Midterm Exam

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Exercise 1

For the live survey estimate, $\hat{\mu}_L$: $\mathbb{E}(\hat{\mu}_L) = a\mu + b$, $VAR(\hat{\mu}_L) = \left(\frac{a^2}{N}\right)\mu(1-\mu)$.

For the IVR survey estimate, $\hat{\mu}_I$: $\mathbb{E}(\hat{\mu}_I) = c\mu + d$, $VAR(\hat{\mu}_I) = \left(\frac{c^2}{N}\right)\mu(1-\mu)$.

Assume that 0 < a < c, 0 < d < b, and $\frac{b-d}{c-a} > 1$. $\mu \in (0,1)$.

(a) What is $BIAS(\hat{\mu}_L)$? What is $BIAS(\hat{\mu}_I)$. Let $i \in \{L, I\}$.

$$BIAS(\hat{\mu}_i, \mu) = \mathbb{E}_{x|\mu}[\hat{\mu}_i] - \mu$$

$$BIAS(\hat{\mu}_L, \mu) = a\mu + b - \mu = \mu(a-1) + b$$

$$BIAS(\hat{\mu}_I, \mu) = c\mu + d - \mu = \mu(c-1) + d$$

Show that $BIAS(\hat{\mu}_L) > BIAS(\hat{\mu}_I)$.

$$\begin{split} \mu(a-1) + b &> \mu(c-1) + d \\ b - d &> \mu(c-1) - \mu(a-1) \\ b - d &> \mu(c-1-a+1) \\ b - d &> \mu(c-a) \\ \frac{b-d}{c-a} &> \mu, \end{split}$$

which is true given that $\mu \in (0,1)$ and $\frac{b-d}{c-a} > 1$ by the statement of the problem, hence, $BIAS(\hat{\mu}_L) > BIAS(\hat{\mu}_I)$, Q.E.D.

(b) Show (trivially) that $VAR(\hat{\mu}_I) > VAR(\hat{\mu}_L)$.

$$VAR(\hat{\mu}_I) > VAR(\hat{\mu}_L)$$

$$\left(\frac{c^2}{N}\right)\mu(1-\mu) > \left(\frac{a^2}{N}\right)\mu(1-\mu)$$

$$c^2 > a^2,$$

which is true given that 0 < a < c by the statement of the problem, hance, $VAR(\hat{\mu}_I) > VAR(\hat{\mu}_I)$, Q.E.D.

(c) An estimator $\hat{\theta}$ is unbiased if $\mathbb{E}(\hat{\theta}) \to \theta$, i.e. the expected value of an estimator converges to the population parameter. An estimator $\hat{\theta}$ is more efficient than another estimator $\tilde{\theta}$ if the variance of $\hat{\theta}$ is less than the variance of $\tilde{\theta}$, i.e. $\hat{\theta}$ returns estimates closer to θ than $\tilde{\theta}$. Since $VAR(\hat{\mu}_I) > VAR(\hat{\mu}_L)$, the estimate $\hat{\mu}_L$ is more efficient than $\hat{\mu}_I$ but it is more biased than $\hat{\mu}_I$, since $BIAS(\hat{\mu}_L) > BIAS(\hat{\mu}_I)$.

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(d) $MSE(\hat{\mu}_i) \equiv VAR(\hat{\mu}_i) + [BIAS(\hat{\mu}_i)]^2$. Consider the case where $a = .9, b = .2, c = 1, d = .05, \mu = .5, N = 1,000.$

$$MSE(\hat{\mu_L}) > MSE(\hat{\mu_I})$$

$$\left(\frac{a^2}{N}\right)\mu(1-\mu) + [\mu(a-1)+b]^2 > \left(\frac{c^2}{N}\right)\mu(1-\mu) + [\mu(c-1)+d]^2$$

$$(\mu(1-\mu))(\frac{a^2-c^2}{N}) + (\mu(a-1)+b)^2 - (\mu(c-1)+d)^2 > 0$$

$$(.5 \cdot .5)(\frac{.9^2-1}{1000}) + (.5(.9-1)+.2)^2 - (.5(1-1)+.05)^2 > 0$$

$$0.02 > 0$$

See the calculations in R:

$$(.5*.5)*((.9^2-1)/1000) + (.5*(.9-1) + .2)^2 - (.05)^2$$

[1] 0.0199525

Since $MSE(\hat{\mu_L}) > MSE(\hat{\mu_I})$, $\hat{\mu_I}$ is the better estimator, given the data.

Exercise 2

- (a) Our estimates of population means using large samples rely heavily on assumptions about the shape of the distribution of the population.
- FALSE. When N is large, the Central Limit Theorem states that no assumptions about the distribution of the population Y are necessary to fully describe the sampling distribution of \bar{Y} , i.e. the estimator \bar{Y} is near Y and its sampling distribution converges to normal when N is large, no matter what the shape of the population distribution.
- (b) Our estimates of population means using large samples rely heavily on assumptions about the process giving rise to the sample.
 - TRUE, the estimates rely on the simple random sample assumption that the sample data is representative of the population and is i.i.d. The i.i.d. sample has also a nice and important for us property that the larger the sample gets, the greater the probability it will closely resemble the population. The definition of consistency of an estimate, for instance, relies heavily on this i.i.d property: the estimator comes close to the parameter (population) value as we gather more data.
- (c) $\frac{\partial \sigma_{\bar{Y}}^2}{\partial \sigma_Y^2} < 0$
- FALSE, because $\sigma_{\tilde{Y}}^2 = \frac{\sigma_Y^2}{N}$, so $\frac{\partial \frac{\sigma_Y^2}{N}}{\partial \sigma_Y^2} = \frac{1}{N}$, and 1/N > 0.
- (d) $\frac{\partial \sigma_{\bar{Y}}^2}{\partial N} < 0$
- TRUE, since $\frac{\partial \frac{\sigma_Y^2}{N}}{\partial N} = -\frac{\sigma_Y^2}{N^2}$, which is less than zero.
- (e) If Y_1 is the first observation of a sample random sample (and thus an independently and identically distributed sample) of size N drawn from the population Y, $\mathbb{E}(Y_1) = Y$.

• FALSE, the expectation of an unbiased OLS sample estimator equals the population parameter, $\mathbb{E}(\bar{Y}) = Y$.

Exercise 3

The population model $y = \beta_0 + \beta_1 x + u$.

$$y = \begin{bmatrix} 3 \\ 9 \\ -2 \\ -4 \end{bmatrix} \quad \text{and} \quad X = \begin{bmatrix} 1 & 6 \\ 1 & 10 \\ 1 & 3 \\ 1 & 0 \end{bmatrix}.$$

(a) Show that the OLS estimates $\hat{\beta}_0 \approx -4.88$ and $\hat{\beta}_1 \approx 1.34$.

$$(X'X)^{-1} \approx \begin{bmatrix} .662 & -.087 \\ -.087 & .018 \end{bmatrix}$$

Recall that $\hat{\beta} = (X'X)^{-1}X'y$. First, let's find X'y:

$$\boldsymbol{X'y} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 6 & 10 & 3 & 0 \end{bmatrix} \begin{bmatrix} 3 \\ 9 \\ -2 \\ -4 \end{bmatrix} = \begin{bmatrix} 6 \\ 102 \end{bmatrix}.$$

Then, multiply $(X'X)^{-1}$ by it:

$$\begin{bmatrix} \hat{\beta}_0 \\ \hat{\beta}_1 \end{bmatrix} = (X'X)^{-1}(X'y) = \begin{bmatrix} .662 & -.087 \\ -.087 & .018 \end{bmatrix} \begin{bmatrix} 6 \\ 102 \end{bmatrix} \approx \begin{bmatrix} -4.88 \\ 1.34 \end{bmatrix}$$

(b) Show that $SER \approx 1.08$.

$$SER = \sqrt{\frac{1}{N-2} \sum \hat{u}_i^2} = \frac{1}{\sqrt{N-2}} (\hat{\boldsymbol{u}}'\hat{\boldsymbol{u}})^{1/2} = \frac{1}{\sqrt{2}} (\hat{\boldsymbol{u}}'\hat{\boldsymbol{u}})^{1/2}$$

First, find $\hat{\boldsymbol{u}}$.

$$\hat{\boldsymbol{u}} = \boldsymbol{y} - \boldsymbol{X}\hat{\boldsymbol{\beta}} = \begin{bmatrix} 3\\9\\-2\\-4 \end{bmatrix} - \begin{bmatrix} 1&6\\1&10\\1&3\\1&0 \end{bmatrix} \begin{bmatrix} -4.88\\1.34 \end{bmatrix} = \begin{bmatrix} 3\\9\\-2\\-4 \end{bmatrix} - \begin{bmatrix} 3.17\\8.52\\-0.86\\-4.88 \end{bmatrix} = \begin{bmatrix} -0.17\\0.48\\-1.14\\0.88 \end{bmatrix}.$$

Then, find $\hat{u}'\hat{u}$:

$$\hat{\boldsymbol{u}}'\hat{\boldsymbol{u}} = \begin{bmatrix} -0.17 & 0.48 & -1.14 & 0.88 \end{bmatrix} \begin{bmatrix} -0.17 \\ 0.48 \\ -1.14 \\ 0.88 \end{bmatrix} = 2.33$$

Finally,

$$SER = \frac{1}{\sqrt{2}} (\hat{\boldsymbol{u}}' \hat{\boldsymbol{u}})^{1/2} = \frac{1}{\sqrt{2}} \sqrt{2.33} \approx 1.08$$

(c) Show that $R^2 \approx .98$.

$$R^{2} = 1 - \frac{SSR}{TSS} = 1 - \frac{\sum (y_{i} - \hat{y}_{i})^{2}}{\sum (y_{i} - \bar{y})^{2}}$$

First, let's find $\sum (y_i - \bar{y})^2$, where $\bar{y} = 1.5$. Denote a demeaned y as \tilde{y} .

$$\tilde{\boldsymbol{y}} = \begin{bmatrix} 3\\9\\-2\\-4 \end{bmatrix} - 1.5 = \begin{bmatrix} 1.5\\7.5\\-3.5\\-5.5 \end{bmatrix}.$$

$$TSS = \sum (y_i - \bar{y})^2 = \tilde{\mathbf{y}}'\tilde{\mathbf{y}} = \begin{bmatrix} 1.5 & 7.5 & -3.5 & -5.5 \end{bmatrix} \begin{bmatrix} 1.5 \\ 7.5 \\ -3.5 \\ -5.5 \end{bmatrix} = 101$$

We already know the value of $SSR = \hat{\boldsymbol{u}}'\hat{\boldsymbol{u}} = 2.33$, hence,

$$R^2 = 1 - \frac{SSR}{TSS} = 1 - \frac{2.33}{101} \approx 0.98$$

Exercise 4

- (a) We simply need to divide the estimates by their standard errors to get a t-score. If the absolute value of the t-score is greater than 1.96, we reject the null hypothesis that $\beta=0$ with more than 95% confidence, i.e. this coefficient estimate is statistically significant at the 95% confidence level. Significant coefficients for all the models are female, white, white x education, LGB, age (years) (when included) and the intercepts.
- (b) According to Model I:
 - Trump rating for a white, straight man with a college education (16 years): $27.6 + 0 \cdot (-6.5) + 1 \cdot 64.5 + 16 \cdot (-0.07) + (16 \cdot 1) \cdot (-3.1) + 0 \cdot (-21.6) = 41.38$
 - Trump rating for a non-white, bisexual woman with a post-graduate degree (20 years): $27.6 + 1 \cdot (-6.5) + 0 \cdot 64.5 + 20 \cdot (-.07) + (20 \cdot 0) \cdot (-3.1) + 1 \cdot (-21.6) = -1.9$, which is zero if want to make it substantially meaningful.
 - the magnitude of $y_i \hat{y}_i = \hat{u}_i$ for the typical observation i, or, equivalently, a typical size of a residual is $SER = \sqrt{\frac{1}{N-2}\sum \hat{u}_i^2} = 32.6$. We can also deduce an average of an estimated typical residual:

$$\sqrt{\frac{1}{3841} \sum \hat{u}_i^2} = 32.6$$

$$\sum \hat{u}_i^2 = \sqrt{32.6} \cdot 3841 = 21,931$$

$$\bar{u}_i^2 = 21,931/3,841 = 5.7$$

$$\sqrt{\bar{u}_i^2} = 2.39$$

- a hypothetical American whose predicted rating of Trump is the intercept 27.6 would be a straight, male, non-white person with no education.
- the approximate predicted difference in ratings given to Trump between a white person with a high-school education (12 years) and a white person with a college education (16 years), holding other regressors constant, is 12.68:

$$64.5 + 12 \cdot (-.07) - 3.1 \cdot (12) - [64.5 + 16 \cdot (-.07) - 3.1 \cdot (16)] = 26.46 - 13.78 = 12.68$$

• the approximate predicted difference in ratings given to Trump between a non-white person with a high-school education (12 years) and a non-white person with a college education (16 years), holding other regressors constant, is 0.28:

$$12 \cdot (-.07) - [16 \cdot (-.07)] = -0.84 - (-1.12) = 0.28.$$

- the strength of the association between education and support for Trump is stronger among white Americans than among non-white Americans: 12.68 > 0.28. A one-unit increase in education is associated with a greater increase in Trump support among whites, than among non-whites, holding other regressors constant.
- (c) TRUE or FALSE:
- The relationship between gender and support for Trump is confounded by age.
 - FALSE, because the female coefficient does not change when we include age in the model (compare Models II and III).
- The relationship between gender and support for Trump is confounded somewhat by either party ID, state of residence, or both.
 - TRUE, because the magnitude of gender coefficient changes significantly once we add fixed effects (compare Model I and Model II: the coefficient increases from -6.5 to -3.4).
- The extent to which variation in support for Trump is explained by our estimated model triples when we add fixed effects for party ID and state of residence to the model.
 - TRUE, because the adjusted R^2 statistics triples: it goes from .14 to .43.
- · After controlling for race, education, age, partisanship and state of residence, the gender gap in support for Trump is larger than the sexuality gap.
 - FALSE, the sexuality gap (27.6-(27.6-21.6)=21.6) is larger than the gender gap (27.6-(27.6-21.6)=21.6)6.5) = 6.5.
- (d) Compare the linear and polynomial estimates of age's assocation with support for Trump in Models III and IV.
- (e) Which of the four models best fits the data? Why?

Models III and IV fit the data best because they have the highest $R^2 = .43$ (i.e. they explain 43% of the sample variation in Trump ratings) and have the smallest SER = 26.5, which is the average distance the observed values fall from the regression line. However, since Model IV differs from Model III only by a quadratic term Age² included in the regression, I would choose Model III in terms of the best fit since this additional term Age² is not statistically significant. As a rule of thumb, we choose a more "compact"/"economical" model between two identical in terms of fit models.

Exercise 5

(a) Replicate Model III := Model A.

```
library(haven)
data <- read_dta("/Users/herrhellana/Dropbox/_NYU studies/Quant I/exam/anes2016forq1.dta")
# Model A
m a <- lm(data=data, trump ft ~ female + white + educyears
          + white:educyears + lgb + age
```

```
+ as.factor(pid3) + as.factor(state))
library(stargazer)
library(sandwich)
# To the Justin's comment:
# I use 'text' because 'latex' outputs a tex code
# (even once you knit the file)
# which is nice if you compile then things in tex
# but I'm using Markdown...
stargazer(m_a, type = 'text', dep.var.caption = 'Model A',
          omit = c("pid3", "state"),
          omit.stat=c('f', 'rsq'),
         digits=2,
          dep.var.labels = 'Trump support',
          covariate.labels = c('Female', 'White',
                               'Education (years)', 'LGB',
                               'Age (years)',
                               'White x Education',
                               'Intercept'),
          add.lines = list(c('Party ID and state of residence FEs',
                             'Yes')),
          se = list(sqrt(diag(vcovHC(m_a, type = 'HC1')))))
```

Trump support Female -3.44*** (0.87)39.99*** White (4.98)Education (years) -0.27 (0.28)-11.48*** LGB (1.75)0.13*** Age (years) (0.03)White x Education -2.17*** (0.34)16.50*** Intercept Party ID and state of residence FEs Yes

Observations

3,843

```
Adjusted R2 0.43
Residual Std. Error 26.48 (df = 3784)
------
Note: *p<0.1; **p<0.05; ***p<0.01
```

(b) Recoding income.

```
library(dplyr)
data <- data %>% mutate(income_new = case_when(income == 1 ~ 2500,
                                                      income == 2 \sim 7500,
                                                      income == 3 \sim 11250,
                                                      income == 4 \sim 13750,
                                                      income == 5 \sim 16250,
                                                      income == 6 \sim 18750,
                                                      income == 7 \sim 21250,
                                                      income == 8 \sim 23750,
                                                      income == 9 \sim 26250,
                                                      income == 10 \sim 28750,
                                                      income == 11 \sim 32500,
                                                      income == 12 \sim 37500,
                                                      income == 13 \sim 42500,
                                                      income == 14 \sim 47500,
                                                      income == 15 \sim 52500,
                                                      income == 16 \sim 57500,
                                                      income == 17 \sim 62500,
                                                      income == 18 \sim 67500,
                                                      income == 19 \sim 72500,
                                                      income == 20 \sim 77500,
                                                      income == 21 \sim 85000,
                                                      income == 22 \sim 95000,
                                                      income == 23 \sim 105000,
                                                      income == 24 \sim 117500,
                                                      income == 25 \sim 137500,
                                                      income == 26 \sim 162500,
                                                      income == 27 \sim 212500,
                                                      income == 28 \sim 375000)
```

(c) Generate a new variable ln_income, estimate a Model B.

ln_income
-1.421774

Interpretation of the ln_i come coefficient: a 1% change in income is associated with a change of $.01 \cdot (-1.42) = -0.0142$ in support for Trump.

(d) Model C: the interaction term white x ln_income.

```
m_c <- lm(data=data, trump_ft ~ female + white + educyears</pre>
          + white:educyears + lgb + age + ln_income
          + white:ln_income
          + as.factor(pid3) + as.factor(state))
stargazer(m_b, m_c, type = 'text', dep.var.caption = 'Models B and C',
          omit = c("pid3", "state"),
          omit.stat=c('f', 'rsq'),
          dep.var.labels = 'Trump support',
          covariate.labels = c('Female', 'White',
                               'Education (years)', 'LGB',
                               'Age (years)', 'Income (log)',
                                'White x Education', 'White x Income (log)',
                               'Intercept'),
          add.lines = list(c('Party ID and state of residence FEs',
                             'Yes', 'Yes')),
          se = list(sqrt(diag(vcovHC(m_b, type = 'HC1'))),
                    sqrt(diag(vcovHC(m_c, type = 'HC1')))))
```

	Models 1	Models B and C		
	Trump : (1)	support		
	(1)	(2) 		
Female	-3.855***	-3.867***		
	(0.884)	(0.883)		
White	41.150***	51.349***		
	(5.035)	(9.501)		
Education (years)	-0.020	-0.159		
	(0.288)	(0.302)		
LGB	-11.615***	-11.650***		
	(1.752)	(1.754)		
Age (years)	0.135***	0.133***		
	(0.026)	(0.026)		
Income (log)	-1.422***	-0.601		
	(0.451)	(0.742)		
White x Education	-2.236***	-2.047***		
	(0.346)	(0.370)		
White x Income (log)		-1.203		
-		(0.921)		
Intercept	28.258***	21.653***		
	(6.415)	(8.059)		

Party ID and state of residence FEs	Yes	Yes
Observations	3,730	3,730
Adjusted R2	0.434	0.434
Residual Std. Error	26.383 (df = 3670)	26.380 (df = 3669)
Note:	*p<0.1;	**p<0.05; ***p<0.01

The interaction term is not statistically significant, and it made the ln_income coefficient insignificant, too. The sign of the interaction term coefficient shows that the relationship between income and Trump support is stronger (and more negative) among white people (-1.8) than among non-white people (just -0.6). But since both the intercation term and the ln_income coefficients are not statistically significant (and thus not different from 0, as we did not reject the null), Model C tells us that white (which is statistically significant in both models) explains support for Trump better than ln_income.

(e) Does income confound the relationship between education and support for Trump among whites? Explain in a few sentences by comparing Model A and Model C.

Models A and C Trump support (1) (2) -3.438*** -3.867*** Female (0.868)(0.883)39.986*** 51.349*** White (4.976)(9.501)Education (years) -0.267-0.159(0.302)(0.277)LGB -11.482*** -11.650*** (1.754)(1.754)Age (years) 0.134*** 0.133*** (0.025)(0.026)-0.601 Income (log) (0.742)

```
-2.165***
White x Education
                                                     -2.047***
                                      (0.343)
                                                        (0.370)
White x Income (log)
                                                         -1.203
                                                        (0.921)
Intercept
                                     16.501***
                                                       21.653***
                                      (5.016)
                                                        (8.059)
Party ID and state of residence FEs
                                       Yes
                                                         Yes
Observations
                                       3,843
                                                         3,730
Adjusted R2
                                       0.431
                                                         0.434
                                 26.482 \text{ (df = 3784) } 26.380 \text{ (df = 3669)}
Residual Std. Error
                             -----
                                           *p<0.1; **p<0.05; ***p<0.01
Note:
```

No, income does not confound the relationship between education and support for Trump among white people, because the coefficient magnitude and its significance do not substantially change (it goes from -2.43 to -2.2, which is less than even a 10% change: $\frac{(-0.267-2.165)-(-0.159-2.047)}{(-0.267-2.165)}\cdot 100 = 9.3\%$).

(f) Model D: including religion.

```
m_d <- lm(data=data, trump_ft ~ female + white + educyears</pre>
          + white:educyears + lgb + age + ln_income
          + white:ln_income + religion
          + as.factor(pid3) + as.factor(state))
stargazer(m_a, m_c, m_d, type = 'text', dep.var.caption = 'Models A, C, and D',
          omit = c("pid3", "state"),
          omit.stat=c('f', 'rsq'),
          dep.var.labels = 'Trump support',
          covariate.labels = c('Female', 'White',
                               'Education (years)', 'LGB',
                                'Age (years)', 'Income (log)',
                                'Religion',
                                'White x Education', 'White x Income (log)',
                                'Intercept'),
          add.lines = list(c('Party ID and state of residence FEs',
                             'Yes', 'Yes', 'Yes')),
          se = list(sqrt(diag(vcovHC(m_a, type = 'HC1'))),
                    sqrt(diag(vcovHC(m_c, type = 'HC1'))),
                    sqrt(diag(vcovHC(m_d, type = 'HC1')))))
```

```
Models A, C, and D

Trump support

(1) (2) (3)

Female -3.438*** -3.867*** -3.909***
```

	(0.868)	(0.883)	(0.882)
White	39.986***	51.349***	51.492***
	(4.976)	(9.501)	(9.467)
Education (years)	-0.267	-0.159	-0.146
•	(0.277)	(0.302)	(0.301)
LGB	-11.482***	-11.650***	-11.194***
	(1.754)	(1.754)	(1.754)
Age (years)	0.134***	0.133***	0.112***
	(0.025)	(0.026)	(0.026)
Income (log)		-0.601	-0.747
-		(0.742)	(0.743)
Religion			-1.012***
_			(0.305)
White x Education	-2.165***	-2.047***	-2.103***
	(0.343)	(0.370)	(0.369)
White x Income (log)		-1.203	-1.150
		(0.921)	(0.919)
Intercept	16.501***	21.653***	27.379***
	(5.016)	(8.059)	(8.271)
Party ID and state of residence FEs	Yes	Yes	Yes
Observations	3,843	3,730	3,730
Adjusted R2	0.431	0.434	0.436
Residual Std. Error			26.343 (df = 3668)
Note:	==================================		**p<0.05; ***p<0.01

The results specified above hold. The effect of education on Trump support among white people fluctuates a bit from -2.432 (Model A) to -2.206 (Model B) to -0.146 - 2.103 = -2.249 (Model C). This means that the effect of education on Trump support among whites is robust. This effect is statistically significant at the 99% confidence level, negative, and large (approx -2.2).