.nxmesh)*test(w)

T_{yx} → (ec.Dx*spf.nxmesh + ec.Dy*spf.nymesh + ec.Dz*spf.nzmesh)*(ec.Ex*spf.nymesh)*test(u)

T_{yz} → (ec.Dx*spf.nxmesh + ec.Dy*spf.nymesh + ec.Dz*spf.nzmesh)*(ec.Ez*spf.nymesh)*test(w)

T_{zx} → (ec.Dx*spf.nxmesh + ec.Dy*spf.nymesh + ec.Dz*spf.nzmesh)*(ec.Ex*spf.nzmesh)*test(u)

T_{zy} → (ec.Dx*spf.nxmesh + ec.Dy*spf.nymesh + ec.Dz*spf.nzmesh)*(ec.Ey*spf.nzmesh)*test(v)

Slip Wall → u*spf.nxmesh + v*spf.nymesh

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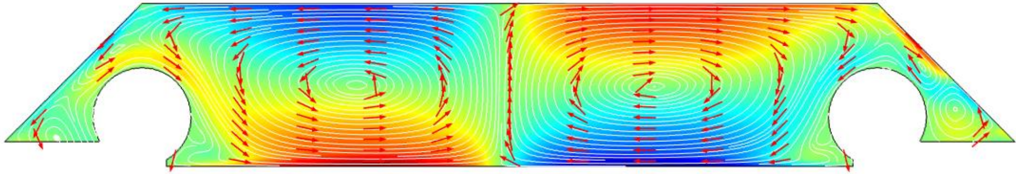
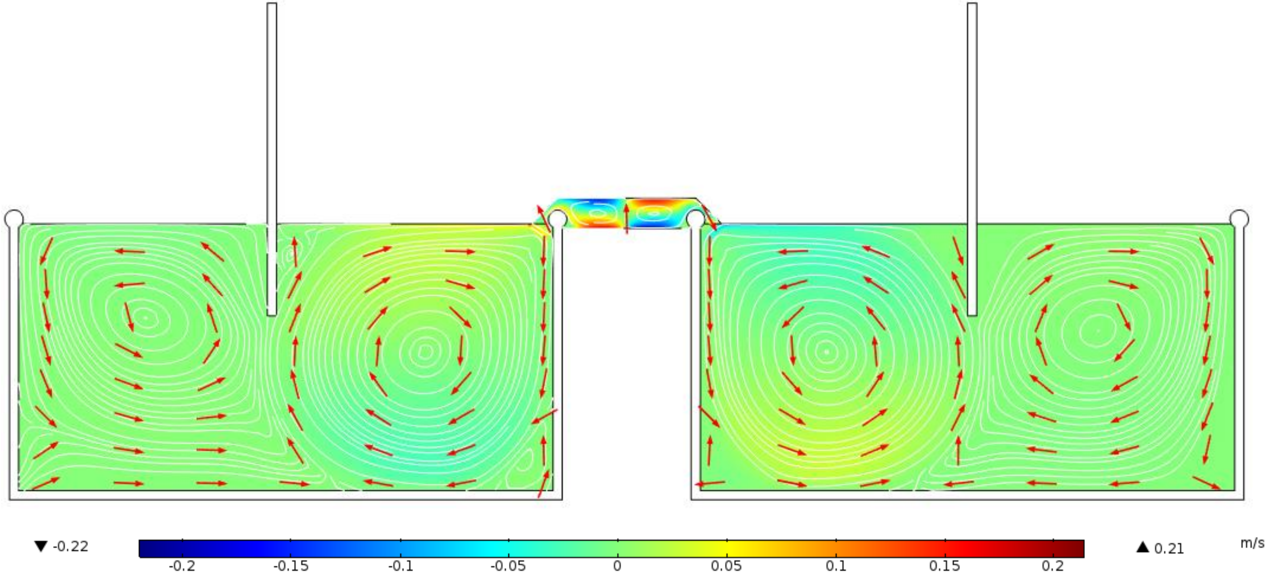
Simplified Bridge Initial Results

20 kV – Water Bridge



Presence of Cellular Convection similar to the one present in the Pellat Experiment and also a recirculating flow pattern as noted by Marin and Lohse [10]



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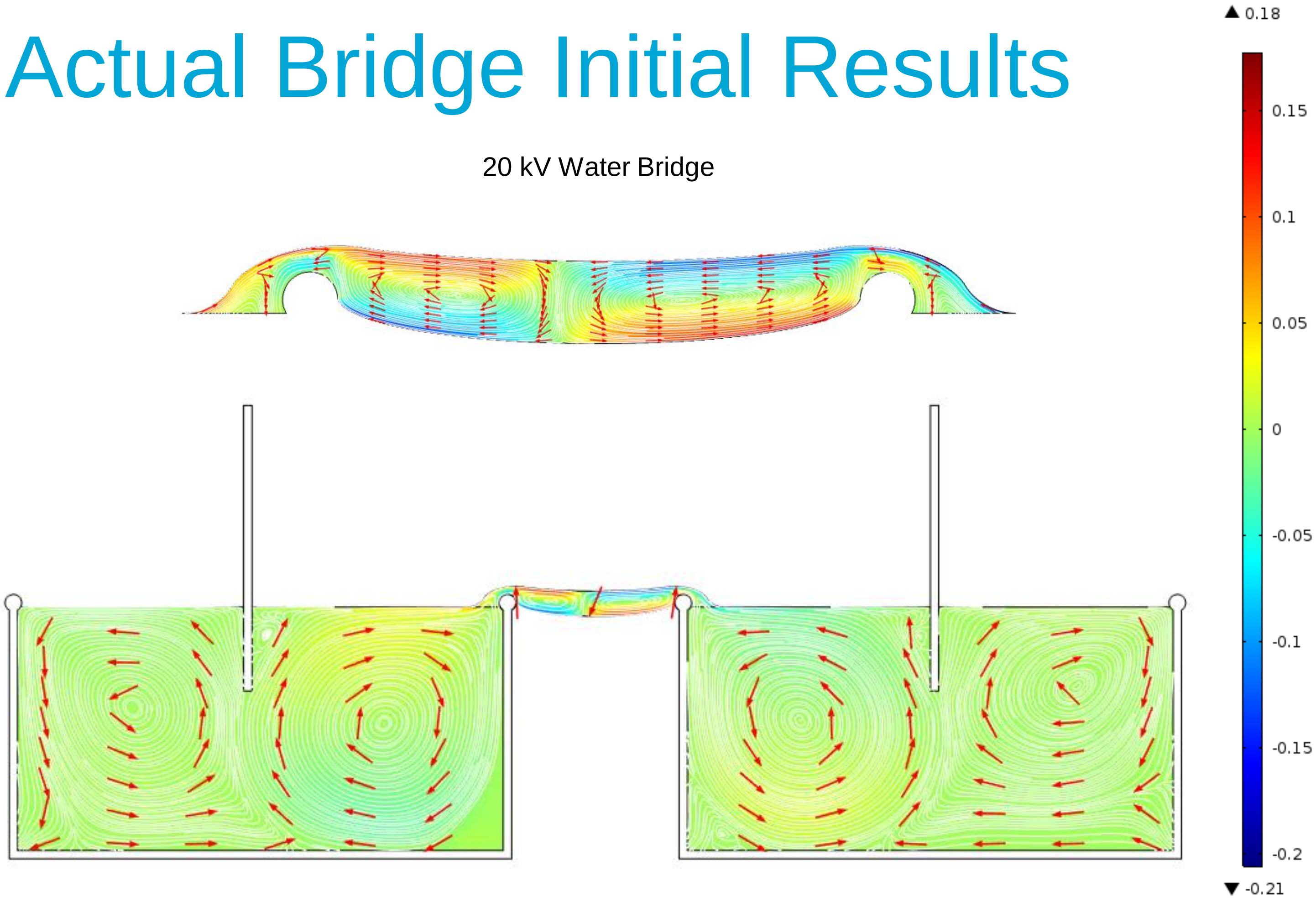
Actual Bridge Initial Results

20 kV Water Bridge



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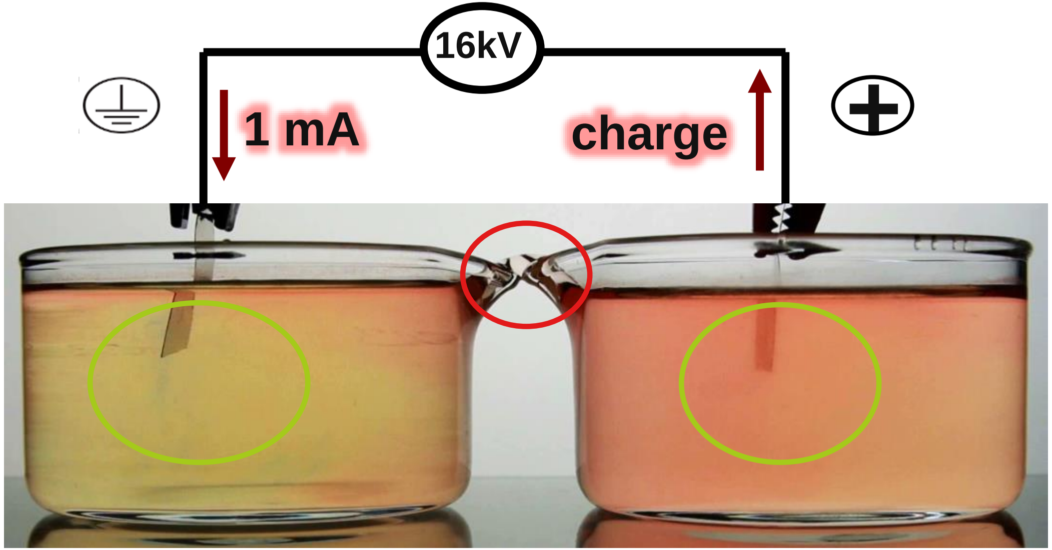
EHD Bridge Experiment



EHD flow regime	**– strong local field discontinuity along the electrode edge; Interfacial electric shear force dominates flow** **– low local field strength in the bulk**
Conduction flow regime	**– high local field strength (5kV/cm) in the bridge** **– proton channels do form in the p+ charged water, causing increased proton mobility**

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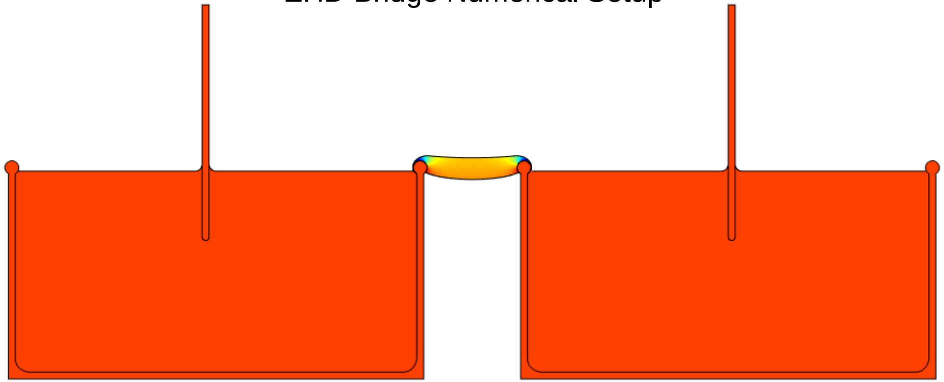
Observations

Horizontal Electric Field Strength in the EHD Bridge Numerical Setup



- As expected, the electric field strength is smaller in the beaker section and higher in the bridge section.
- However, the interfacial shear stresses at the air-water interface seem to be larger compared to the water-electrode interface.

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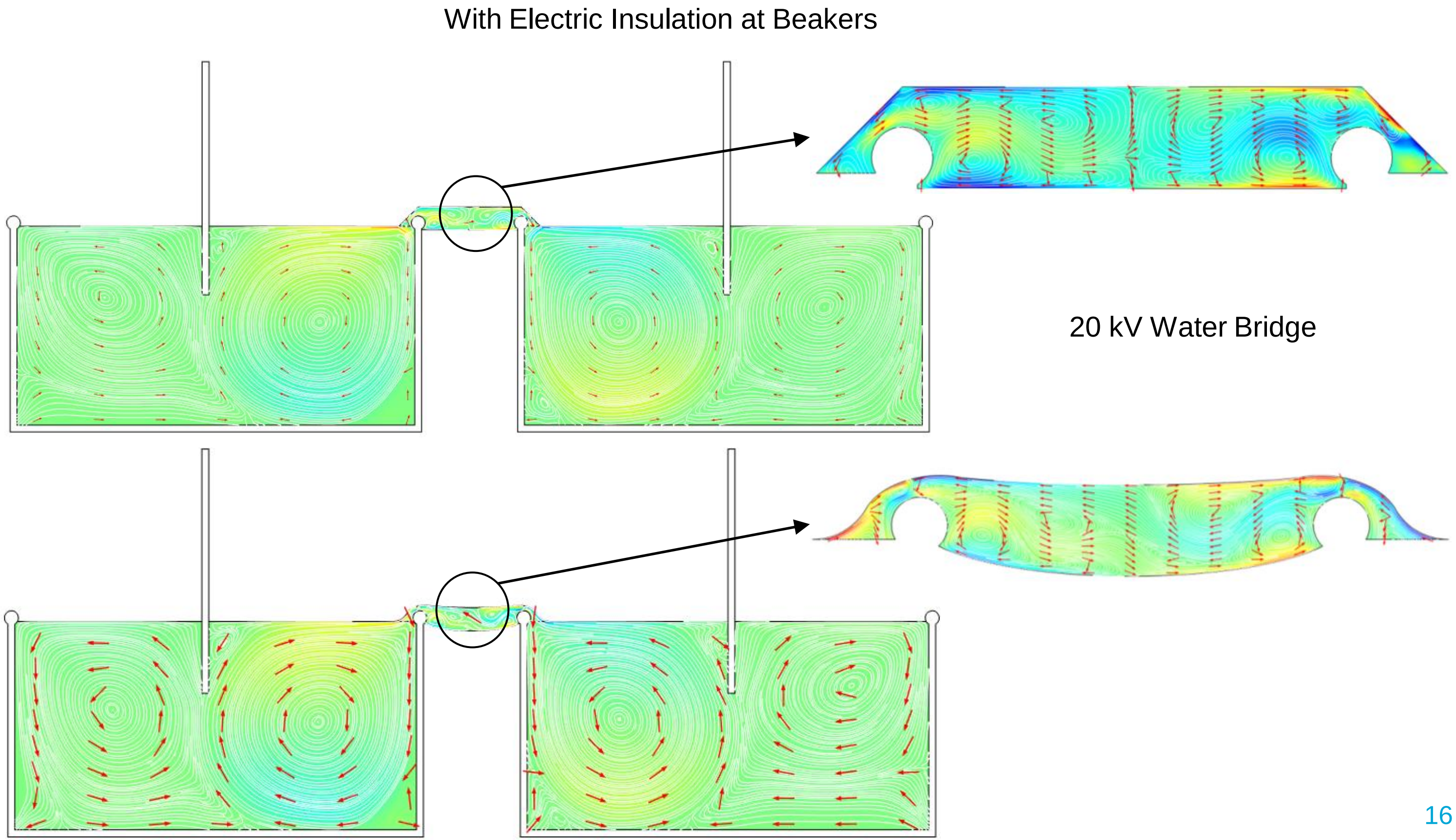
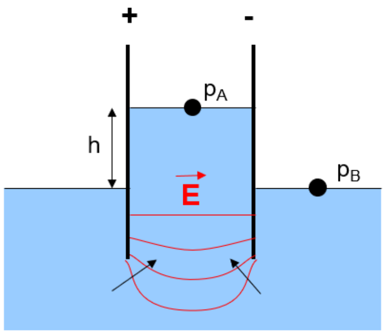
Corrected Flow Field Results



Dielectric Pressure in Pellat Experiment



- In a non-uniform electric field, the dipoles in a fluid experiences a net electric force stated as Kelvin Polarization force.
- The Kelvin Polarization force density results in a pressure gradient in the dielectric fluid inducing motion.

$$f_{kelvin} = -\nabla p = \frac{1}{2}\varepsilon_0(\varepsilon_r - 1)\nabla.\left(\vec{E}.\vec{E}\right)$$

- The analytical expression does not take into consideration the capillary effects due to surface tension forces.

- This dielectric pressure concept is first verified through simulations on a simple experiment like **Modified Pellat Experiment** before studying complex phenomenon like water bridges.

- This pressure gradient causes the fluid to rise between the two electrodes similar to capillary rise as shown here.
- The height risen by the fluid due to the dielectric pressure is calculated analytically using Bernoulli equation as,

$$p_A + \rho g h - \frac{1}{2}\varepsilon_0(\varepsilon_r - 1)E^2 = p_B$$

$$h = \frac{1}{2}\frac{\varepsilon_0(\varepsilon_r - 1)E^2}{\rho g}$$

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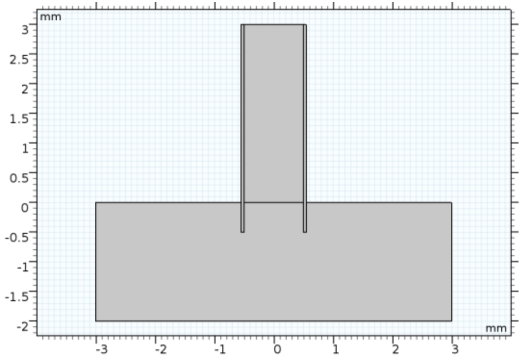
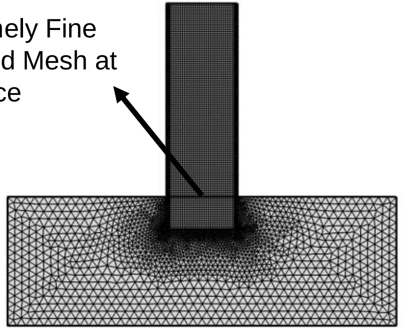
Simulation of Modified Pellat Experiment



Test Case Geometry → 2 Platinum Electrodes placed 1mm apart in a beaker containing dielectric fluid (water or glycerol)

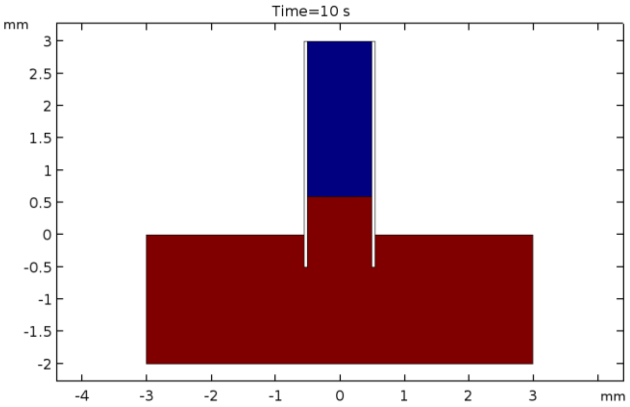
- Electric Potential (Anode) – 200V
- Electric Potential (Cathode) – 0V



Tracking Physics – 2 Phase Moving Mesh interface method

- Since physical interfaces are usually much thinner than practical mesh resolutions, this method offers the most accurate representation of the interface.
- Since, no additional transport equations are solved, the computational requirement is low than other interface tracking methods.

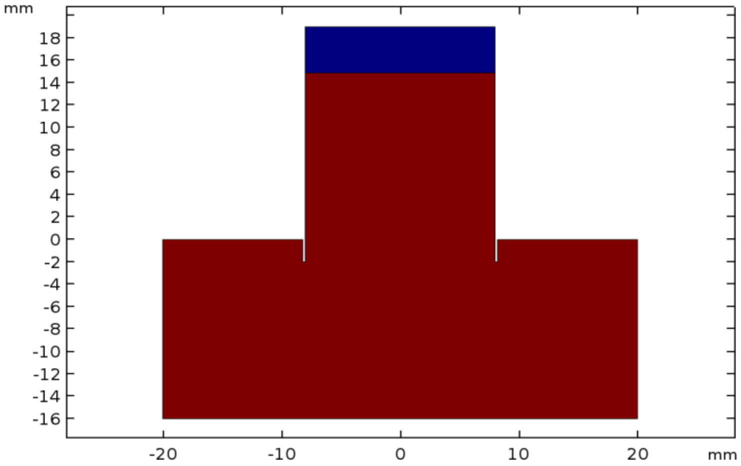
Simulation Results



Snapshot of the Final Interface position obtained using the TPFMM (Two-Phase flow moving mesh) method

Potential Difference (V)	Interface Height (mm)	
	Analytical	Numerical
200	0.595	0.593
300	1.339	1.338

Snapshot of the Final Interface position in the Modified Pellat Experiment where the Platinum electrodes are placed 16 mm apart



- Since the initial test case model is very small, the capillary effects would completely overwhelm the effects of dielectric pressure.
- Hence, the effects of surface tension has been neglected for this simulation.

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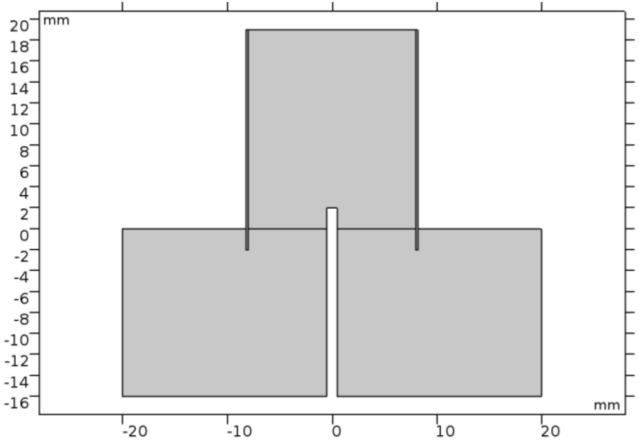

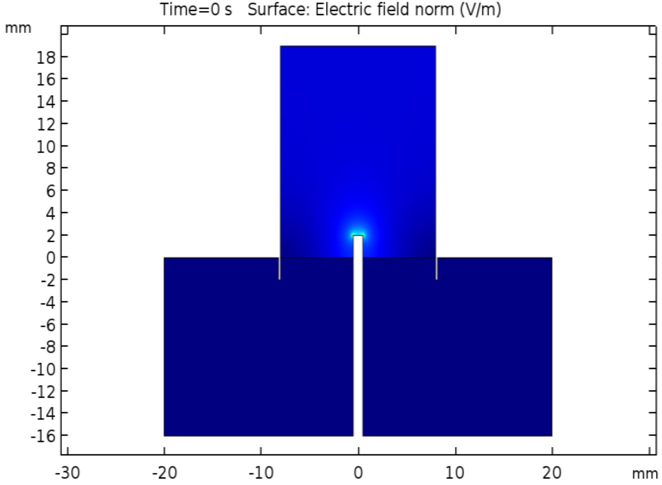

- In the previous simulations, a uniform electric field was maintained between the electrodes and non-uniform electric field was observed only at the edges of the electrode.
- In the case of liquid bridges, there's uniform electric field only in the beaker section and non-uniform electric field is observed in the edges of the beaker.
- As a test case for the actual water bridge model, a small obstacle is introduced between the two electrodes thereby creating a non-uniform electric field similar to that of a liquid bridge.
- This non-uniform electric field results in ***Kelvin Polarization Force density*** which induces a dielectric pressure on the fluid and causes it to rise above the obstacle and form a bridge similar to that of the liquid bridge experiment.

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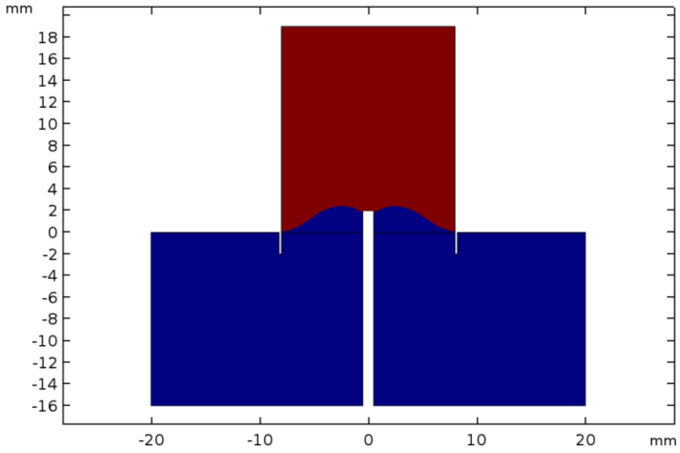
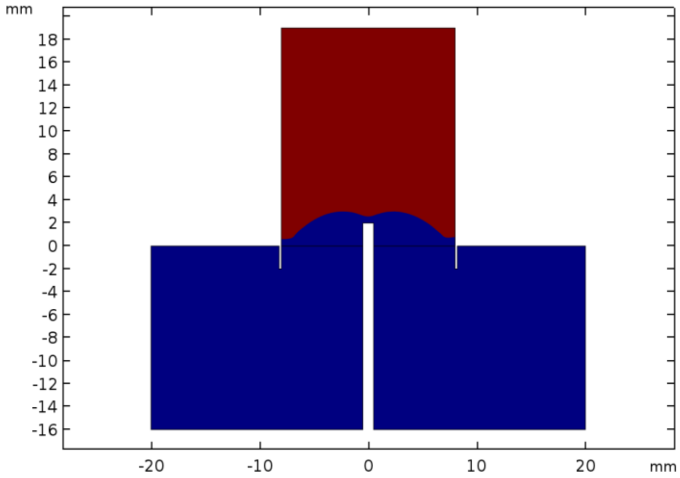
Snapshot of the Final Interface position obtained using the TPFMM method

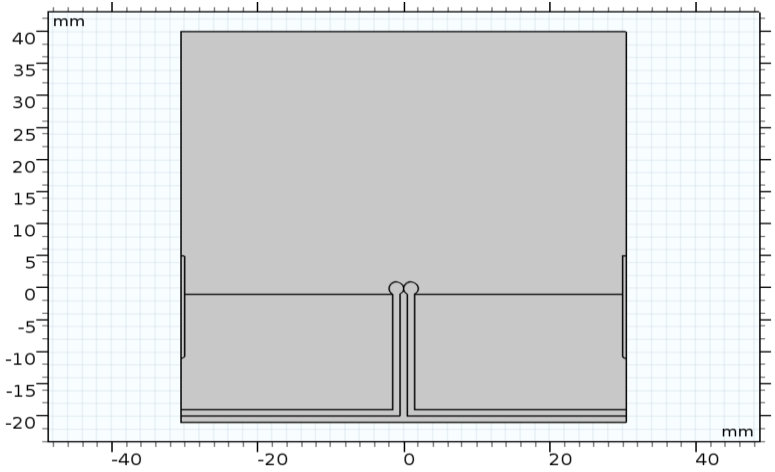




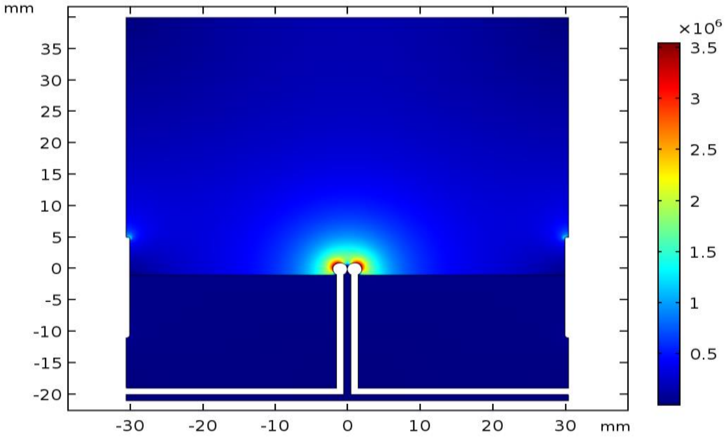
Snapshot of the Final Interface position obtained using the Level set method

- The numerical setup established earlier using the ***Moving Mesh*** method failed to simulate the formation of the liquid bridge since the method cannot be used to simulate the topological changes in a model.
- Therefore, the numerical setup was modified to include the ***Level Set*** method in which The interface is tracked using a colour function on a fixed mesh. The colour function is tracked by solving one additional transport equation for the Level set method
- All the fluid properties such as density and viscosity are scaled according to the colour function.
- The *Level set* method uses considerably more physics than the moving mesh method. Hence, it's less robust and more computationally expensive than the *Moving Mesh* method.
- Surface tension effects included

Simplified Domain of the Liquid bridge experiment



Electric Field Intensity plot of the simplified model of the Liquid Bridge experiment with high values at the edges of the beaker walls



- The high value of the Electric field intensity is along the edges of the beaker above the dielectric fluid.

$$f_{kelvin} = -\nabla p = \frac{1}{2}\varepsilon_0(\varepsilon_r - 1)\nabla.(\vec{E}.\vec{E})$$

- From the equation, it can be inferred that the ***Kelvin polarisation force density*** due to the change in Electric field intensity in air is zero (since the relative permittivity of air is 1).
- The force density only acts on the dielectric fluid along the walls of the beaker.
- This force density is applied on the dielectric fluid in a transient simulation where the time period is set at 5 seconds at which point the liquid bridge reaches a steady state.

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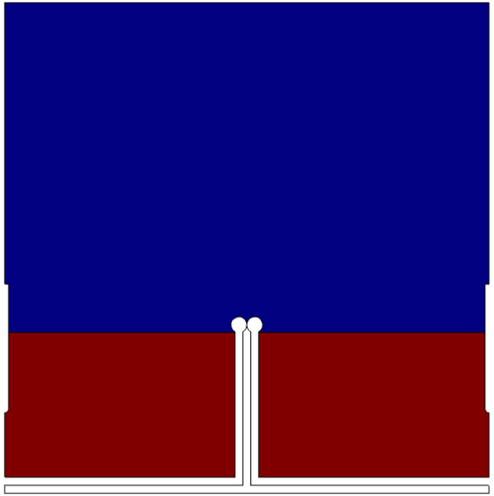
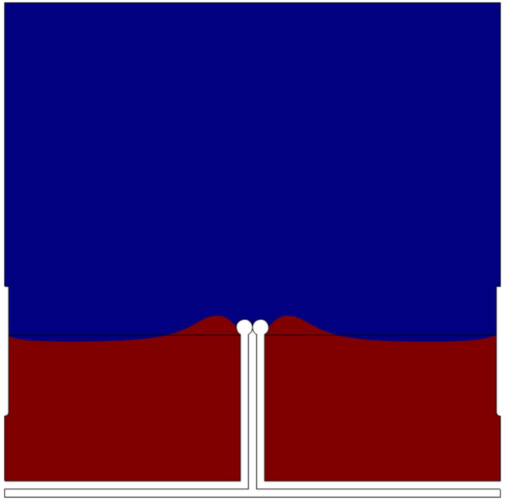
Snapshots of the formation of Liquid bridge at different times using the Level set method with red for Glycerol and blue for air

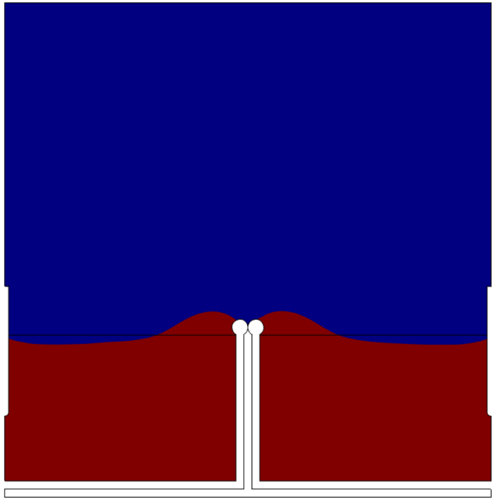
Time = 0 secs



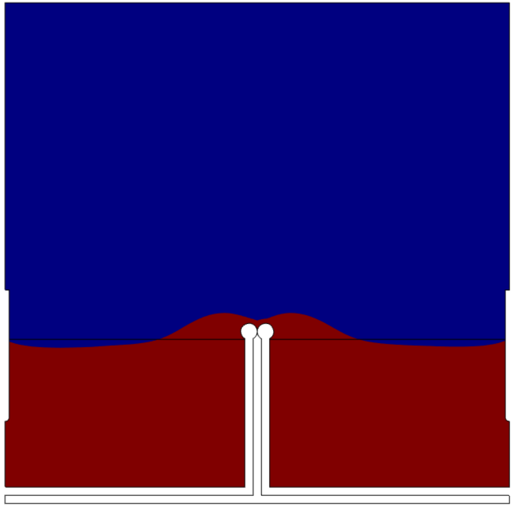
Time = 0.5 secs



Time = 0.05 secs



Time = 2 secs



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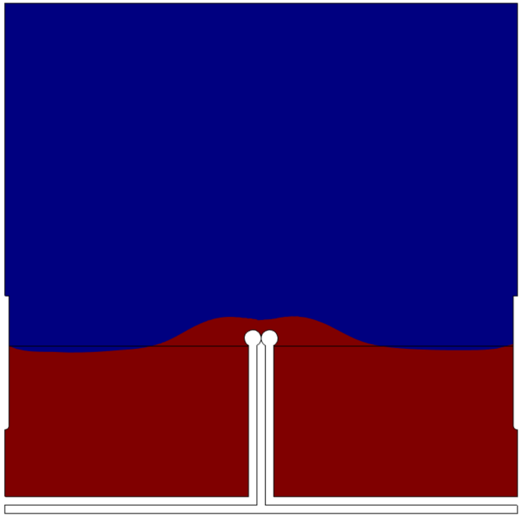
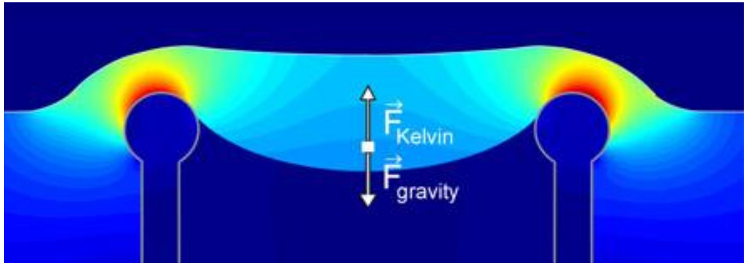
- The dielectric liquid within the beaker rises along the walls due to the force density applied.
- This force density counteracts the effects of gravity on the fluid.
- A liquid bridge forms at around 1.8 seconds and it reaches a steady state at around 4.5 seconds.

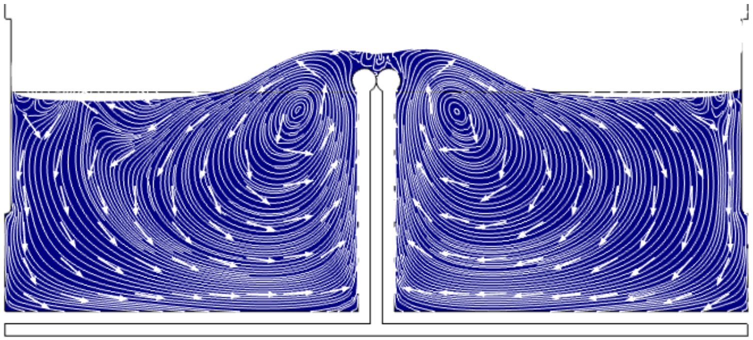


Snapshot of the Liquid bridge at t = 5 secs

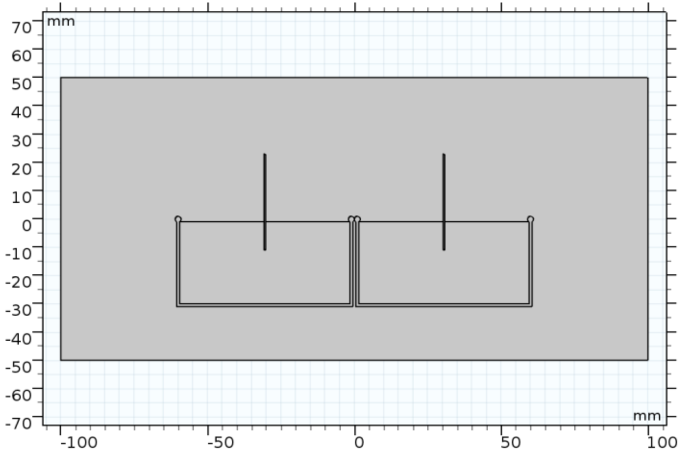


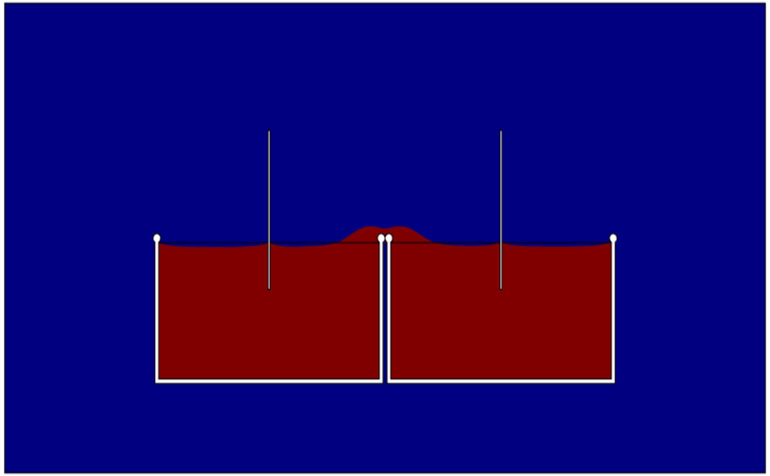
Representation of Kelvin Force counteracting Gravity within the Liquid bridge from the Numerical simulations done by Woisetschlager et al.

- This Kelvin Polarization force density is coupled to the interfacial shear force densities (which have been discussed before) acting on the interface between the dielectric fluid and air due to the discontinuity in the Electric field displacement field at the interface.
- These shear force densities are added as weak contribution along the interface.
- The charge convection along the interface is also assumed as negligible.

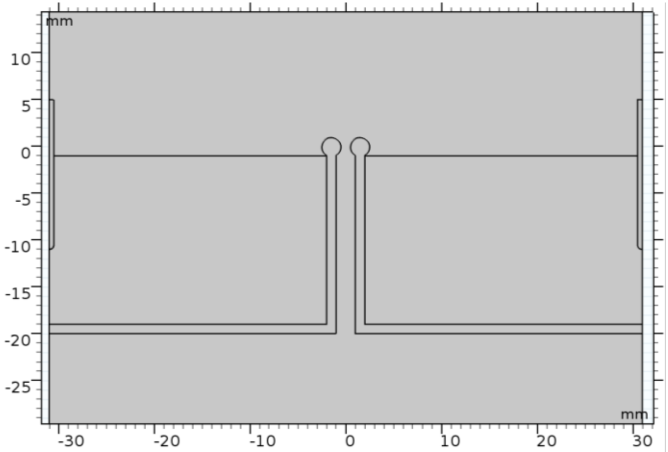


Velocity field in the Liquid bridge at t = 5 secs illustrated using Streamlines and Arrows with blue indicating Glycerol

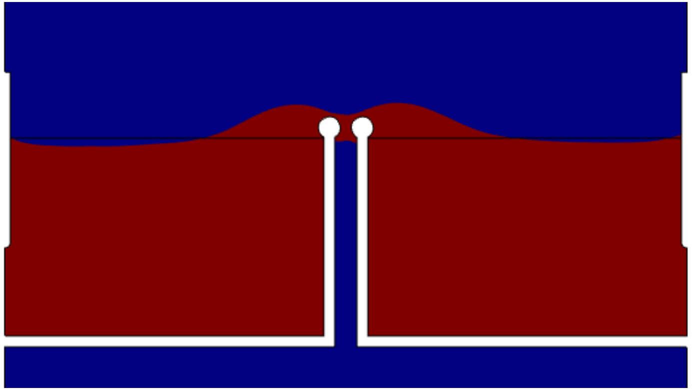




- The flow field in the beaker is similar to the one seen in the experiments.
- Initially, the flow field in the beaker is chaotic with several vortices in each beaker. As the flow field reaches a steady state, the secondary vortices disappear resulting in a single vortex in each beaker.
- Now that the formation of liquid bridge due to the effects of the force density has been simulated, the beakers are slightly moved apart to see whether the *Kelvin Polarization force density* can support the liquid bridge against the effects of gravity between the walls of the beaker



Simplified Domain of the Liquid bridge experiment where the beakers are placed 1mm apart



Snapshot of the Liquid bridge balanced against gravity when the beakers are placed 1mm apart

- The beakers are moved 1 mm apart.
- The potential difference between the electrodes is maintained at 20 kV as in the previous experiments.
- It can be seen that the liquid bridge is balanced against the effects of gravity by the applied force density
- We can also see that the liquid bridge slightly sags due to its weight.
- This results in distortions in the Electric field lines within the bridge.
- When the fluid tends to drip this distortion will counteract the dripping and stabilize the liquid under gravity through the gradient of the dielectric pressure and the resulting force density.

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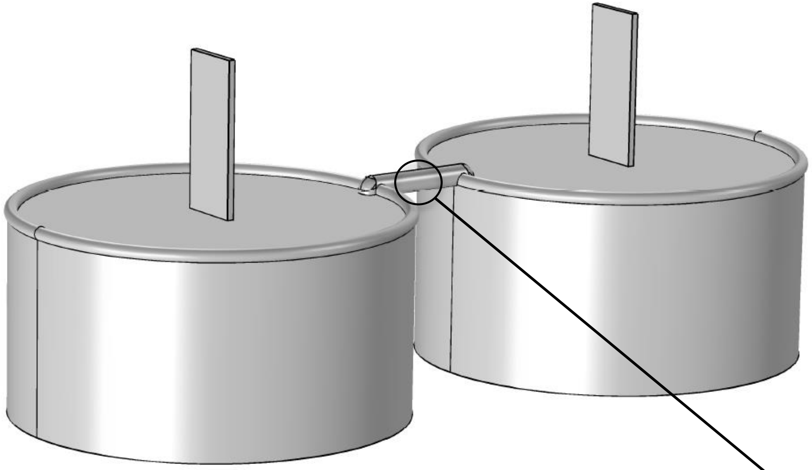
Future Work

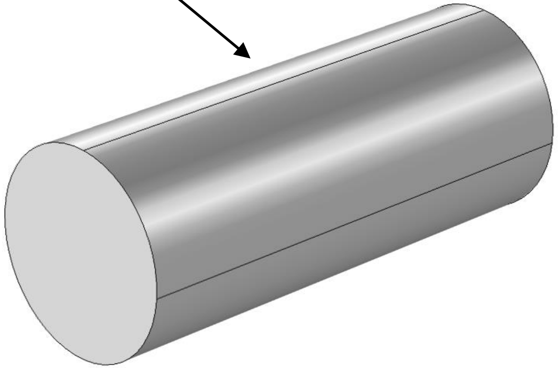


Simulations yet to be done for 3D after verifying the results for 2D liquid bridges

Geometry → PTC Creo

Livelink Interface ↓

Analysis → COMSOL Multiphysics



Test Case

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