

Frankle Revisited
(Finite Familys with finite Members)

$A^s, B^s, C^s, \dots Z^s :=$ Set of Set's.

$A, B, C, \dots Z :=$ Set. ex. D is member of A^s .

$a, b, c, \dots z :=$ Building Block. ex. e is member of A.

A Family, M^s is Set such that:

$$\forall A, B \in M^s : A \cup B \in M^s$$

Each Family has a Basis. Meaning there are Members of a Family that can not be „created“ by others. This members „created“ all other Elements by uniting with each other and/or uniting with other Set's of the Family (Elements of that Family). The Basis of a Family M^s we call $B^s = \beta(M^s)$. To create a Family u don't need to start with a Basis. The Basis is just the Smallest Set of Sets u need to create a given Family. $\varphi(F^s) = M^s$. Where M^s is the Family which emerges from a Set of Sets F^s .

$$\beta(F^s) \subseteq K^s \wedge \forall A \in K^s : A \in F^s \Rightarrow \varphi(K^s) = F^s$$

For an building Block to be abundant it means it is in half or more of the Sets in M^s . It does not need to be in half or more of the Sets of the Basis of that Family. But if that's the case inevitably that Building Block is abundant in that Family. In the Extrem there is only the Basis. Meaning there are no other Elements in that Familie.

In this case it means:

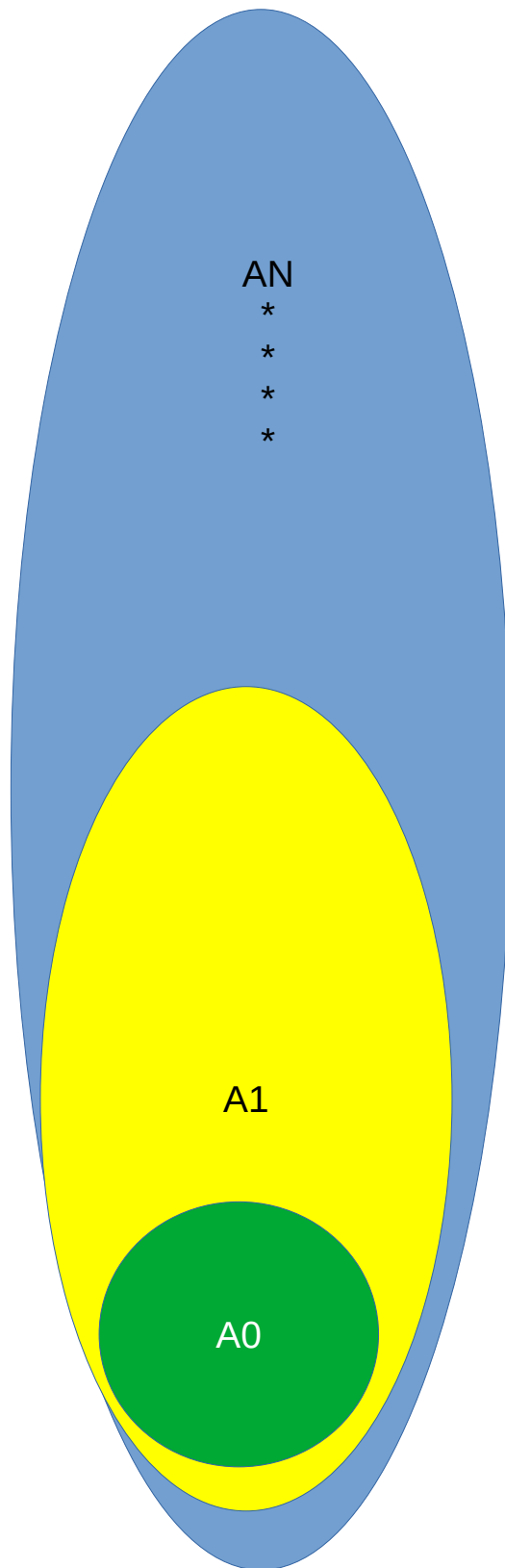
$$\forall A_i \in M^s \exists A_j \in M^s : A_i \subset A_j \vee A_j \subset A_i$$

$$M^s := \{A_1, A_2, A_3, \dots, A_n\}$$

Venn-Diagramm

Extrem
Streamlined Basis.

*



If $|M^s| = n$ then there are exactly $n(n-2)/2$ subset relations and i call that case a maximal absorbing base.

If we define $r > s \Rightarrow A_r$ is a superset of A_s .

Than in that extrem case only the member of the A_k 's are abundant. Where k is smaller or equal to: $(n/2)+1$ (rounding down).

The other Extrem case is when the Basis $\beta(F^s)=B^s$ has only paarwise completely distinct members.

Such that:

$$\forall A_i, A_j \in \beta(F^s) : A_i \neq A_j \Rightarrow A_i \cap A_j = \emptyset.$$

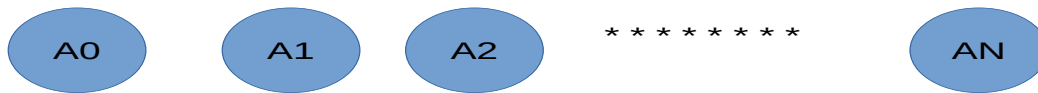
That case is the case of a extrem parallel Basis. The resulting Familie is of Size: β

$$S = 2^{|\beta(F^s)|} - 1$$

Every Building Block is abundant in that Family.

It is the case the more subset relations a Basis has, among it' members, the smaller the resulting Family. Also the lesser the number of Sets that have abundant elements.

Venn-Diagramm Extrem Parallel Basis.



Induction

Given a Set Z , and a Family ${}_1M^s$. Z is defined not to be a Member of ${}_1M^s$. If one extends ${}_1M^s$ with Z . You get another Set of Sets K^s . K^s is not necessarily a Family. But if we define:

$${}_2M^s = \varphi \left({}_1M^s \cup \{ Z \} \right)$$

${}_2M^s$ is a Family. Now we show that regardless how u choose Z the abundance state of Elements of ${}_1M^s$ is preserved to ${}_2M^s$ or new

abundance is created. Given that ${}_1M^s$ has some abundant Building Blocks. If i am correct that means Frankles Conjecture is right.

Here we go:

A.) Z contains Elements that are abundant in ${}_1M^s$. Those Elements are in every Set which is not in ${}_1M^s$ but in ${}_2M^s$. So half or more of ${}_1M^s$ has them and the „new“ ones too. Abundance of those is preserved.

B.) Z is parallel to ${}_1M^s$. Meaning the intersection of any Set of ${}_1M^s$ with Z is empty. That means every Union of Set of ${}_1M^s$ with Z is unique. There can't be „Collisions“.

$$|{}_2M^s| = 2|{}_1M^s| + 1.$$

Every element of Z is abundant in ${}_2M^s$. Also abundance is preserved.

C.) Z contains only Elements which are not abundant in ${}_1M^s$. Let's call the Set of Set's that contain abundant Elements in ${}_1M^s$, ${}_aM^s$.

$$\text{Further } {}_bM^s = {}_1M^s \setminus {}_aM^s.$$

$${}_aM^s \supseteq {}_bM^s.$$

$$\varphi({}_aM^s) \supseteq \varphi({}_bM^s).$$

${}_aM^s$ „creates“ a bigger or equal big Family then ${}_bM^s$ because it is emerging Family has less or equal subset relations in it's Basis. Thank's to the abundant Building Blocks of ${}_1M^s$ (Need's to be proven). Also ${}_aM^s$ contains **all** Set's that contain abundant Elements of ${}_1M^s$. And that means $\varphi({}_aM^s \cup \{Z\}) \supseteq \varphi({}_bM^s \cup \{Z\})$.

$$\varphi({}_aM^s \cup {}_bM^s \cup \{Z\}) = {}_2M^s.$$

Which means Abundance of Building Blocks in ${}_1M^s$ is preserved in ${}_2M^s$.