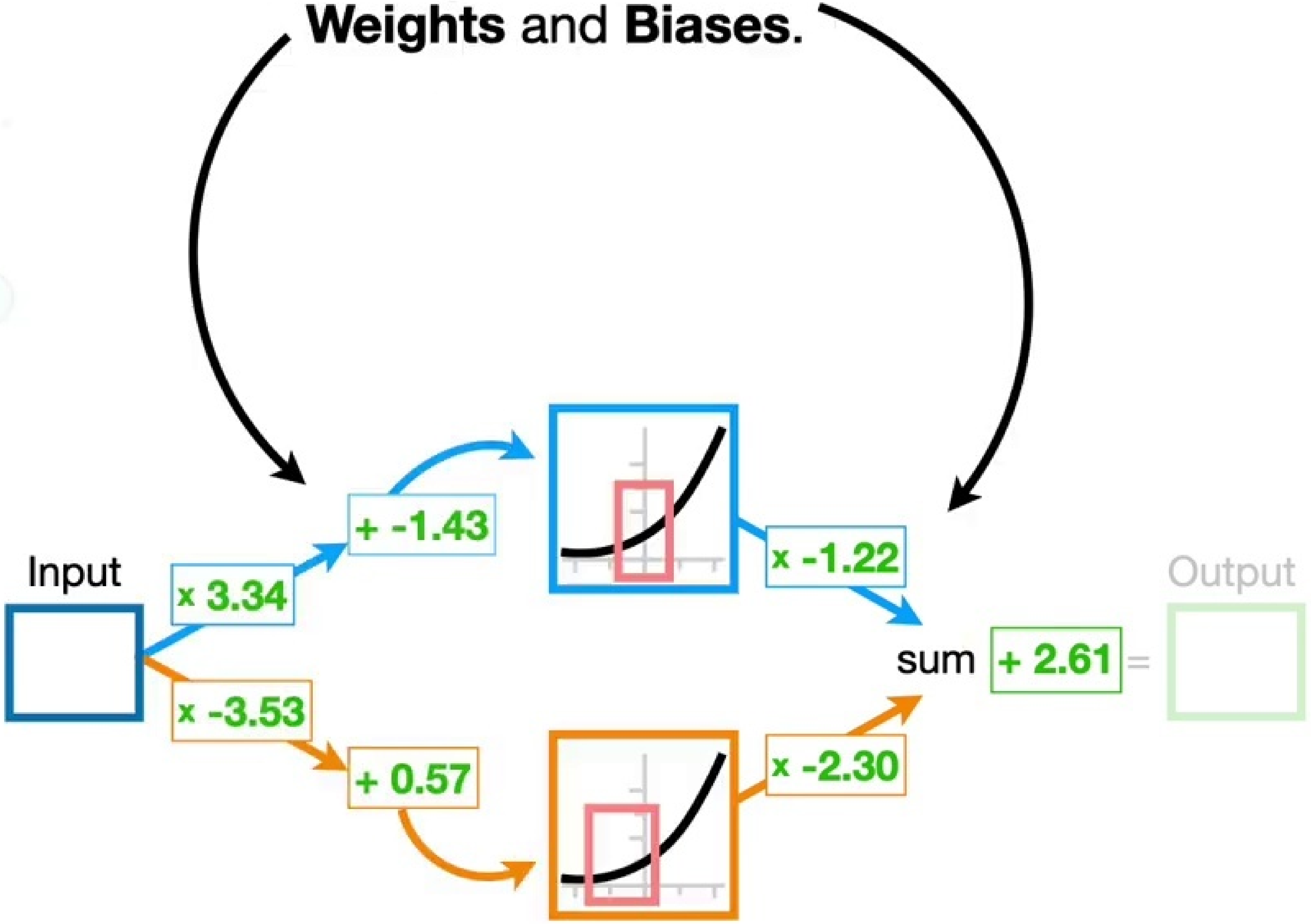
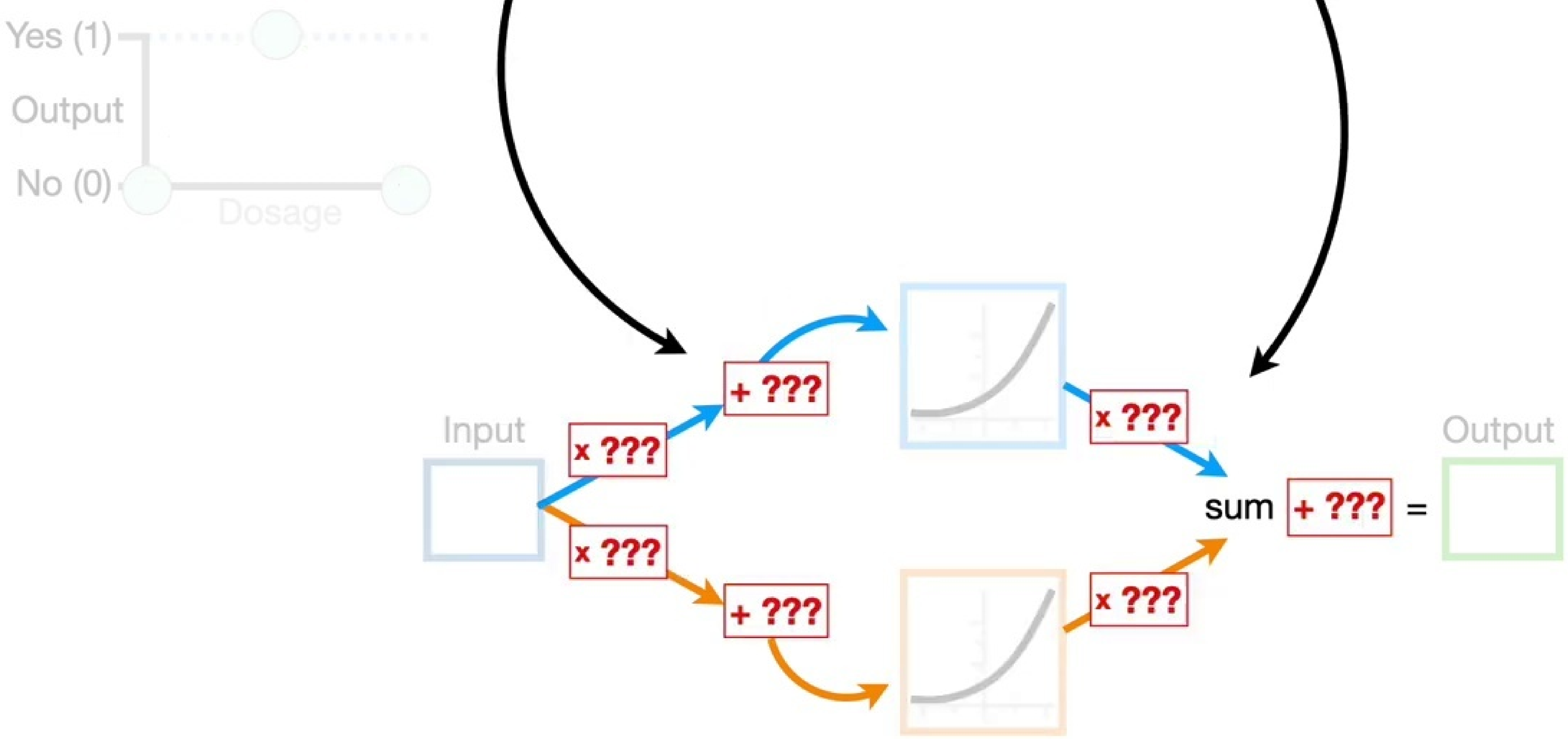
However, we did not talk about how to estimate the **Weights** and **Biases**.



So let's talk about how **Backpropagation** optimizes the **Weights** and **Biases** in this and other **Neural Networks**.



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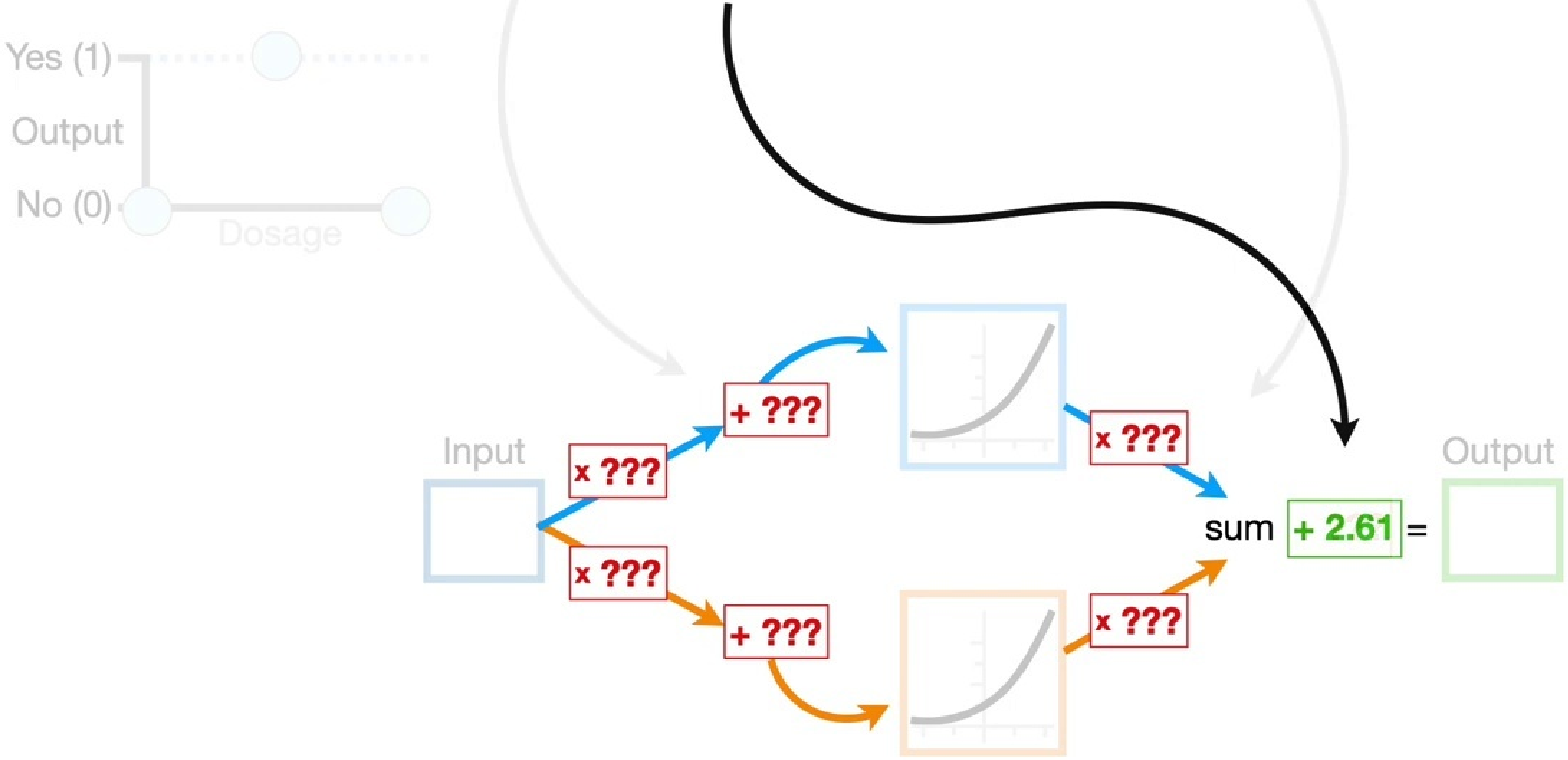

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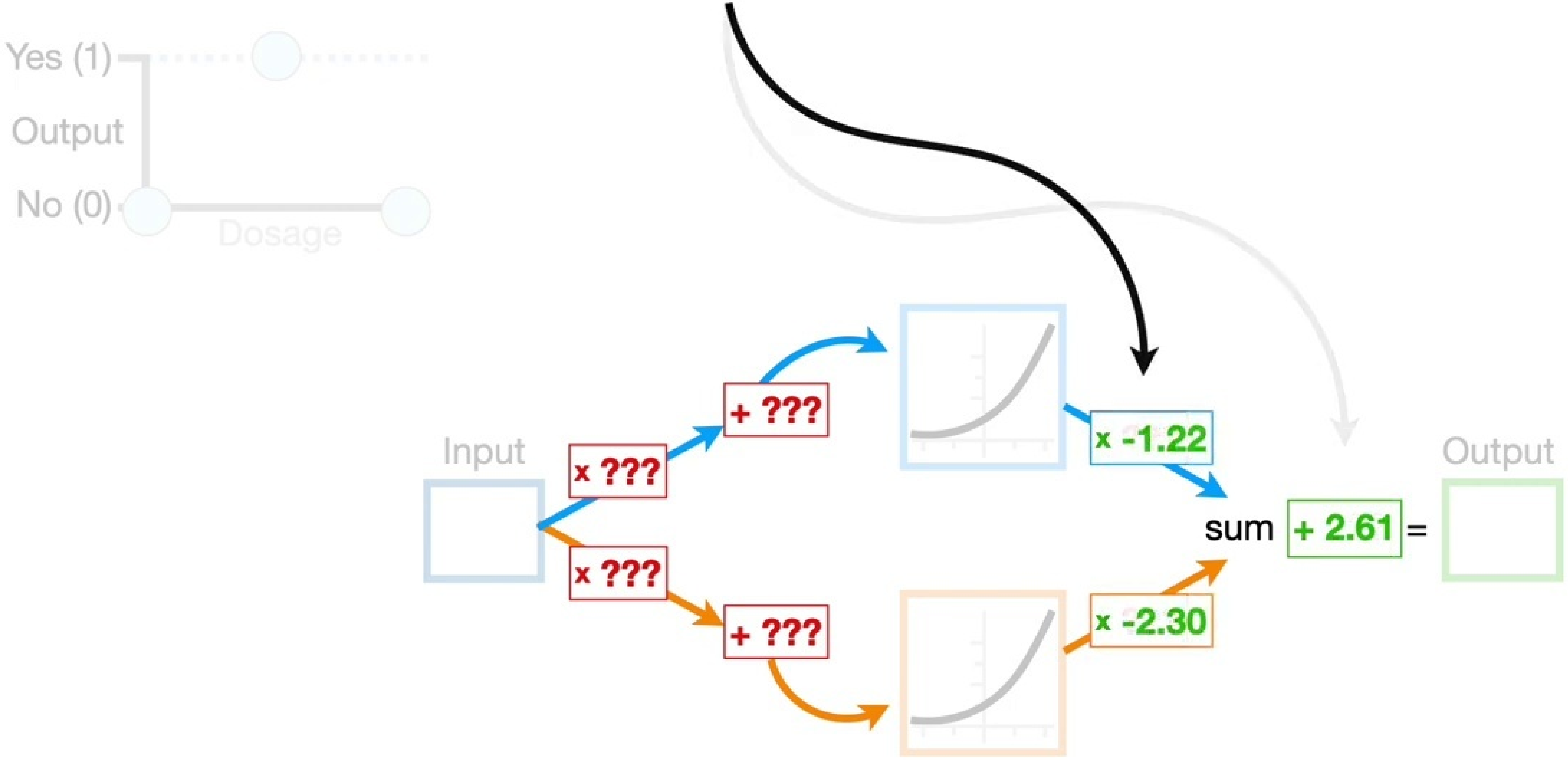

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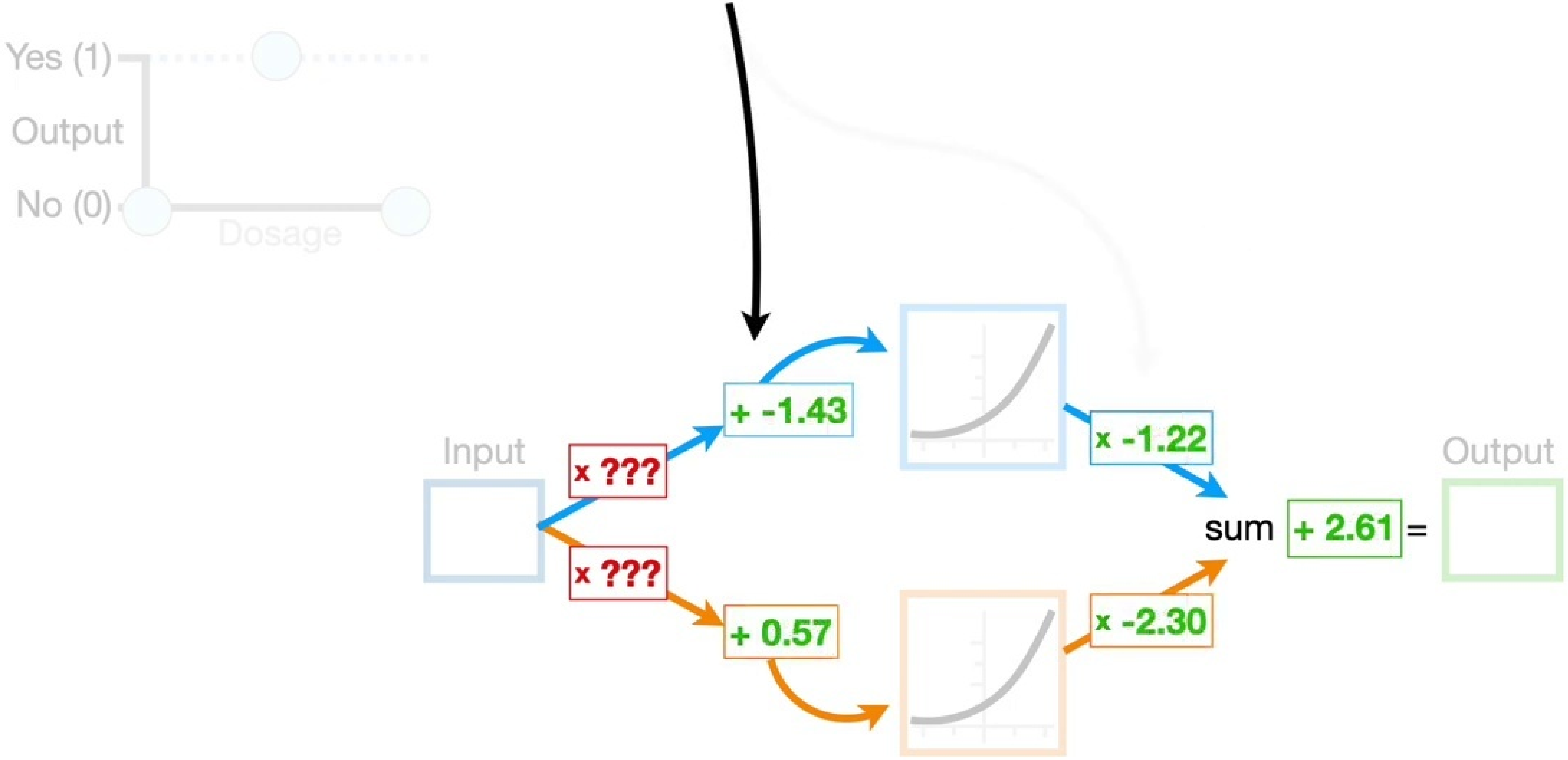

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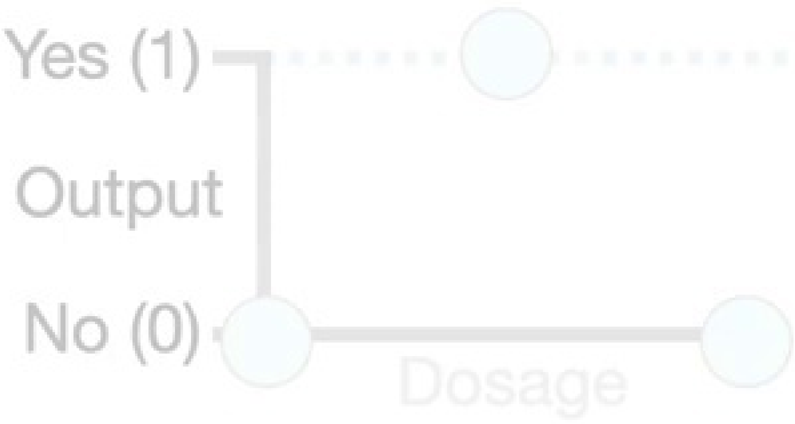
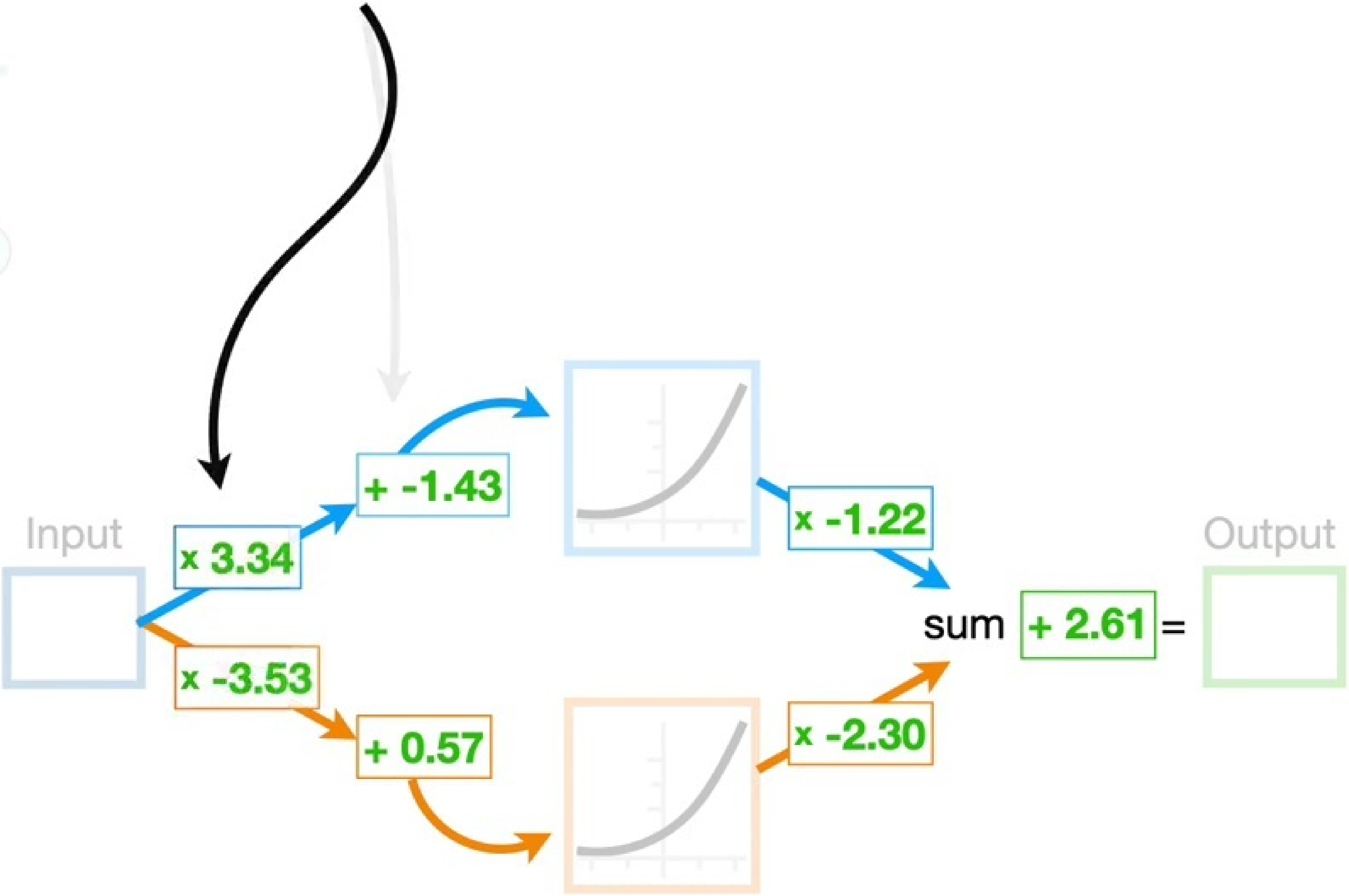
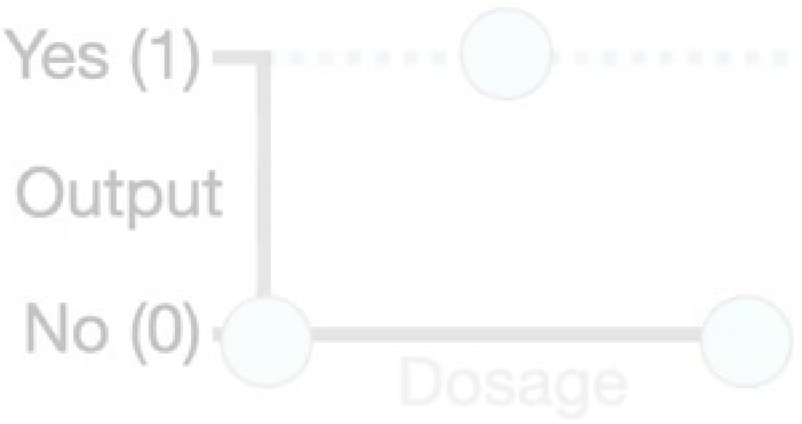
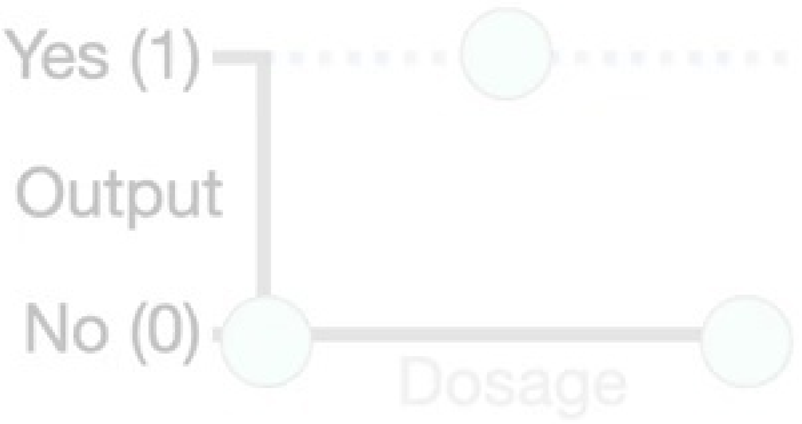




NOTE: Backpropagation is relatively simple, but there are a ton of details. So I've spit it up into bite sized pieces.



In this part, we talk about the **Main Ideas** of **Backpropagation:**



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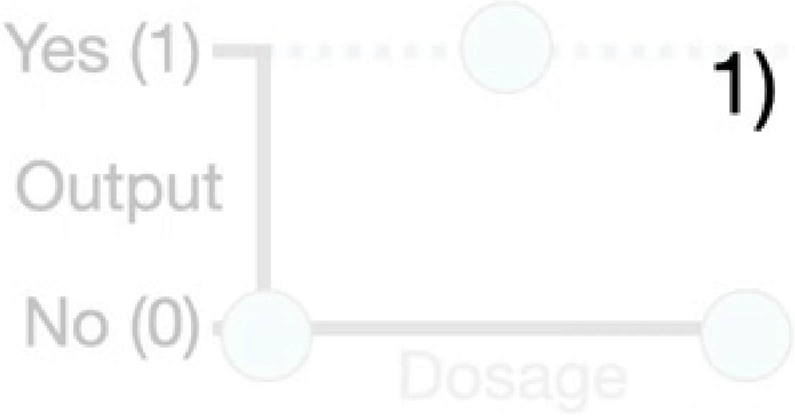
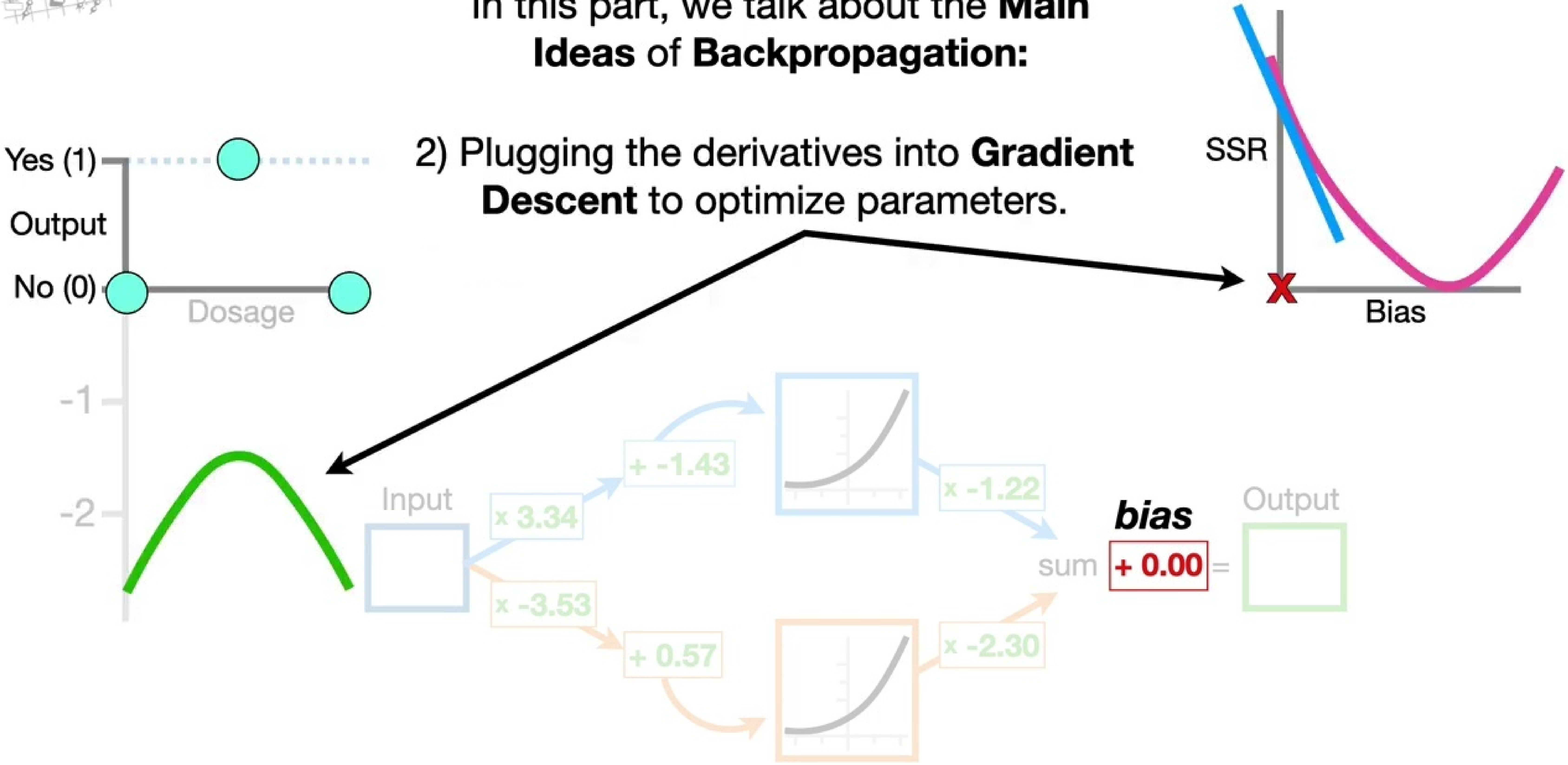


1) Using the **Chain Rule** to calculate derivatives…

$$\frac{d\ SSR}{d\ bias} = \frac{d\ SSR}{d\ Predicted} \times \frac{d\ Predicted}{d\ bias}$$

In this part, we talk about the **Main Ideas** of **Backpropagation:**

2) Plugging the derivatives into **Gradient Descent** to optimize parameters.



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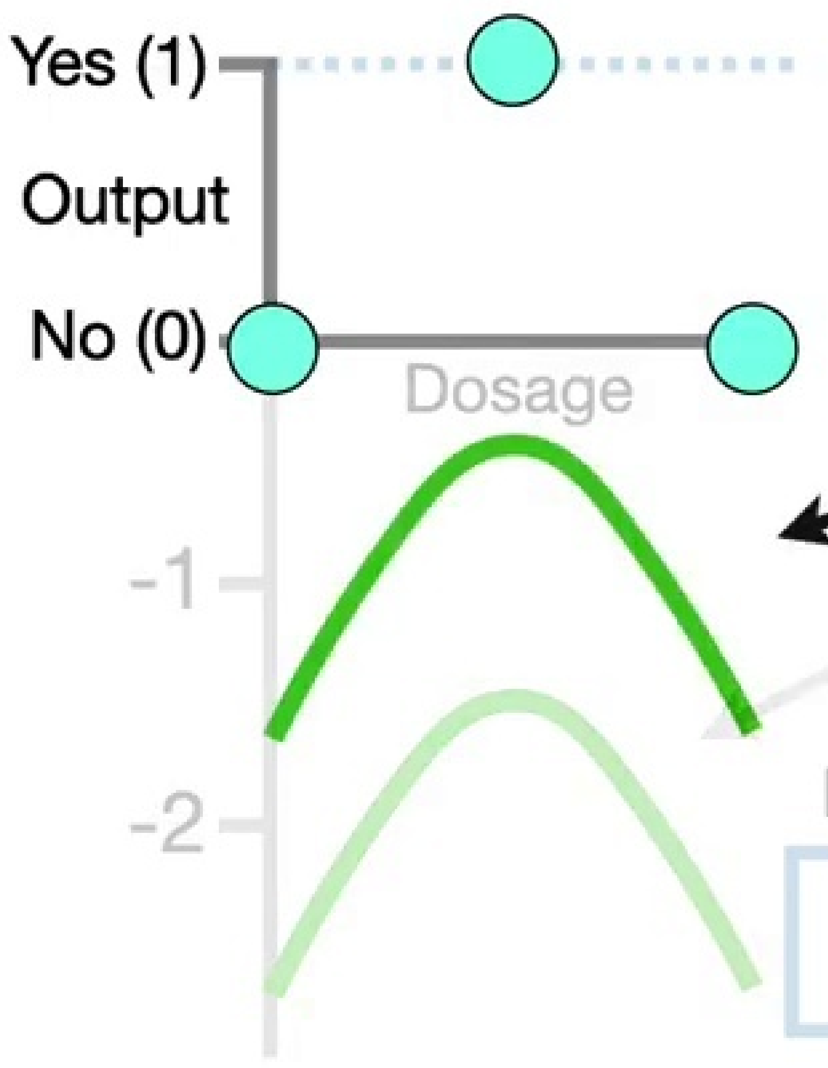

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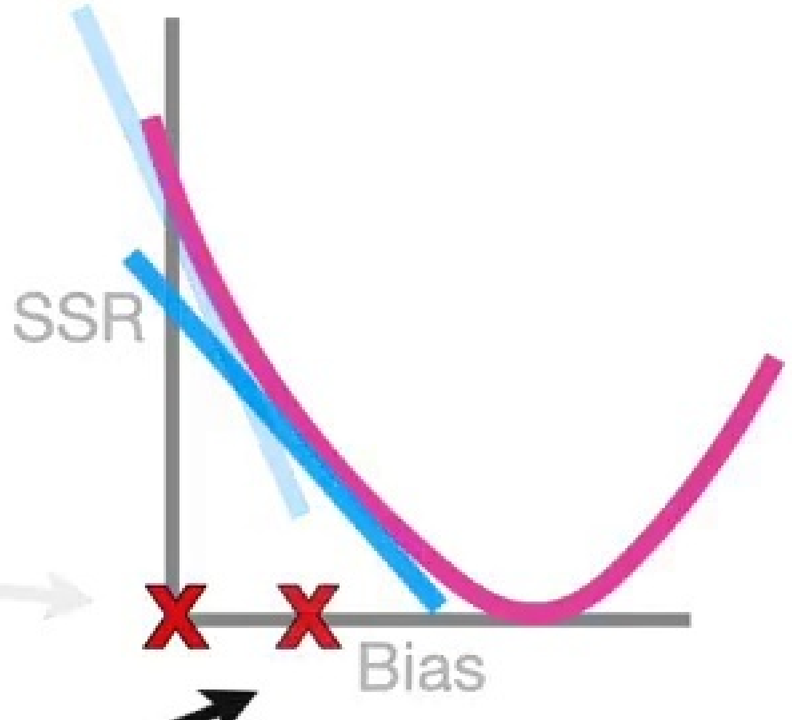

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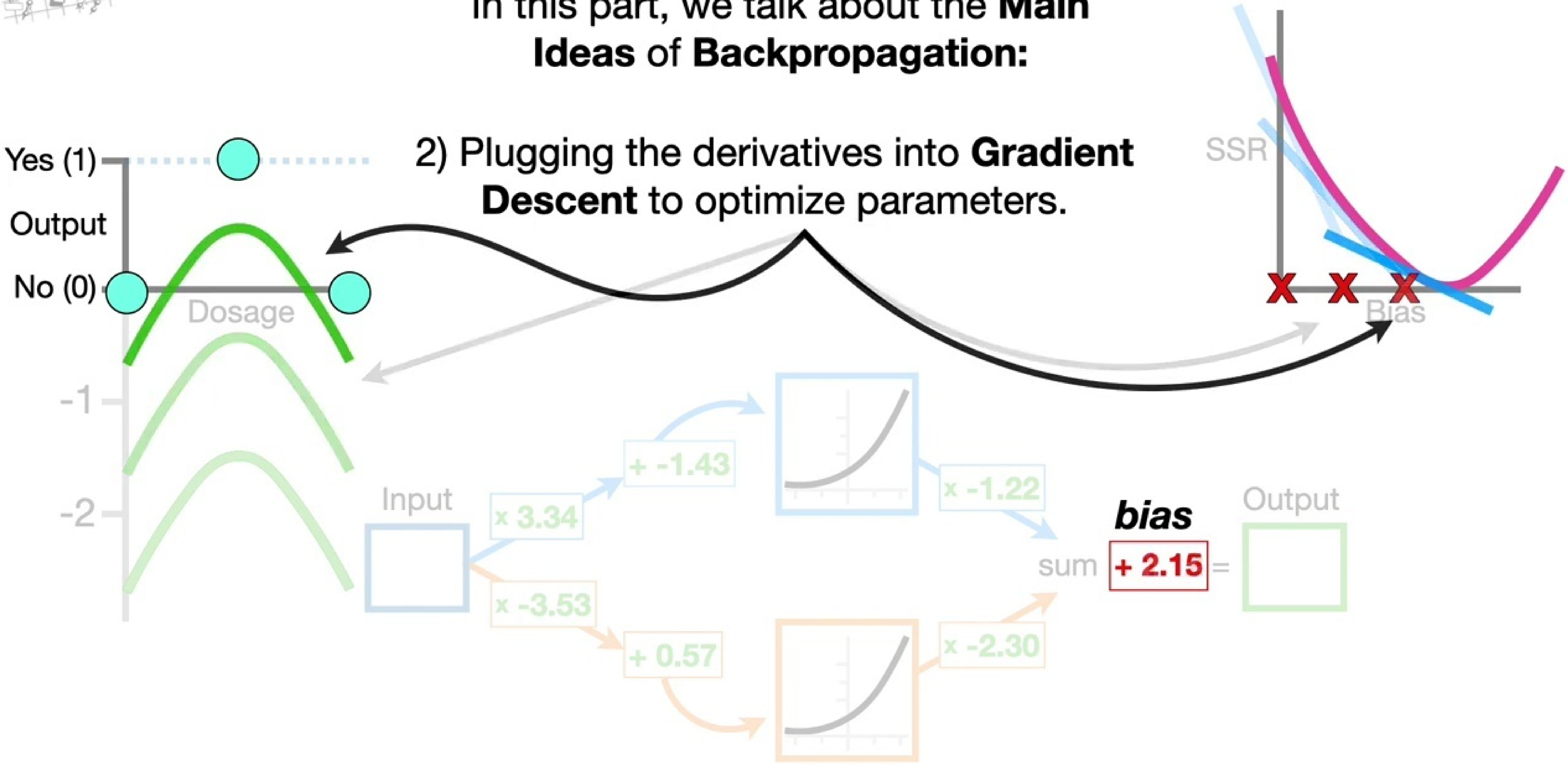

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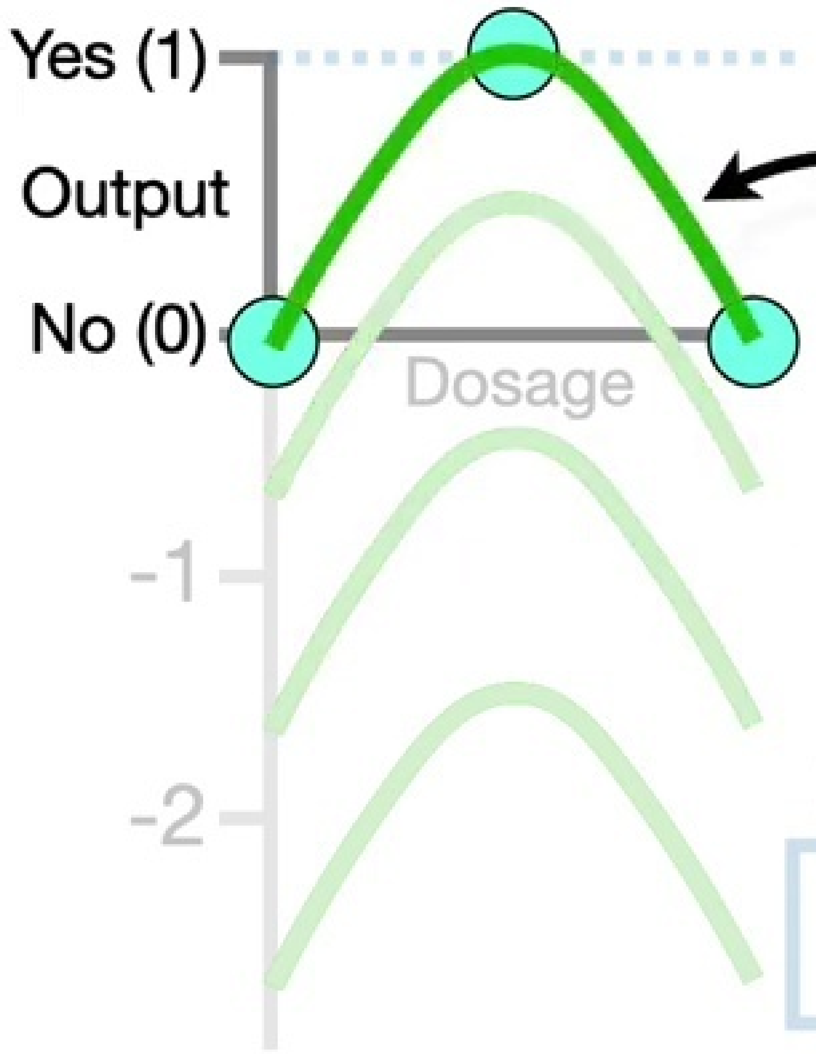
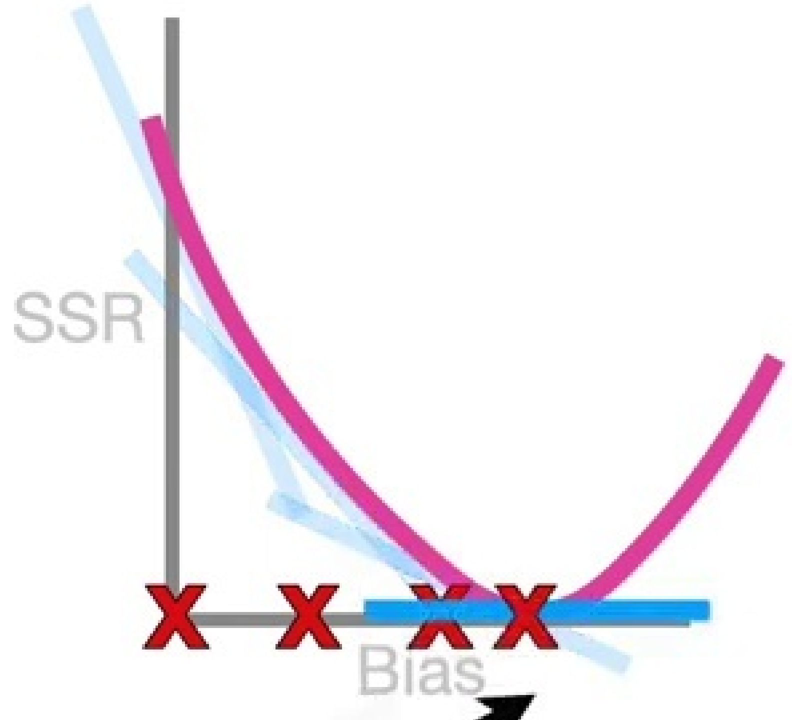
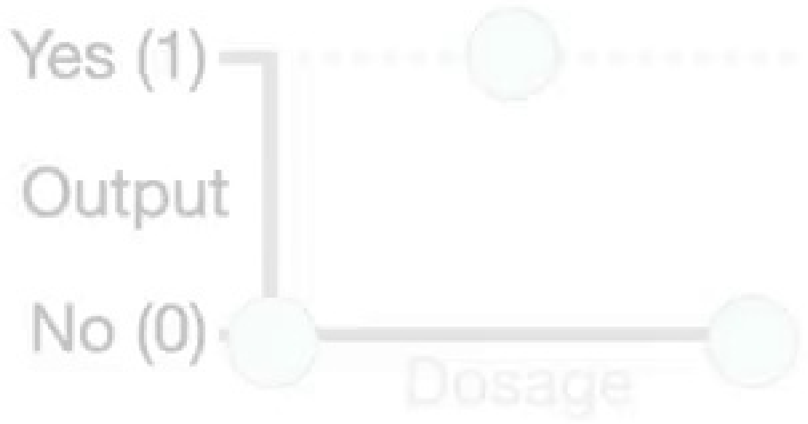




First, so we can be clear about which specific **Weights** we are talking about, let's give each one a name.



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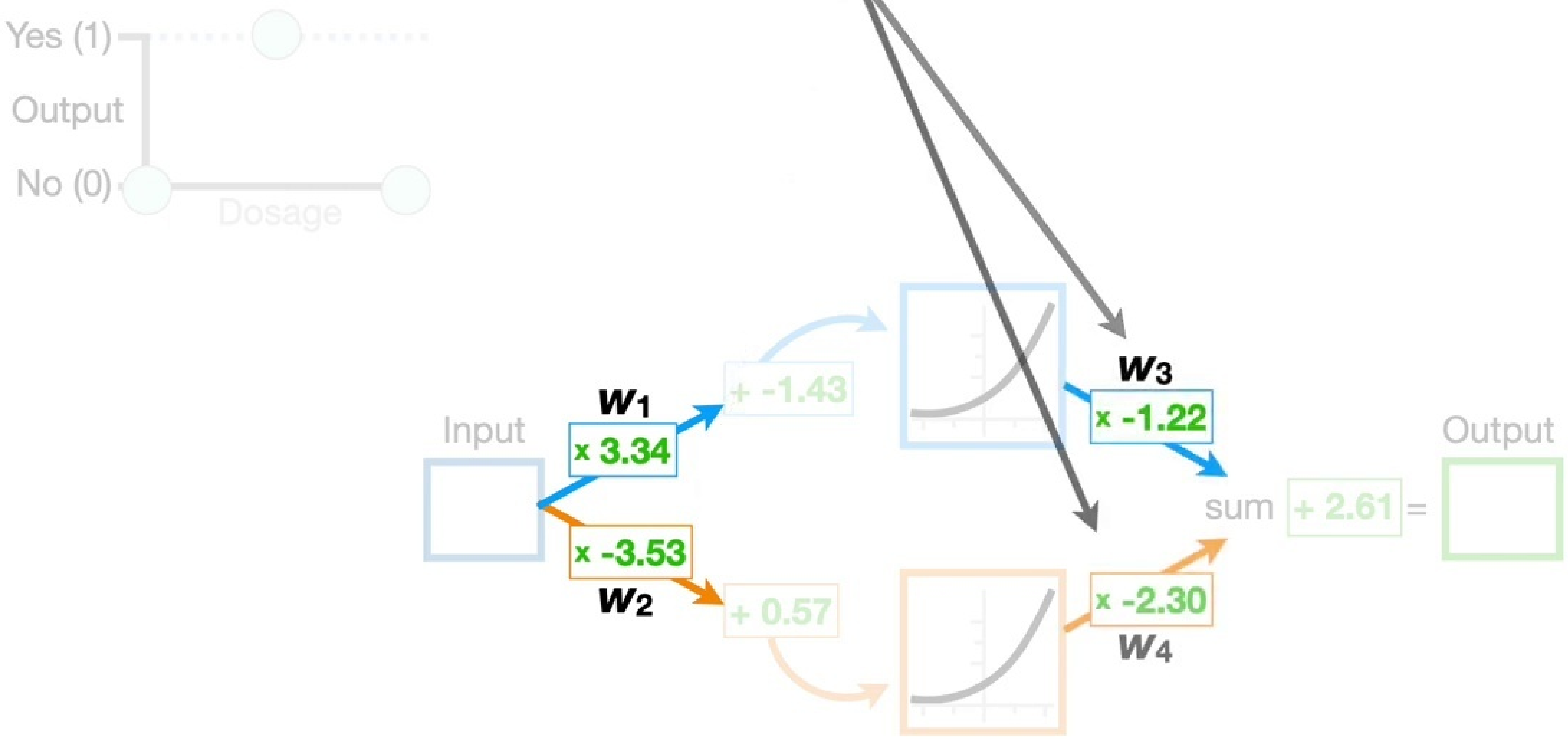
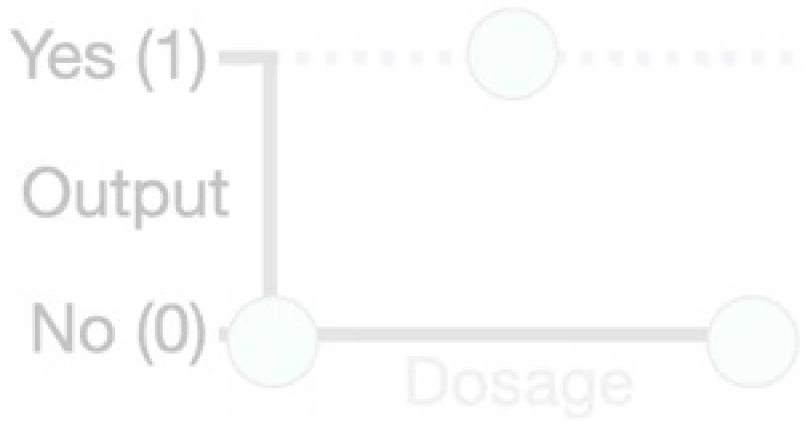
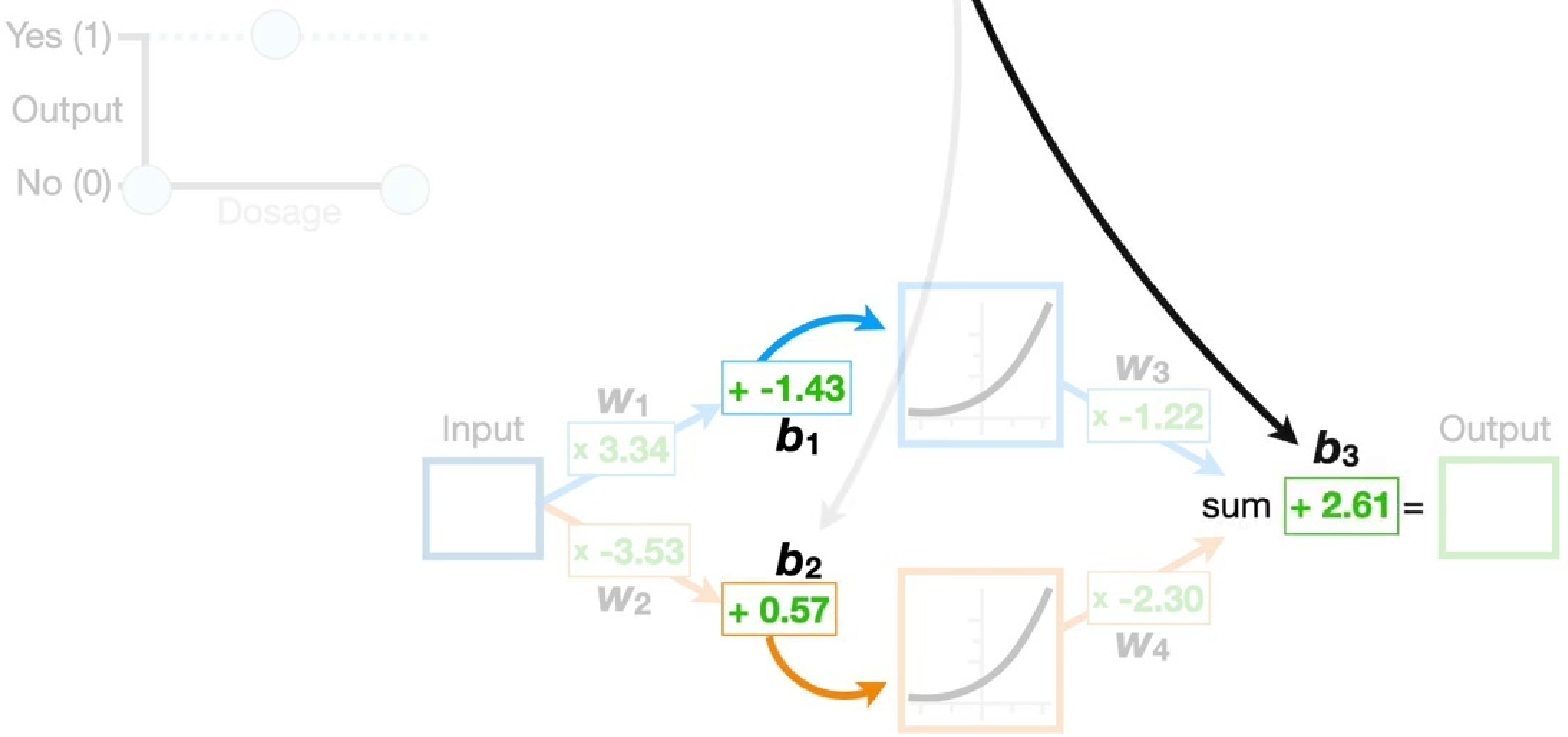


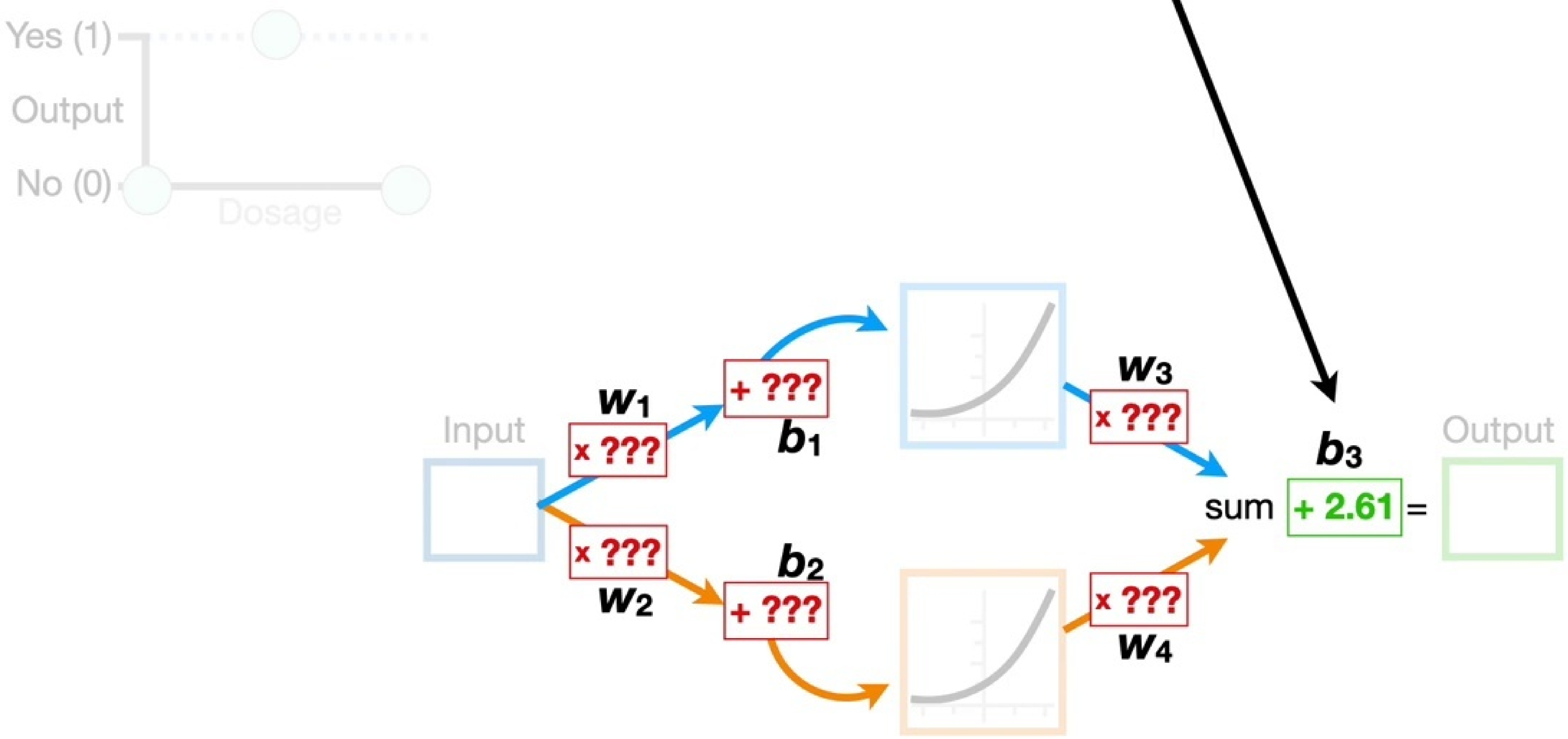
And let's name
each **Bias.**



And let's name each **Bias.**



...and works its way backwards to estimate all of the other parameters.



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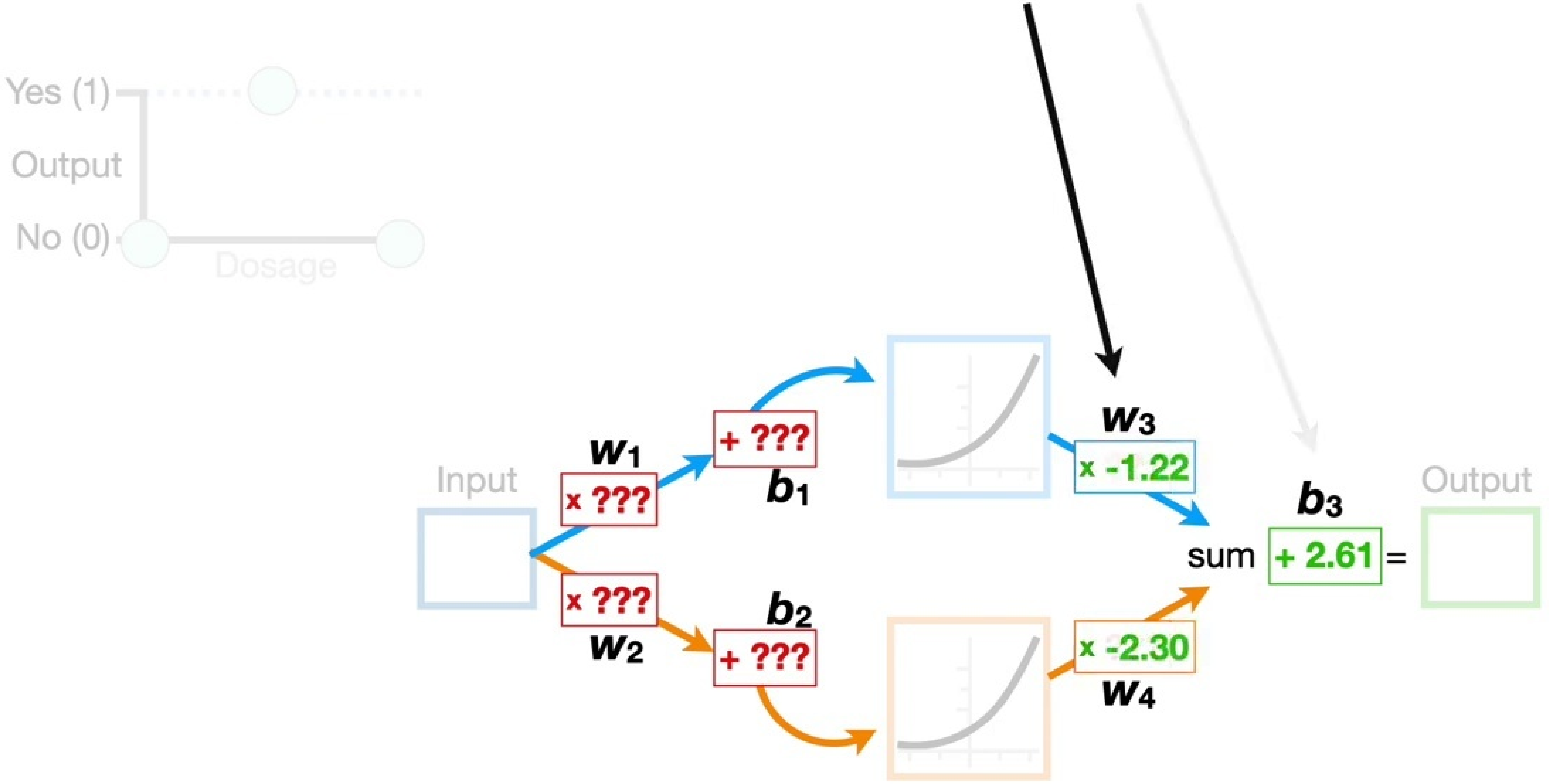

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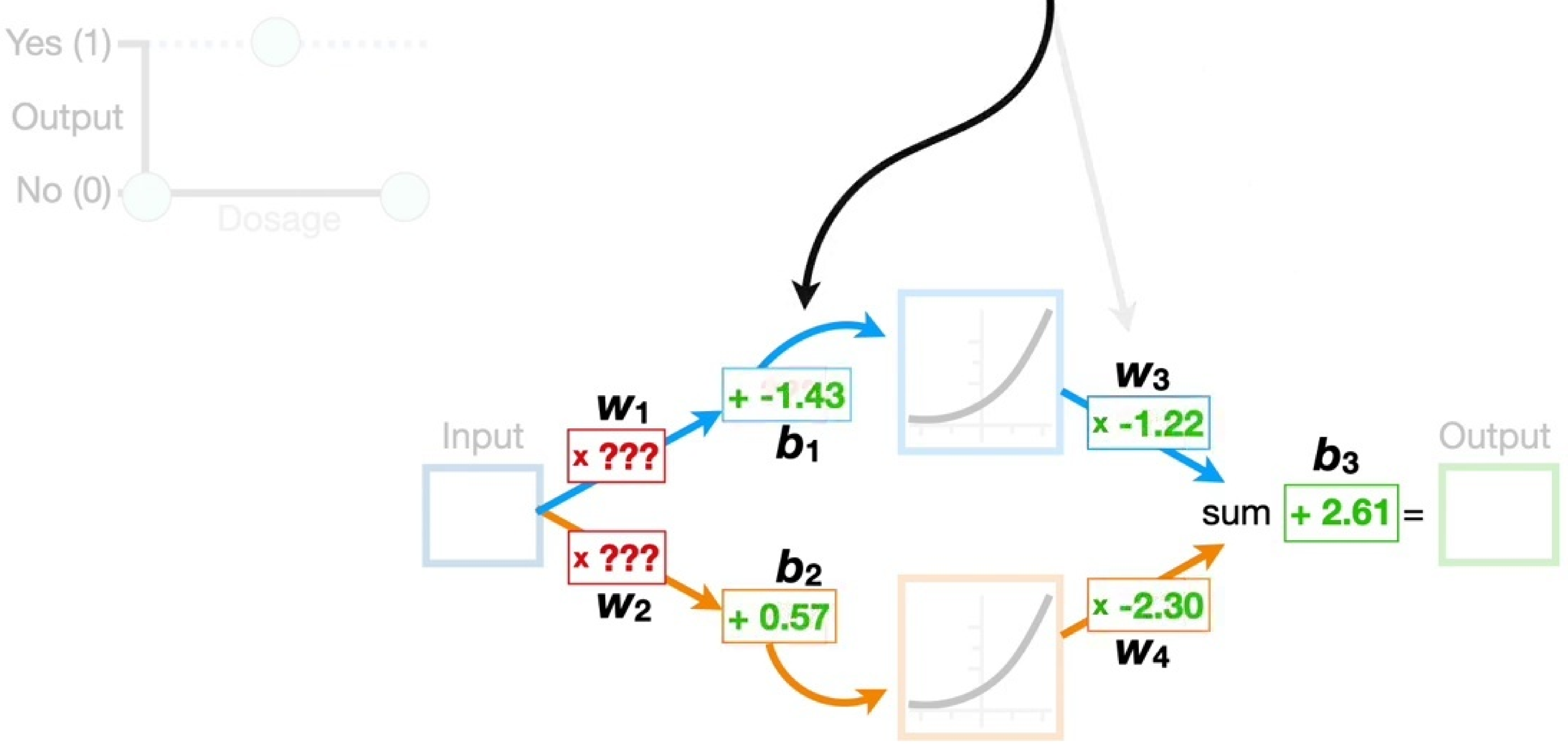

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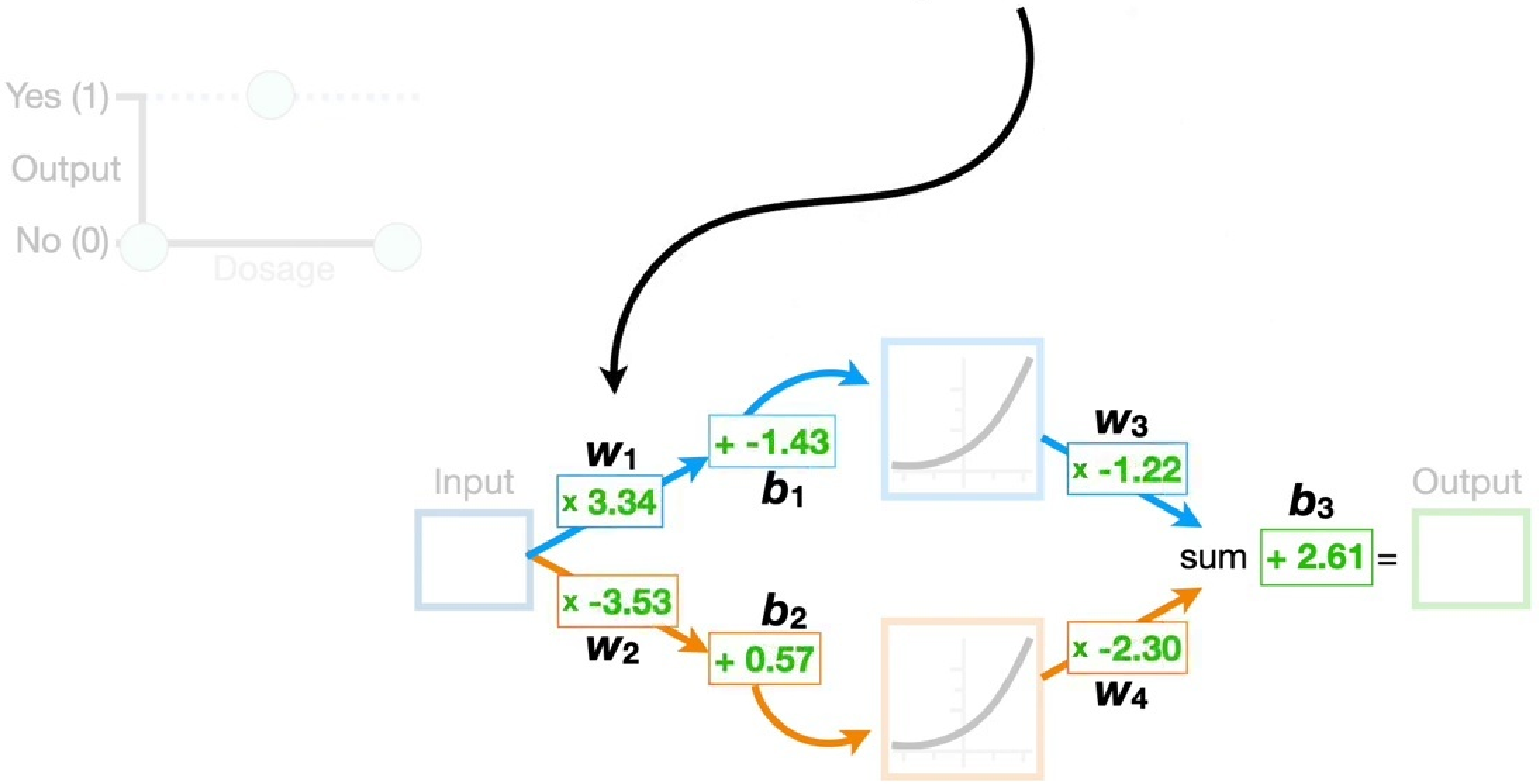
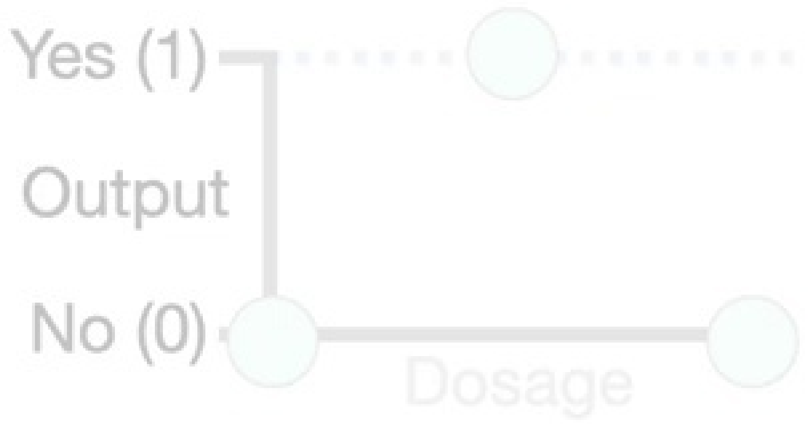
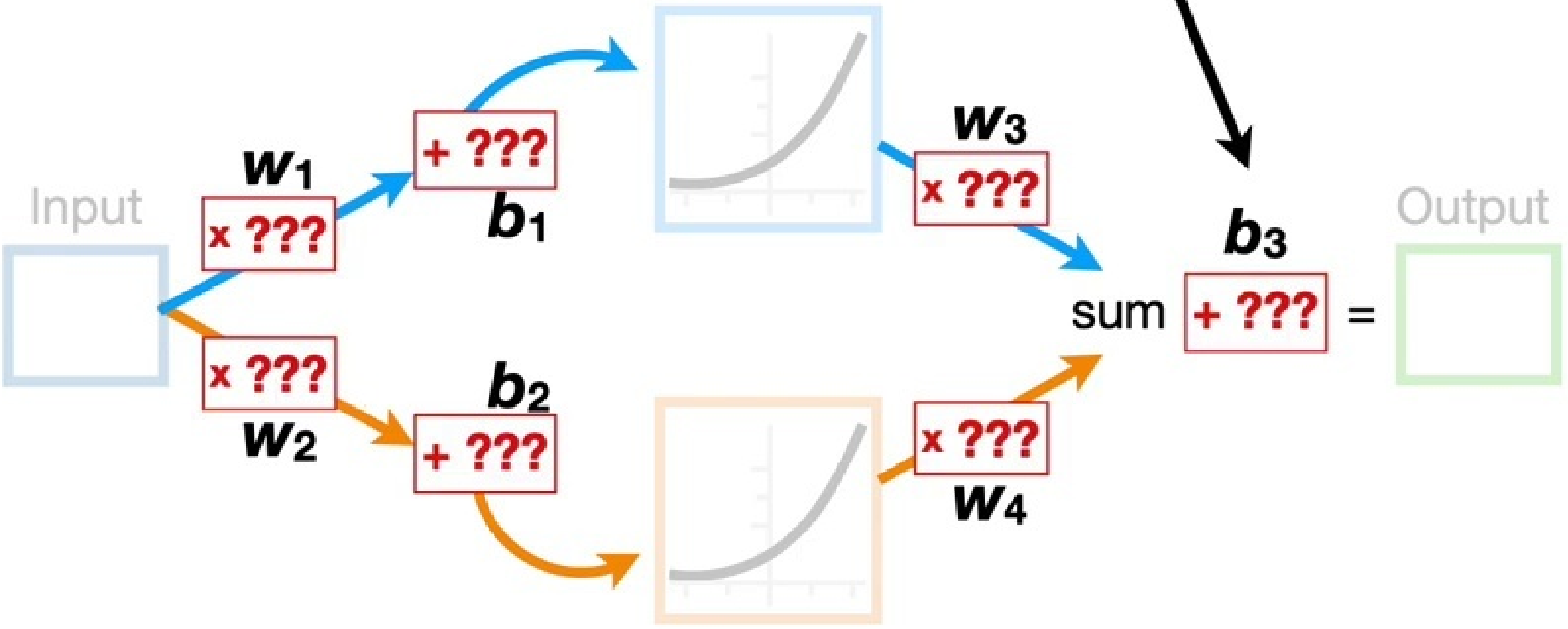
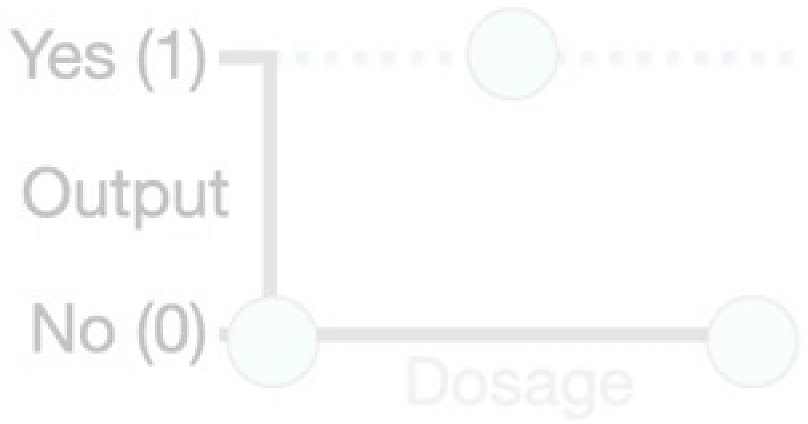


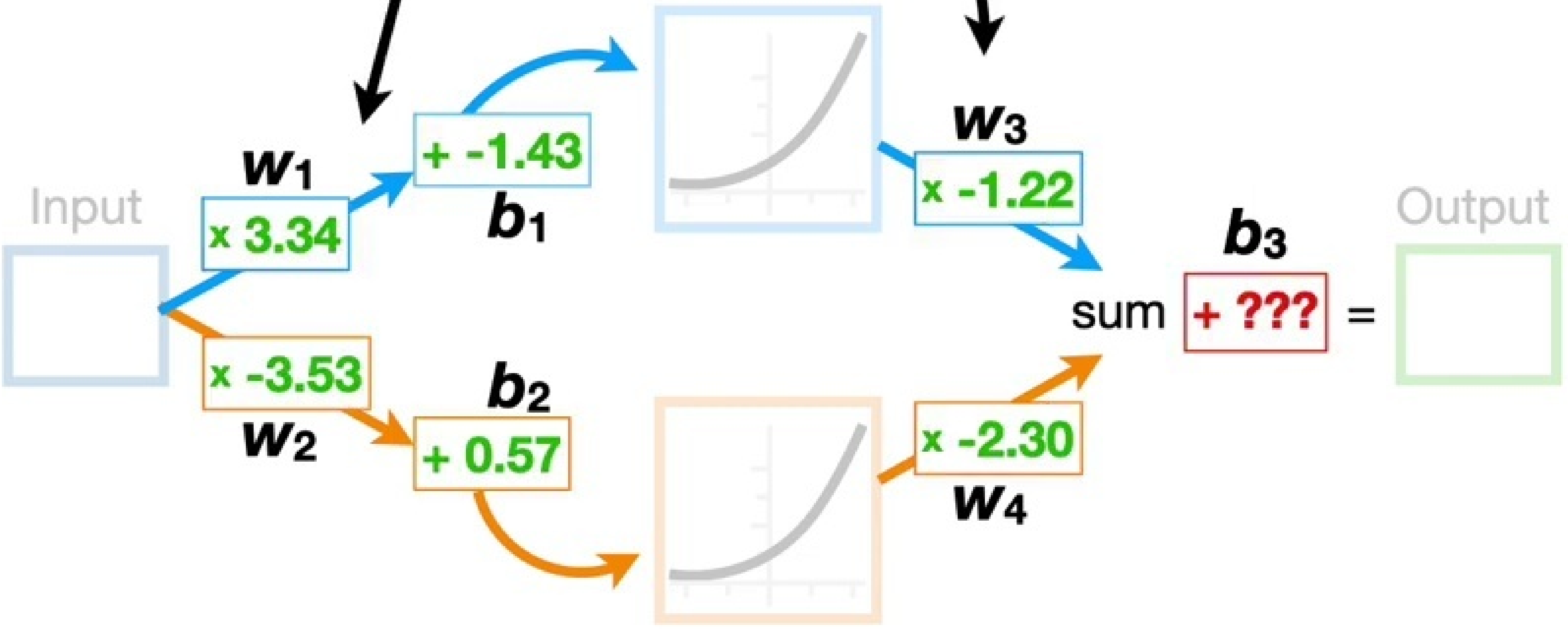
However, we can discuss all of the **Main Ideas** behind **Backpropagation** by just estimating the last **Bias**, ***b*₃**.





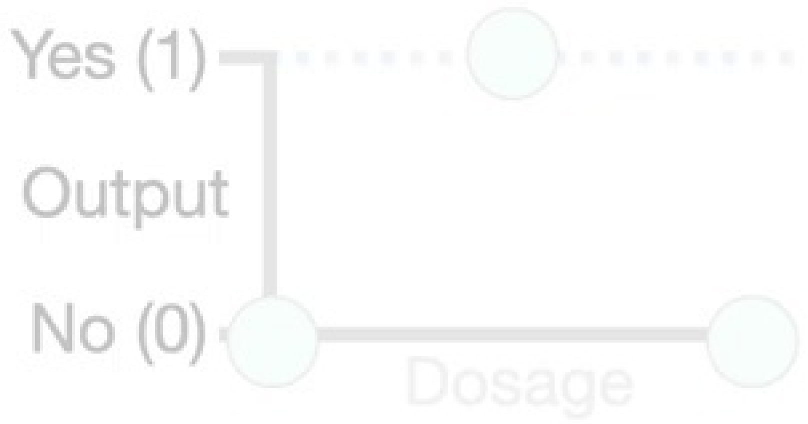
So, in order to start from the back, let's assume that we already have **optimal values** for all of the parameters…

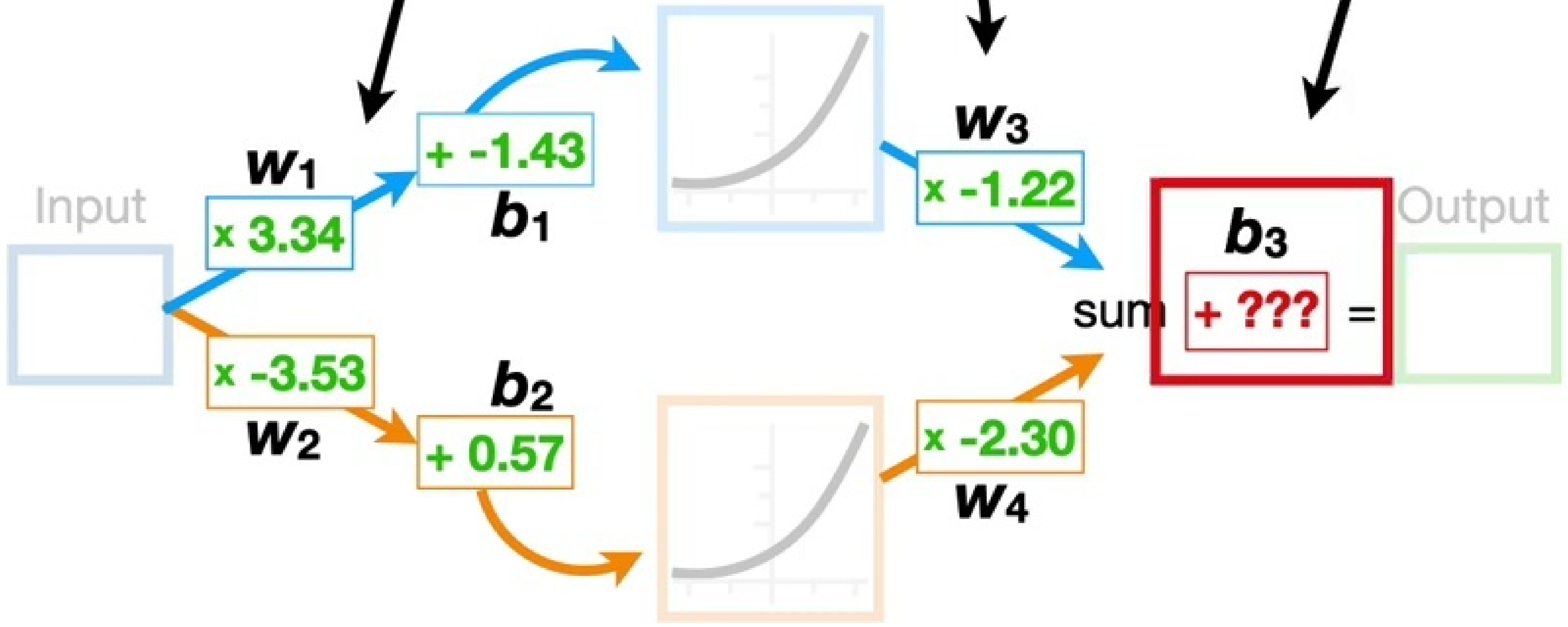




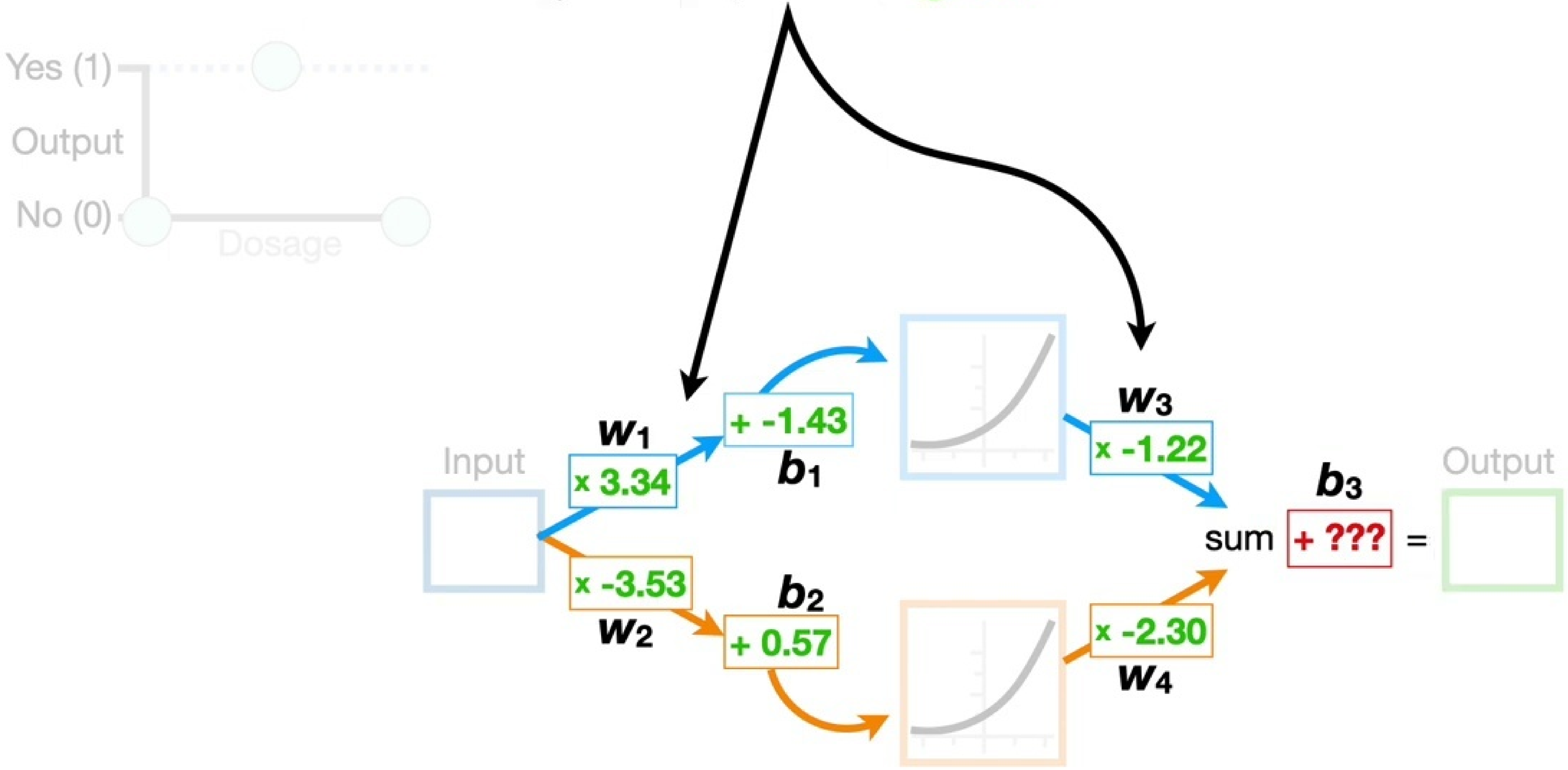
So, in order to start from the back, let's assume that we already have **optimal values** for all of the parameters…

…except for the last **Bias** term, ***b*₃**.



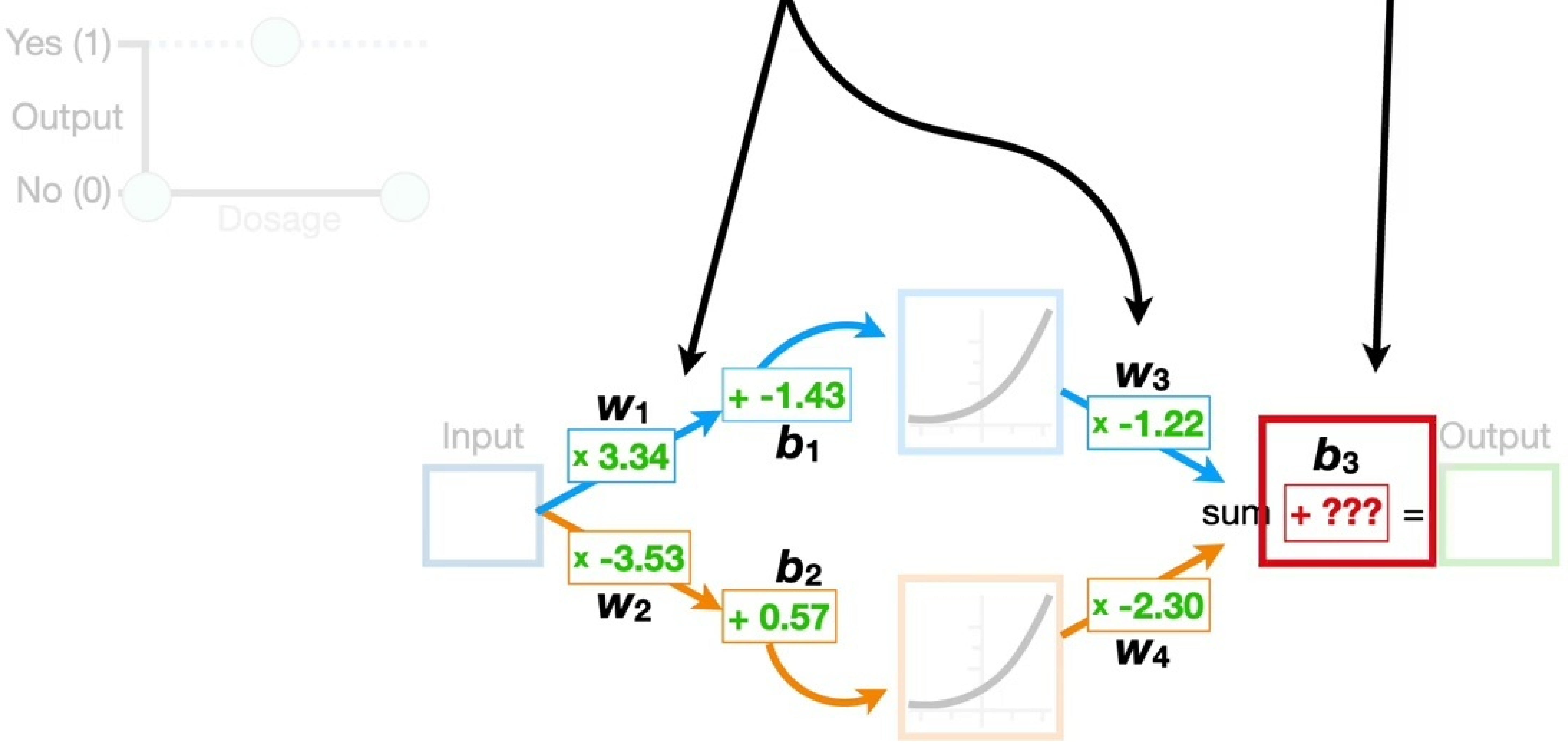


NOTE: Throughout this and the next **StatQuests**, I will make parameter values that have already been optimized **green**…

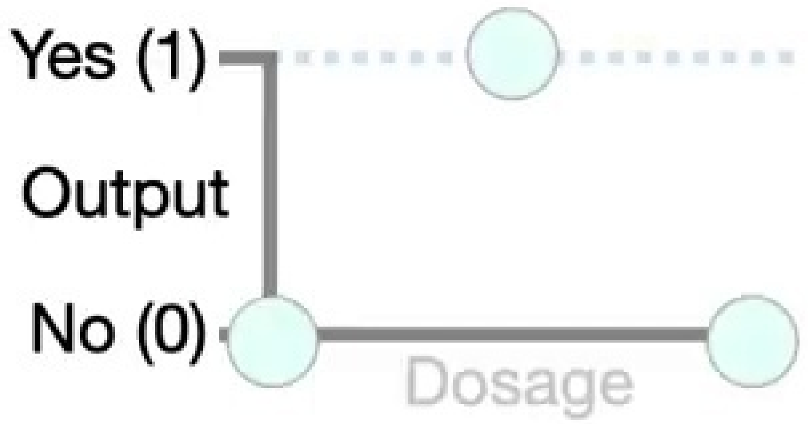


NOTE: Throughout this and the next **StatQuests**, I will make parameter values that have already been optimized green…

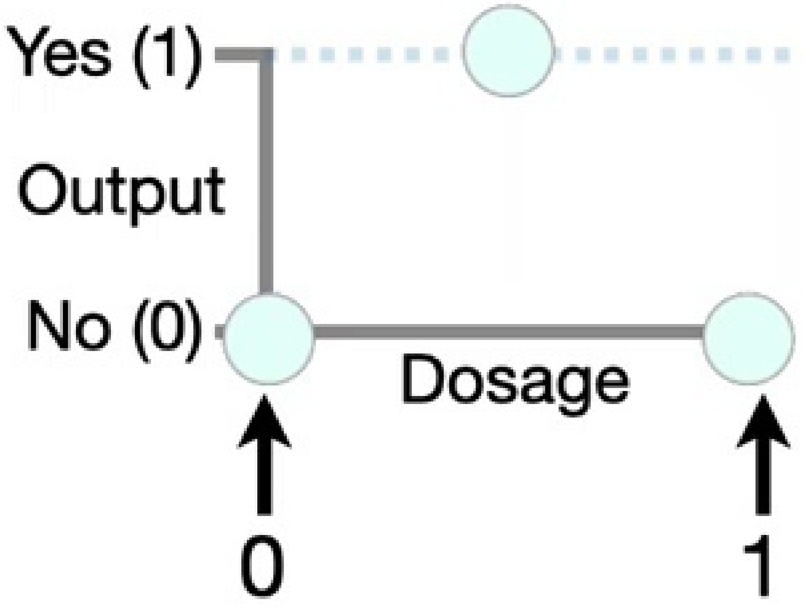
…and unoptimized parameters will be red.



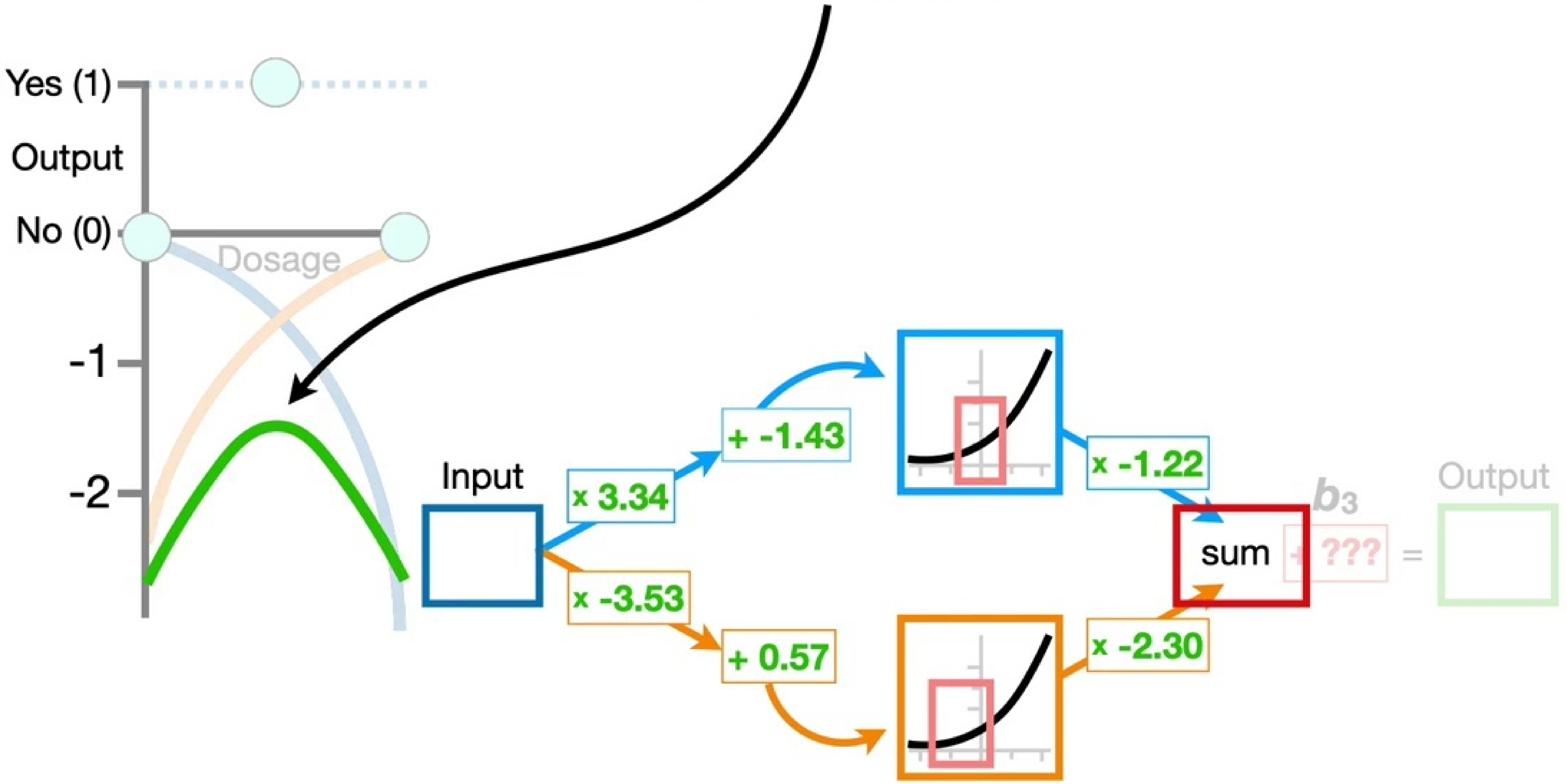
ALSO NOTE: To keep the math simple, let's assume **Dosages** go from **0** (low) to **1** (high).



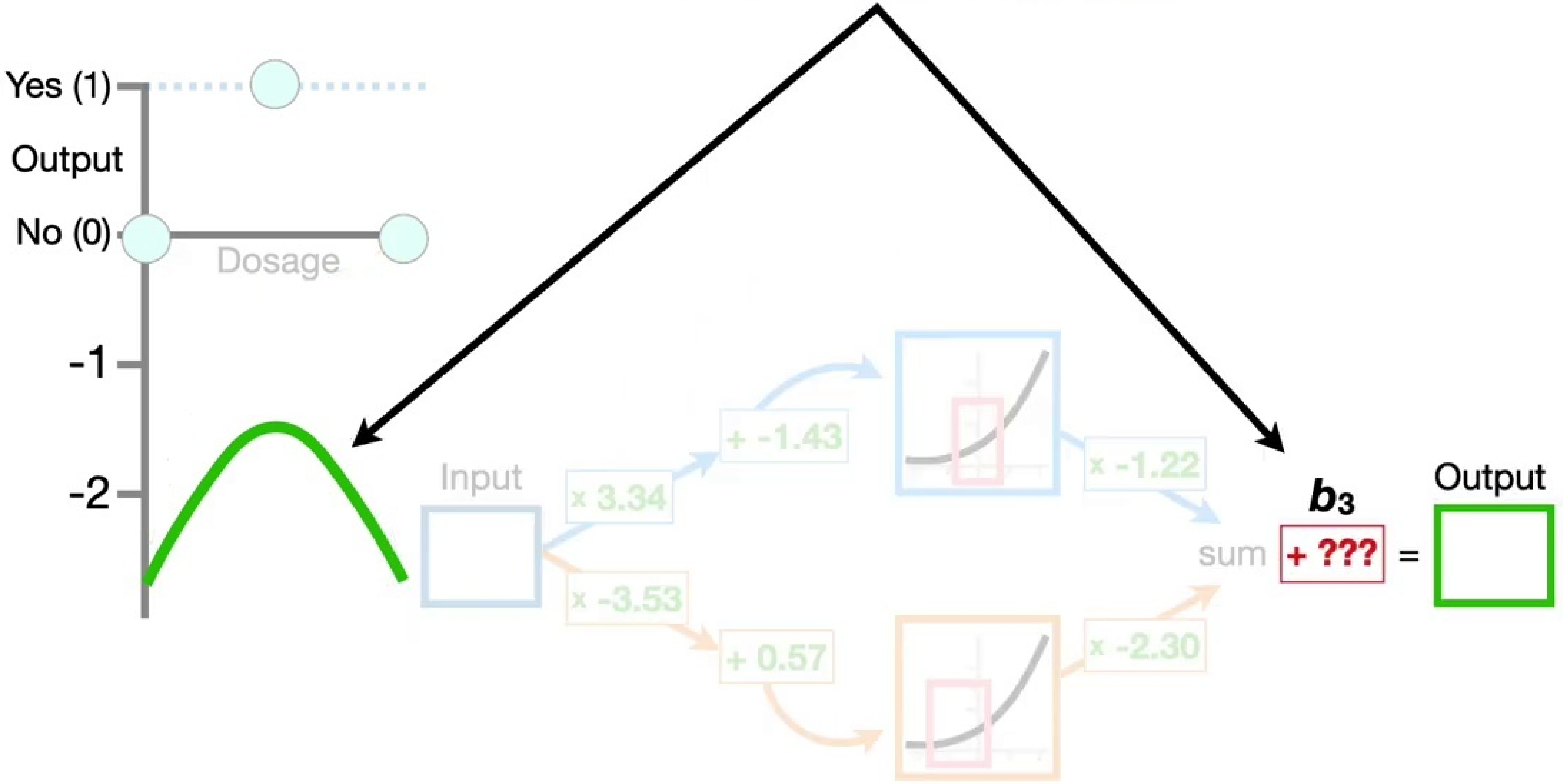
Now, if we run
Dosages from **0** to **1**…



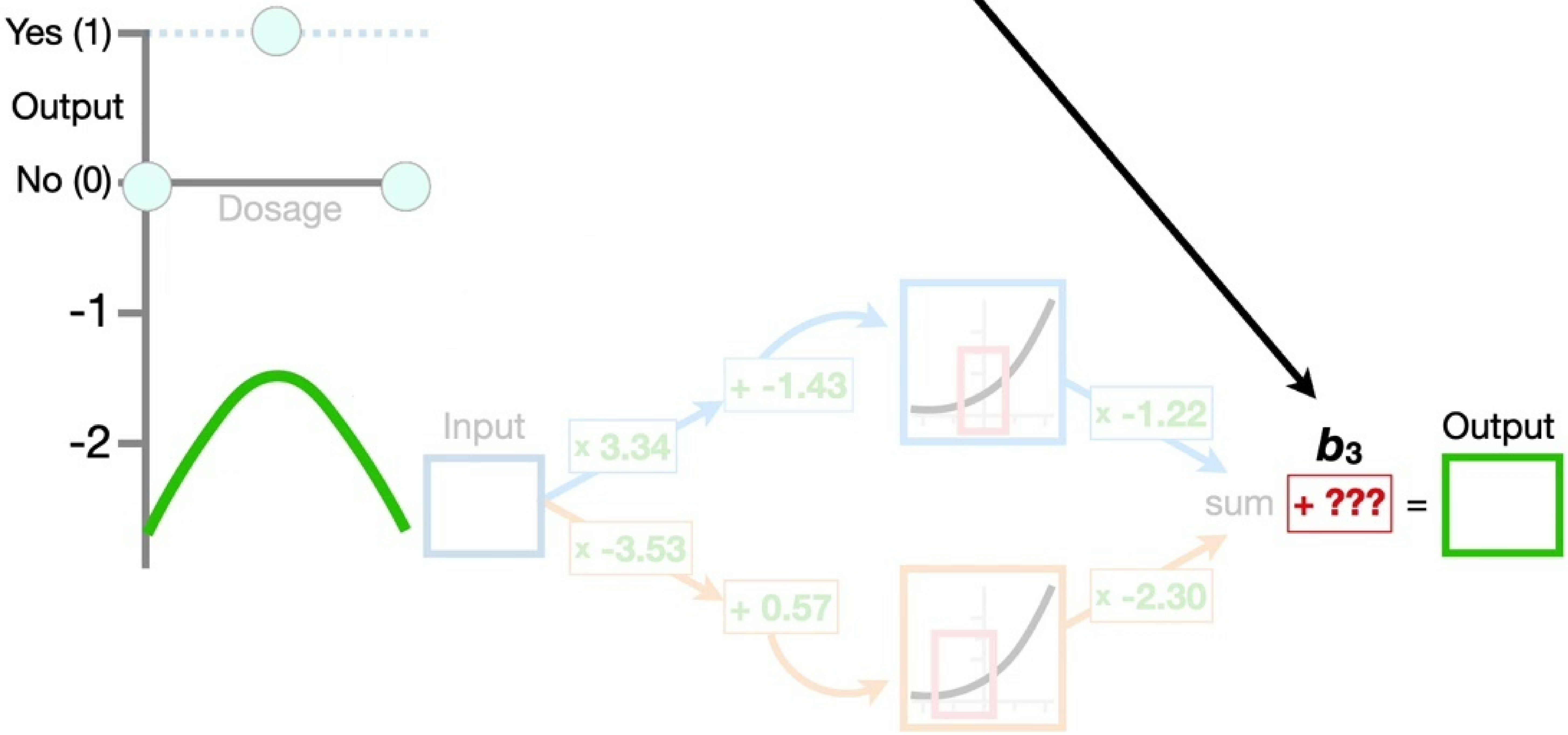
…to get this
green squiggle.



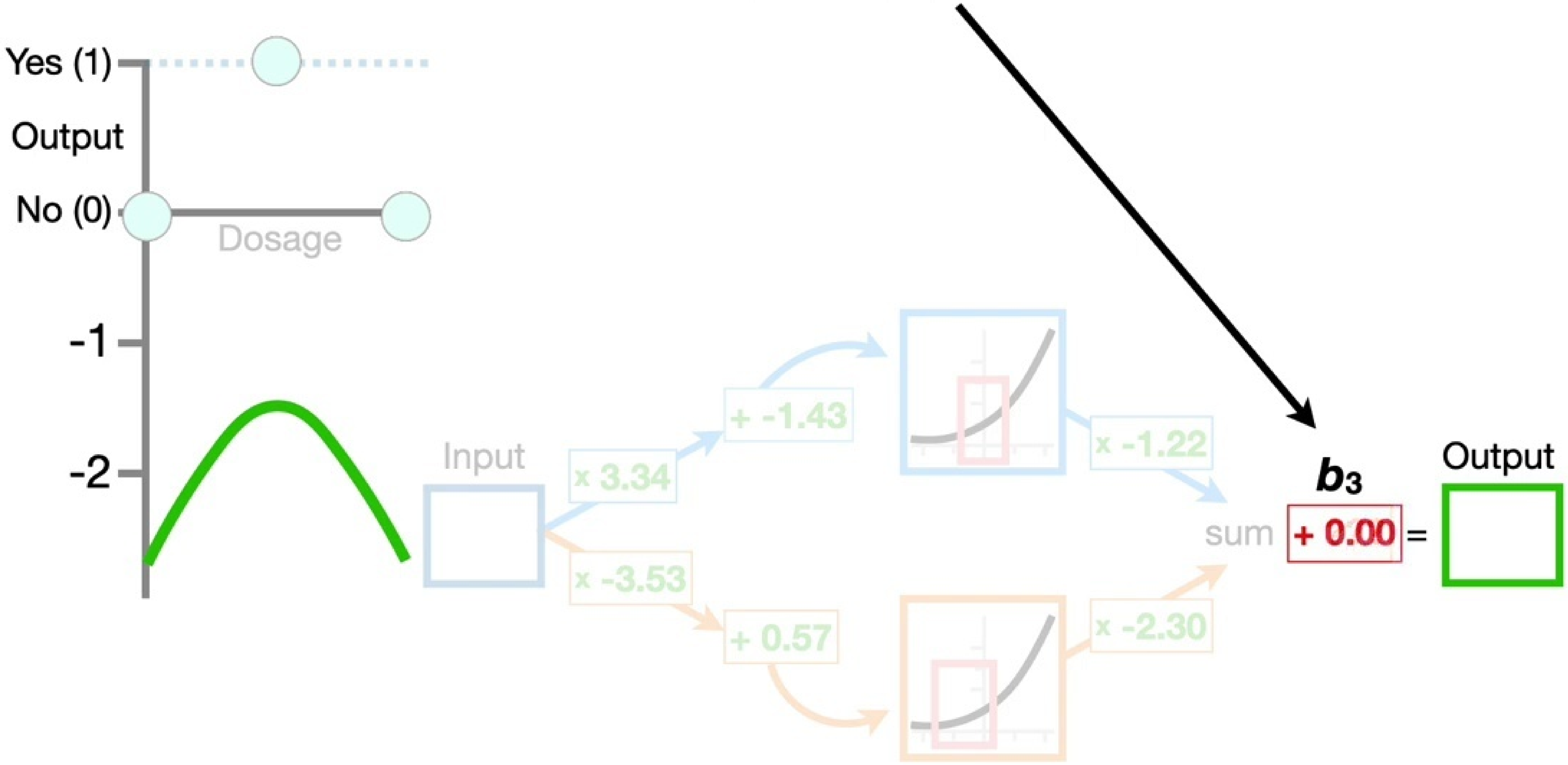
Now we are ready to add the final **Bias, b_3,** to the **green squiggle**.



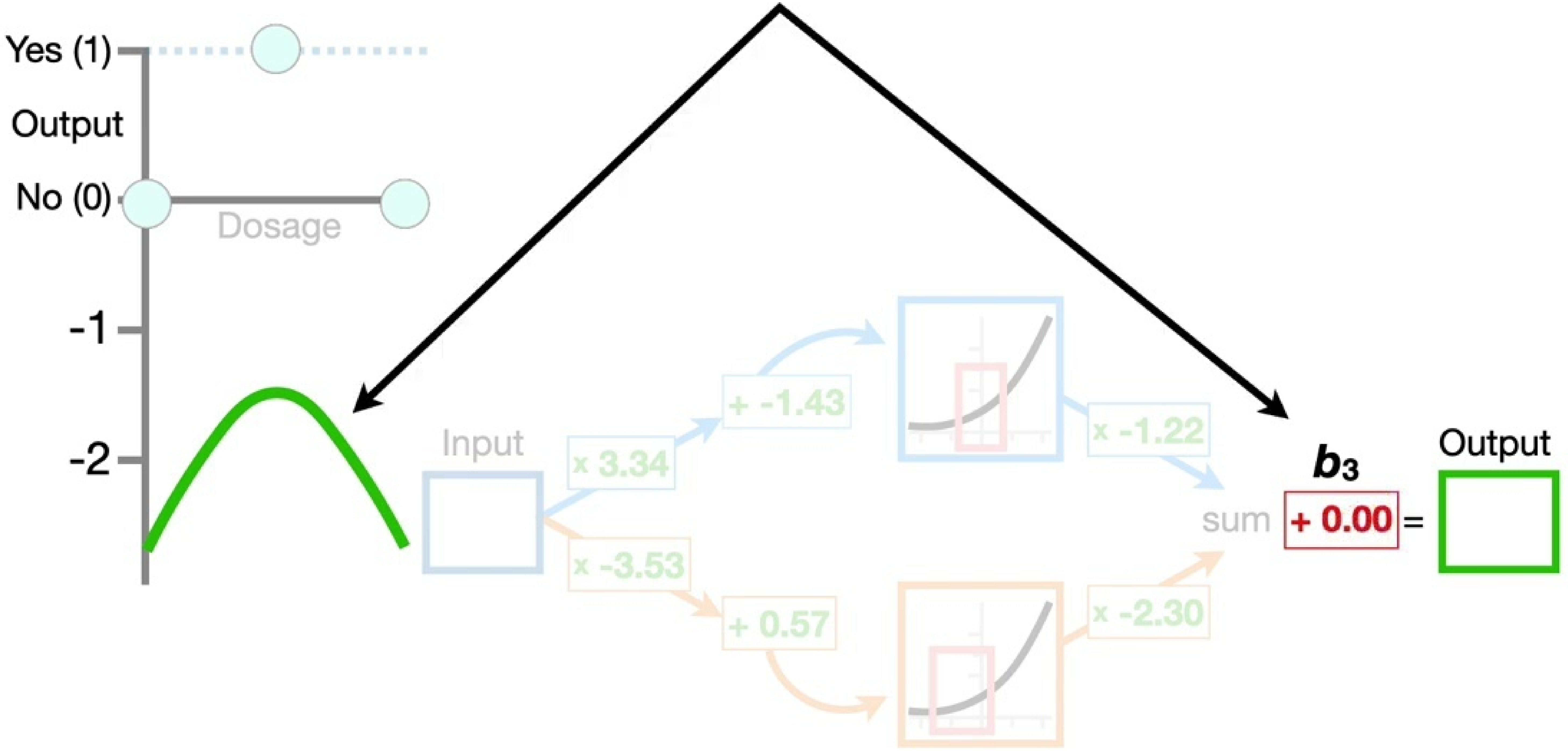
Because we don't yet know the optimal value for b_3, we have to give it an initial value…



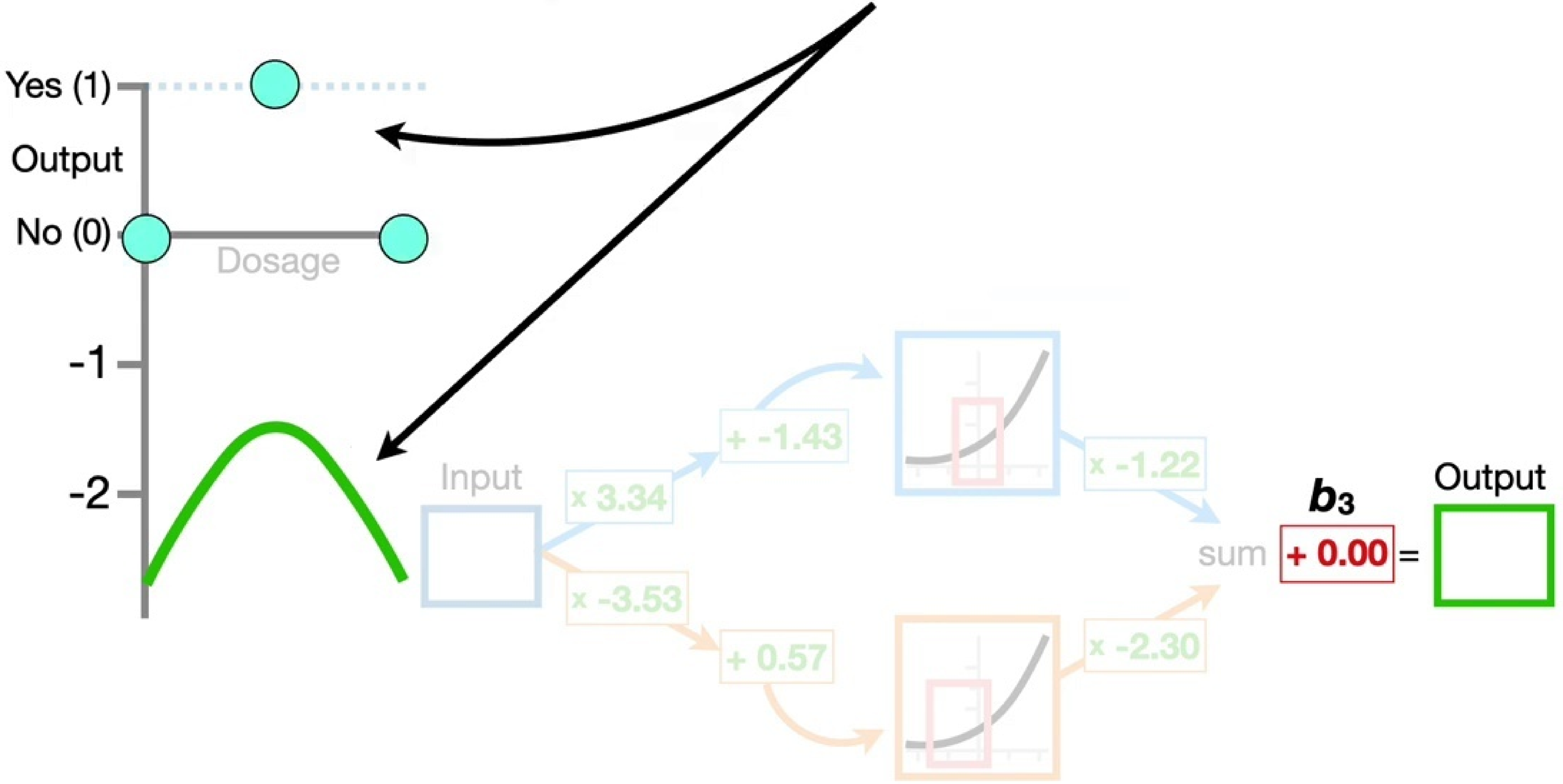
…and because **Bias** terms are frequently initialized to **0**, we will set $\boldsymbol{b_3} = \mathbf{0}$.



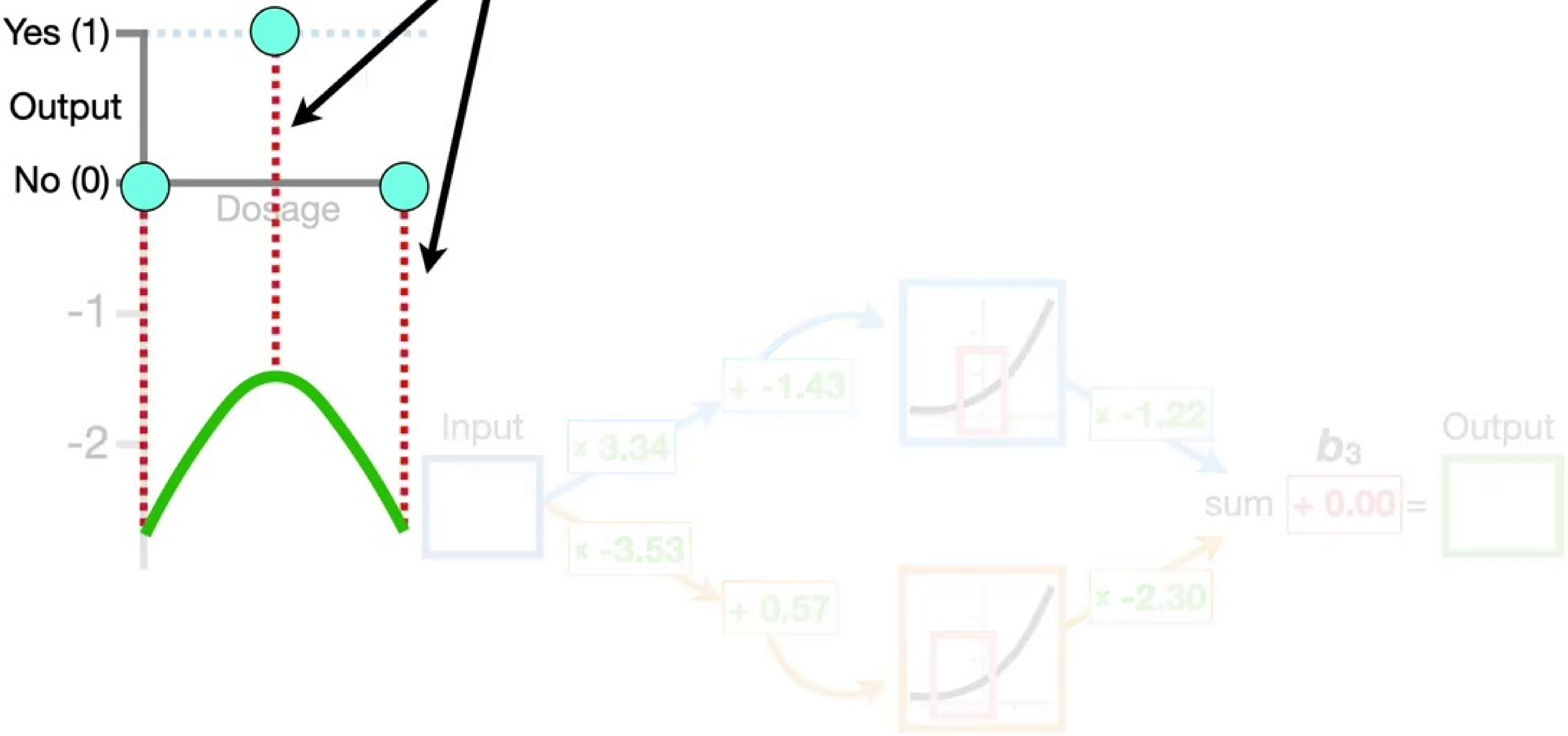
Now, adding **0** to all of the y-axis coordinates on the **green squiggle** leaves it right where it is.



However, that means the **green squiggle** is pretty far from the data that we observed.

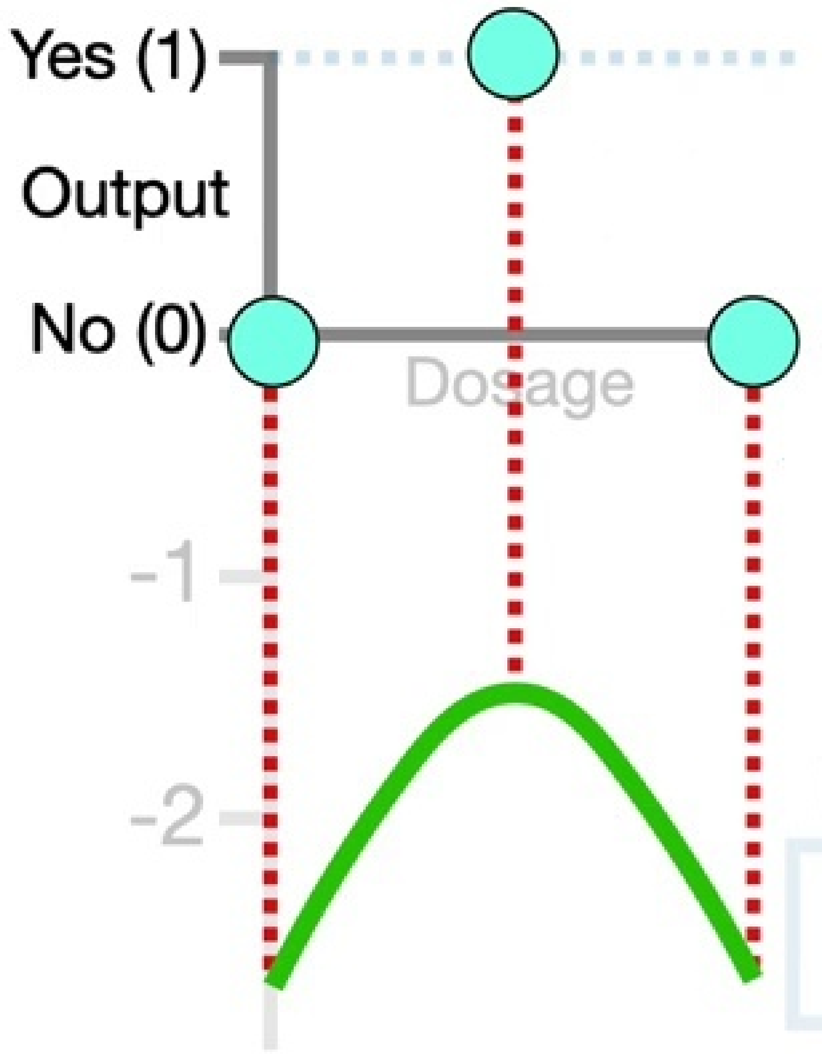


We can quantify how good the **green squiggle** fits the data by calculating the **Sum of the Squared Residuals.**

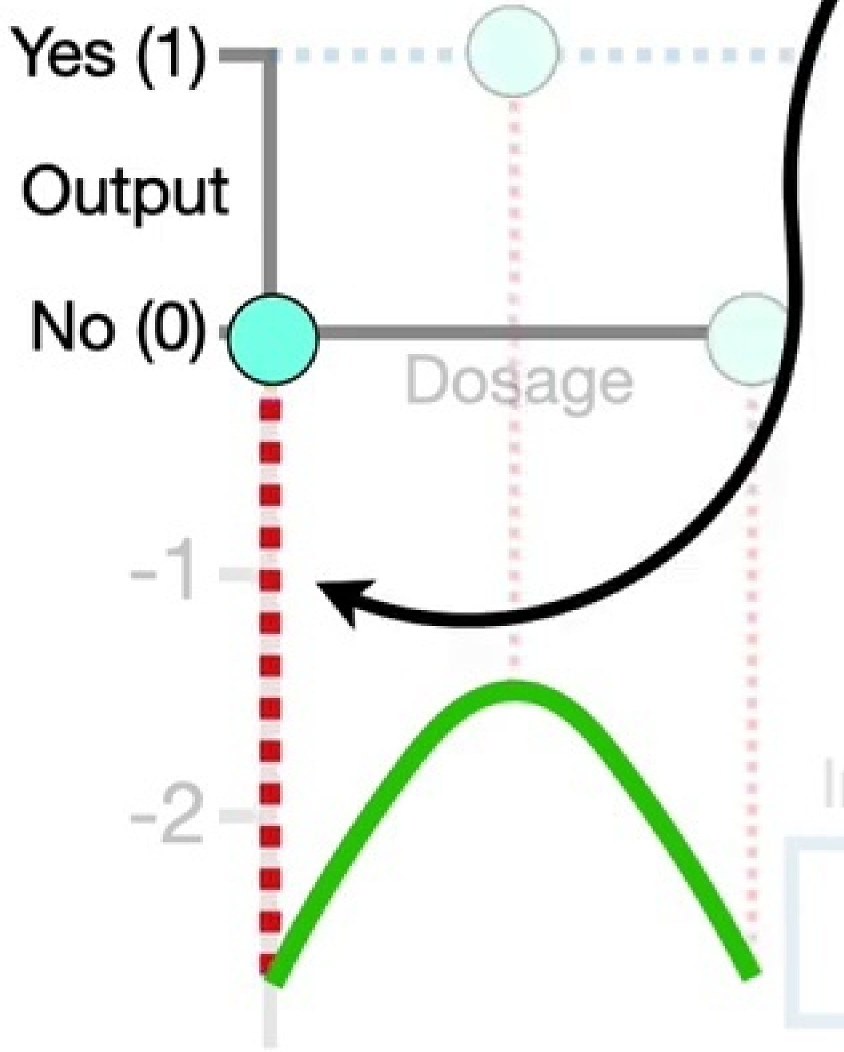


A **Residual** is the difference between the **Observed** and **Predicted** values.

Residual = (Observed - Predicted)



For example,
this **Residual**…

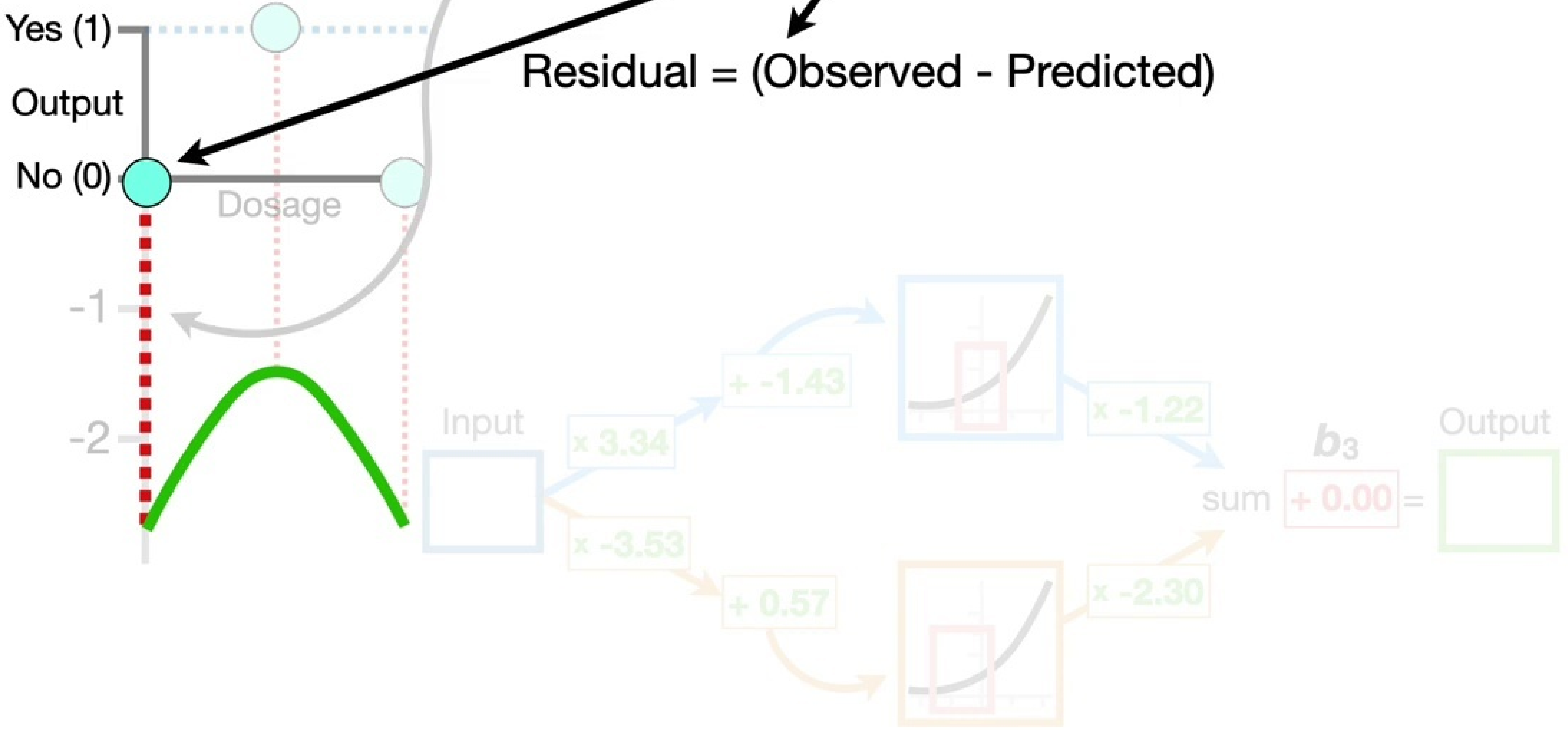


Residual = (Observed - Predicted)

For example, this **Residual**…

…is the **Observed** value, **0**…

Residual = (Observed - Predicted)

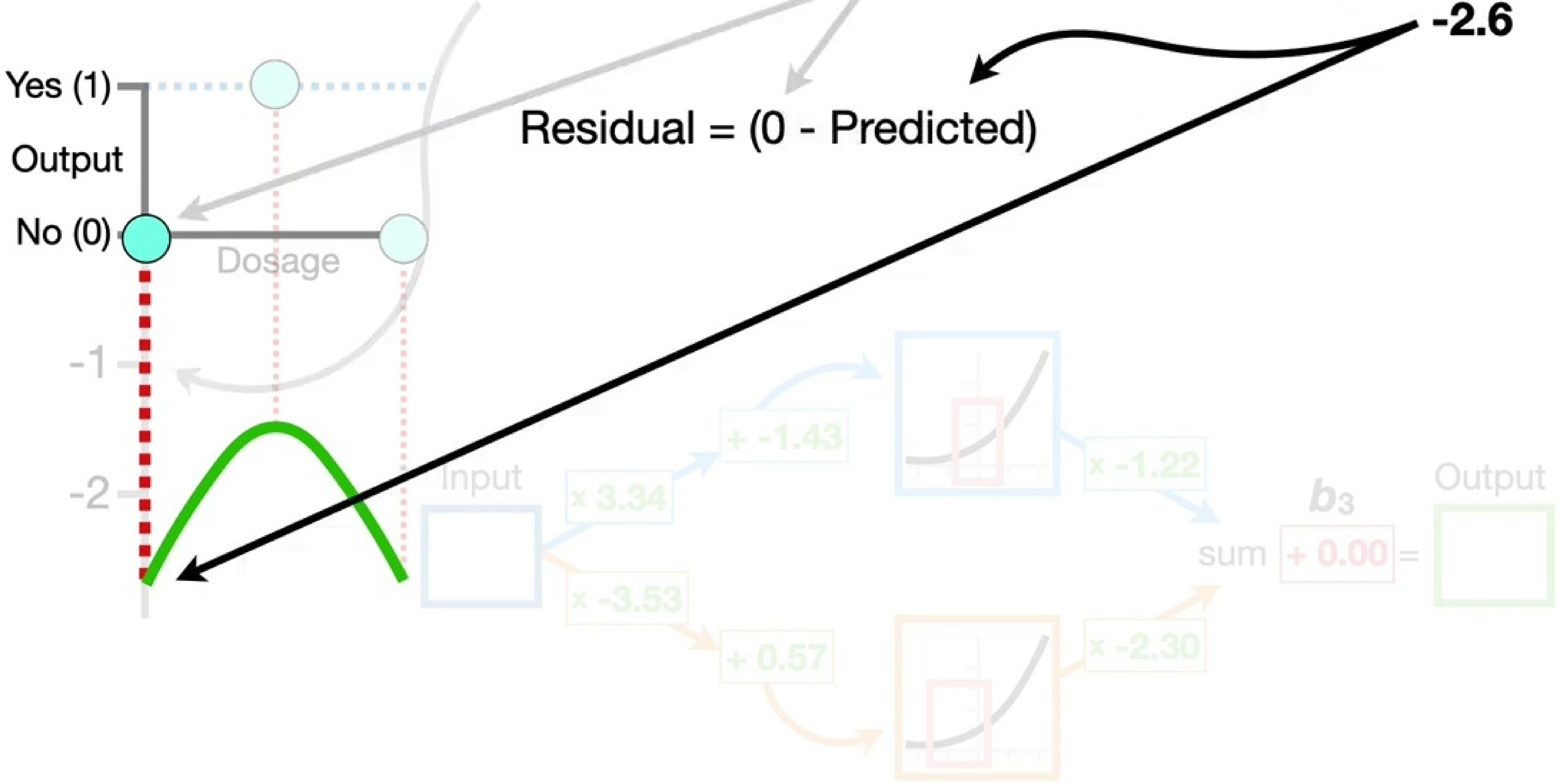


For example, this **Residual**…

…is the **Observed** value, **0**…

…minus the **Predicted** value from the **green squiggle**, **-2.6**

Residual = (0 - Predicted)

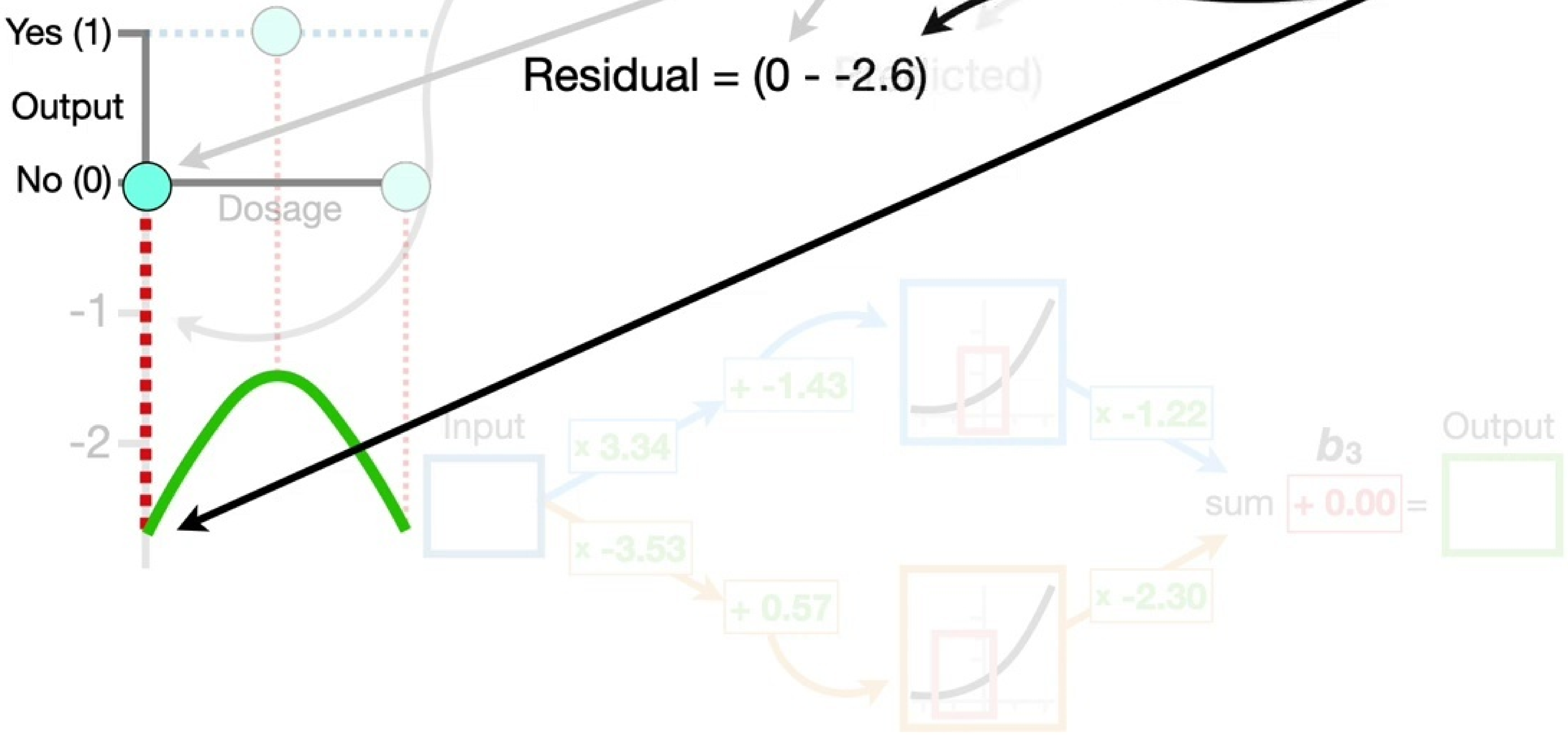


For example, this **Residual**…

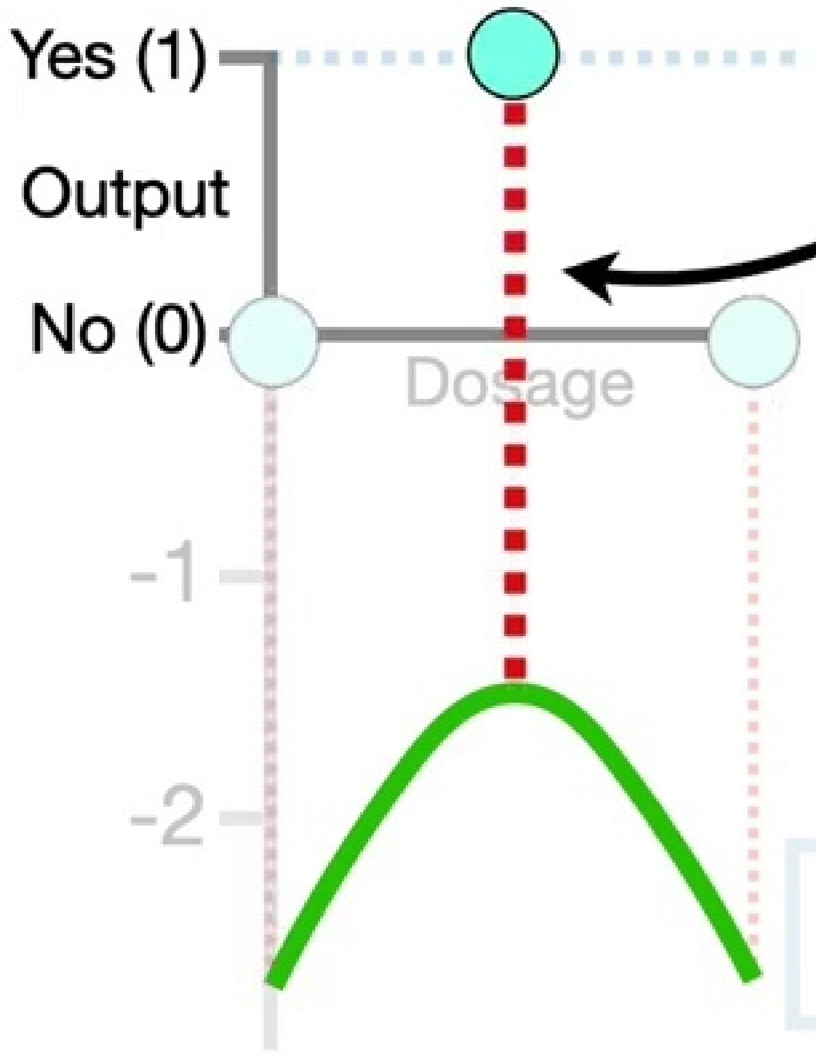
…is the **Observed** value, **0**…

…minus the **Predicted** value from the **green squiggle**, **-2.6**

Residual = (0 - -2.6)



This **Residual**…



(0 - -2.6)

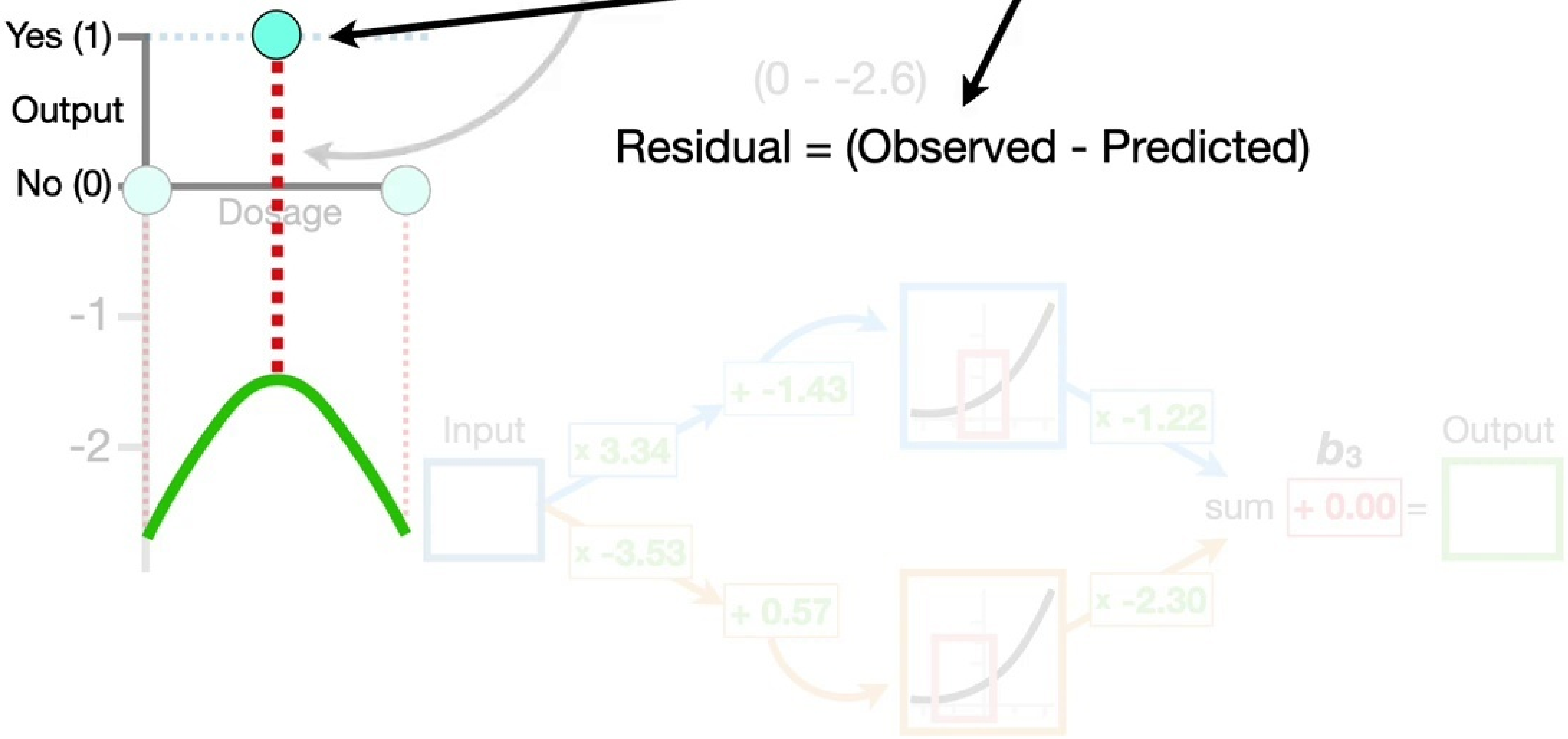
Residual = (Observed - Predicted)

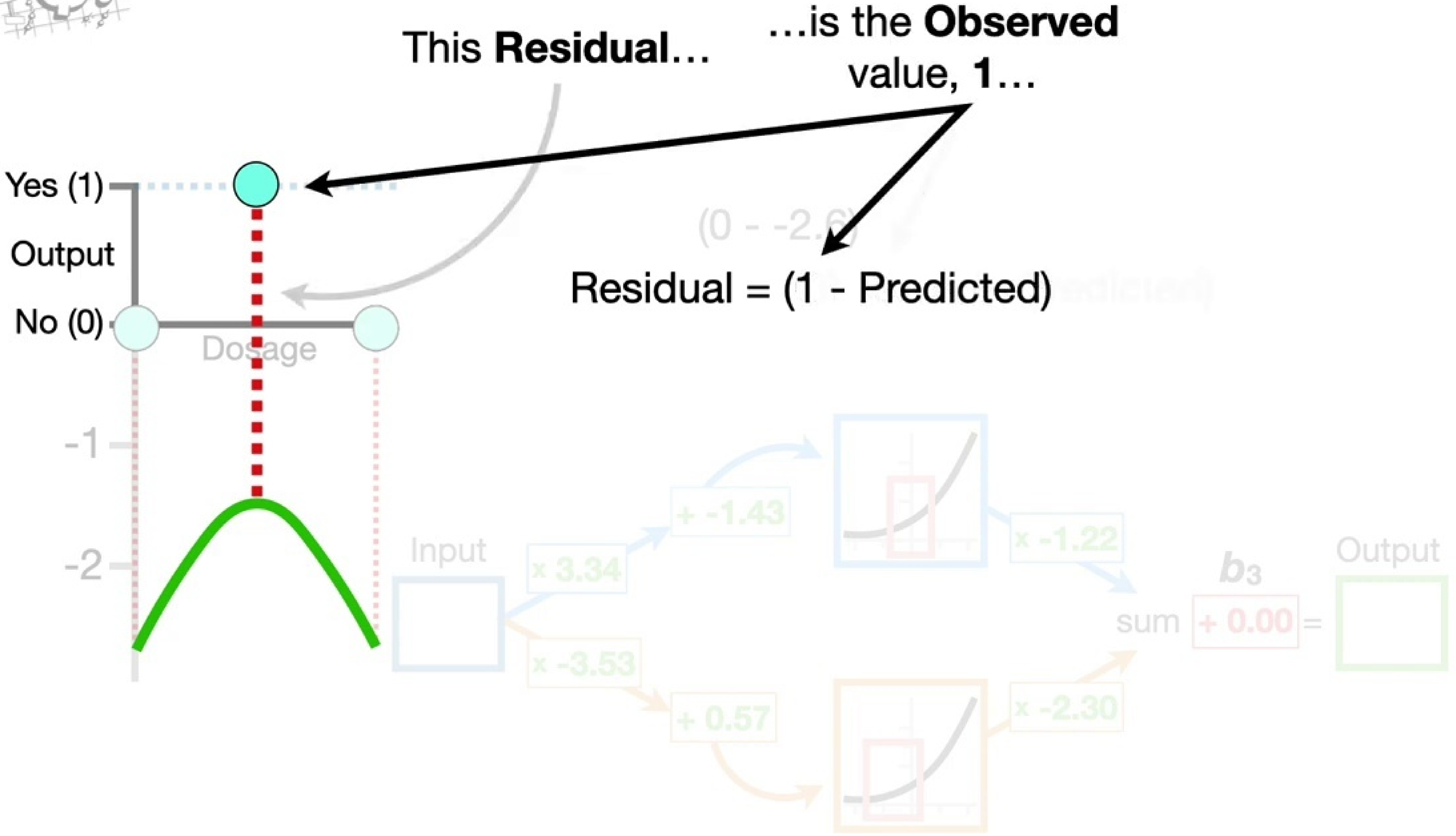
This **Residual**…

…is the **Observed** value, **1**…

(0 - -2.6)

Residual = (Observed - Predicted)



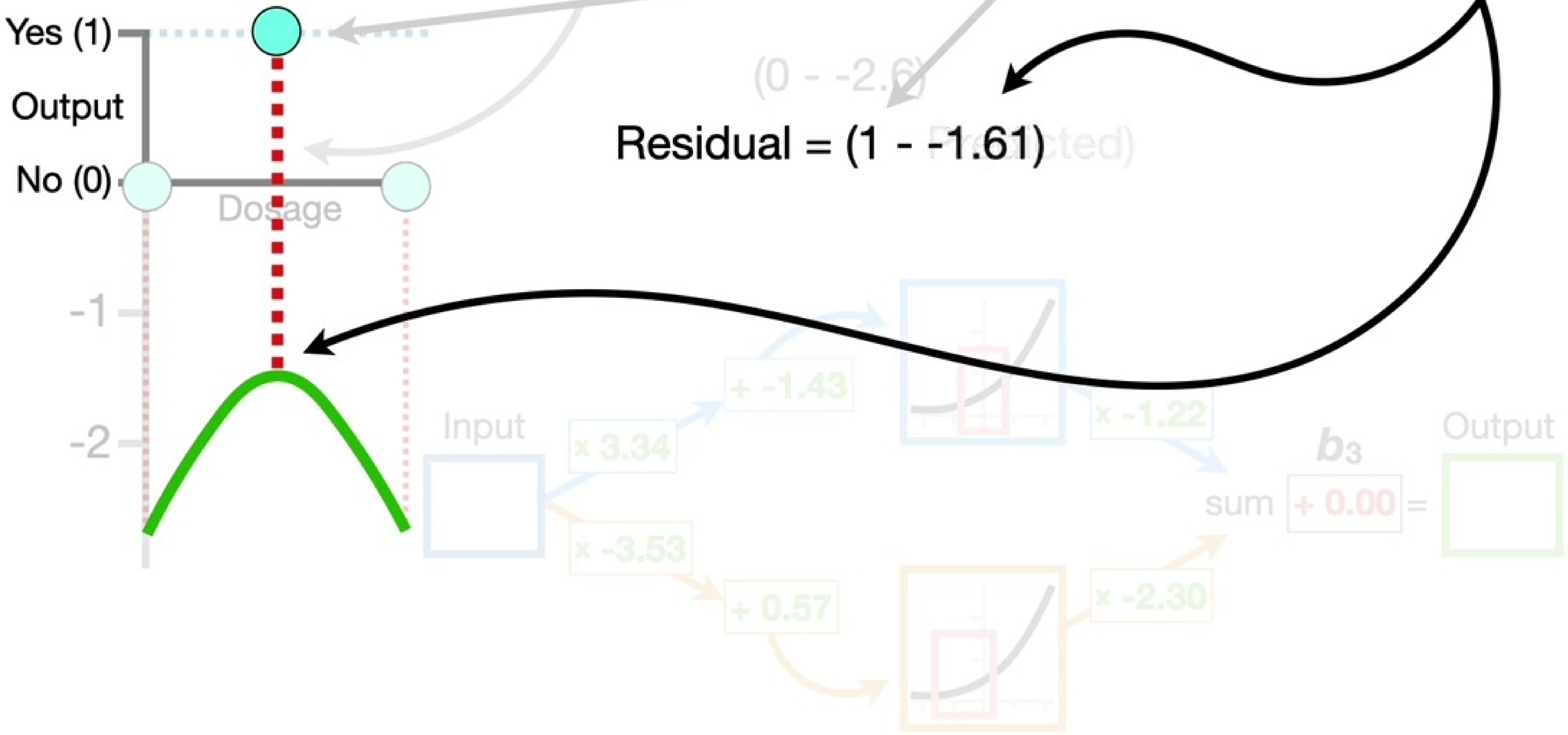

This **Residual**…
…is the **Observed** value, **1**…
Residual = (1 - Predicted)
Yes (1)
Output
No (0)
Dosage
-1
-2
Input
x 3.34
+ -1.43
x -1.22
x -3.53
+ 0.57
x -2.30
sum
b_3
+ 0.00 =
Output

This **Residual**…

…is the **Observed** value, **1**…

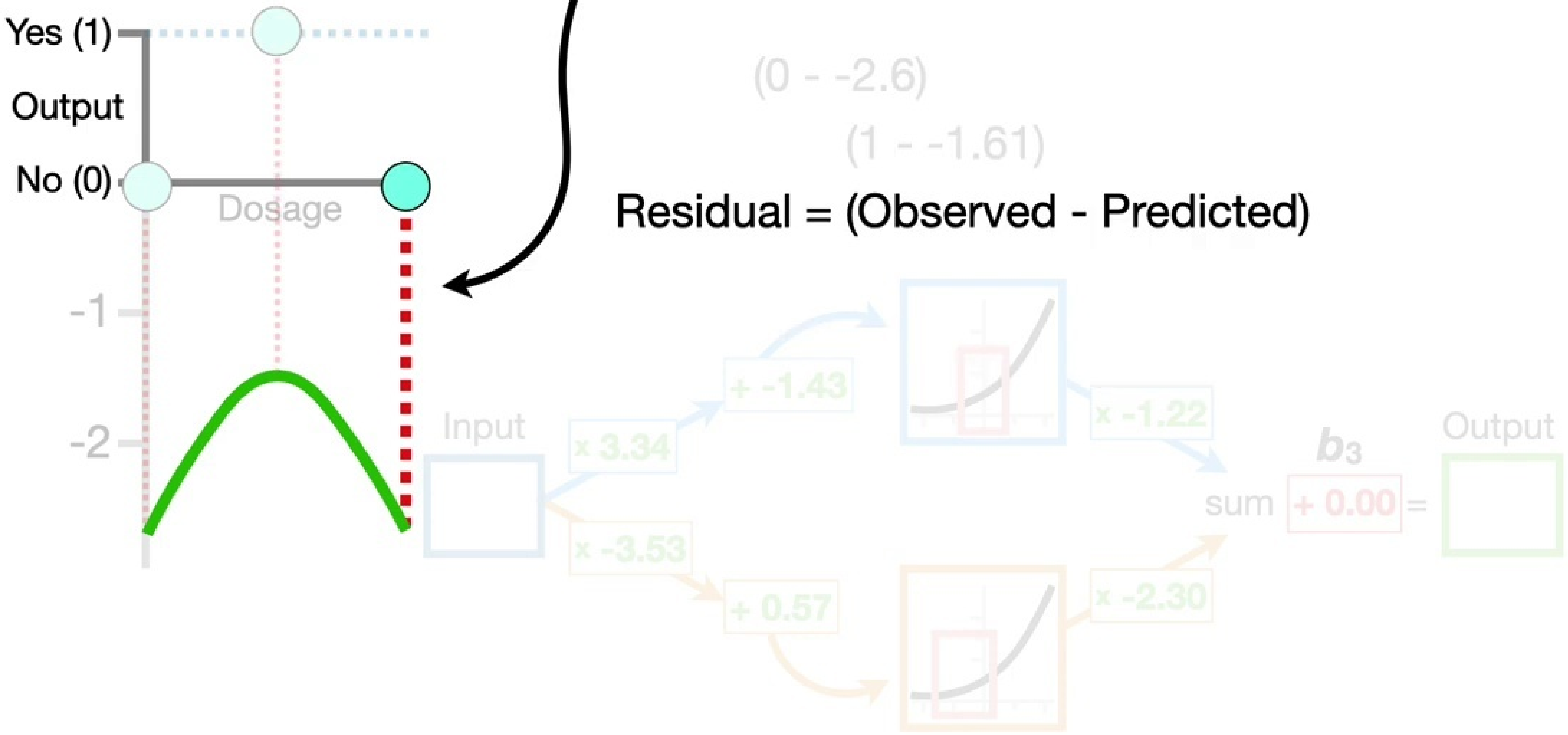
…minus the **Predicted** value from the green squiggle, **-1.61**

Residual = (1 - -1.61)



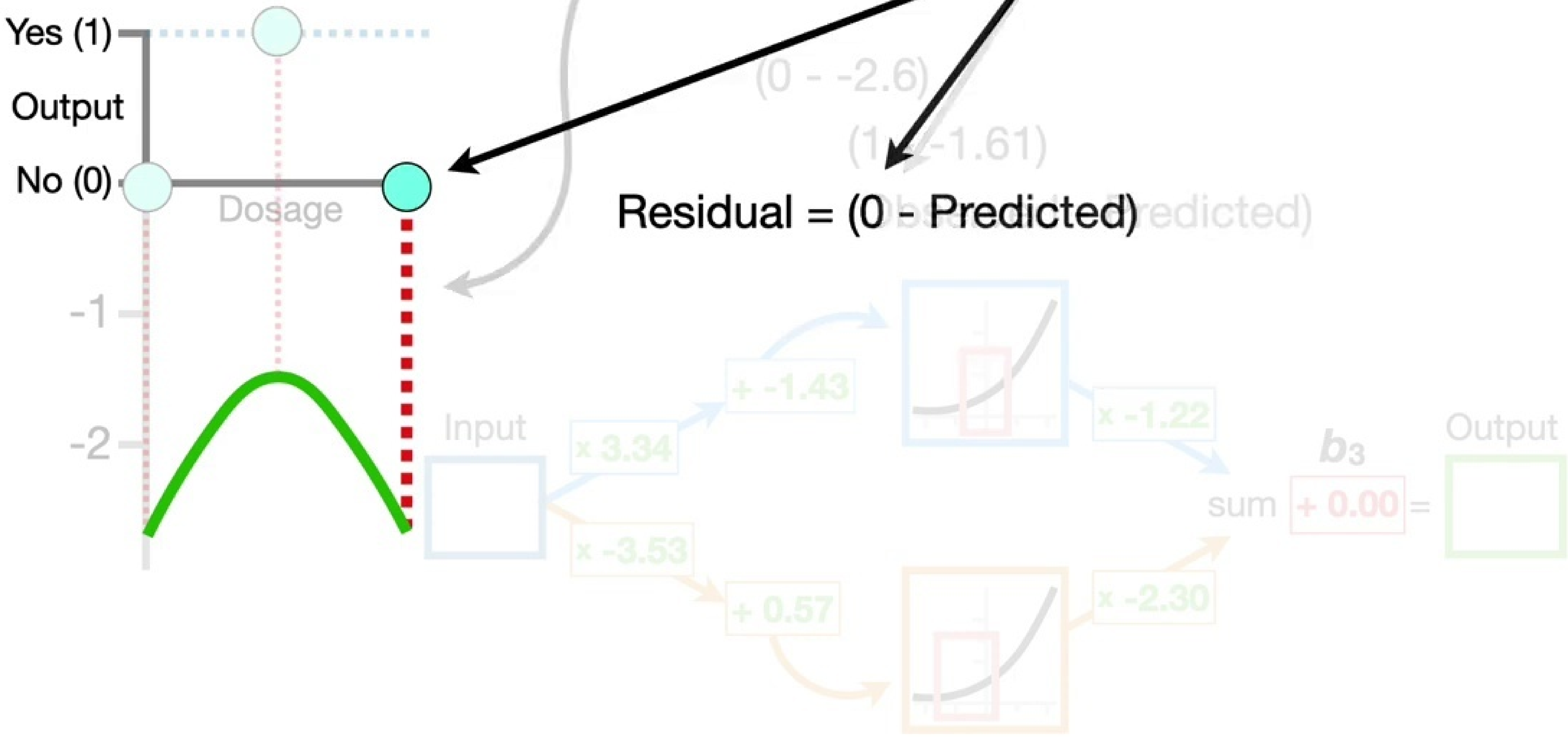
Lastly, this **Residual**…

Residual = (Observed - Predicted)

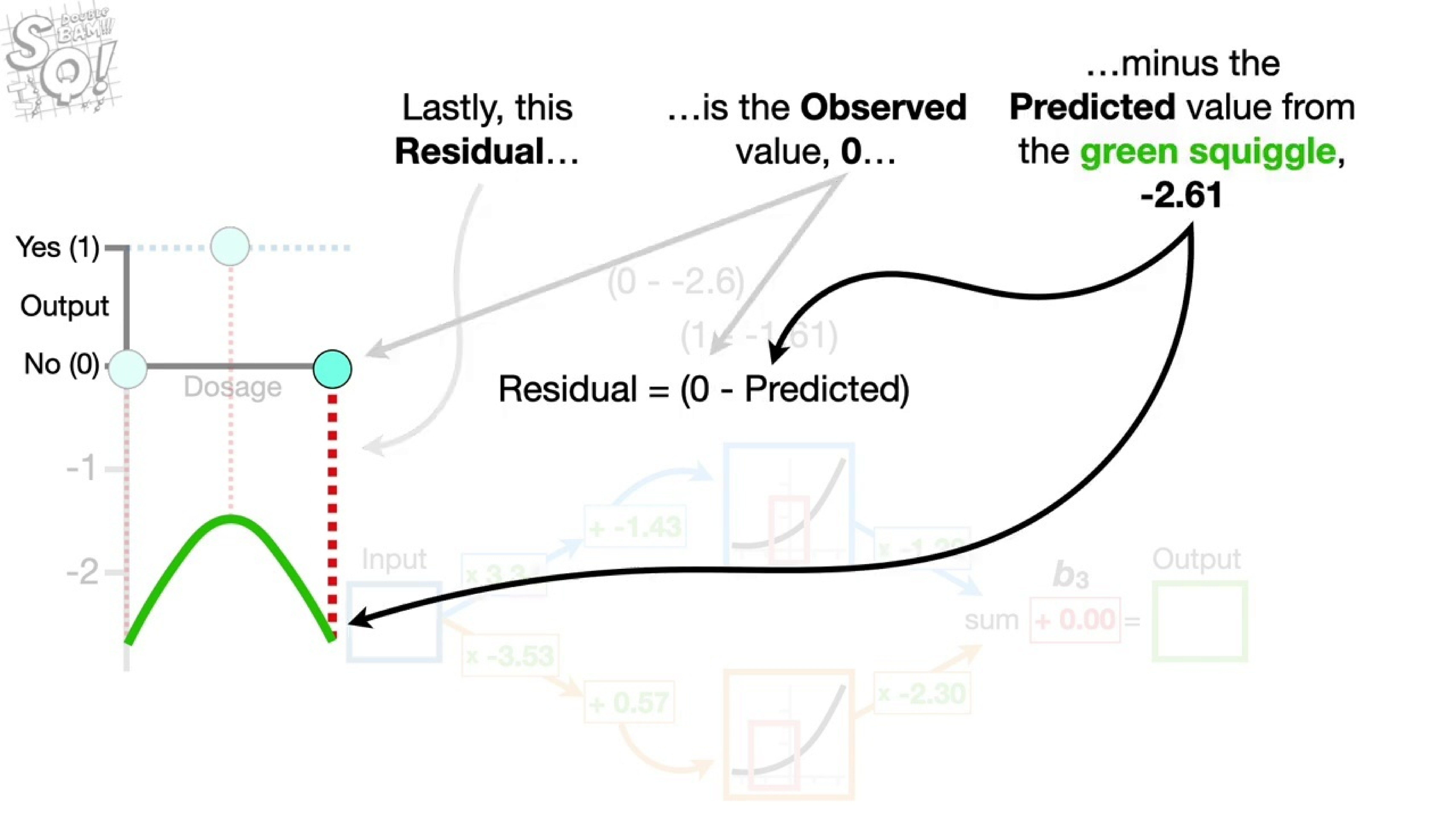


Lastly, this **Residual**…

…is the **Observed** value, **0**…



Residual = (0 - Predicted)

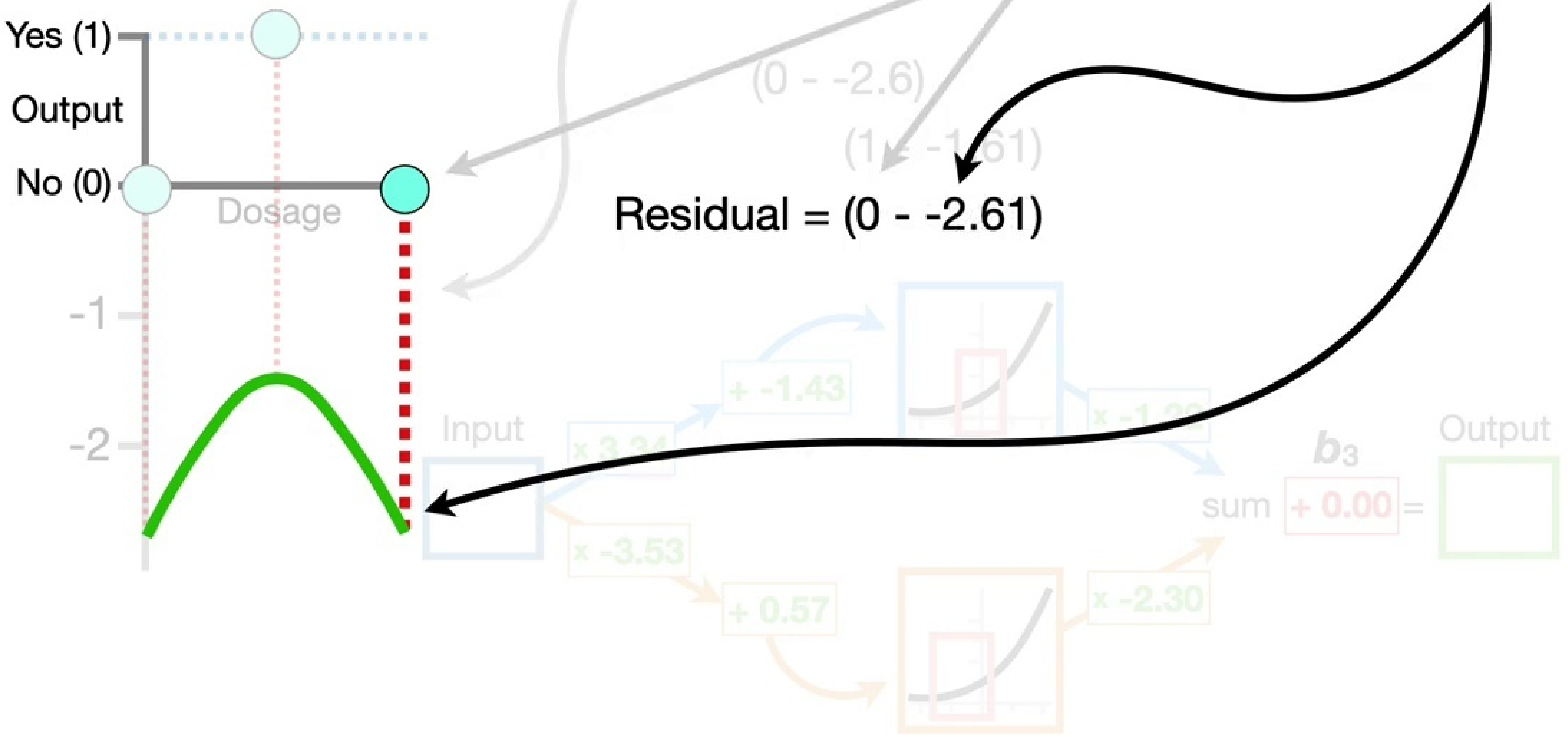

Lastly, this **Residual**…
…is the **Observed** value, **0**…
…minus the **Predicted** value from the **green squiggle**, **-2.61**
Yes (1)
Output
No (0)
Dosage
-1
-2
Residual = (0 - Predicted)
(0 - -2.6)
Input
+ -1.43
x -3.53
+ 0.57
x -2.30
sum
+ 0.00
=
Output

Lastly, this **Residual**…

…is the **Observed** value, **0**…

…minus the **Predicted** value from the **green squiggle**, **-2.61**

Residual = (0 - -2.61)

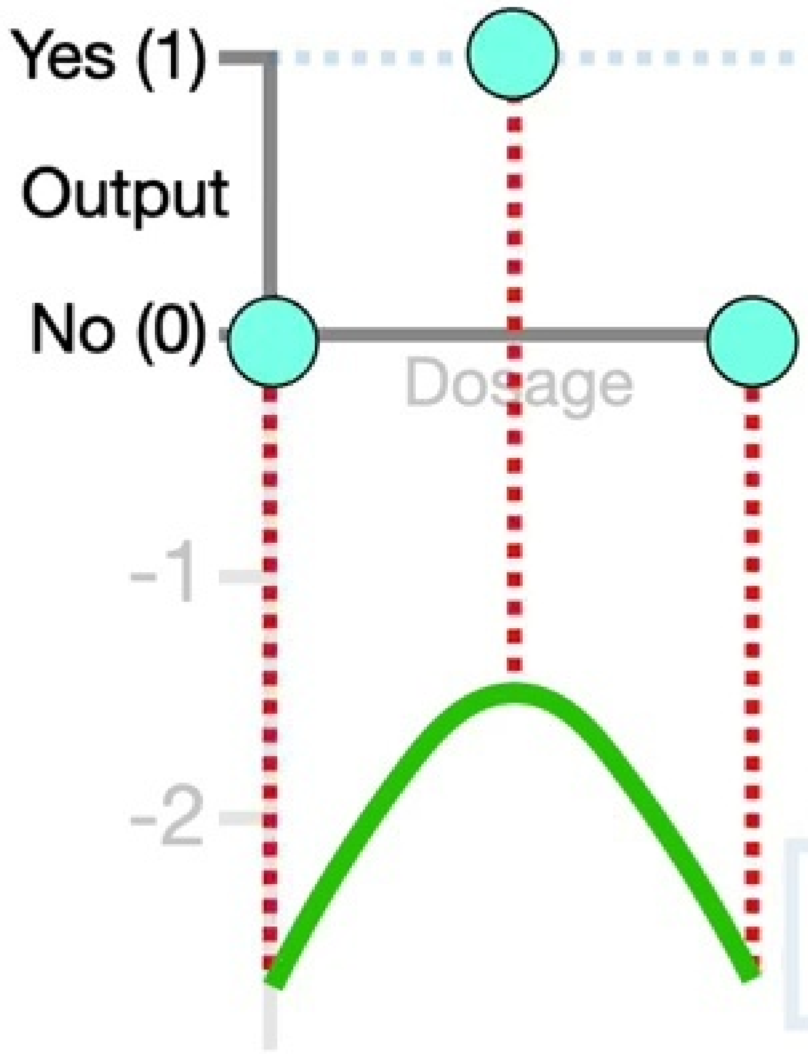


Now we square each **Residual**…

(0 - -2.6)

(1 - -1.61)

(0 - -2.61)

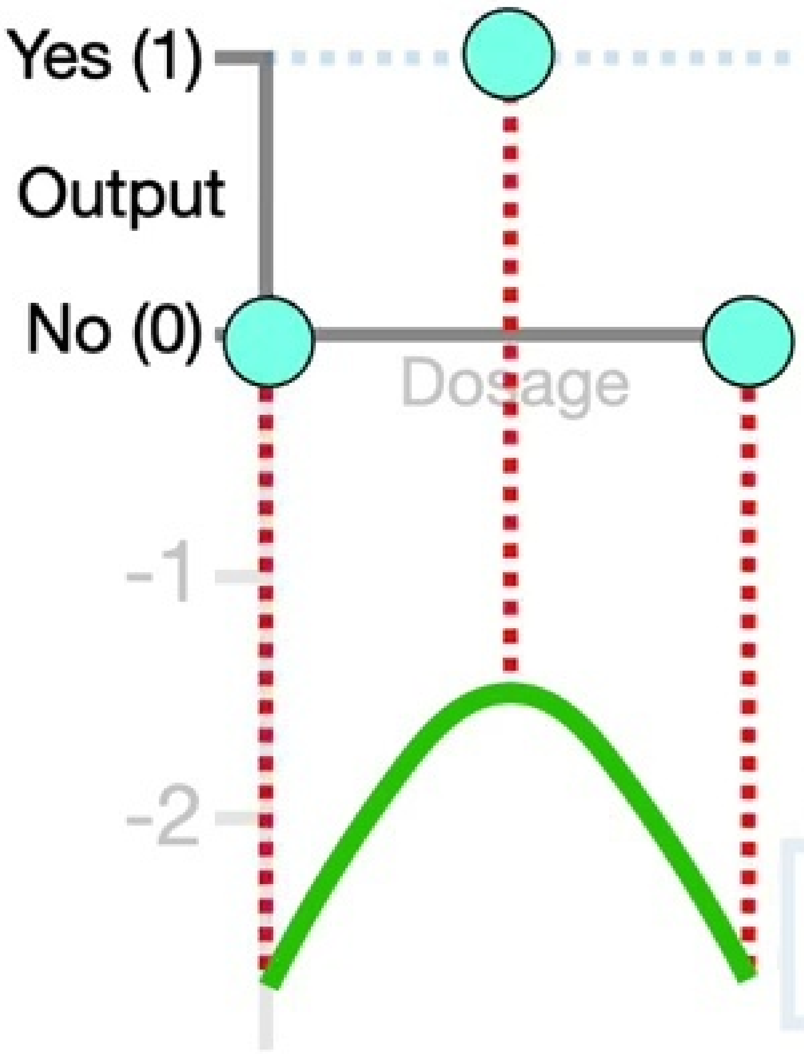


Now we square each **Residual**…

$(0 - -2.6)^2$

$(1 - -1.61)^2$

$(0 - -2.61)^2$

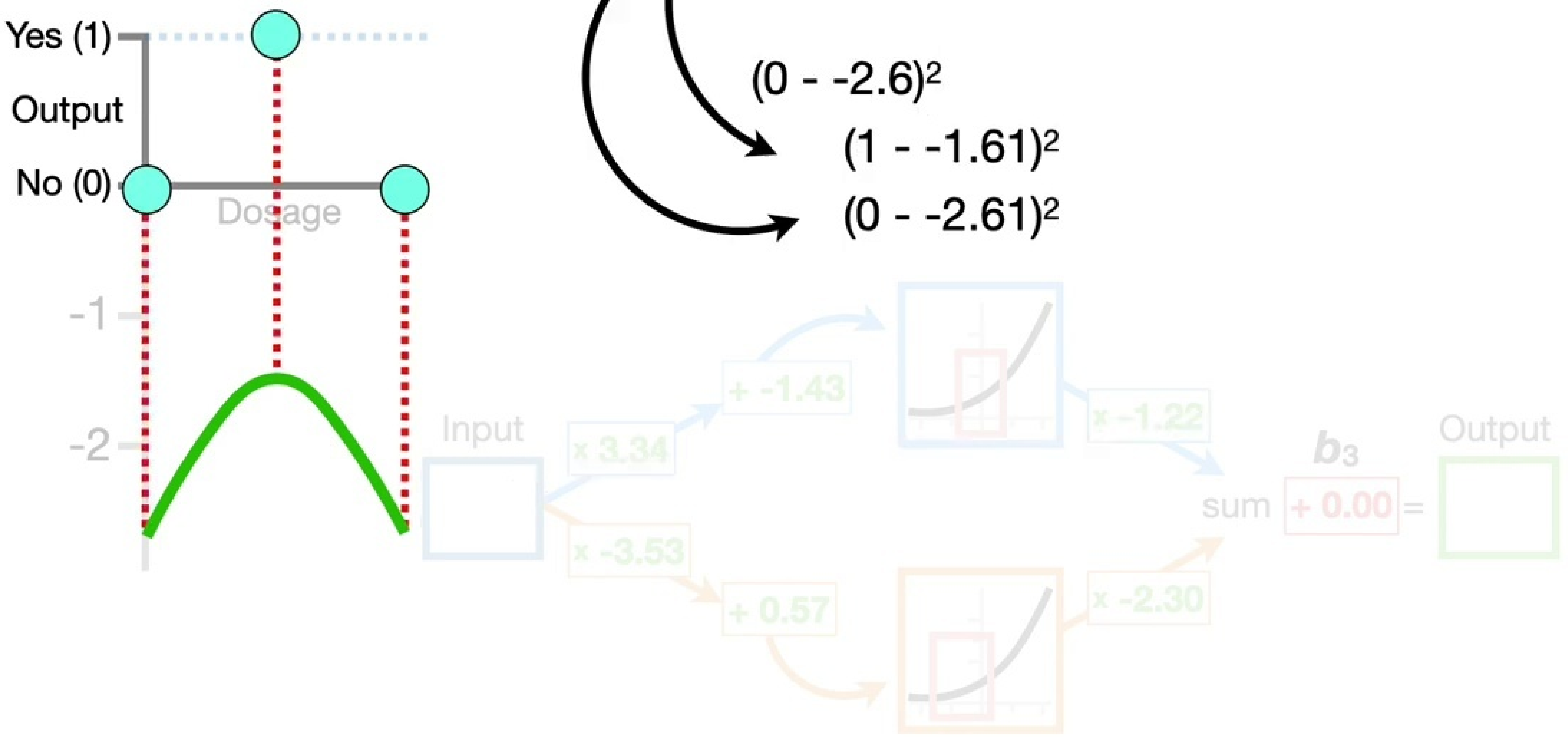


...and add them all together...

$(0 - -2.6)^2$

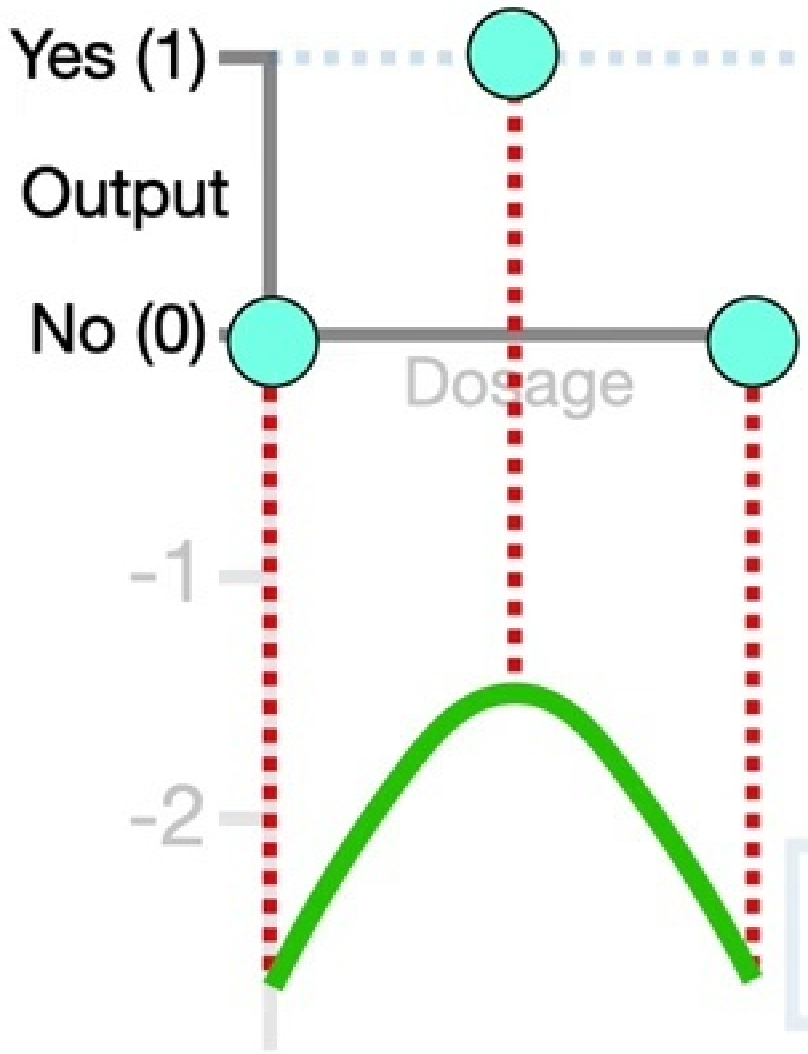
$(1 - -1.61)^2$

$(0 - -2.61)^2$



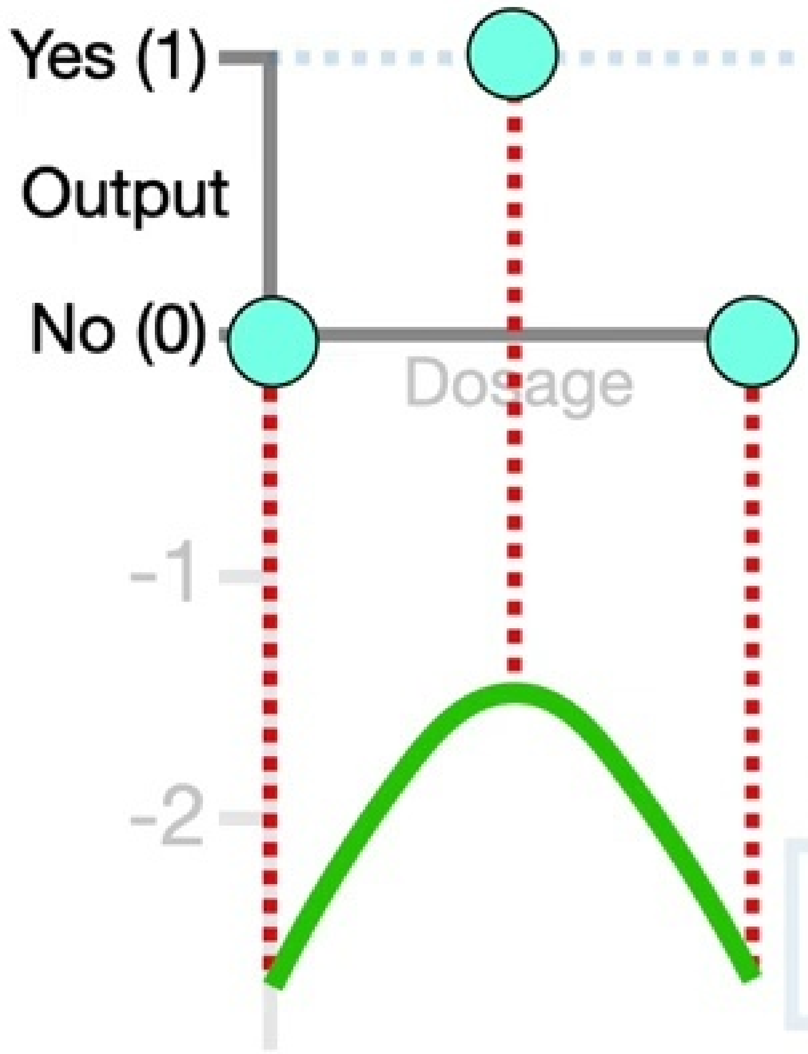
…and add them all together…

$$(0 - -2.6)^2$$
$$+ (1 - -1.61)^2$$
$$+ (0 - -2.61)^2$$

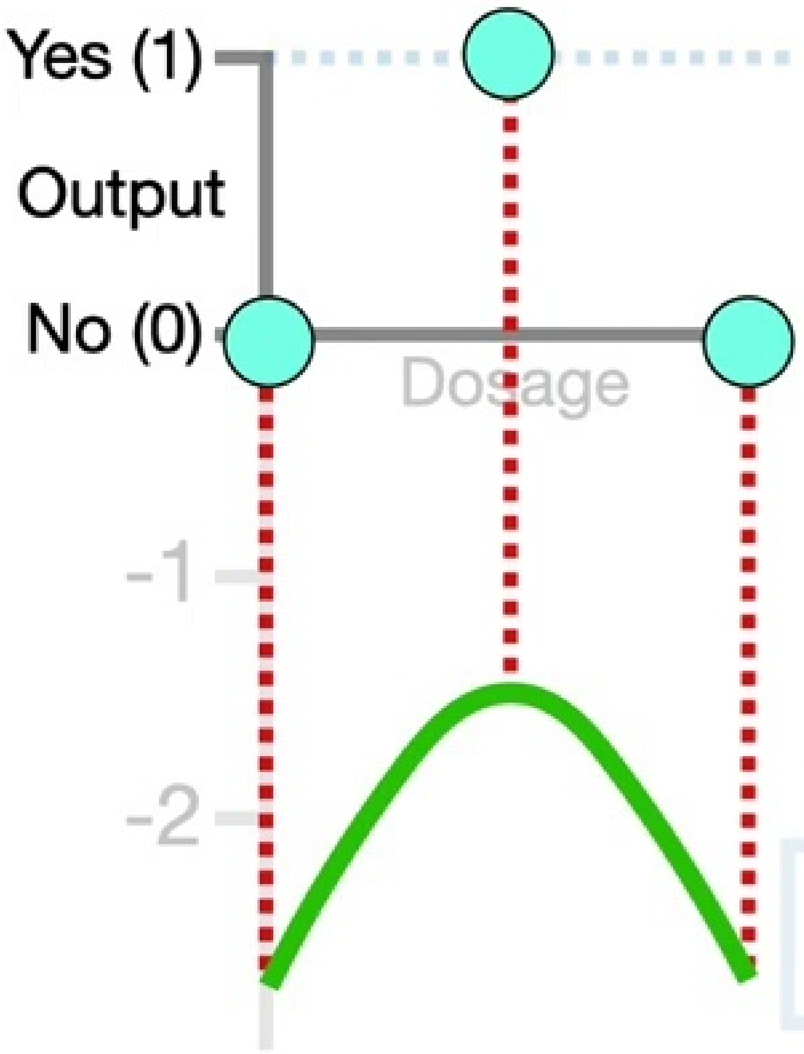


...to get **20.4** for the **Sum of the Squared Residuals (SSR)**.

$$SSR = (0 - -2.6)^2 + (1 - -1.61)^2 + (0 - -2.61)^2 \boxed{= 20.4}$$

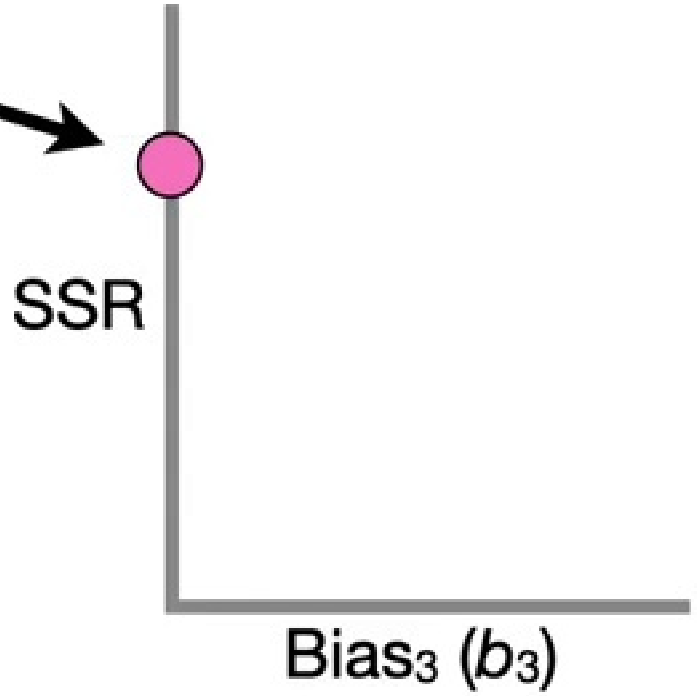


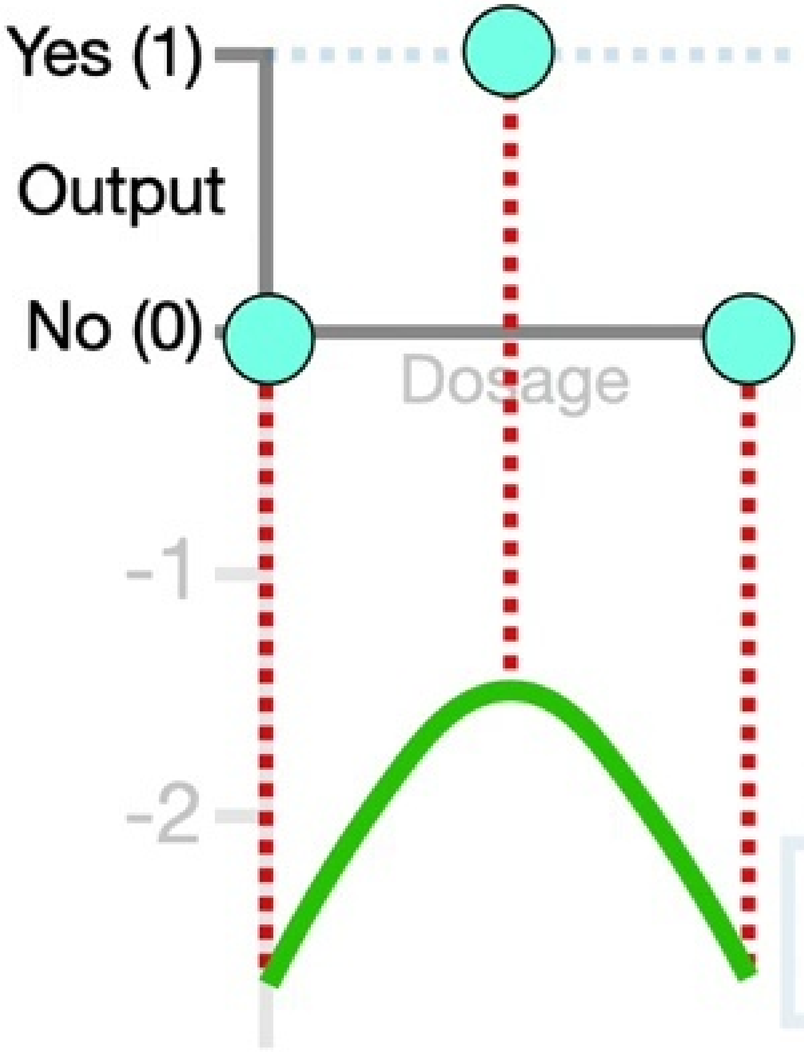
So, when $b_3 = 0$, the **SSR = 20.4**…



$$SSR = (0 - -2.6)^2 + (1 - -1.61)^2 + (0 - -2.61)^2 \boxed{= 20.4}$$

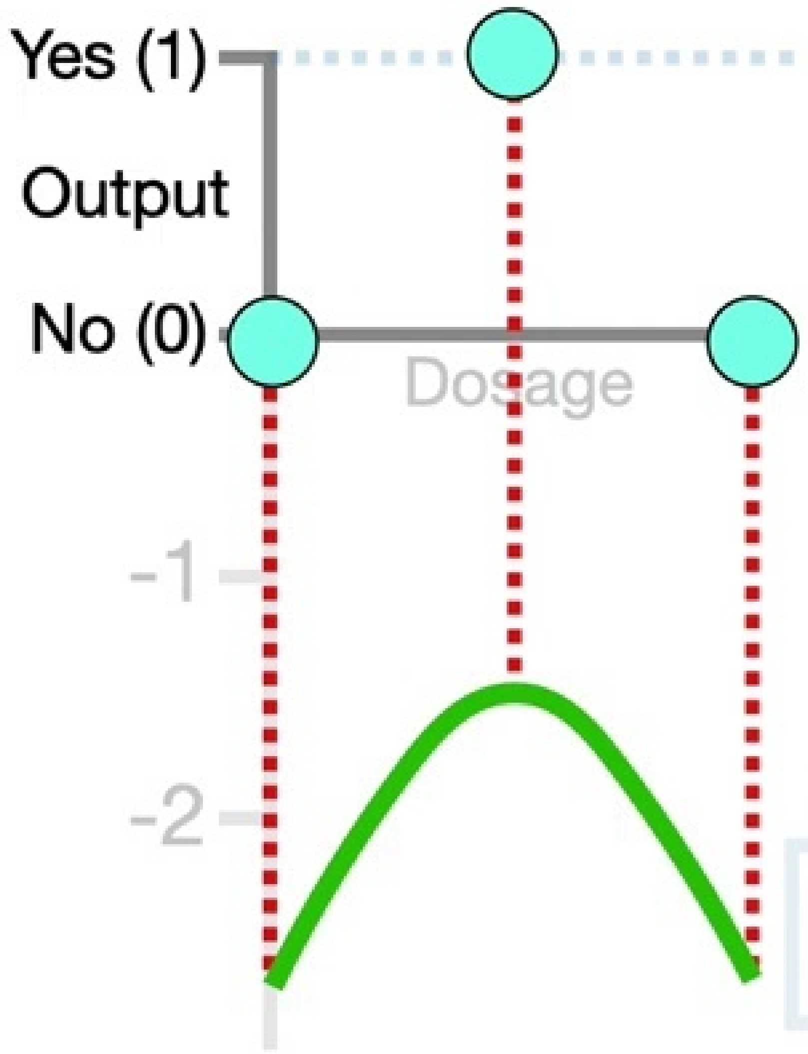
…and that corresponds to this location on this graph…



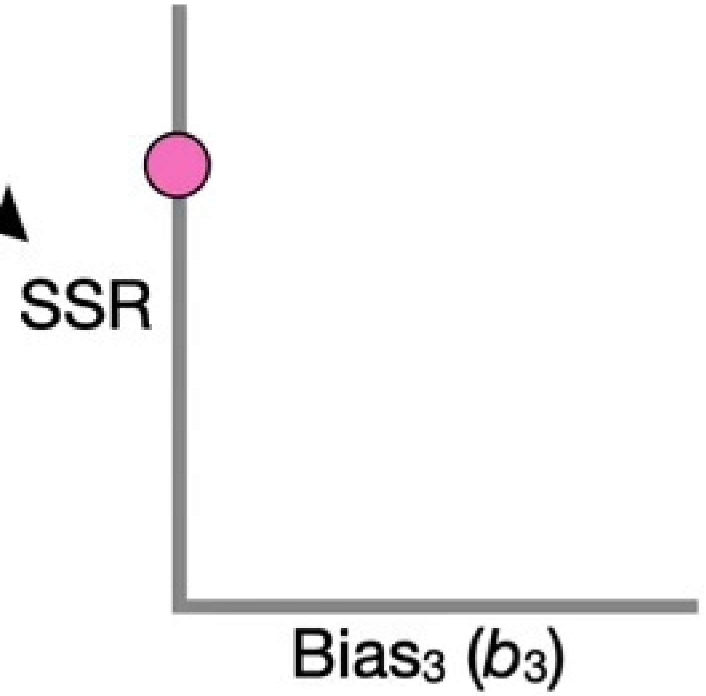


$$SSR = (0 - -2.6)^2 + (1 - -1.61)^2 + (0 - -2.61)^2 = 20.4$$

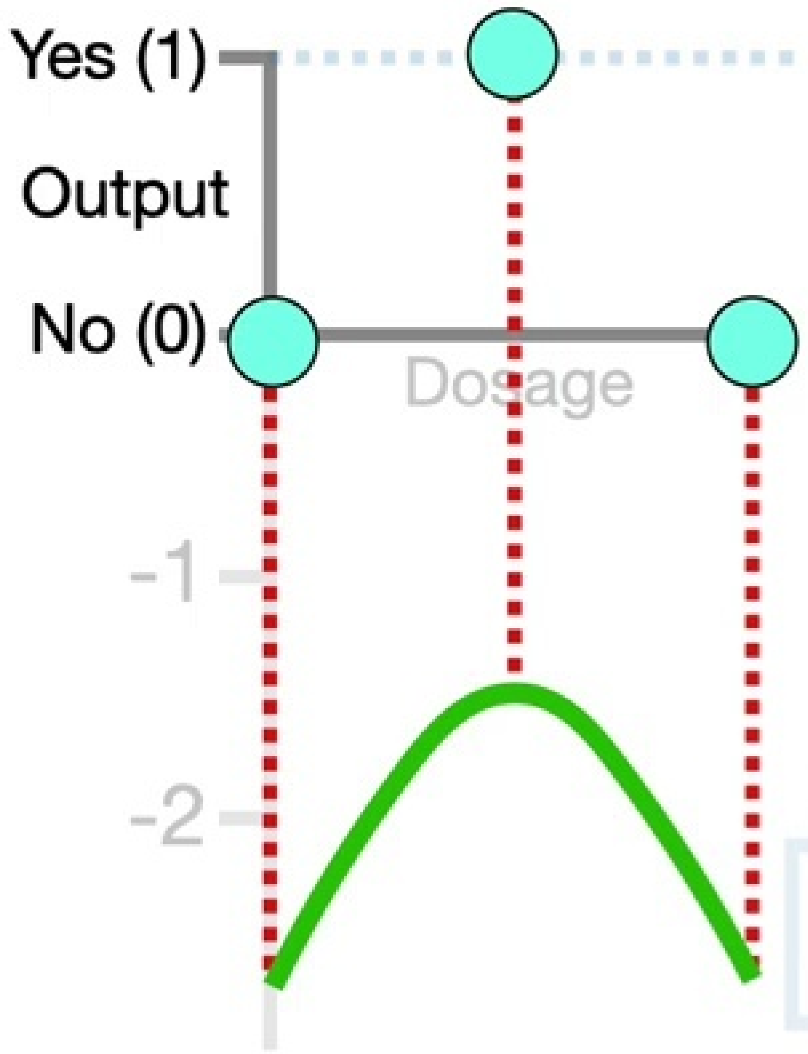
…that has the **SSR** on the y-axis…



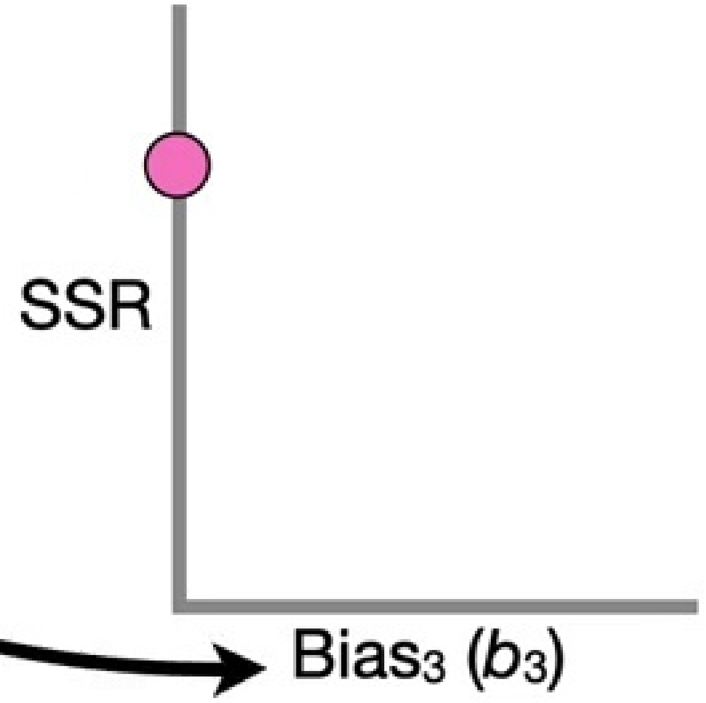
$$\text{SSR} = (0 - -2.6)^2 + (1 - -1.61)^2 + (0 - -2.61)^2 = 20.4$$



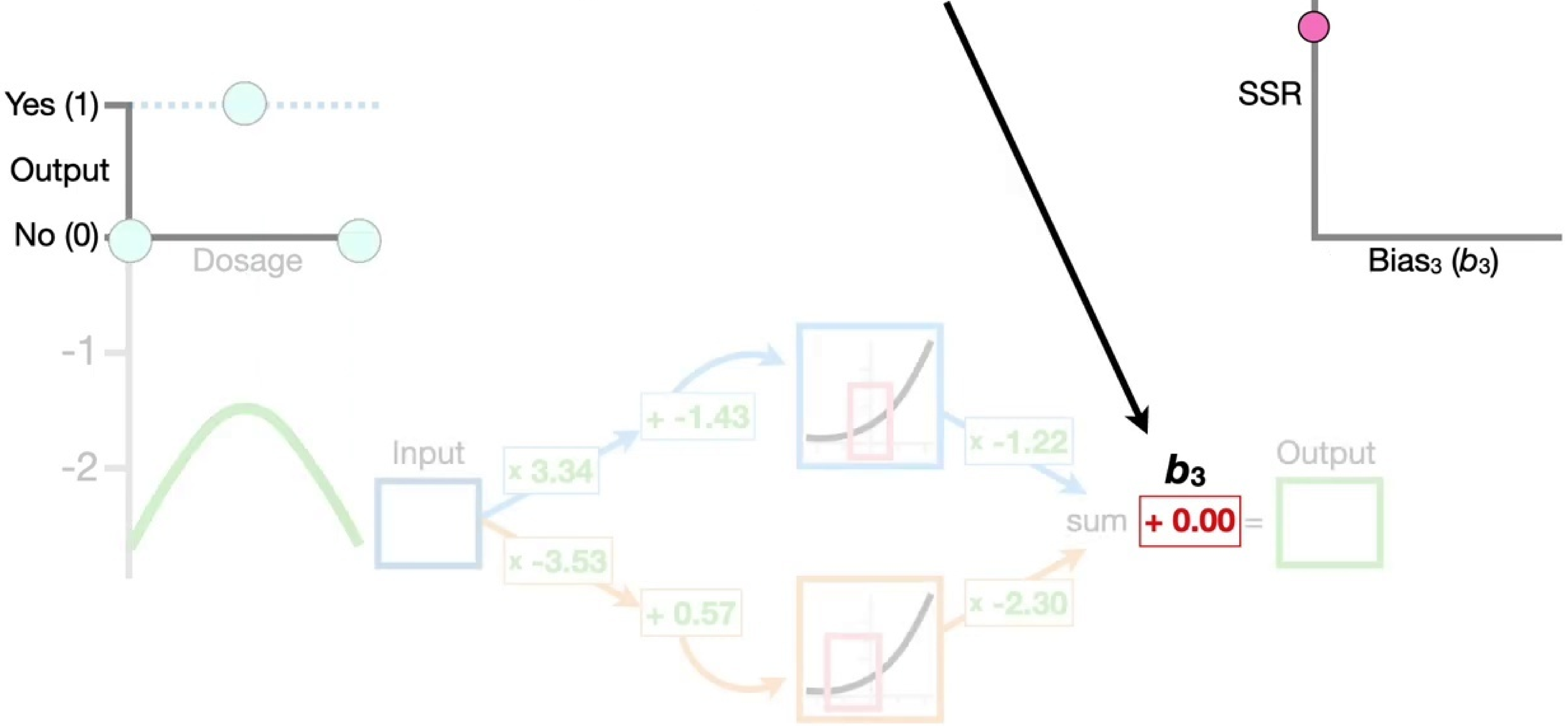
...and the **Bias, *b_3*,** on the x-axis.



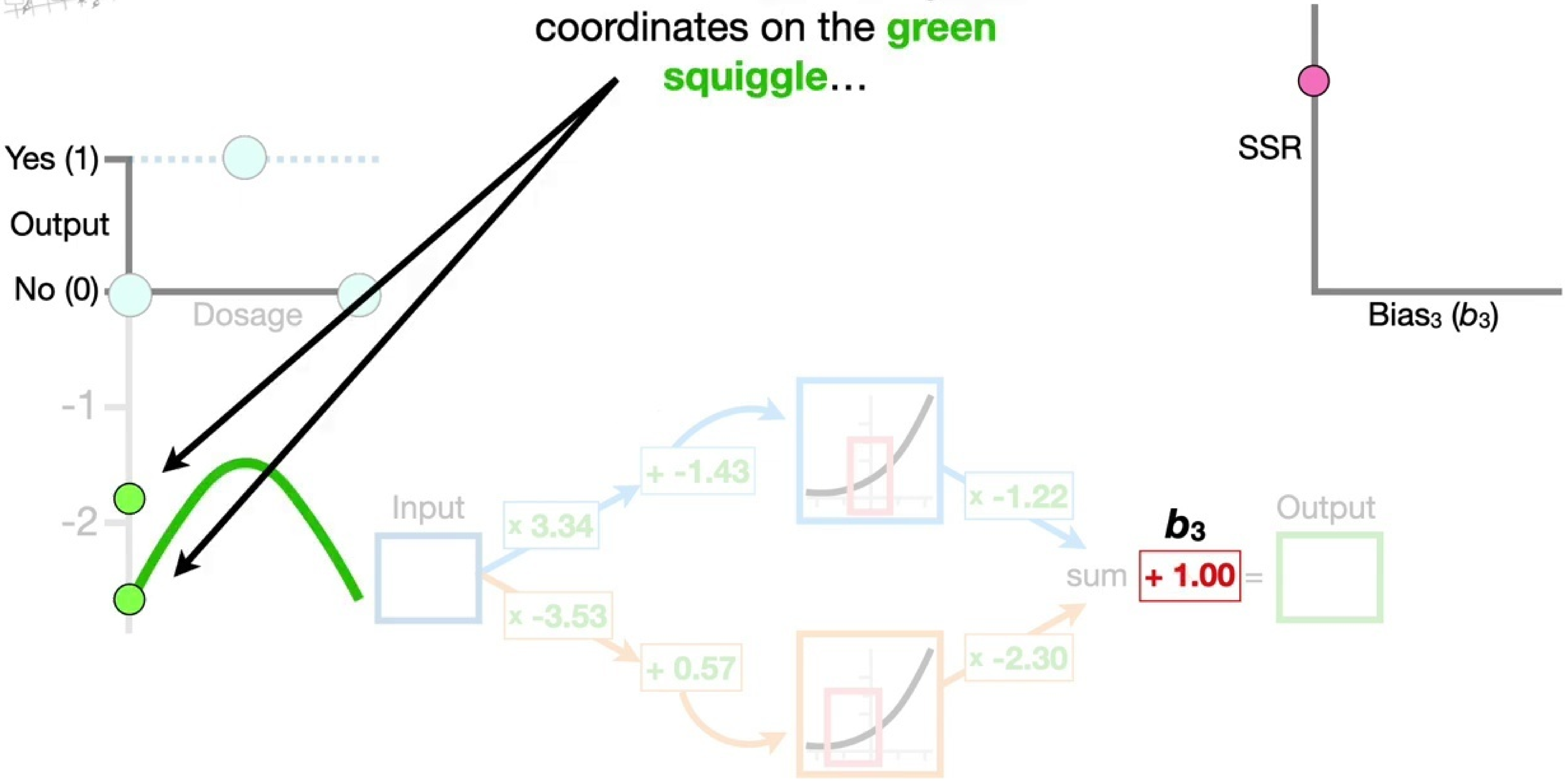
$$SSR = (0 - -2.6)^2 + (1 - -1.61)^2 + (0 - -2.61)^2 = 20.4$$



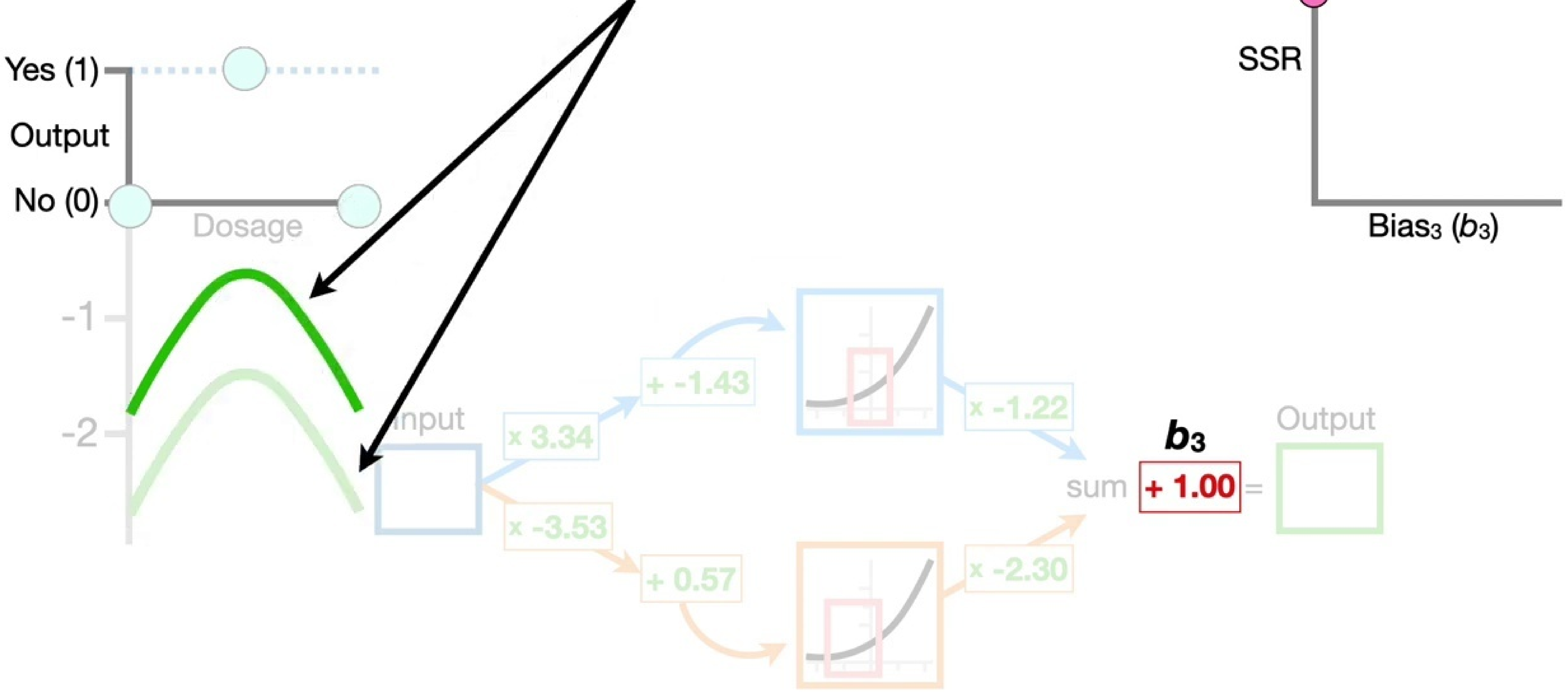
Now, if we increase b_3 to 1…



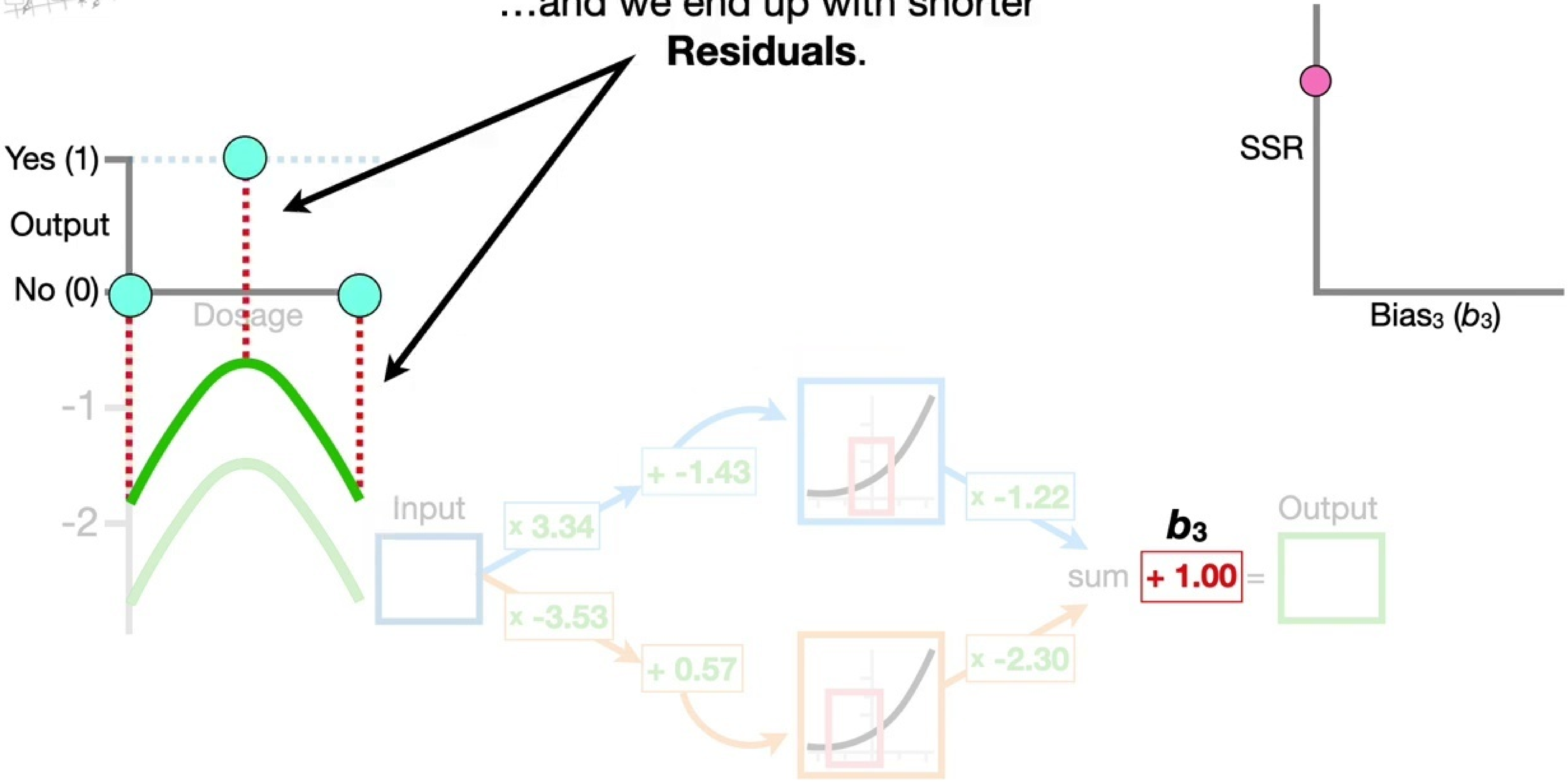
…then we add **1** to the y-axis coordinates on the **green squiggle**…



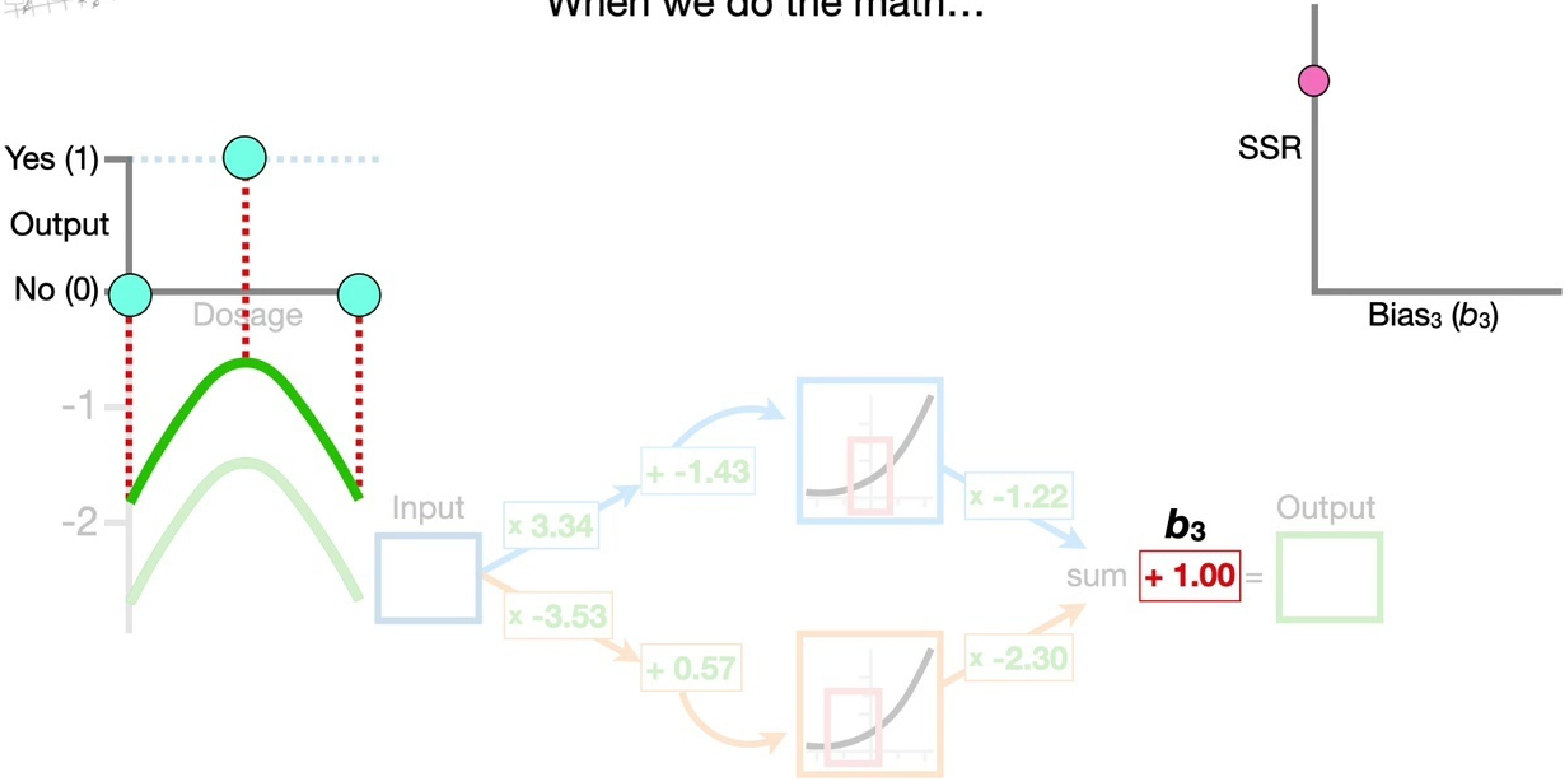
...and shift the **green squiggle** up **1**...



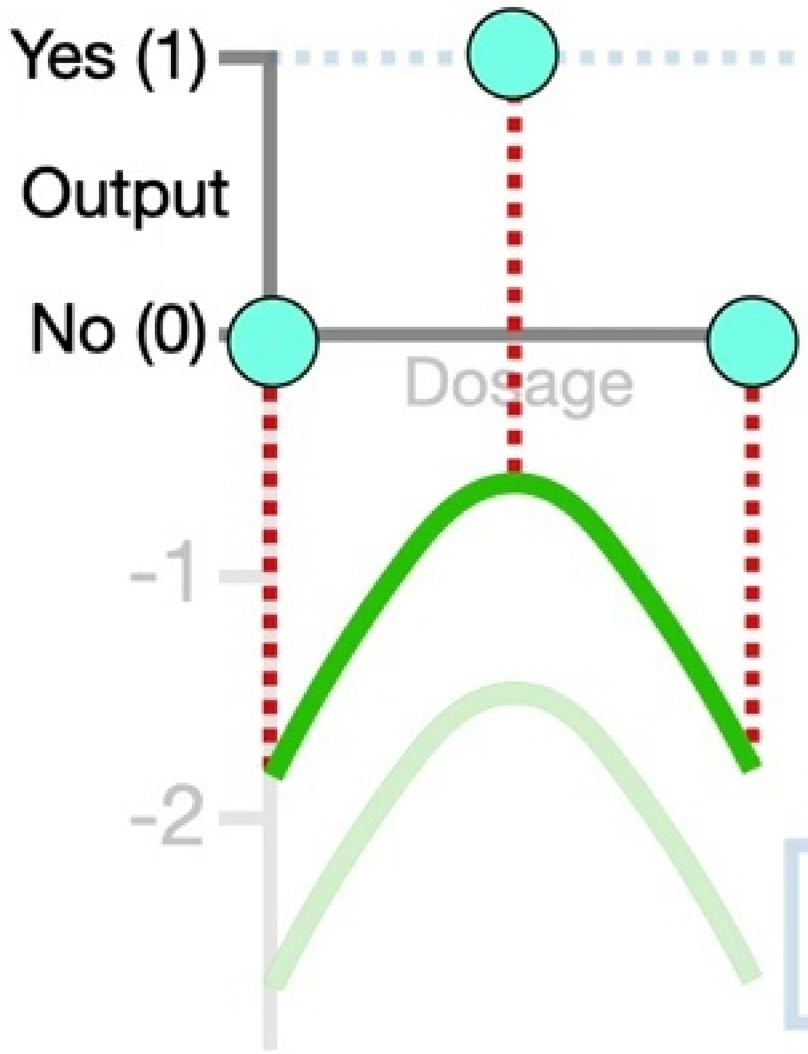
...and we end up with shorter **Residuals.**



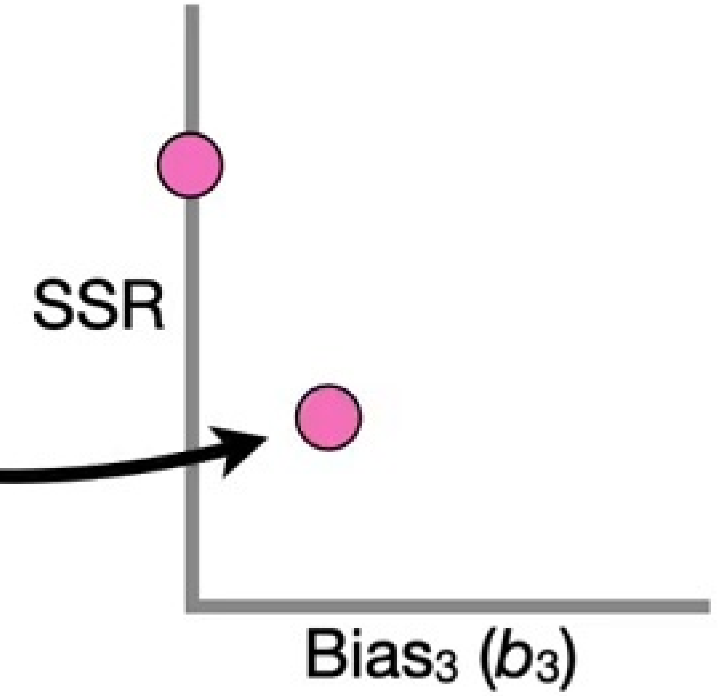
When we do the math…



...and that corresponds to this point on our graph.

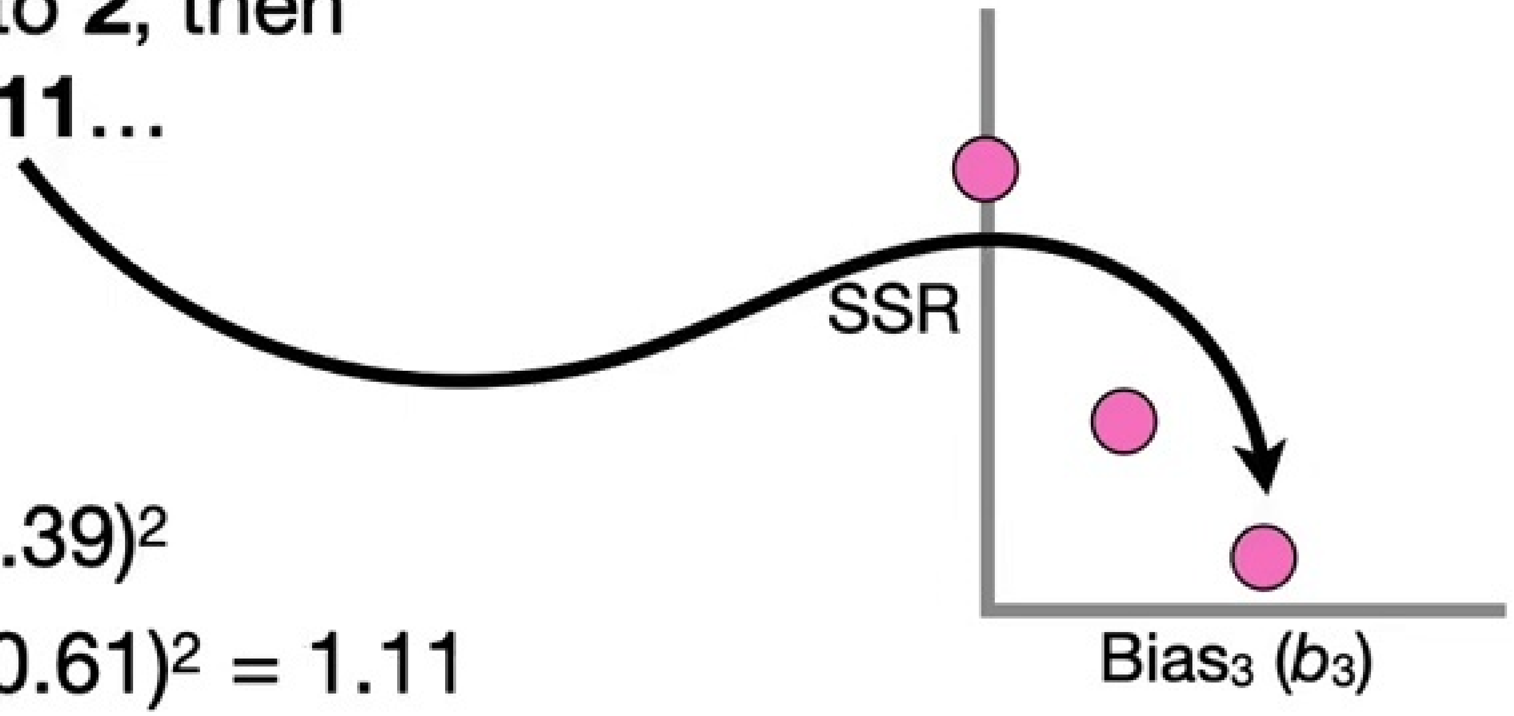


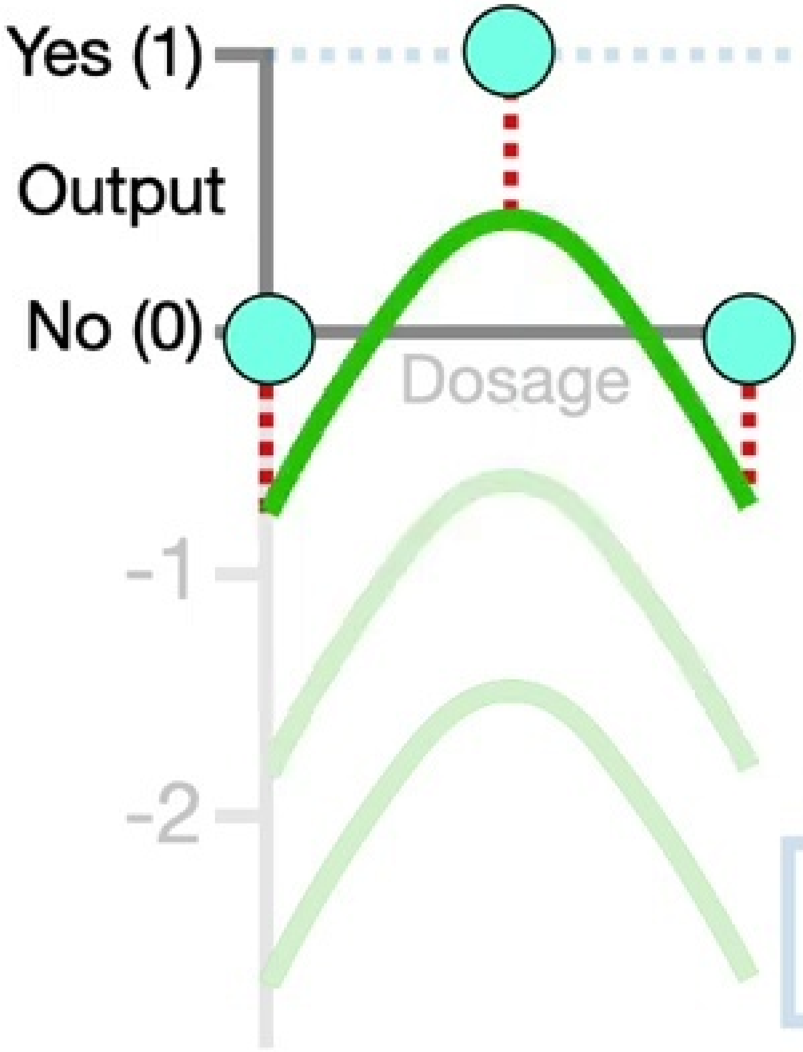
$$SSR = (0 - -1.6)^2 + (1 - -0.61)^2 + (0 - -1.61)^2 = 7.8$$



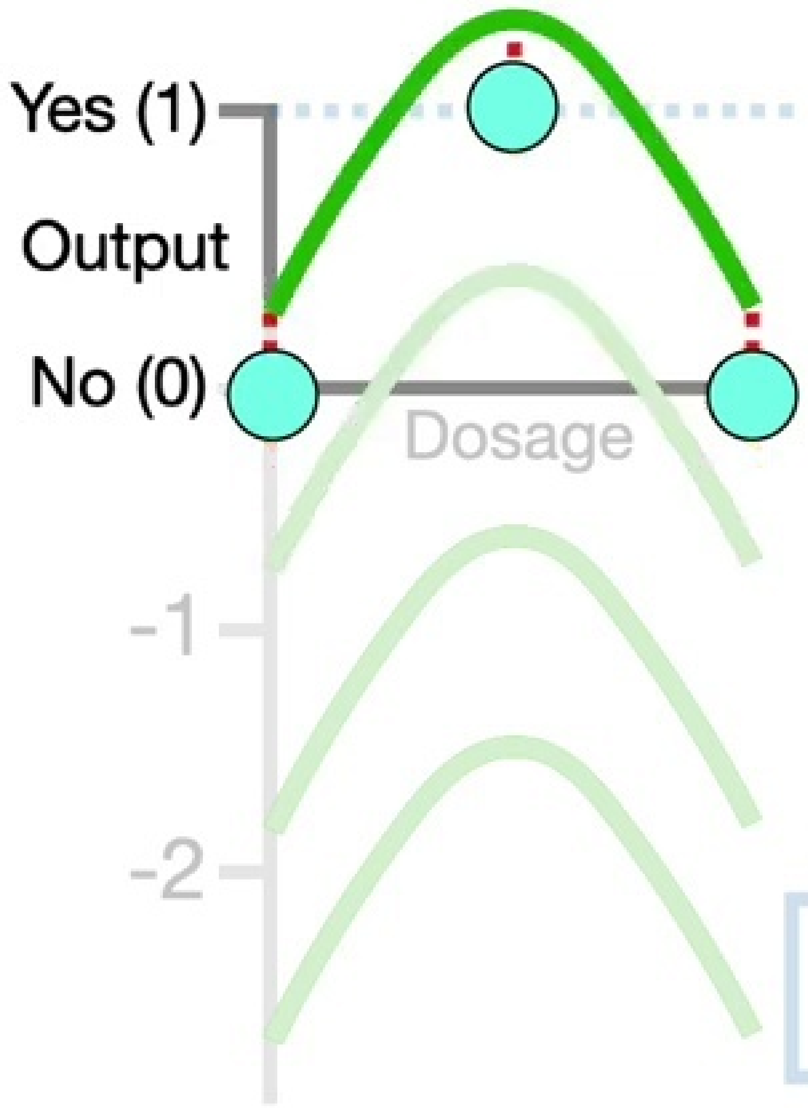
If we increase b_3 to 2, then the SSR = 1.11…

$$SSR = (0 - -0.6)^2 + (1 - 0.39)^2 + (0 - -0.61)^2 = 1.11$$

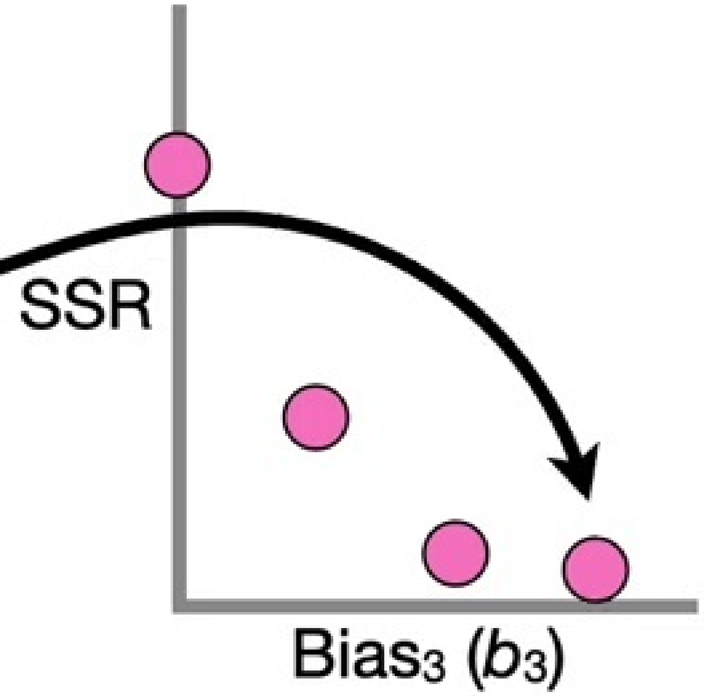




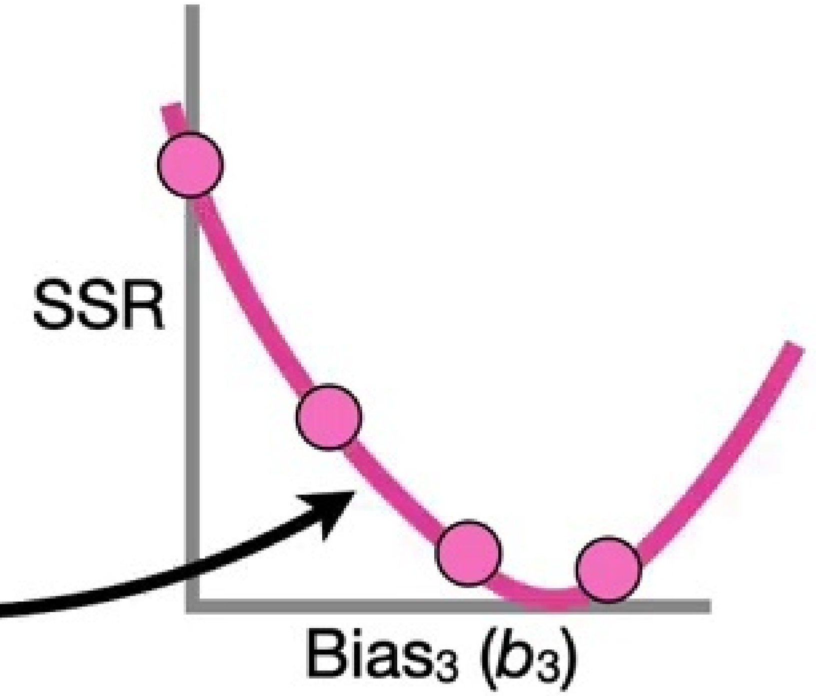
…and if we increase $\boldsymbol{b_3}$ to **3**, then the **SSR = 0.46.**

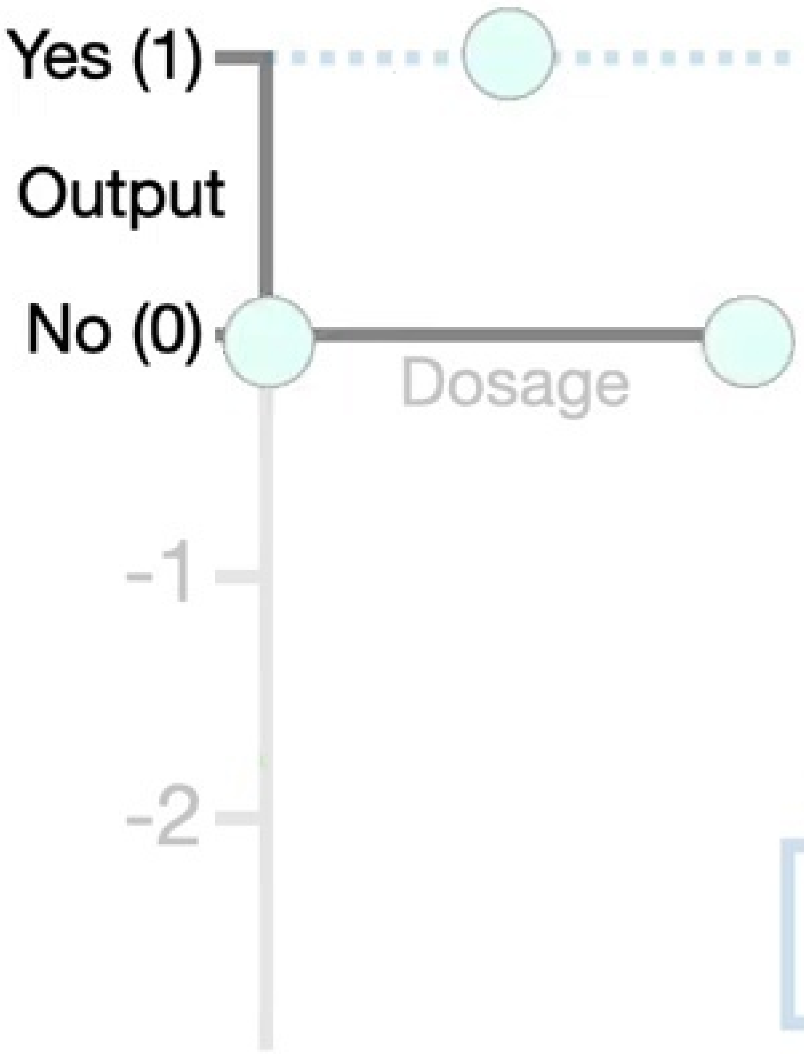


$$\mathrm{SSR} = (0 - 0.4)^2 + (1 - 1.39)^2 + (0 - 0.39)^2 = 0.46$$

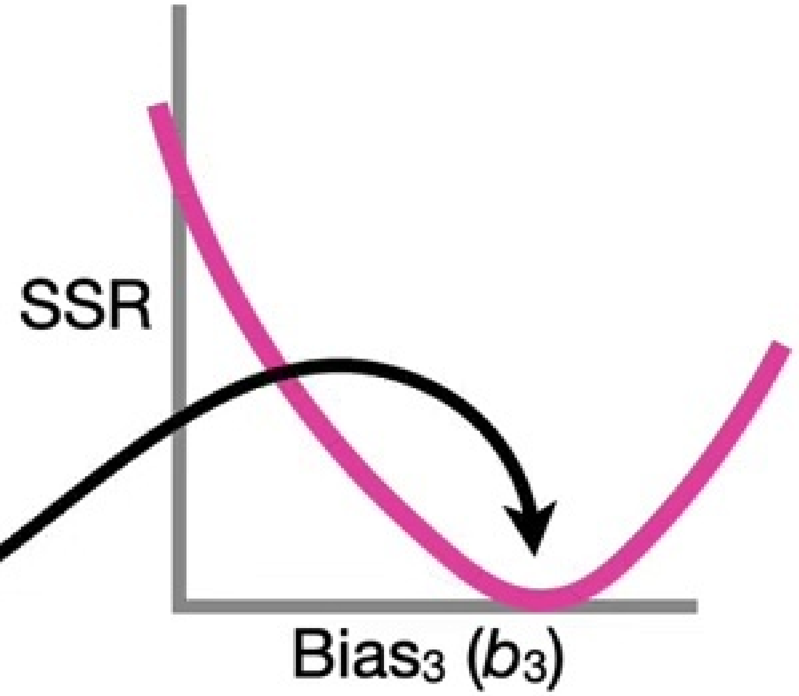


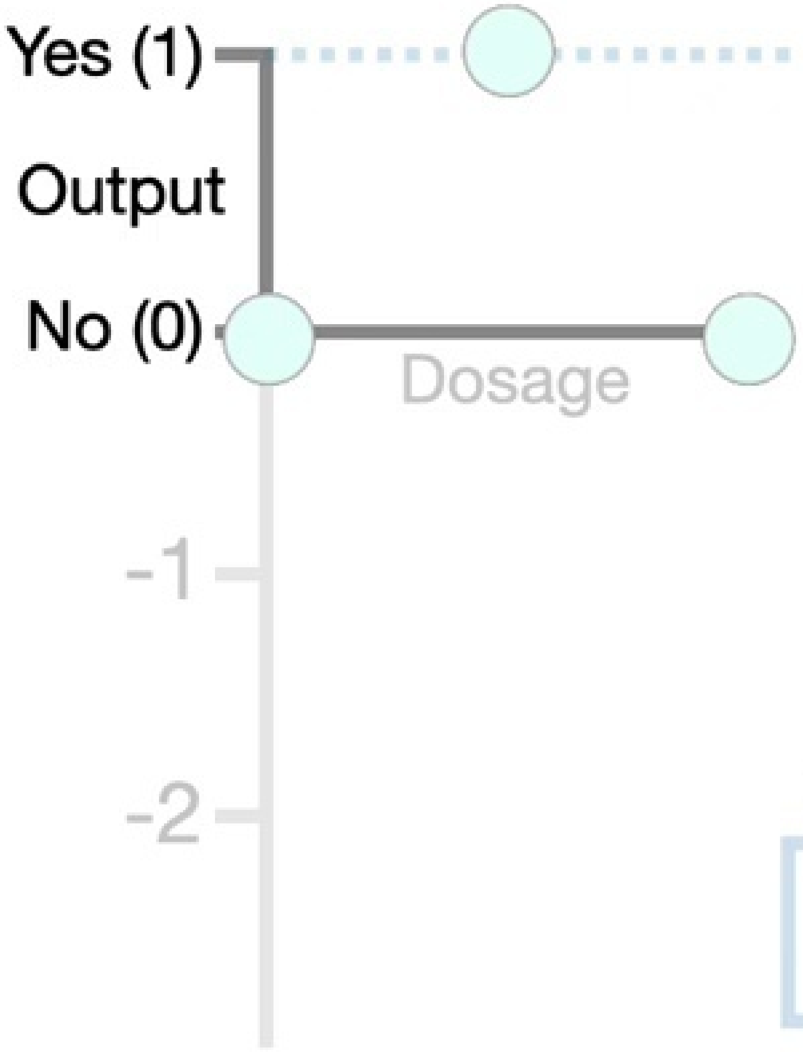
And if we had time to plug in tons of values for $\boldsymbol{b_3}$, we would get this **pink curve**…



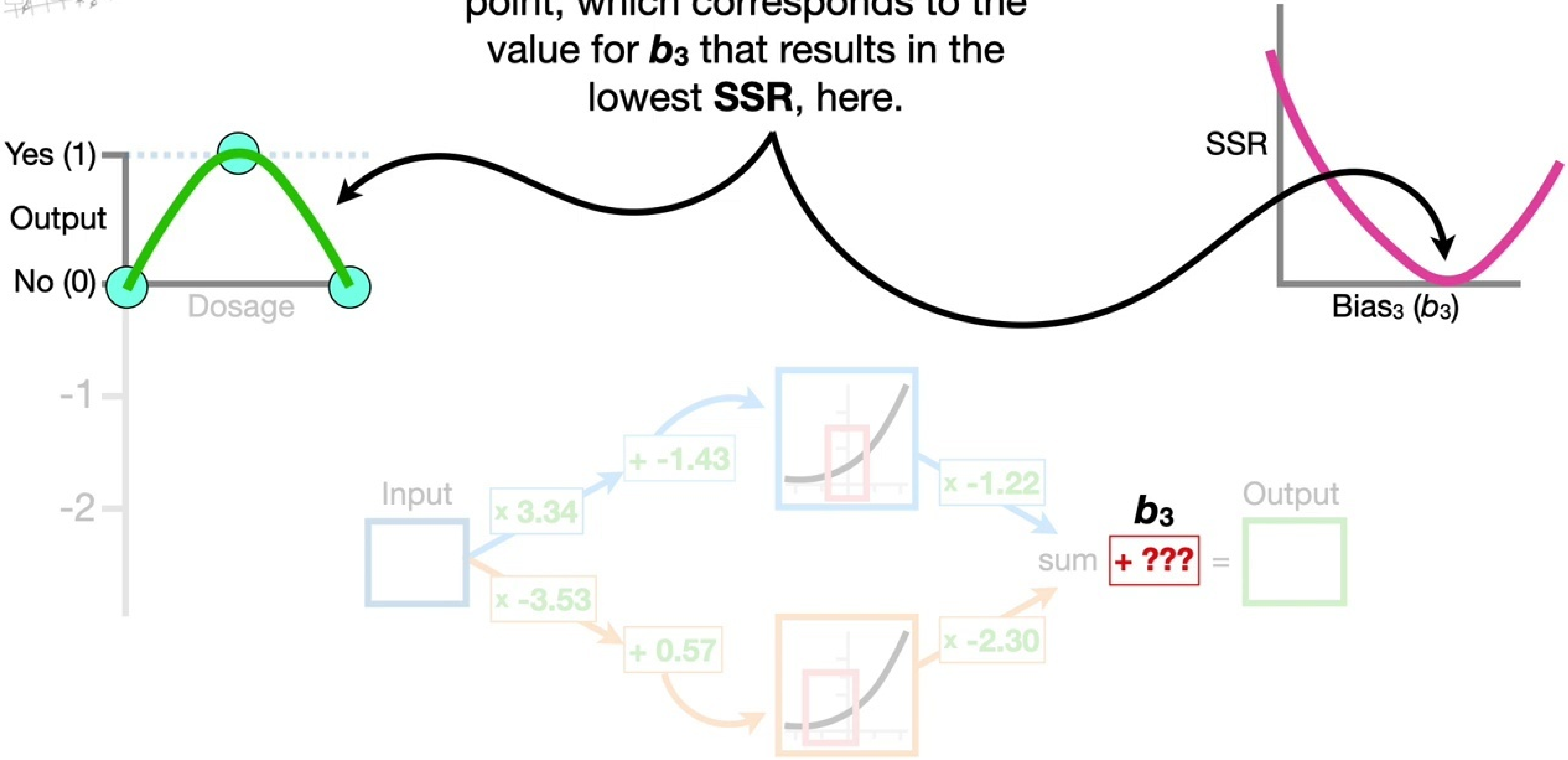


...and we could find the lowest point, which corresponds to the value for b_3 that results in the lowest **SSR**, here.

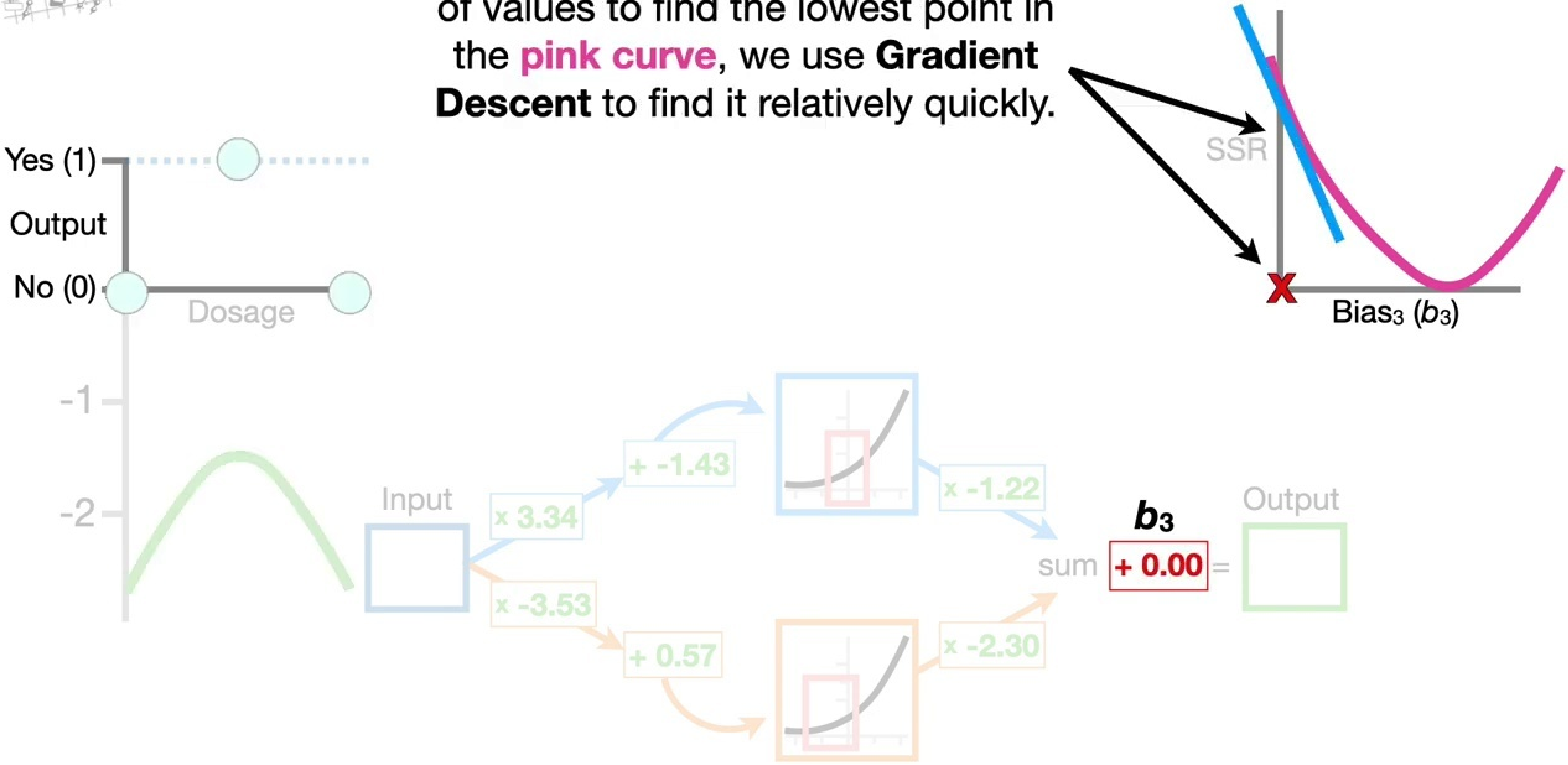




…and we could find the lowest point, which corresponds to the value for $\boldsymbol{b_3}$ that results in the lowest **SSR**, here.



However, instead of plugging in tons of values to find the lowest point in the **pink curve**, we use **Gradient Descent** to find it relatively quickly.



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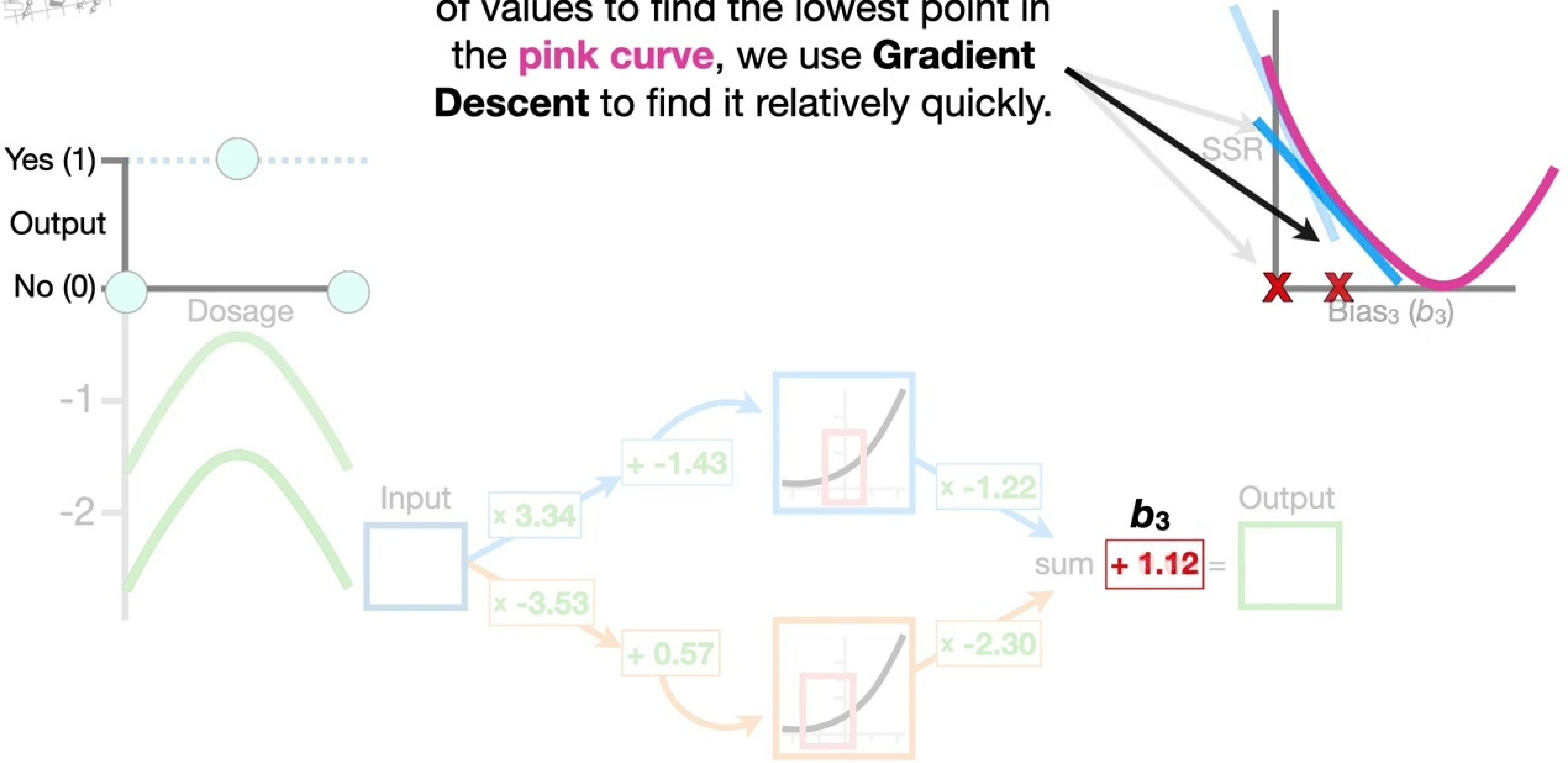
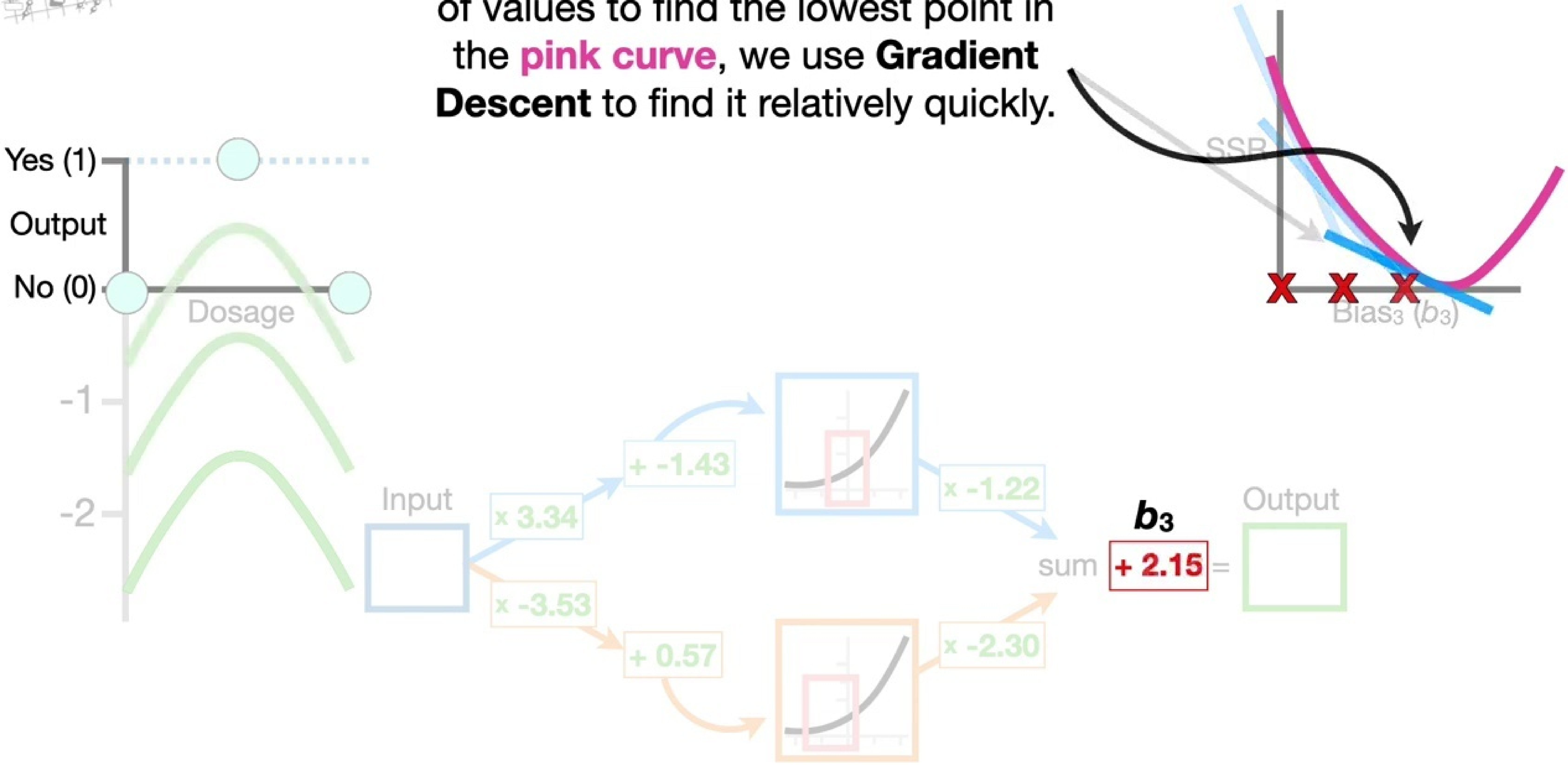
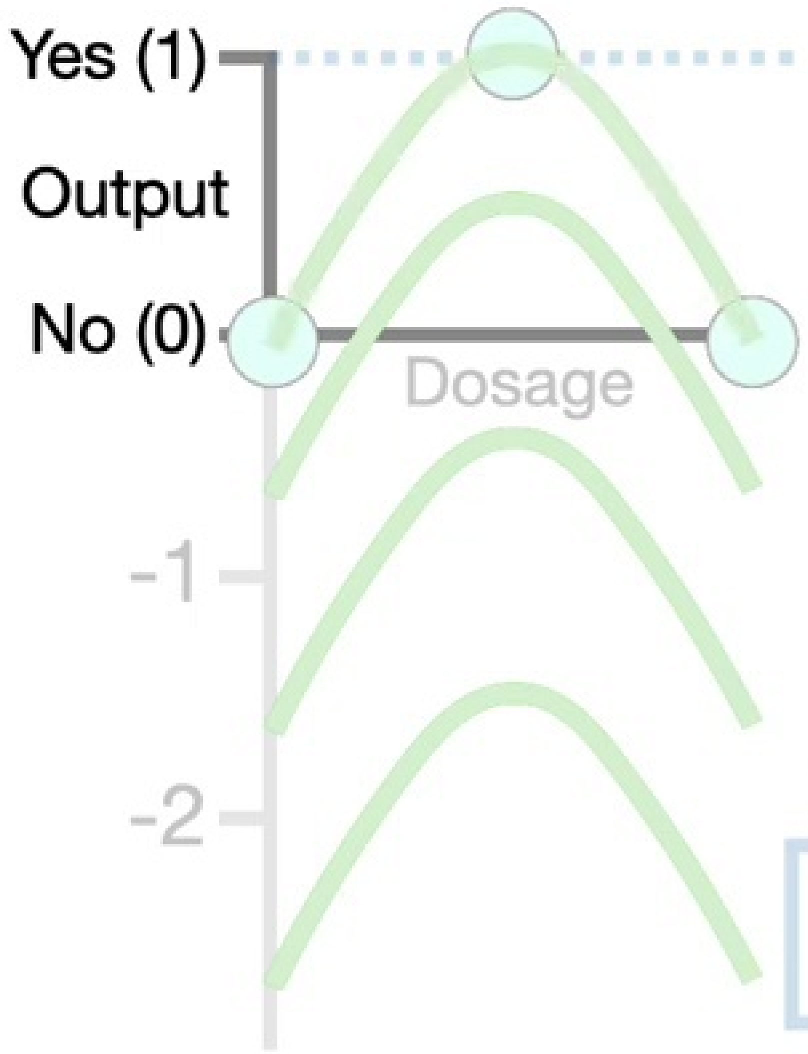
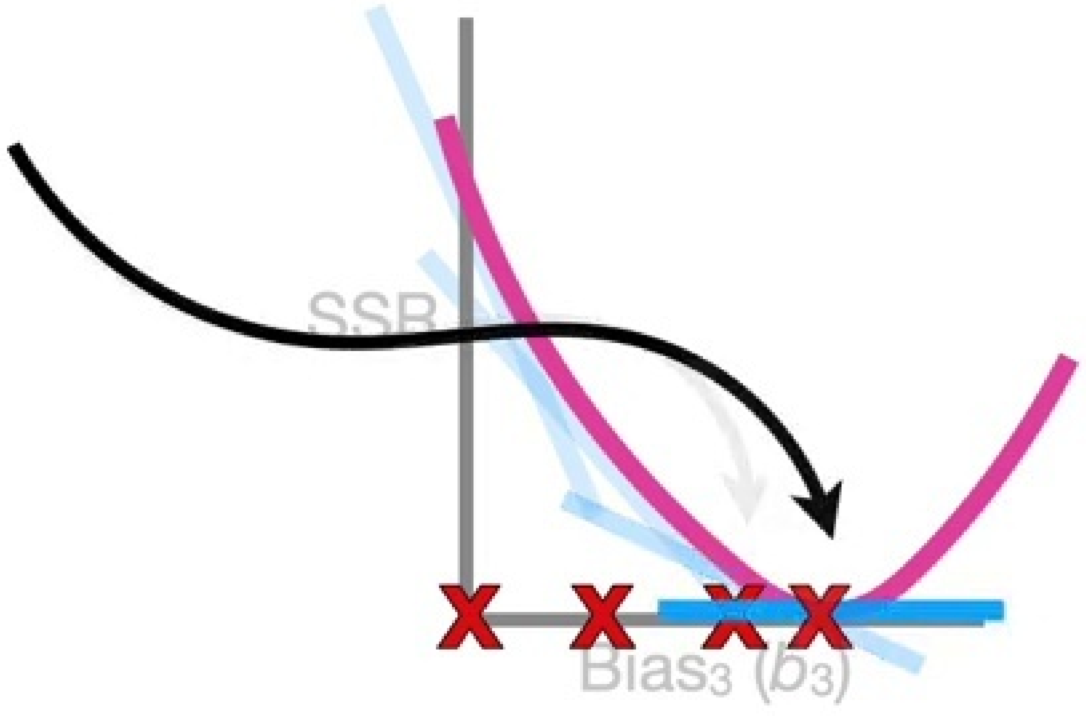


However, instead of plugging in tons of values to find the lowest point in the **pink curve**, we use **Gradient Descent** to find it relatively quickly.

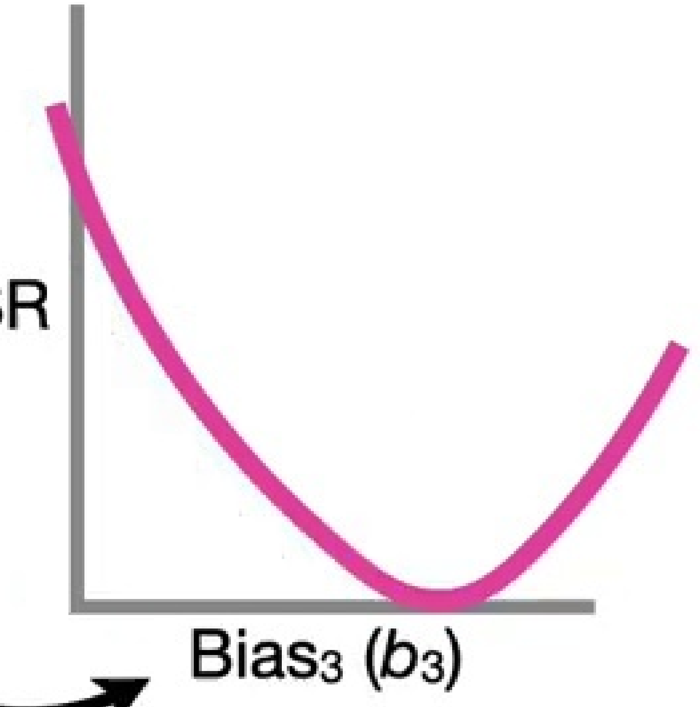


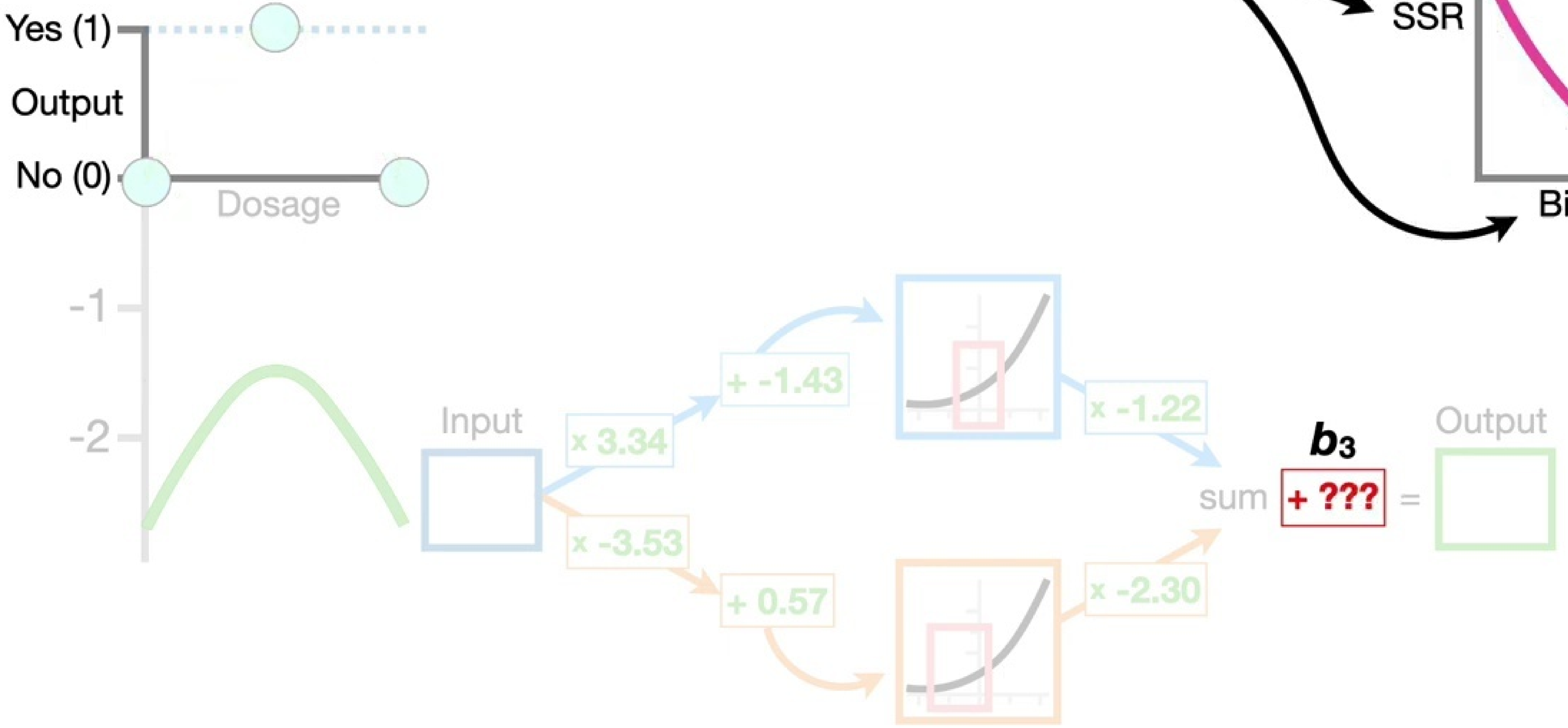
However, instead of plugging in tons of values to find the lowest point in the **pink curve**, we use **Gradient Descent** to find it relatively quickly.





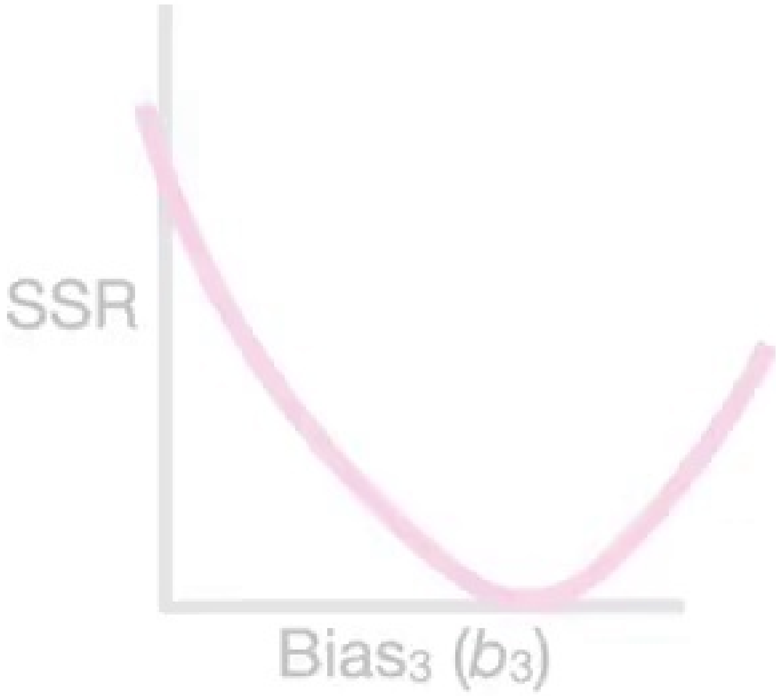
And that means we need to find the derivative of the **Sum of the Squared Residuals** with respect to ***b*₃**.





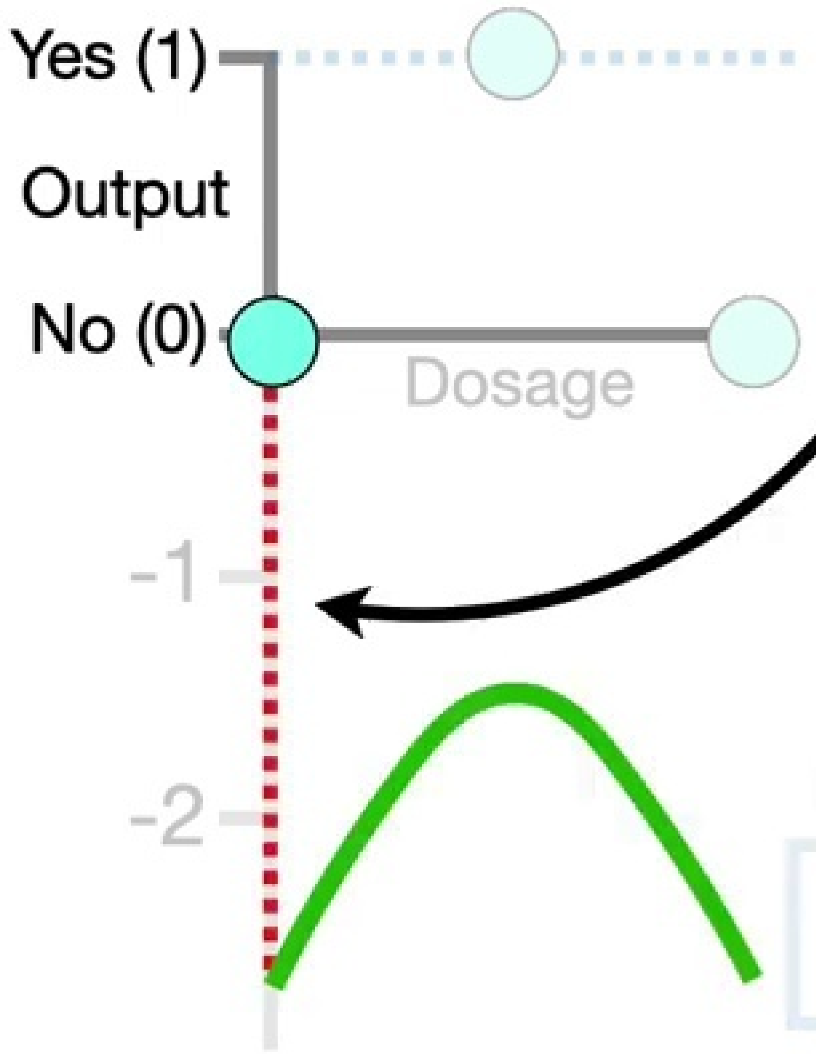
Now, remember the **Sum of the Squared Residuals** equals…

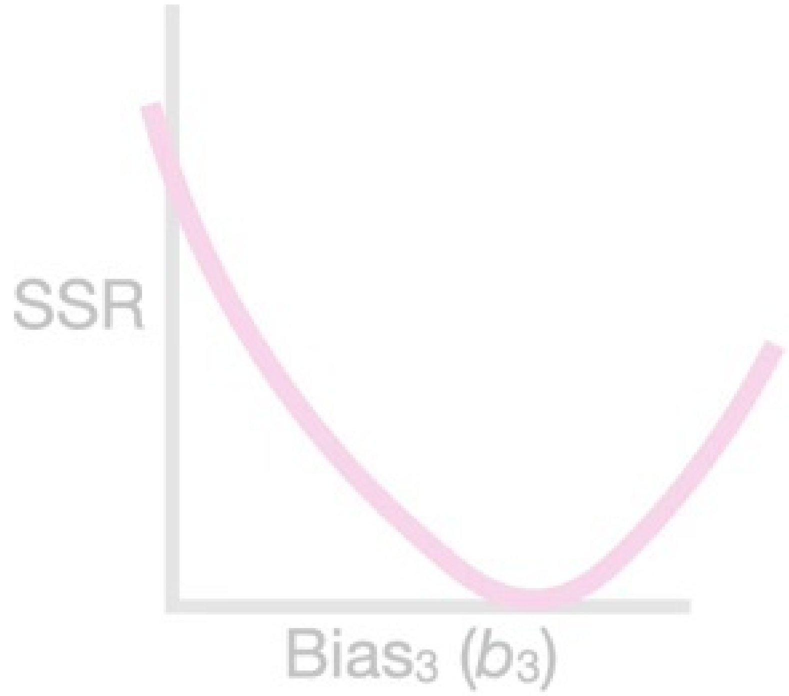
SSR =



…the first **Residual** squared…

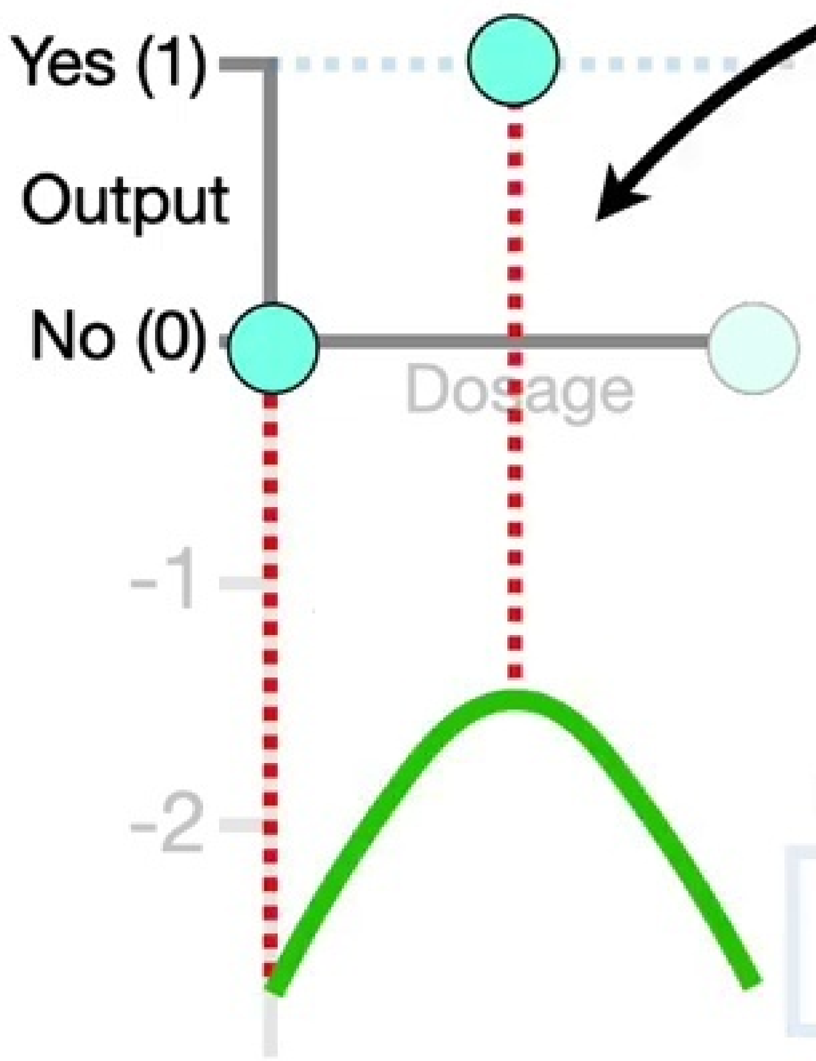
$$SSR = (Observed_1 - Predicted_1)^2$$

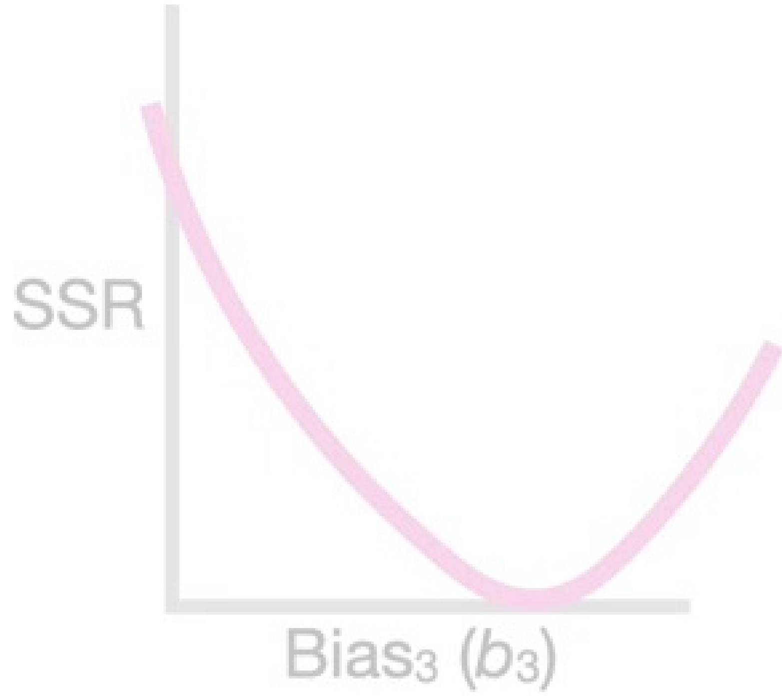




...plus all of the other squared **Residuals.**

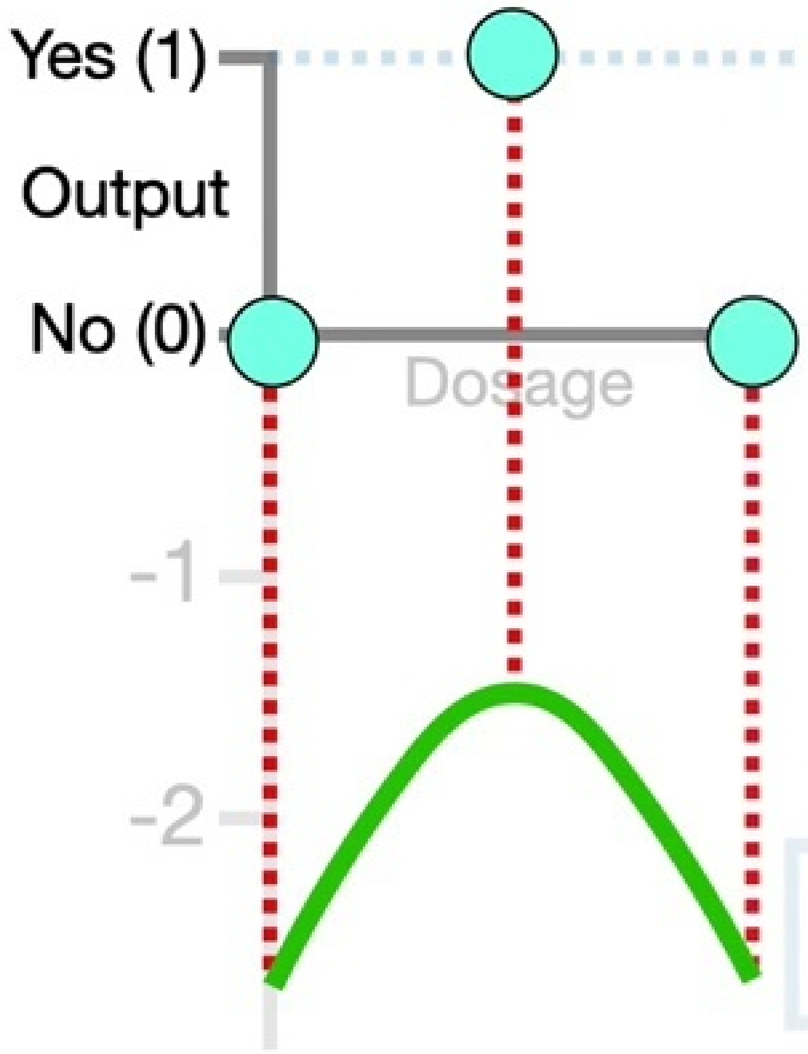
$$SSR = (Observed_1 - Predicted_1)^2$$
$$+ (Observed_2 - Predicted_2)^2$$

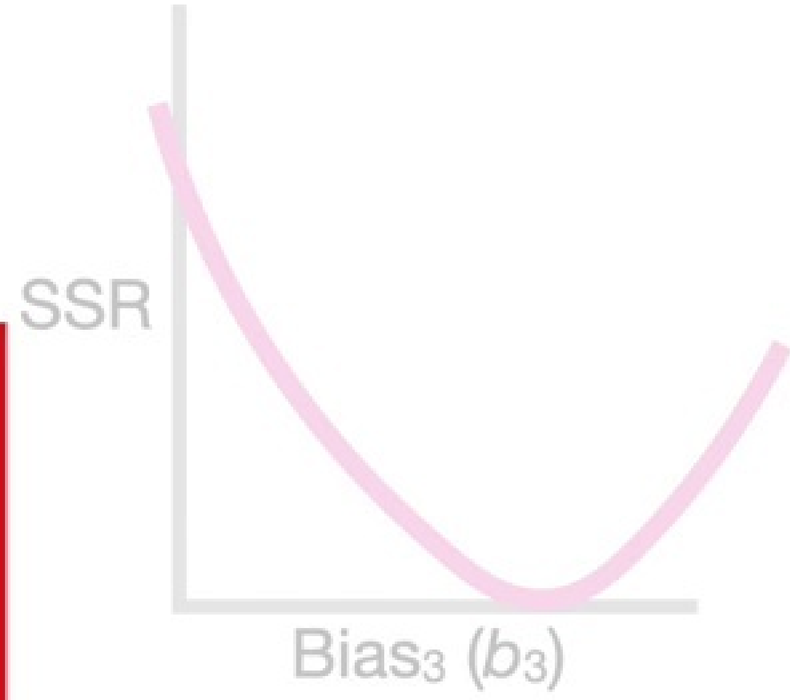




Now, because this equation takes up a lot of space…

$$
\begin{aligned}
\text{SSR} = &(\text{Observed}_1 - \text{Predicted}_1)^2 \\
&+ (\text{Observed}_2 - \text{Predicted}_2)^2 \\
&+ (\text{Observed}_3 - \text{Predicted}_3)^2
\end{aligned}
$$

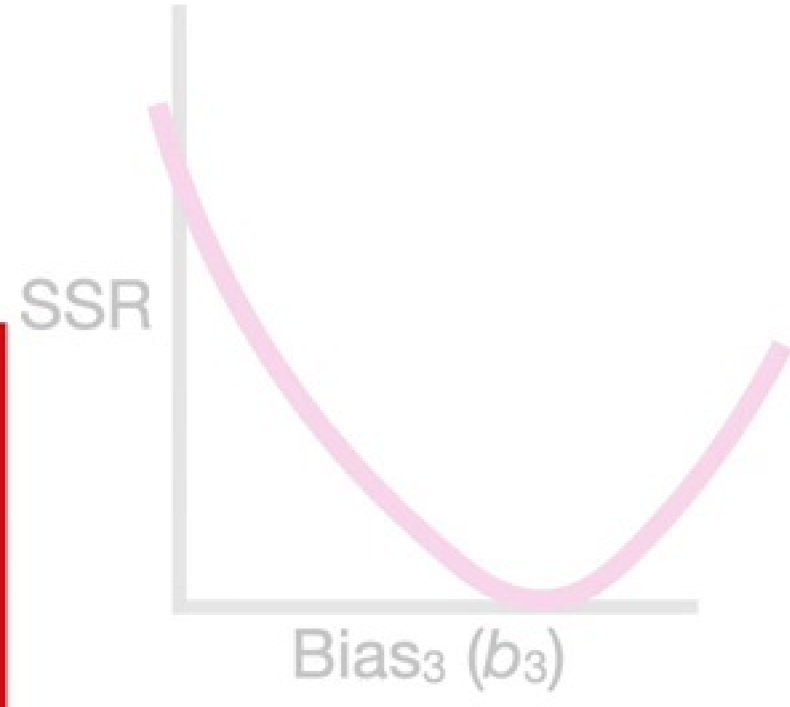


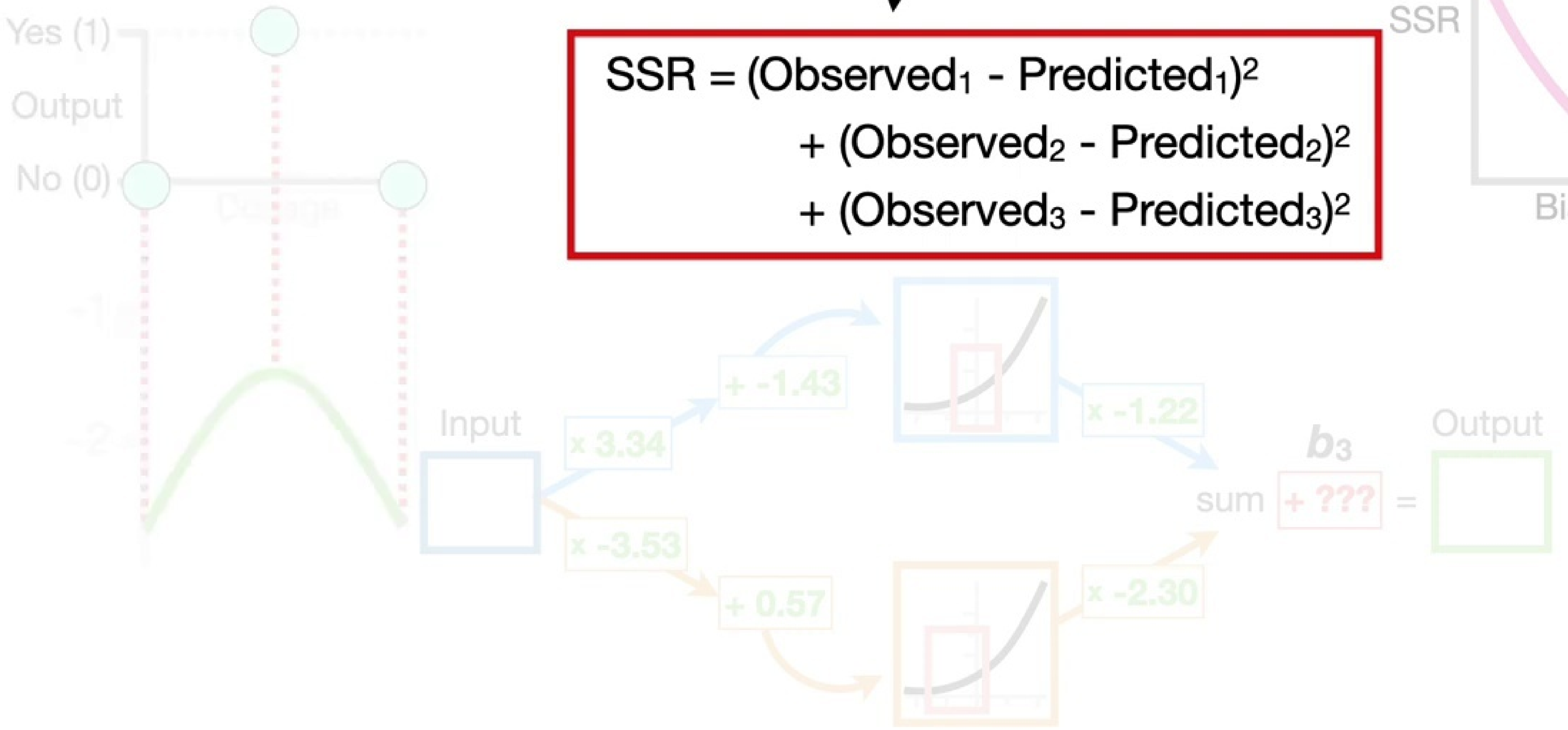


…we can make it smaller by using **summation notation.**

$$\text{SSR} = \sum_{i=1}^{n=3} (\text{Observed}_i - \text{Predicted}_i)^2$$

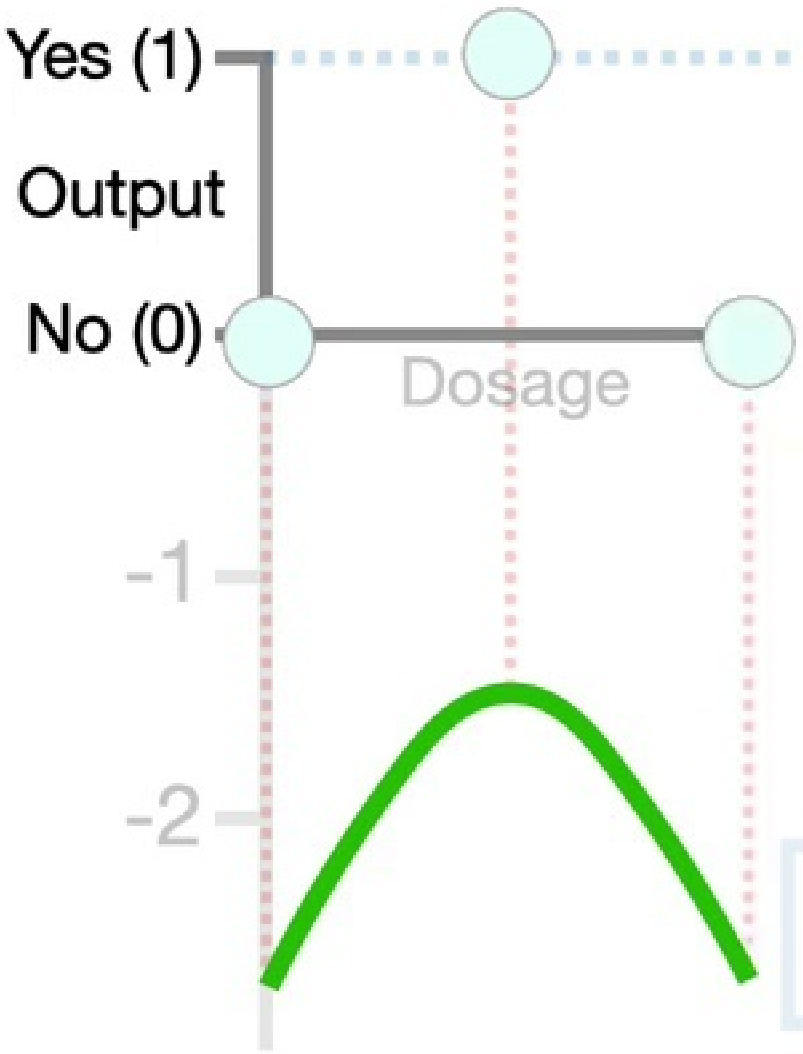
$$\boxed{\begin{aligned}\text{SSR} = &(\text{Observed}_1 - \text{Predicted}_1)^2 \\ &+ (\text{Observed}_2 - \text{Predicted}_2)^2 \\ &+ (\text{Observed}_3 - \text{Predicted}_3)^2\end{aligned}}$$

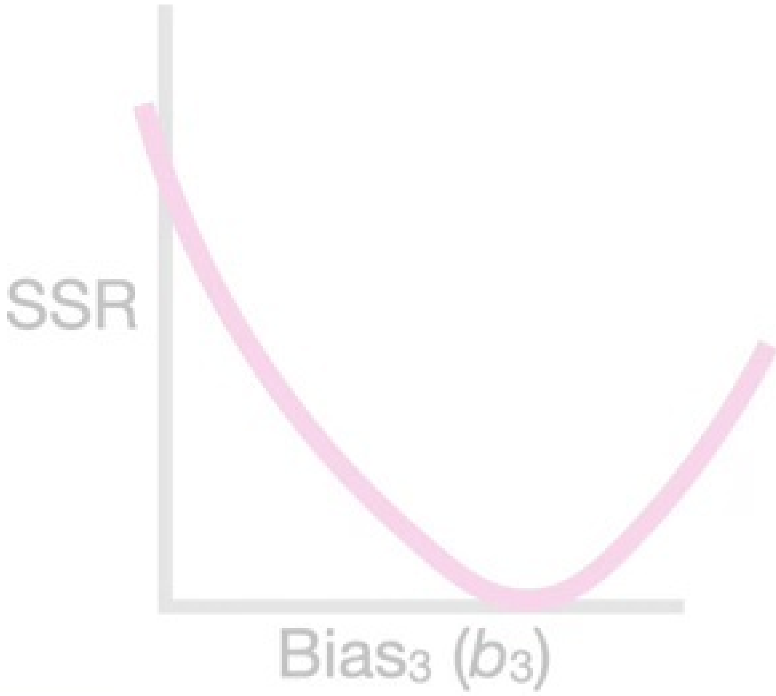




Now let's talk a little bit more about the **Predicted** values.

$$\text{SSR} = \sum_{i=1}^{n=3} (\text{Observed}_i - \boxed{\text{Predicted}_i})^2$$

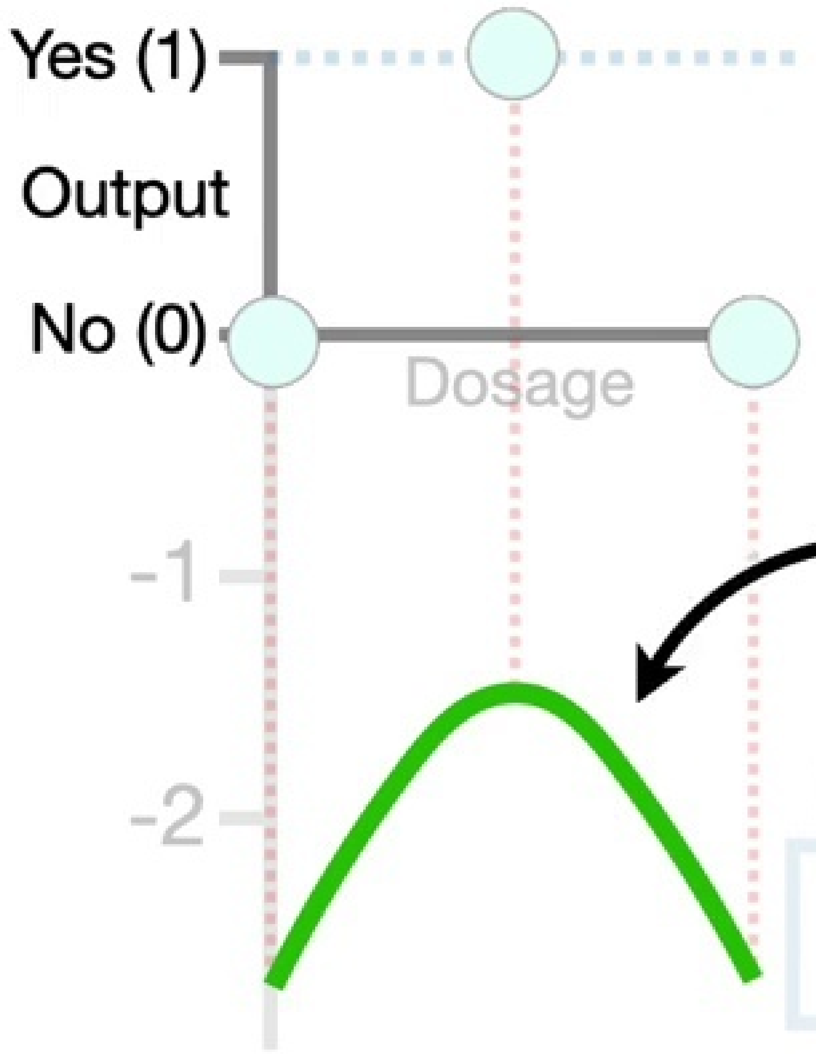


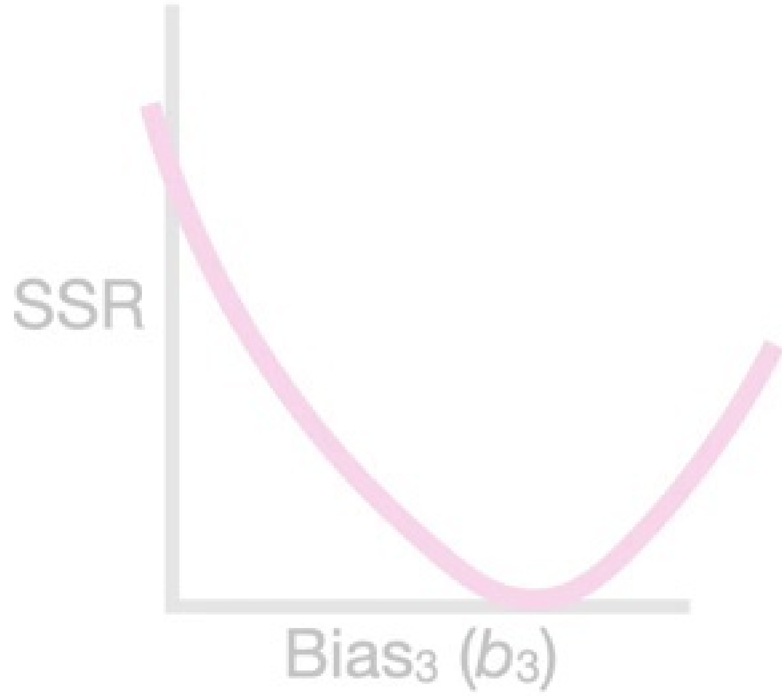


Each **Predicted** value comes from the **green squiggle**…

$$SSR = \sum_{i=1}^{n=3} (\text{Observed}_i - \boxed{\text{Predicted}_i})^2$$

$$\boxed{\text{Predicted}_i = \text{green squiggle}_i}$$

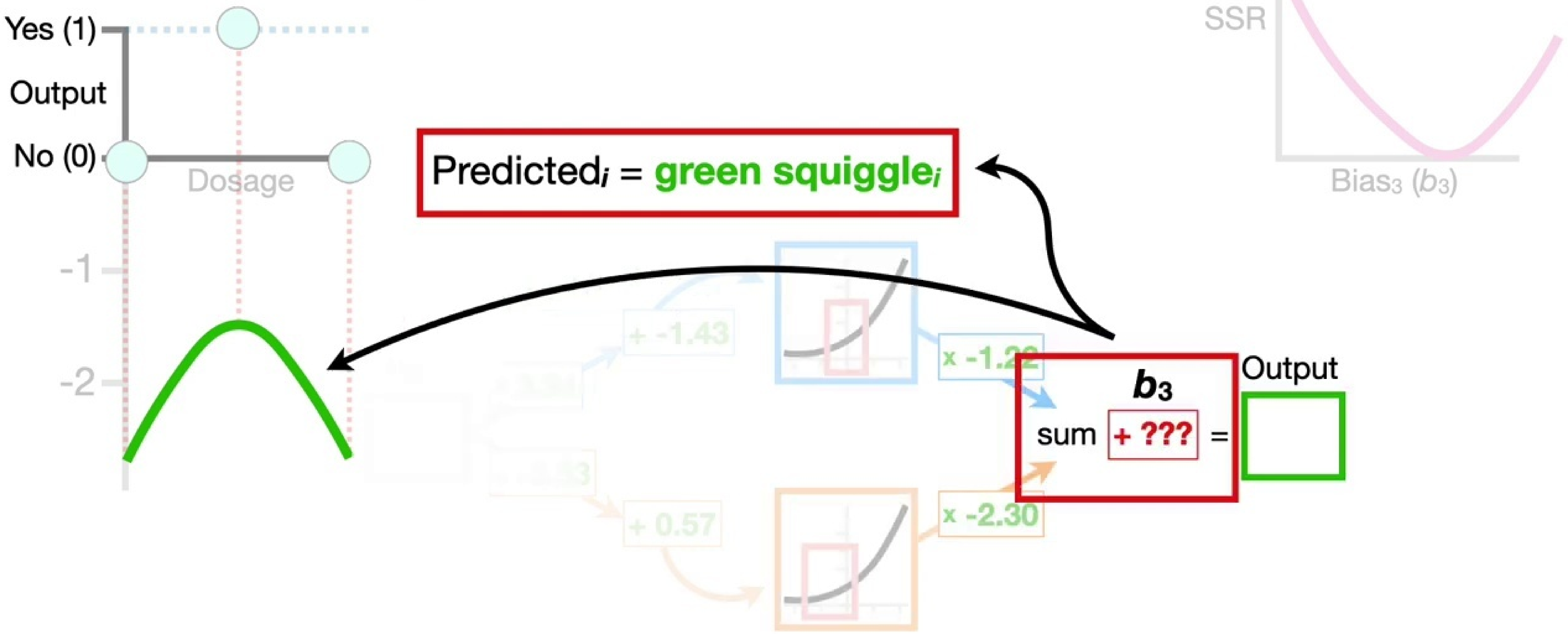




...and the **green squiggle** comes from the last part of the **Neural Network.**

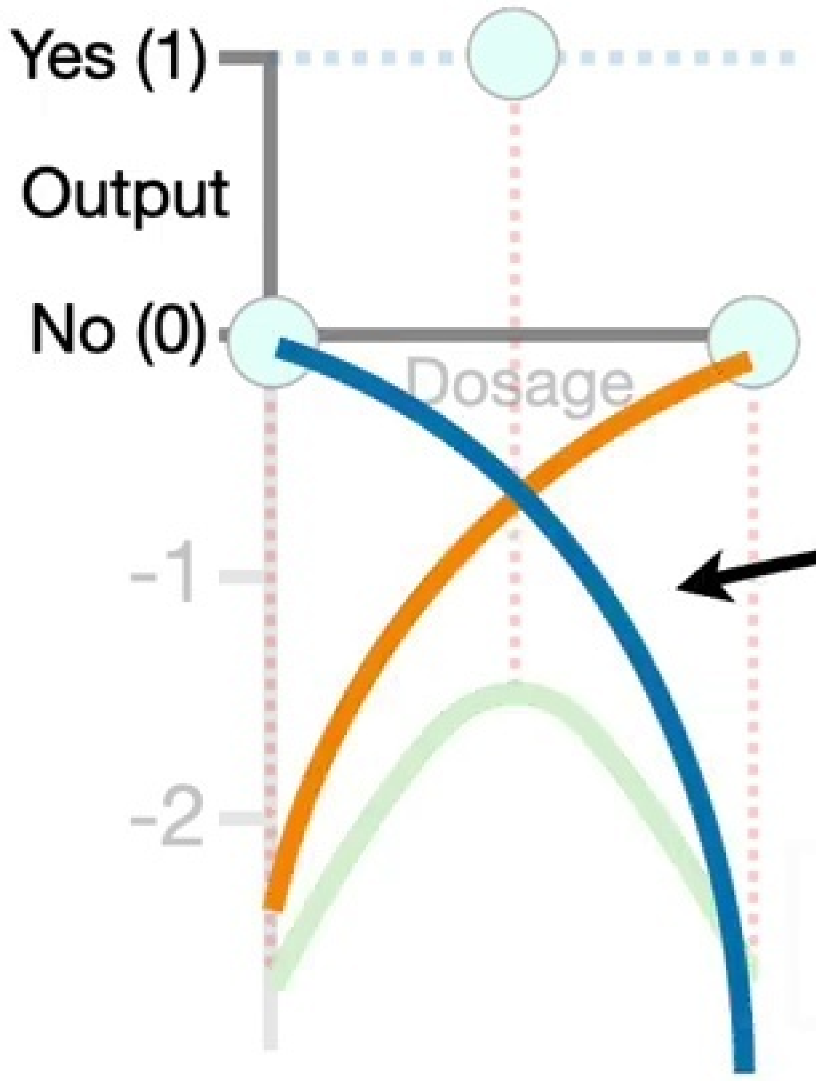
$$SSR = \sum_{i=1}^{n=3}(\text{Observed}_i - \text{Predicted}_i)^2$$

$$\text{Predicted}_i = \text{green squiggle}_i$$

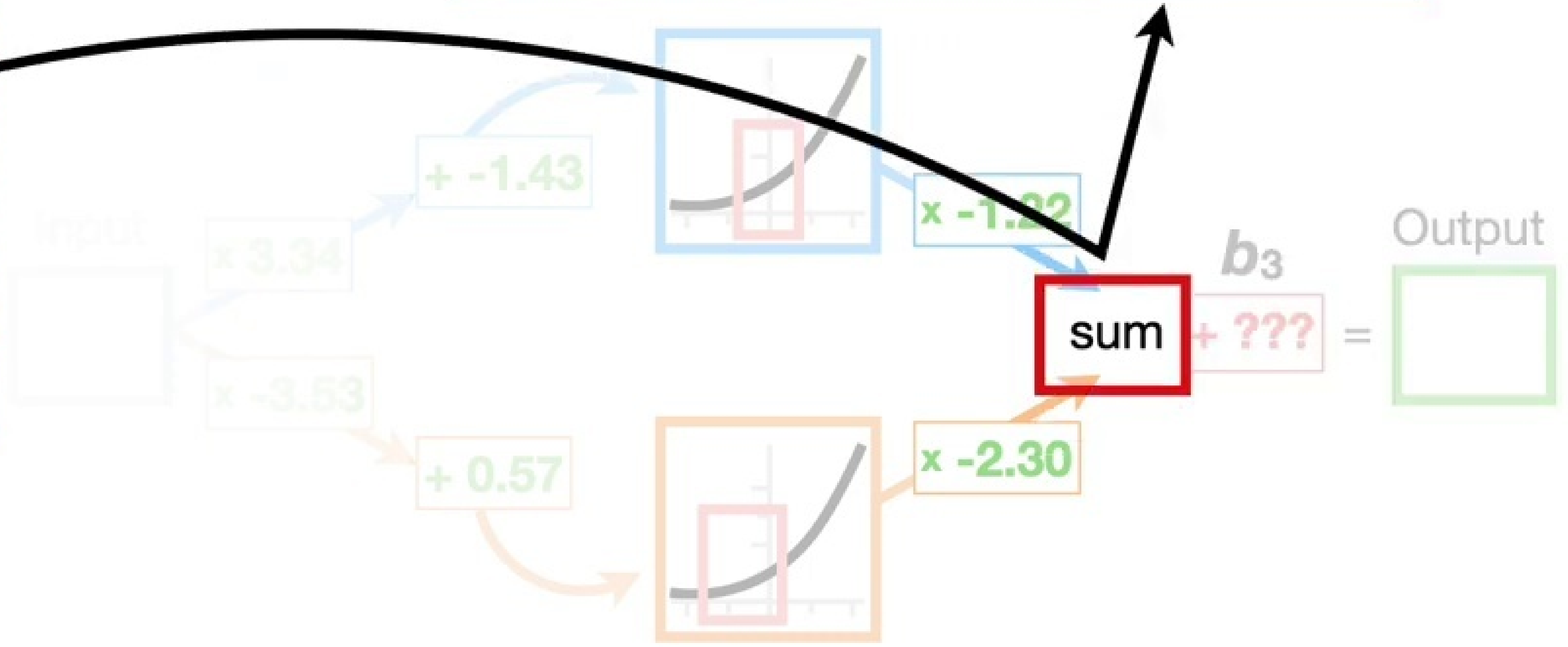


In other words, the **green squiggle** is the **sum** of the **blue** and **orange** curves…

$$SSR = \sum_{i=1}^{n=3} (\text{Observed}_i - \text{Predicted}_i)^2$$

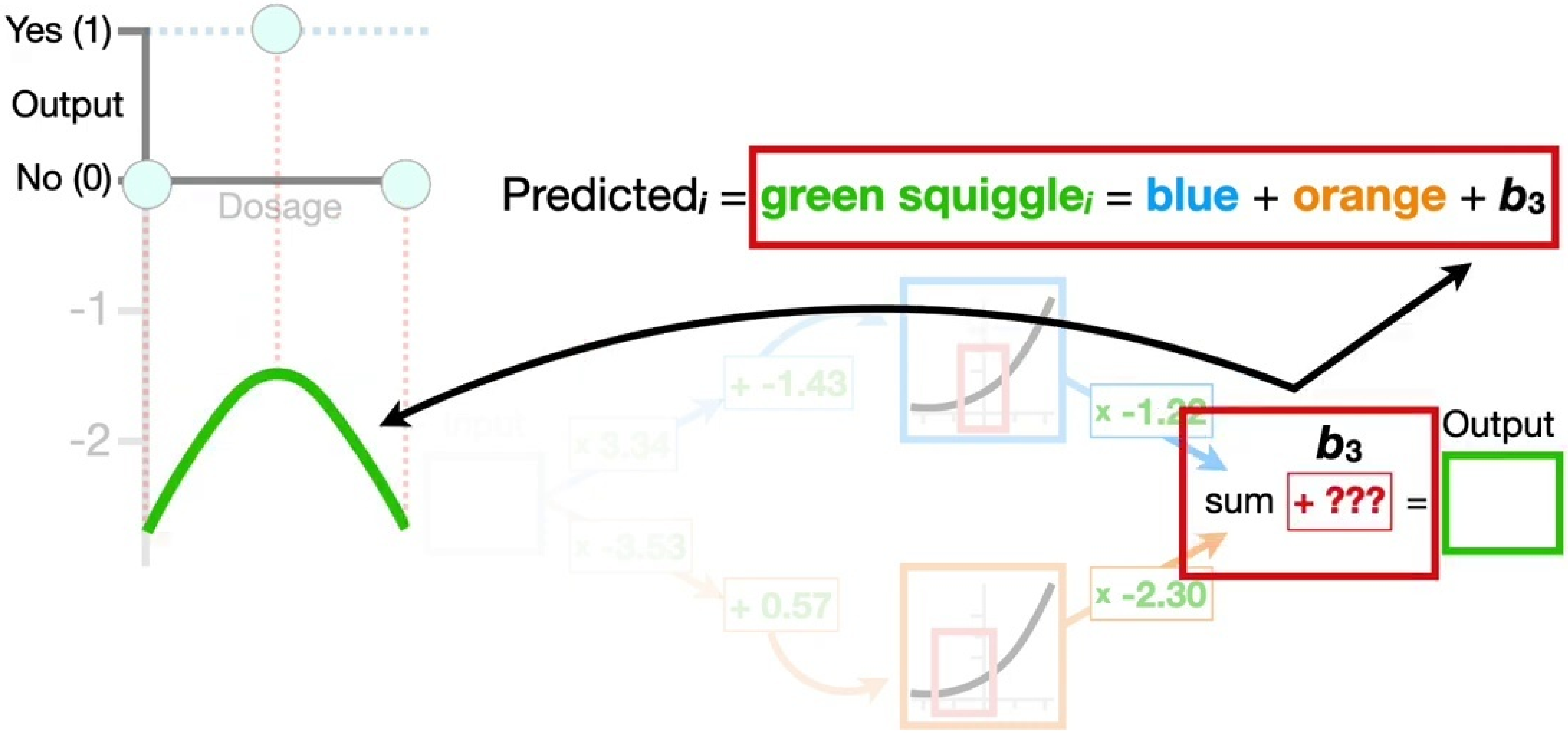


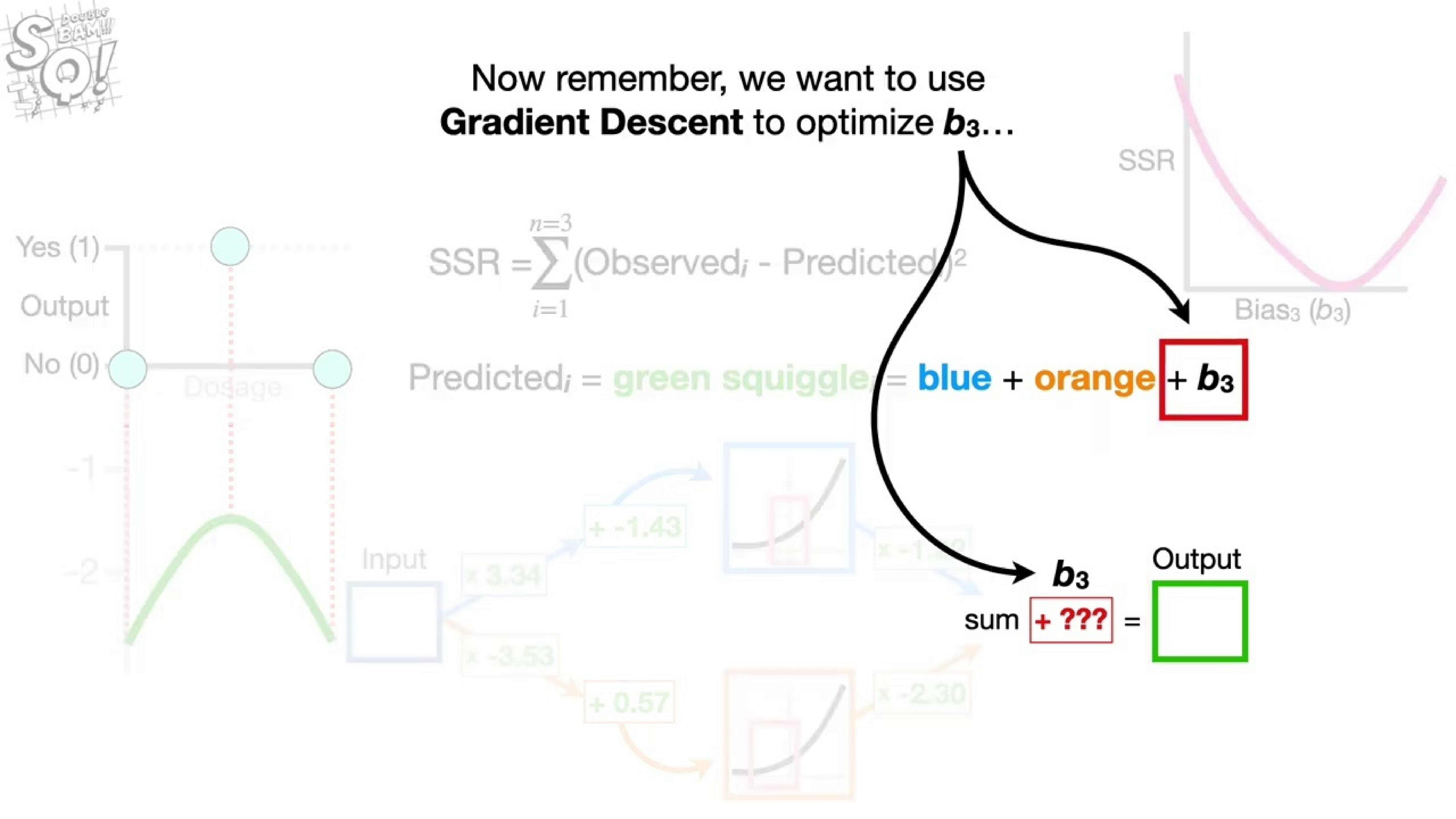
$$\text{Predicted}_i = \boxed{\text{green squiggle}_i = \text{blue} + \text{orange}} + b_3$$



…plus b_3.

$$\text{SSR} = \sum_{i=1}^{n=3} (\text{Observed}_i - \text{Predicted}_i)^2$$

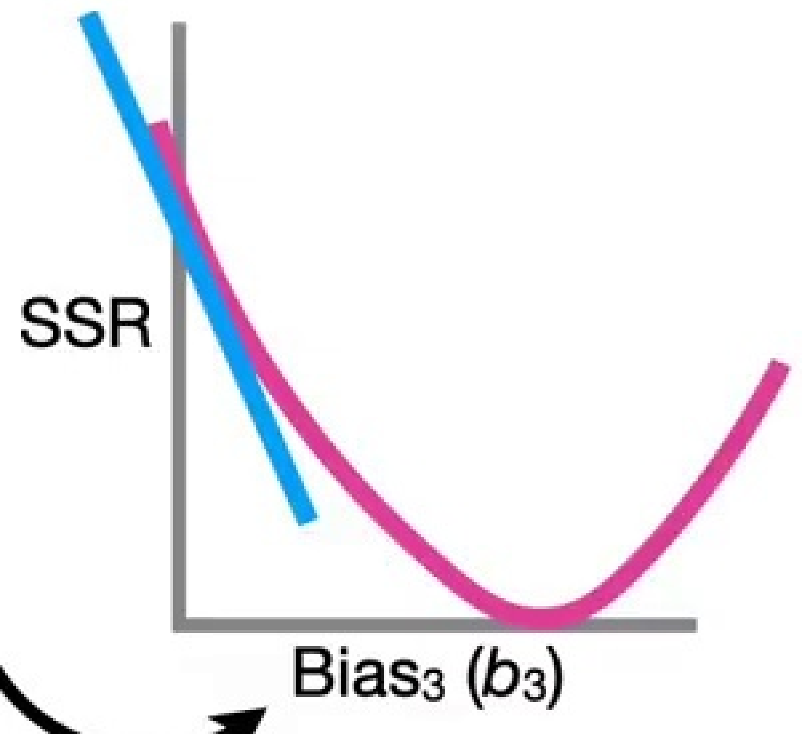


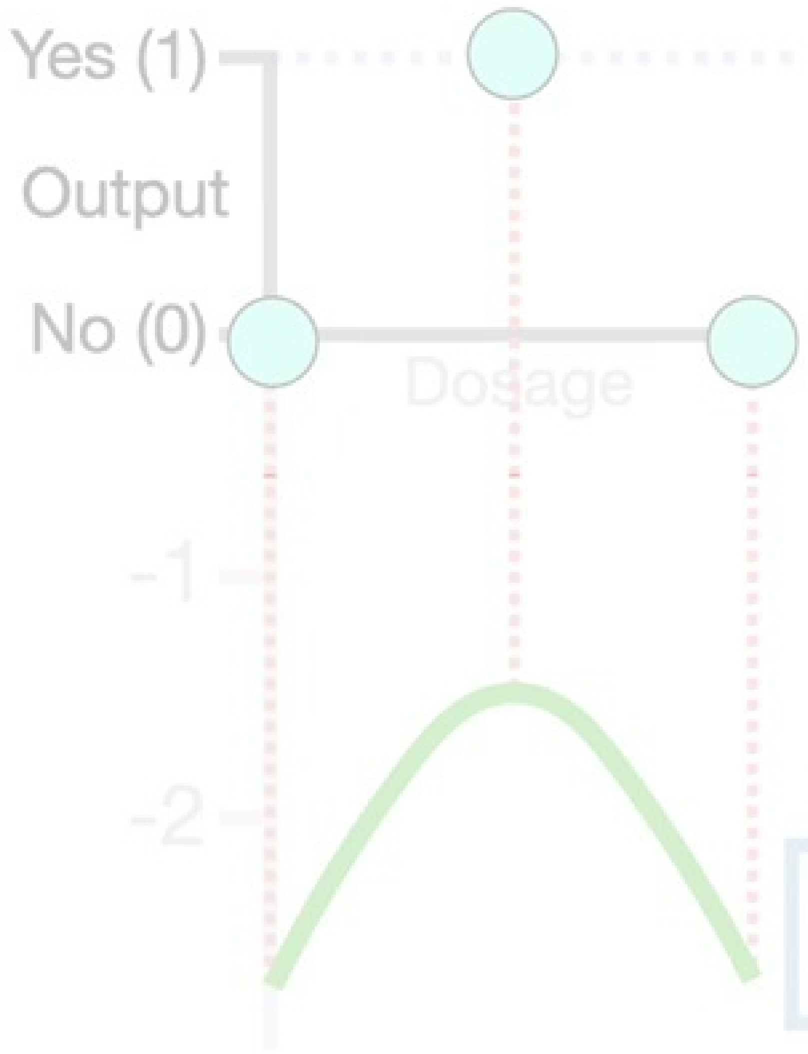


Now remember, we want to use
Gradient Descent to optimize b_3…
SSR
Bias₃ (b_3)
Yes (1)
Output
No (0)
Dosage
$$SSR = \sum_{i=1}^{n=3} (\text{Observed}_i - \text{Predicted}_i)^2$$
Predicted$_i$ = green squiggle$_i$ = blue + orange $\boxed{+ b_3}$
-1
-2
Input
x 3.34
+ -1.43
x -3.53
+ 0.57
x -2.30
b_3
Output
sum $\boxed{+ ???}$ =

$$\frac{d\ SSR}{d\ b_3}$$

...and that means we need to take the derivative of the **SSR** with respect to $\boldsymbol{b_3}$.



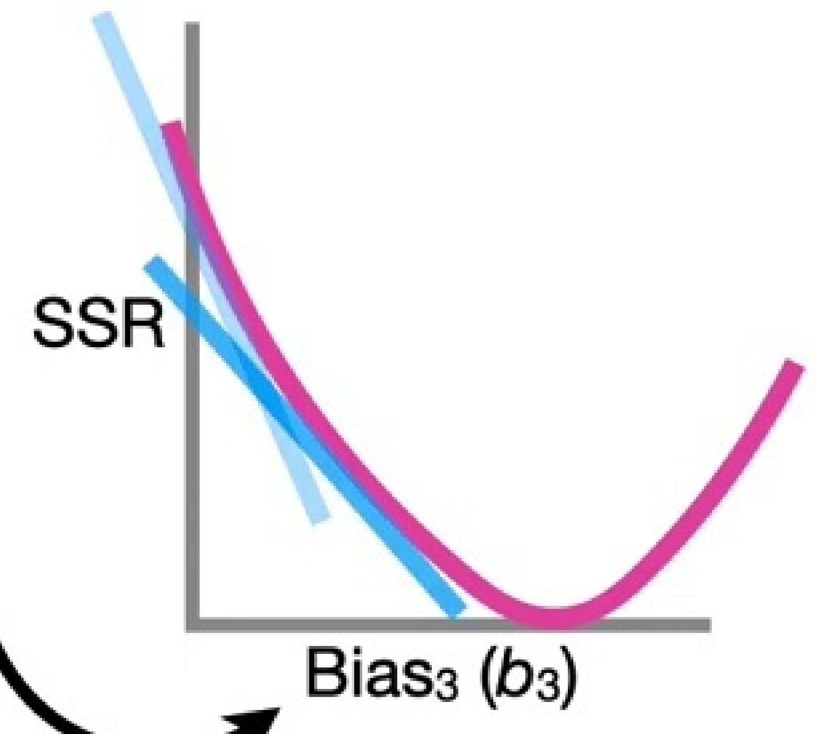


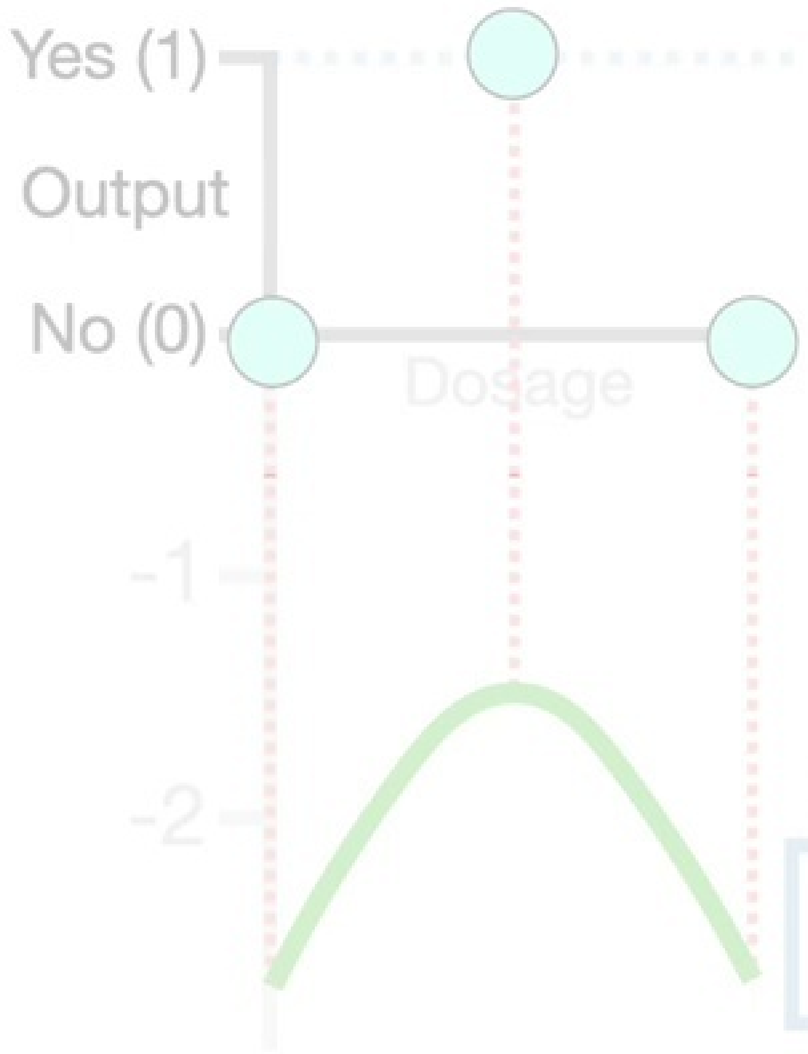
$$\text{SSR} = \sum_{i=1}^{n=3}(\text{Observed}_i - \text{Predicted}_i)^2$$

$$\text{Predicted}_i = \text{green squiggle}_i = \text{blue} + \text{orange} + b_3$$

$$\frac{d\ SSR}{d\ b_3}$$

…and that means we need to take the derivative of the **SSR** with respect to b_3.



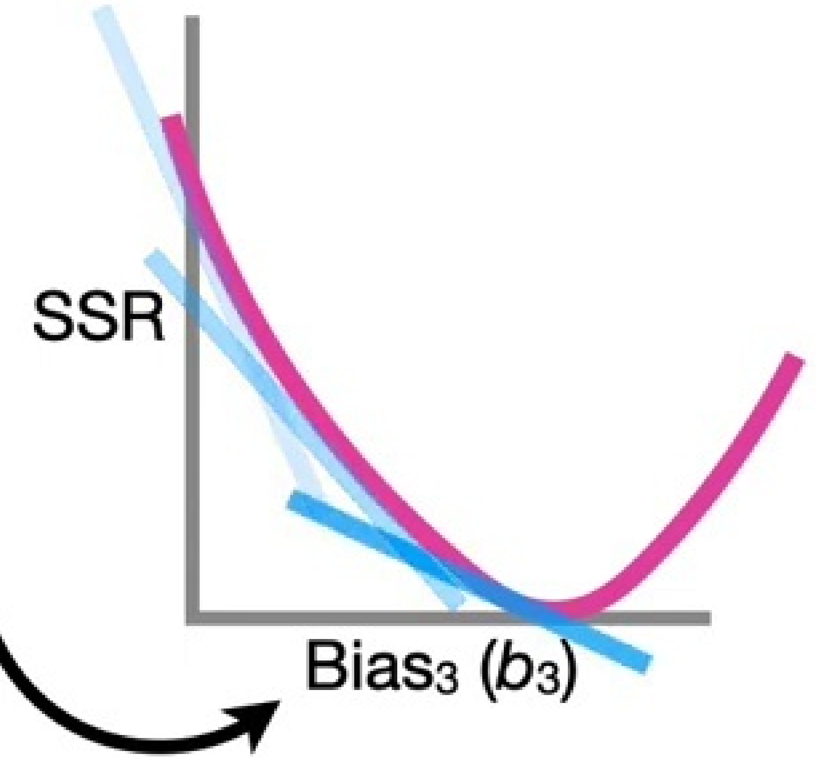


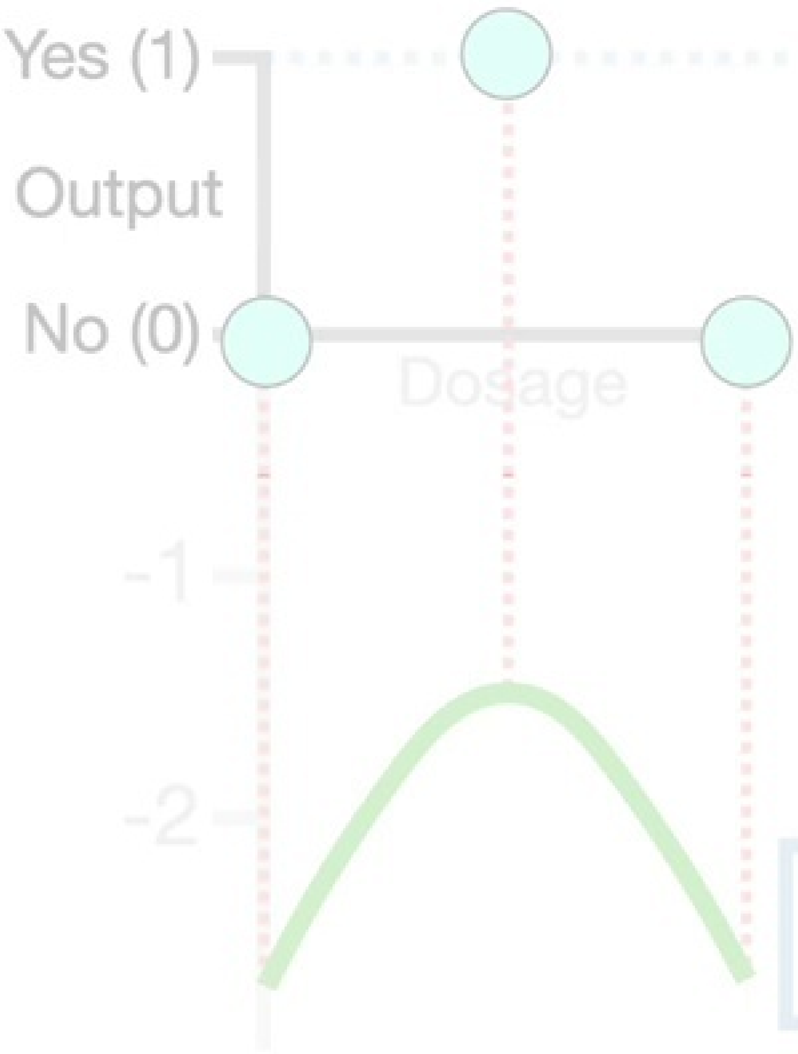
$$SSR = \sum_{i=1}^{n=3}(Observed_i - Predicted_i)^2$$

$$Predicted_i = \text{green squiggle}_i = \text{blue} + \text{orange} + b_3$$

$$\frac{d\ SSR}{d\ b_3}$$

…and that means we need to take the derivative of the **SSR** with respect to b_3.



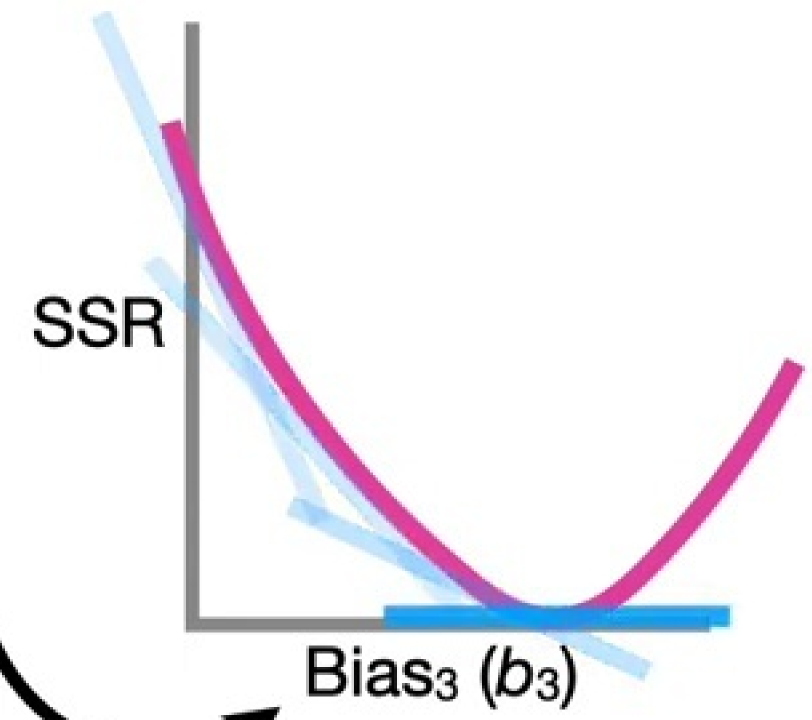


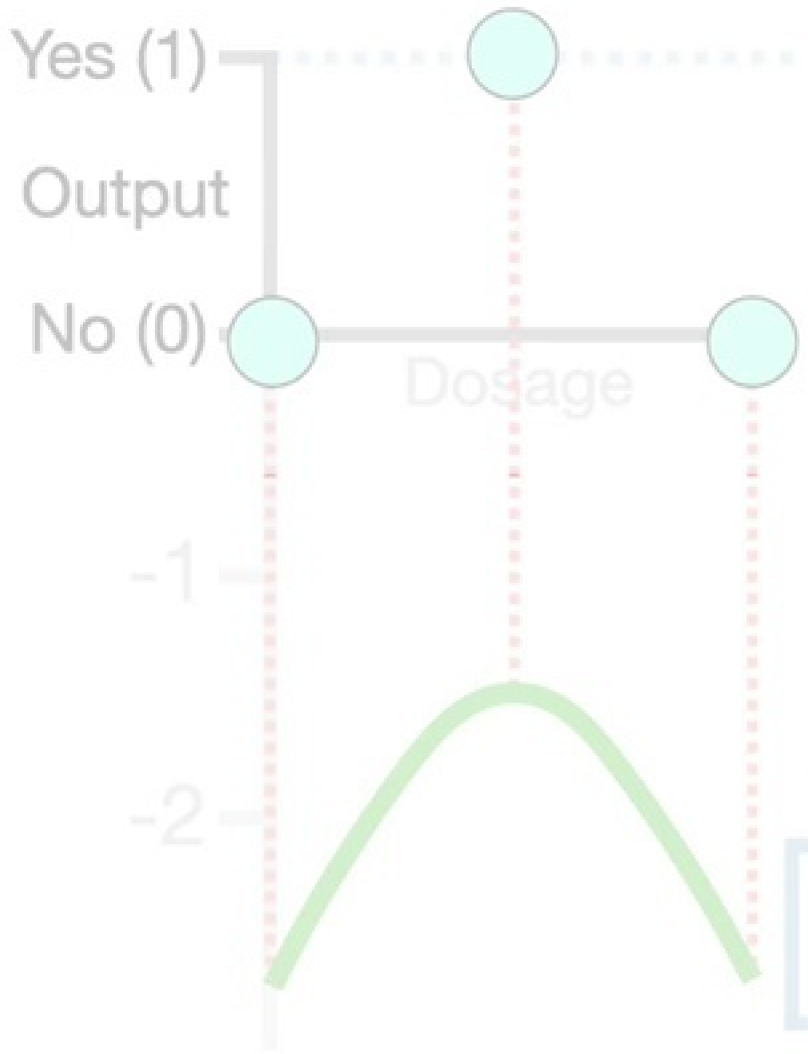
$$\text{SSR} = \sum_{i=1}^{n=3} (\text{Observed}_i - \text{Predicted}_i)^2$$

$$\text{Predicted}_i = \textbf{green squiggle}_i = \textbf{blue} + \textbf{orange} + b_3$$

$$\frac{d\ SSR}{d\ b_3}$$

…and that means we need to take the derivative of the **SSR** with respect to $\boldsymbol{b_3}$.





$$SSR = \sum_{i=1}^{n=3} (\text{Observed}_i - \text{Predicted}_i)^2$$

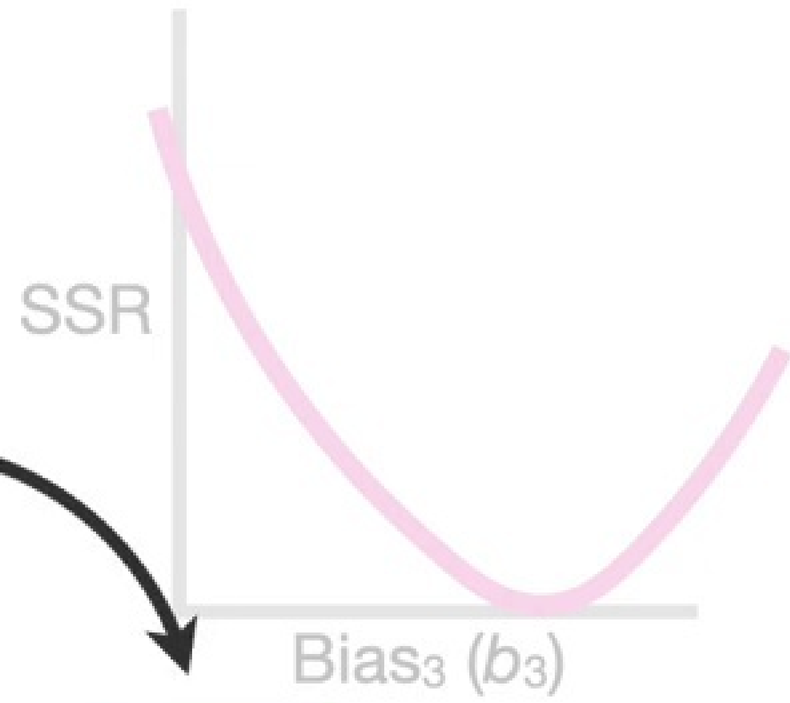
$$\text{Predicted}_i = \text{green squiggle}_i = \text{blue} + \text{orange} + b_3$$

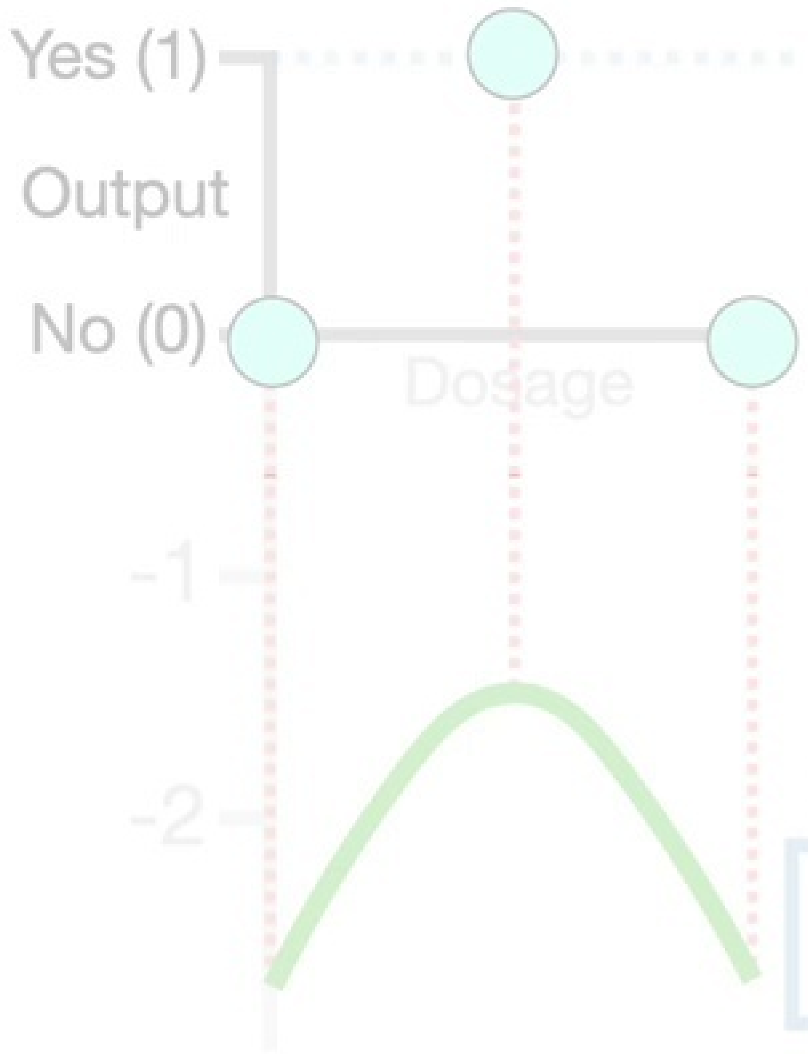
$$\frac{d\ SSR}{d\ b_3}$$

And because the **SSR** are linked to ***b*₃**…

$$\boxed{\text{SSR}} = \sum_{i=1}^{n=3} (\text{Observed}_i - \text{Predicted}_i)^2$$

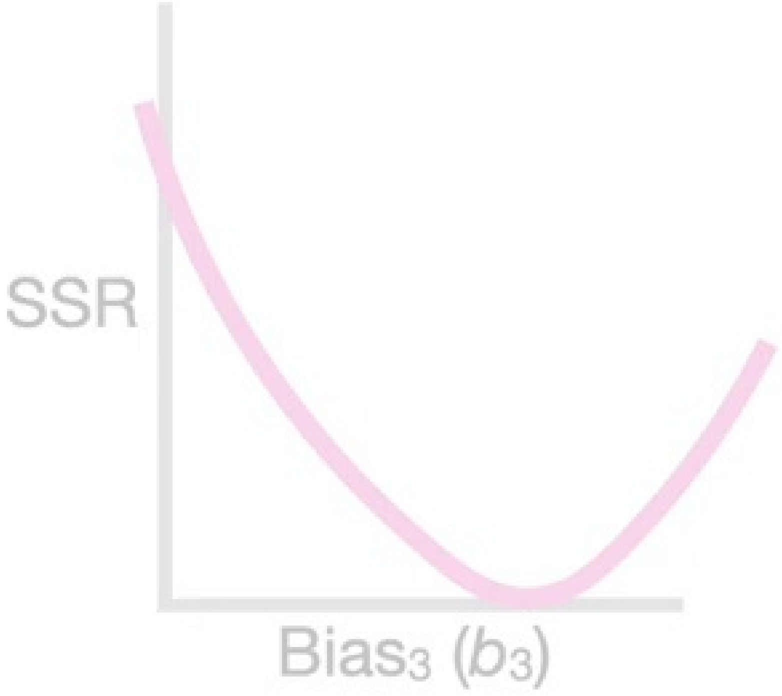
$$\text{Predicted}_i = \text{green squiggle}_i = \text{blue} + \text{orange} \boxed{+\ b_3}$$





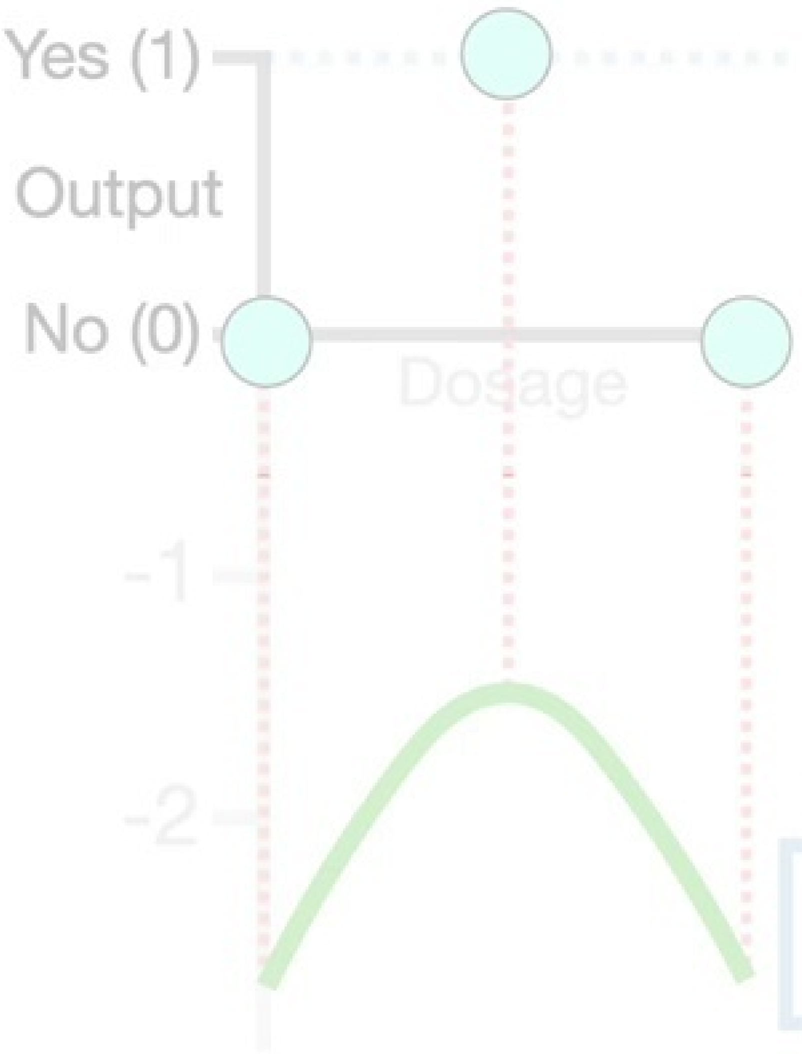
$$\frac{d\ SSR}{d\ b_3}$$

...by the **Predicted** values...



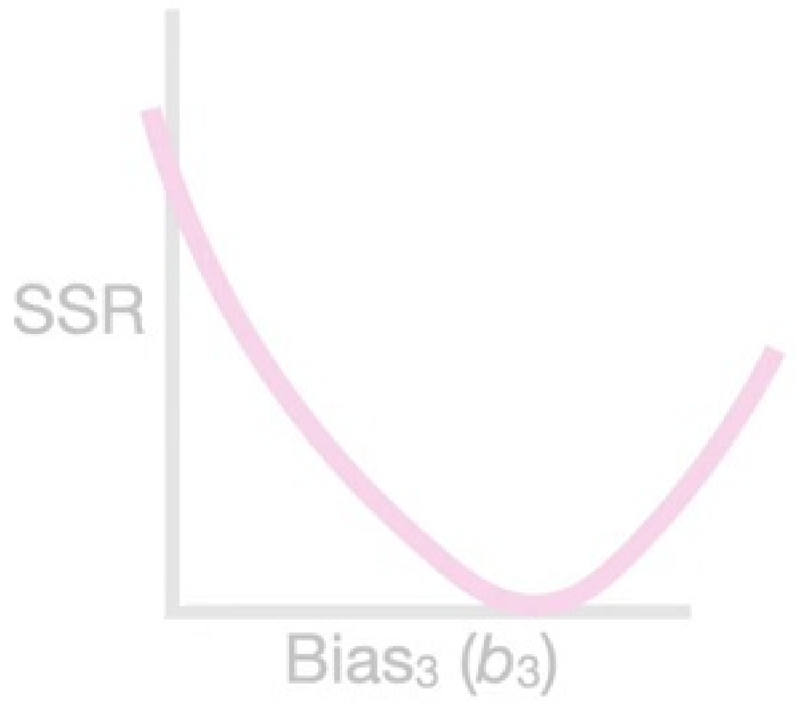
$$SSR = \sum_{i=1}^{n=3} (\text{Observed}_i - \boxed{\text{Predicted}_i})^2$$

$$\boxed{\text{Predicted}_i} = \text{green squiggle}_i = \text{blue} + \text{orange} + b_3$$



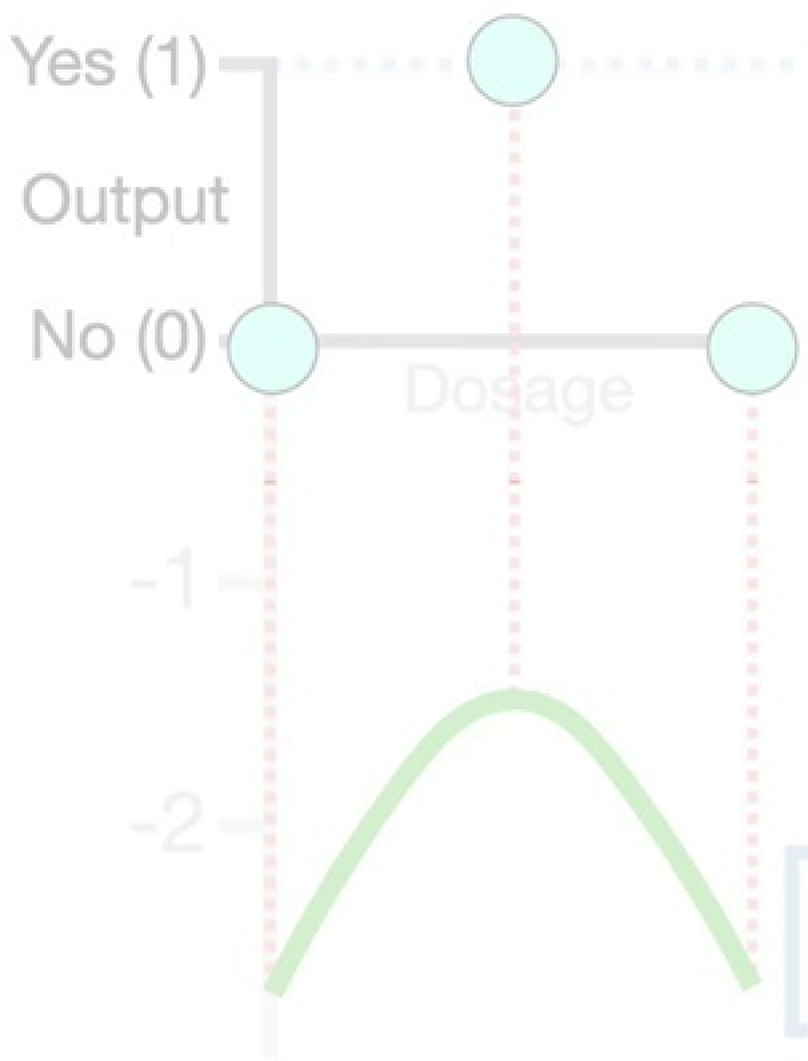
$$\frac{d\ SSR}{d\ b_3}$$

...we can use
The Chain Rule...



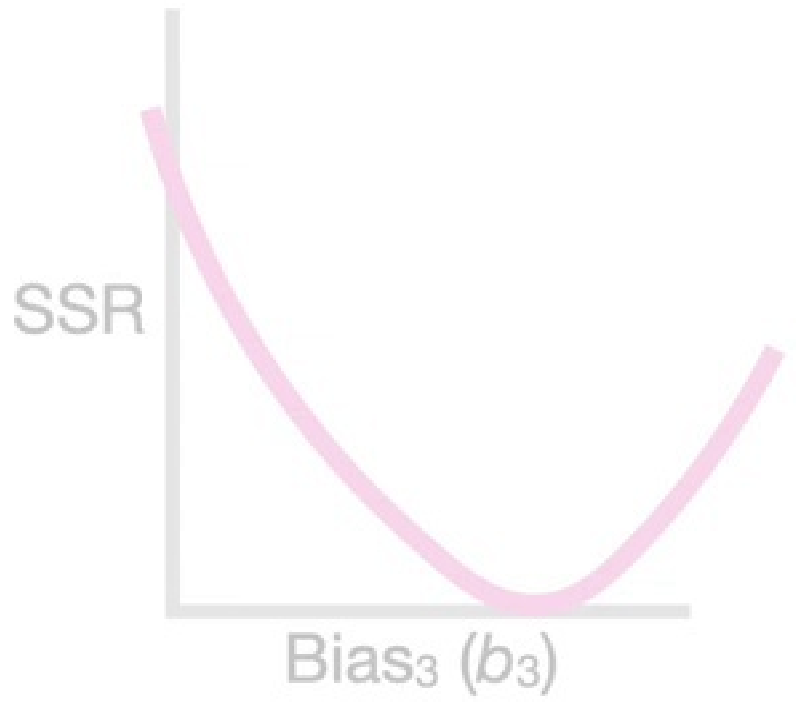
$$\text{SSR} = \sum_{i=1}^{n=3} (\text{Observed}_i - \boxed{\text{Predicted}_i})^2$$

$$\boxed{\text{Predicted}_i} = \text{green squiggle}_i = \text{blue} + \text{orange} + b_3$$



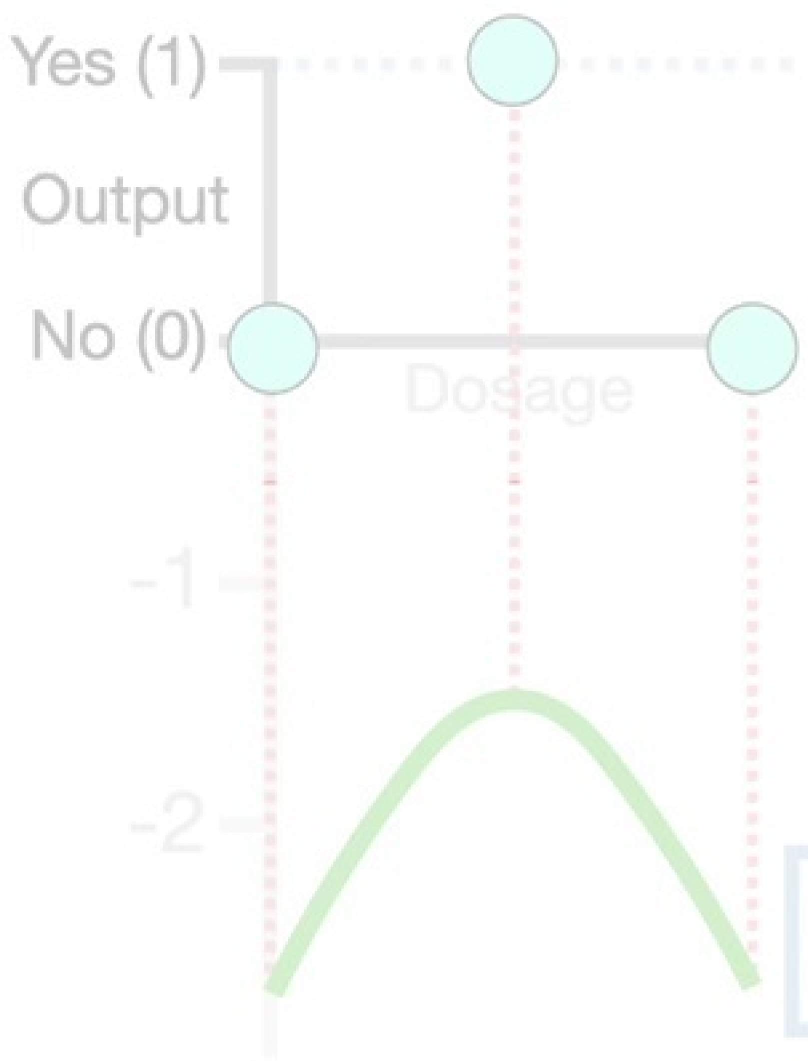
$$\frac{d\ SSR}{d\ b_3} =$$

...to solve for the derivative of the **SSR** with respect to b_3.



$$SSR = \sum_{i=1}^{n=3} (\text{Observed}_i - \boxed{\text{Predicted}_i})^2$$

$$\boxed{\text{Predicted}_i} = \text{green squiggle}_i = \text{blue} + \text{orange} + b_3$$

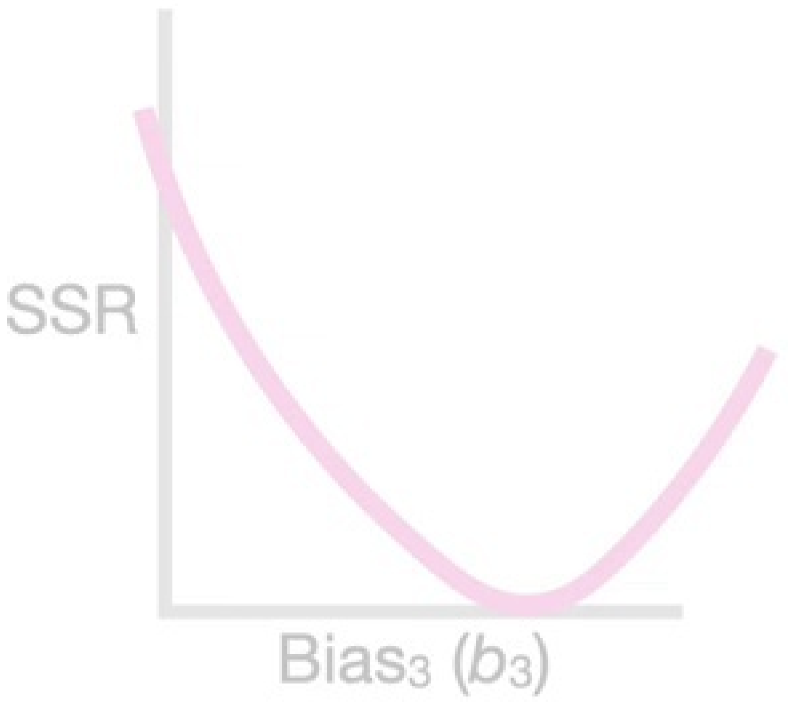


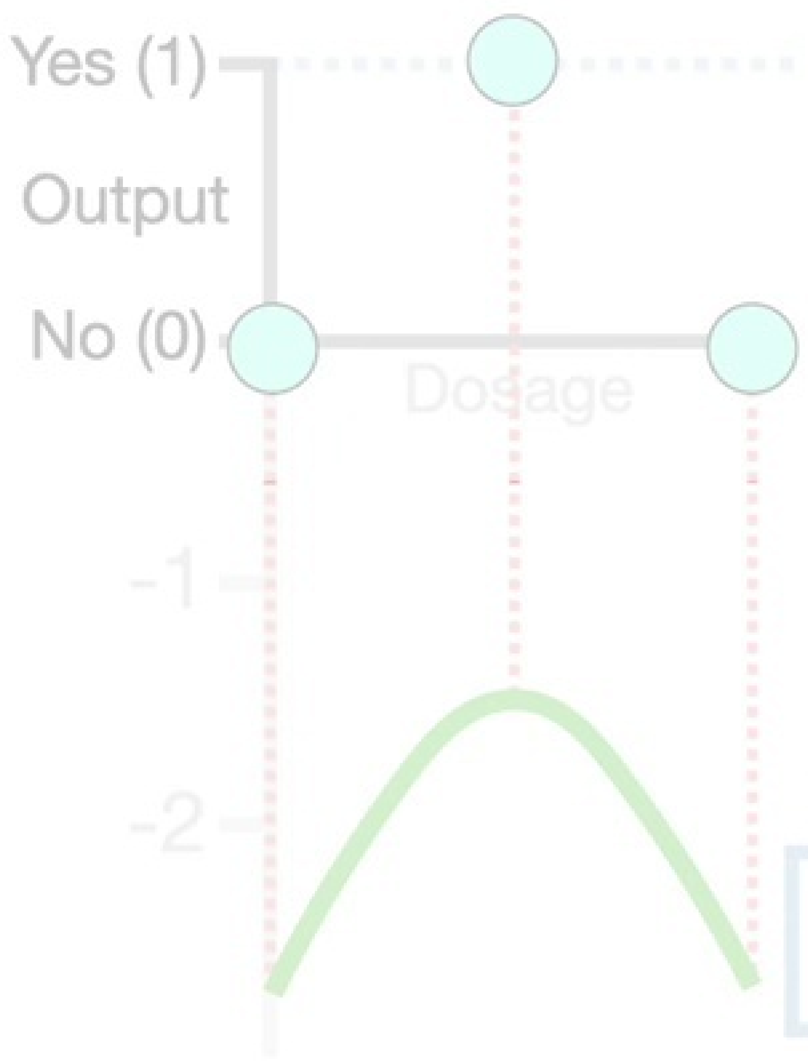
$$\frac{d\,SSR}{d\,b_3} =$$

The Chain Rule says that the derivative of the **SSR** with respect to ***b*₃**...

$$SSR = \sum_{i=1}^{n=3} (\text{Observed}_i - \boxed{\text{Predicted}_i})^2$$

$$\boxed{\text{Predicted}_i} = \text{green squiggle}_i = \text{blue} + \text{orange} + b_3$$

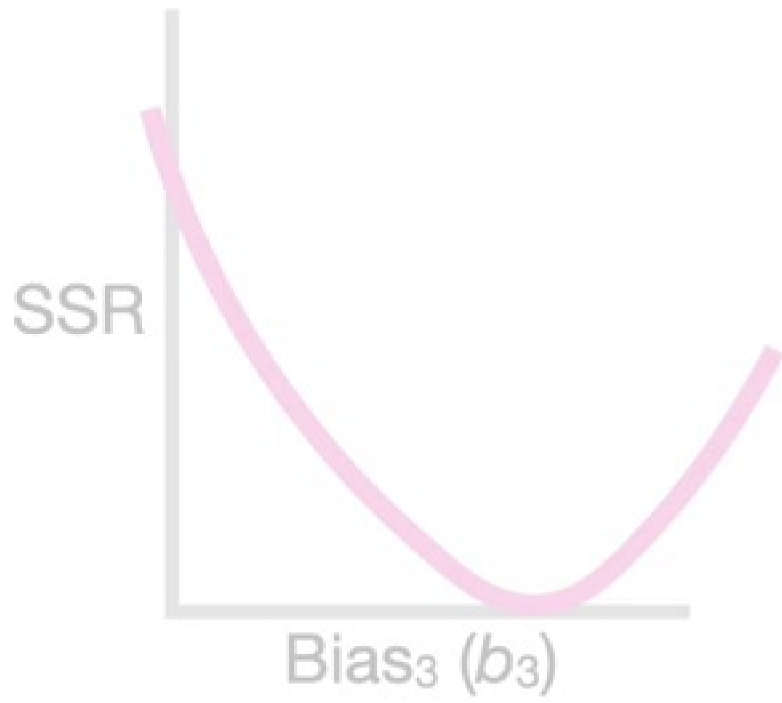




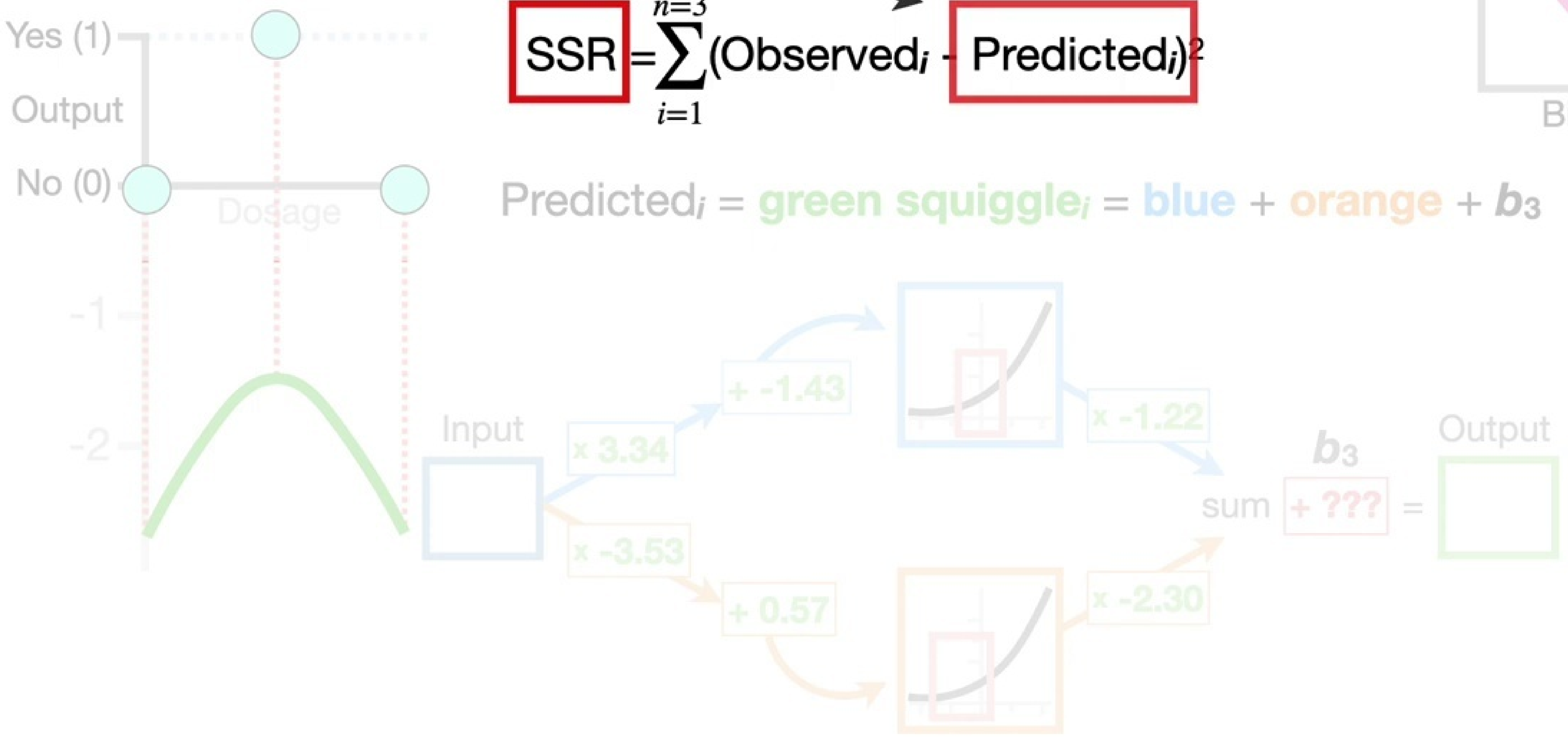
$$\frac{d\,SSR}{d\,b_3} = \frac{d\,SSR}{d\,Predicted}$$

...is the derivative of the **SSR** with respect to the **Predicted** values...

$$\boxed{SSR} = \sum_{i=1}^{n=3} (\text{Observed}_i - \boxed{\text{Predicted}_i})^2$$

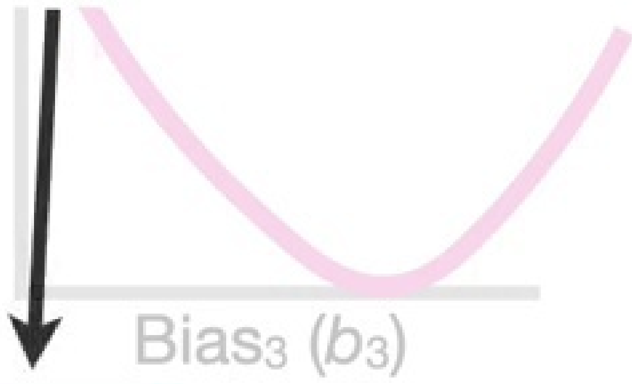


Predicted$_i$ = green squiggle$_i$ = blue + orange + b_3



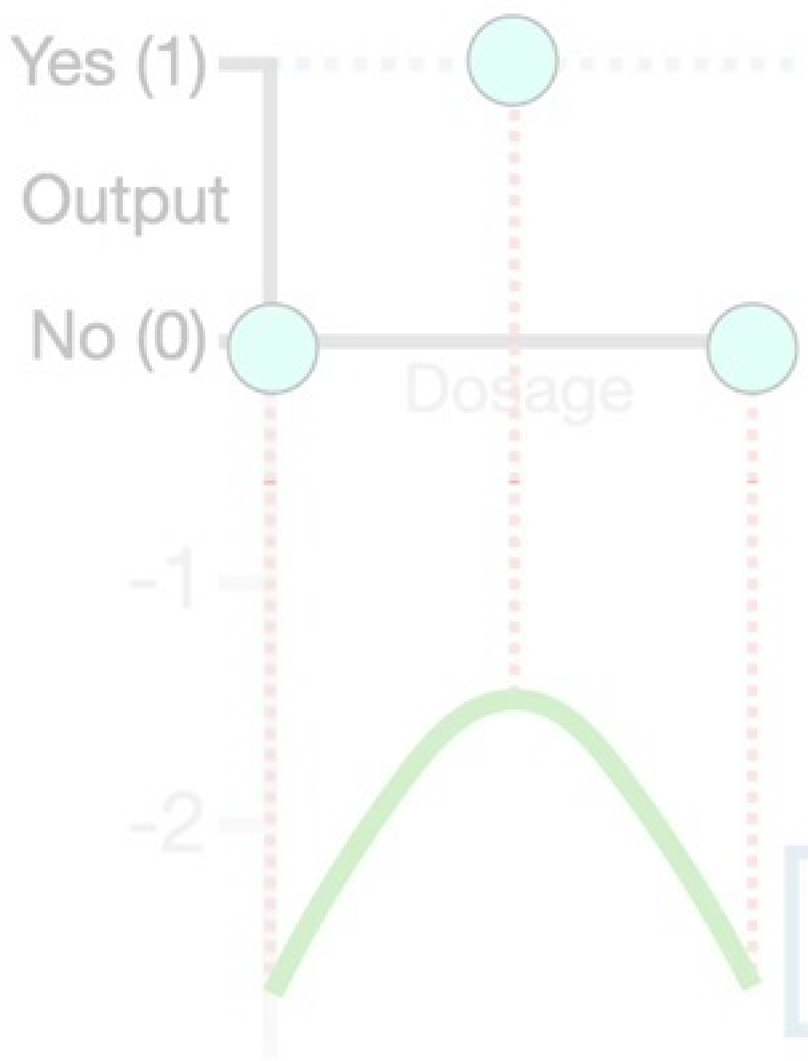
$$\frac{d\,SSR}{d\,b_3} = \frac{d\,SSR}{d\,Predicted} \times \frac{d\,Predicted}{d\,b_3}$$

…times the derivative of the **Predicted** values with respect to $\boldsymbol{b_3}$.



$$SSR = \sum_{i=1}^{n=3}(Observed_i - \boxed{Predicted_i})^2$$

$$\boxed{Predicted_i} = \text{green squiggle}_i = \text{blue} + \text{orange} \boxed{+\, b_3}$$



$$\frac{d\ SSR}{d\ b_3} = \frac{d\ SSR}{d\ Predicted} \times \frac{d\ Predicted}{d\ b_3}$$

Now, we can solve for the derivative of the **SSR** with respect to the **Predicted** values by first substituting in the equation…

$$\frac{d\ SSR}{d\ Predicted} =$$

$$\text{SSR} = \sum_{i=1}^{n=3}(\text{Observed}_i - \text{Predicted}_i)^2 \quad \text{Predicted}_i = \text{green squiggle}_i = \text{blue} + \text{orange} + b_3$$

$$\frac{d\ SSR}{d\ b_3} = \frac{d\ SSR}{d\ Predicted} \times \frac{d\ Predicted}{d\ b_3}$$

Now, we can solve for the derivative of the **SSR** with respect to the **Predicted** values by first substituting in the equation…

$$\frac{d\ SSR}{d\ Predicted} = \frac{d}{d\ Predicted} \sum_{i=1}^{n=3} (\text{Observed}_i - \text{Predicted}_i)^2$$

$$\boxed{\text{SSR} = \sum_{i=1}^{n=3} (\text{Observed}_i - \text{Predicted}_i)^2}$$

$\text{Predicted}_i = \text{green squiggle}_i = \text{blue} + \text{orange} + b_3$

$$\frac{d\ SSR}{d\ b_3} = \frac{d\ SSR}{d\ Predicted} \times \frac{d\ Predicted}{d\ b_3}$$

...and then use **The Chain Rule**...

$$\frac{d\ SSR}{d\ Predicted} = \frac{d}{d\ Predicted} \sum_{i=1}^{n=3} (\text{Observed}_i - \text{Predicted}_i)^2$$

$$\text{SSR} = \sum_{i=1}^{n=3} (\text{Observed}_i - \text{Predicted}_i)^2 \quad \text{Predicted}_i = \text{green squiggle}_i = \text{blue} + \text{orange} + b_3$$

$$\frac{d\ SSR}{d\ b_3} = \frac{d\ SSR}{d\ Predicted} \times \frac{d\ Predicted}{d\ b_3}$$

...and we have the derivative of the **SSR** with respect to the **Predicted** values.

$$\frac{d\ SSR}{d\ Predicted} = \frac{d}{d\ Predicted} \sum_{i=1}^{n=3} (\text{Observed}_i - \text{Predicted}_i)^2 = \boxed{\sum_{i=1}^{n=3} -2 \times (\text{Observed}_i - \text{Predicted}_i)}$$

$$\text{SSR} = \sum_{i=1}^{n=3} (\text{Observed}_i - \text{Predicted}_i)^2 \quad \text{Predicted}_i = \textbf{green squiggle}_i = \textbf{blue} + \textbf{orange} + b_3$$

$$\frac{d\ SSR}{d\ b_3} = \frac{d\ SSR}{d\ Predicted} \times \frac{d\ Predicted}{d\ b_3}$$

So let's move that up here…

$$= \sum_{i=1}^{n=3} -2 \times (\text{Observed}_i - \text{Predicted}_i)$$

$$\frac{d\ SSR}{d\ Predicted} = \frac{d}{d\ Predicted} \sum_{i=1}^{n=3} (\text{Observed}_i - \text{Predicted}_i)^2 \boxed{= \sum_{i=1}^{n=3} -2 \times (\text{Observed}_i - \text{Predicted}_i)}$$

$$\text{SSR} = \sum_{i=1}^{n=3} (\text{Observed}_i - \text{Predicted}_i)^2 \quad \text{Predicted}_i = \text{green squiggle}_i = \text{blue} + \text{orange} + b_3$$

$$\frac{d\ SSR}{d\ b_3} = \boxed{\frac{d\ SSR}{d\ Predicted}} \times \frac{d\ Predicted}{d\ b_3}$$

...and now we are done with the first part.

$$\boxed{= \sum_{i=1}^{n=3} -2 \times (\text{Observed}_i - \text{Predicted}_i)}$$

$$SSR = \sum_{i=1}^{n=3} (\text{Observed}_i - \text{Predicted}_i)^2 \quad \text{Predicted}_i = \text{green squiggle}_i = \text{blue} + \text{orange} + b_3$$

$$\frac{d\ SSR}{d\ b_3} = \frac{d\ SSR}{d\ Predicted} \times \frac{d\ Predicted}{d\ b_3}$$

Now let's solve for the second part, the derivative of the **Predicted** values with respect to b_3.

$$= \sum_{i=1}^{n=3} -2 \times (\text{Observed}_i - \text{Predicted}_i)$$

$$\text{SSR} = \sum_{i=1}^{n=3} (\text{Observed}_i - \text{Predicted}_i)^2 \quad \text{Predicted}_i = \text{green squiggle}_i = \text{blue} + \text{orange} + b_3$$

$$\frac{d\ SSR}{d\ b_3} = \frac{d\ SSR}{d\ Predicted} \times \frac{d\ Predicted}{d\ b_3}$$

We start by plugging in the equation for the **Predicted** values.

$$= \sum_{i=1}^{n=3} -2 \times (\text{Observed}_i - \text{Predicted}_i)$$

$$\frac{d\ Predicted}{d\ b_3}$$

$$\text{SSR} = \sum_{i=1}^{n=3} (\text{Observed}_i - \text{Predicted}_i)^2$$

Predicted_i = **green squiggle$_i$** = **blue** + **orange** + b_3

$$\frac{d\ SSR}{d\ b_3} = \frac{d\ SSR}{d\ Predicted} \times \frac{d\ Predicted}{d\ b_3}$$

We start by plugging in the equation for the **Predicted** values.

$$= \sum_{i=1}^{n=3} -2 \times (Observed_i - Predicted_i)$$

$$\frac{d\ Predicted}{d\ b_3} = \frac{d}{d\ b_3}\ \text{green squiggle}$$

$$SSR = \sum_{i=1}^{n=3} (Observed_i - Predicted_i)^2$$

$$\boxed{Predicted_i = \text{green squiggle}_i = \text{blue} + \text{orange} + b_3}$$

$$\frac{d\ SSR}{d\ b_3} = \frac{d\ SSR}{d\ Predicted} \times \frac{d\ Predicted}{d\ b_3}$$

We start by plugging in the equation for the **Predicted** values.

$$= \sum_{i=1}^{n=3} -2 \times (\text{Observed}_i - \text{Predicted}_i)$$

$$\frac{d\ Predicted}{d\ b_3} = \frac{d}{d\ b_3}\ \text{green squiggle} = \frac{d}{d\ b_3}(\text{blue} + \text{orange} + b_3)$$

$$\text{SSR} = \sum_{i=1}^{n=3} (\text{Observed}_i - \text{Predicted}_i)^2$$

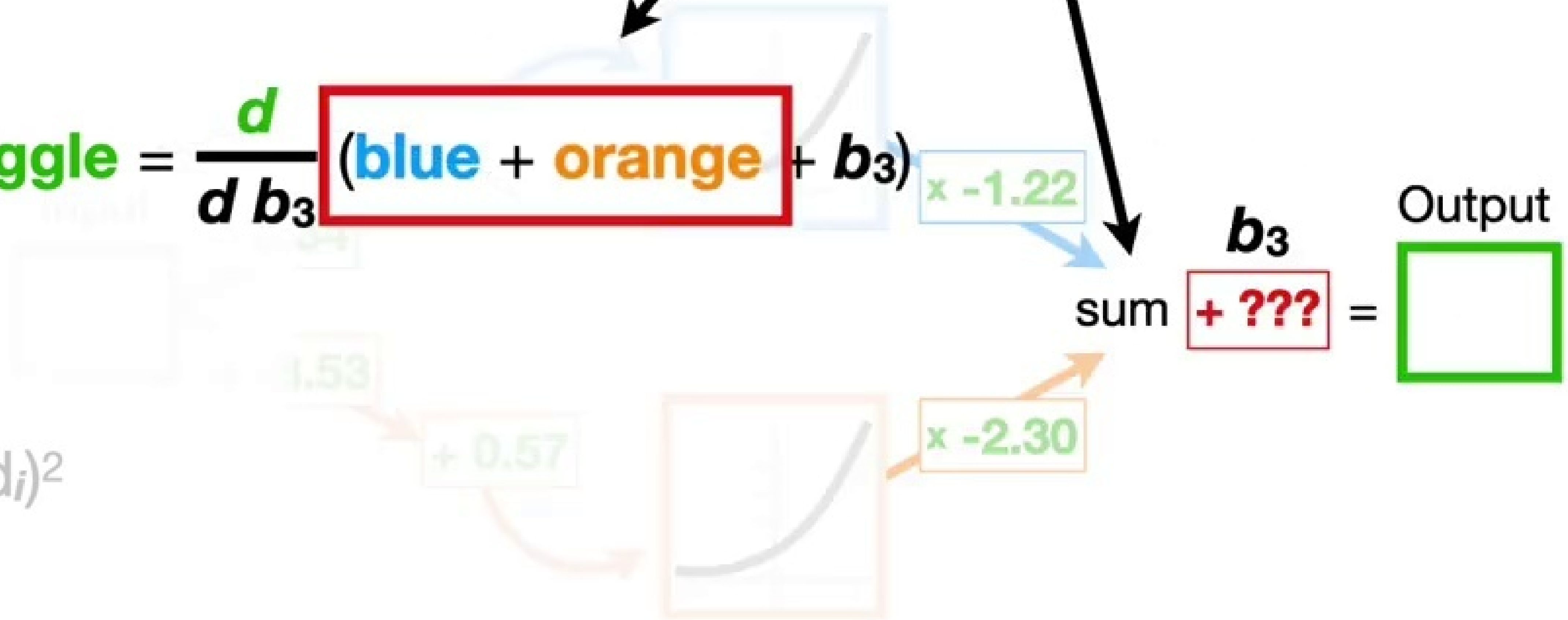
$$\boxed{\text{Predicted}_i = \text{green squiggle}_i = \text{blue} + \text{orange} + b_3}$$

$$\frac{d\ SSR}{d\ b_3} = \frac{d\ SSR}{d\ Predicted} \times \frac{d\ Predicted}{d\ b_3}$$

Remember, the **blue** and **orange** curves were created before we got to b_3…

$$= \sum_{i=1}^{n=3} -2 \times (\text{Observed}_i - \text{Predicted}_i)$$

$$\frac{d\ Predicted}{d\ b_3} = \frac{d}{d\ b_3}\ \text{green squiggle} = \frac{d}{d\ b_3}(\boxed{\text{blue} + \text{orange}} + b_3)$$



$$\text{SSR} = \sum_{i=1}^{n=3} (\text{Observed}_i - \text{Predicted}_i)^2$$

$$\frac{d\,SSR}{d\,b_3} = \frac{d\,SSR}{d\,Predicted} \times \frac{d\,Predicted}{d\,b_3}$$

…so the derivative of the **blue curve** with respect to ***b*₃** is **0**, because the **blue curve** is independent of ***b*₃**…

$$= \sum_{i=1}^{n=3} -2 \times (\text{Observed}_i - \text{Predicted}_i)$$

$$\frac{d\,Predicted}{d\,b_3} = \frac{d}{d\,b_3}\,\text{green squiggle} = \frac{d}{d\,b_3}(\boxed{\text{blue}} + \text{orange} + b_3) \boxed{= 0}$$

$$SSR = \sum_{i=1}^{n=3} (\text{Observed}_i - \text{Predicted}_i)^2 \quad \text{Predicted}_i = \text{green squiggle}_i = \text{blue} + \text{orange} + b_3$$

$$\frac{d\ SSR}{d\ b_3} = \frac{d\ SSR}{d\ Predicted} \times \frac{d\ Predicted}{d\ b_3}$$

...and the derivative of the **orange curve** with respect to b_3 is also **0**.

$$= \sum_{i=1}^{n=3} -2 \times (Observed_i - Predicted_i)$$

$$\frac{d\ Predicted}{d\ b_3} = \frac{d}{d\ b_3}\ \text{green squiggle} = \frac{d}{d\ b_3}(\text{blue} \boxed{+ \text{orange}} + b_3) = 0 \boxed{+ 0}$$

$$SSR = \sum_{i=1}^{n=3}(Observed_i - Predicted_i)^2 \quad Predicted_i = \text{green squiggle}_i = \text{blue} + \text{orange} + b_3$$

$$\frac{d\ SSR}{d\ b_3} = \frac{d\ SSR}{d\ Predicted} \times \frac{d\ Predicted}{d\ b_3}$$

Lastly, the derivative of b_3 with respect to b_3 is **1**.

$$= \sum_{i=1}^{n=3} -2 \times (\text{Observed}_i - \text{Predicted}_i)$$

$$\frac{d\ Predicted}{d\ b_3} = \frac{d}{d\ b_3}\ \text{green squiggle} = \frac{d}{d\ b_3}(\text{blue} + \text{orange}\ \boxed{+\ b_3}) = 0 + 0\ \boxed{+\ 1}$$

$$\text{SSR} = \sum_{i=1}^{n=3} (\text{Observed}_i - \text{Predicted}_i)^2 \quad \text{Predicted}_i = \text{green squiggle}_i = \text{blue} + \text{orange} + b_3$$

$$\frac{d\ SSR}{d\ b_3} = \frac{d\ SSR}{d\ Predicted} \times \frac{d\ Predicted}{d\ b_3}$$

Now we just add everything up…

$$= \sum_{i=1}^{n=3} -2 \times (\text{Observed}_i - \text{Predicted}_i)$$

$$\frac{d\ Predicted}{d\ b_3} = \frac{d}{d\ b_3}\ \text{green squiggle} = \frac{d}{d\ b_3}(\text{blue} + \text{orange} + b_3) \boxed{= 0 + 0 + 1}$$

$$\text{SSR} = \sum_{i=1}^{n=3} (\text{Observed}_i - \text{Predicted}_i)^2 \quad \text{Predicted}_i = \text{green squiggle}_i = \text{blue} + \text{orange} + b_3$$

$$\frac{d\ SSR}{d\ b_3} = \frac{d\ SSR}{d\ Predicted} \times \frac{d\ Predicted}{d\ b_3}$$

...and the derivative of the **Predicted** values with respect to ***b*₃** is **1**.

$$= \sum_{i=1}^{n=3} -2 \times (\text{Observed}_i - \text{Predicted}_i)$$

$$\frac{d\ Predicted}{d\ b_3} = \frac{d}{d\ b_3}\ \text{green squiggle} = \frac{d}{d\ b_3}(\text{blue} + \text{orange} + b_3) \boxed{= 1}$$

$$\text{SSR} = \sum_{i=1}^{n=3} (\text{Observed}_i - \text{Predicted}_i)^2 \quad \text{Predicted}_i = \text{green squiggle}_i = \text{blue} + \text{orange} + b_3$$

$$\frac{d\ SSR}{d\ b_3} = \frac{d\ SSR}{d\ Predicted} \times \frac{d\ Predicted}{d\ b_3}$$

So we multiply the derivative of the **SSR** with respect to the **Predicted** values by **1**.

$$= \sum_{i=1}^{n=3} -2 \times (\text{Observed}_i - \text{Predicted}_i) \boxed{\times\ 1}$$

$$\frac{d\ Predicted}{d\ b_3} = \frac{d}{d\ b_3} \text{ green squiggle} = \frac{d}{d\ b_3} (\text{blue} + \text{orange} + b_3) = 1$$

$$\text{SSR} = \sum_{i=1}^{n=3} (\text{Observed}_i - \text{Predicted}_i)^2 \quad \text{Predicted}_i = \text{green squiggle}_i = \text{blue} + \text{orange} + b_3$$

$$\frac{d\ SSR}{d\ b_3} = \frac{d\ SSR}{d\ Predicted} \times \frac{d\ Predicted}{d\ b_3}$$

NOTE: This "**times 1**" part in the equation doesn't do anything…

$$= \sum_{i=1}^{n=3} -2 \times (\text{Observed}_i - \text{Predicted}_i) \boxed{\times 1}$$

$$\text{SSR} = \sum_{i=1}^{n=3} (\text{Observed}_i - \text{Predicted}_i)^2 \quad \text{Predicted}_i = \text{green squiggle}_i = \text{blue} + \text{orange} + b_3$$

$$\frac{d\ SSR}{d\ b_3} = \frac{d\ SSR}{d\ Predicted} \times \frac{d\ Predicted}{d\ b_3}$$

$$= \sum_{i=1}^{n=3} -2 \times (\text{Observed}_i - \text{Predicted}_i) \times \mathbf{1}$$

…but I'm leaving it in to remind us that the derivative of the **SSR** with respect to b_3 consists of two parts.

$$\text{SSR} = \sum_{i=1}^{n=3} (\text{Observed}_i - \text{Predicted}_i)^2 \quad \text{Predicted}_i = \text{green squiggle}_i = \text{blue} + \text{orange} + b_3$$

$$\frac{d\ SSR}{d\ b_3} = \frac{d\ SSR}{d\ Predicted} \times \frac{d\ Predicted}{d\ b_3}$$

$$= \sum_{i=1}^{n=3} -2 \times (\text{Observed}_i - \text{Predicted}_i) \times 1$$

The derivative of the **SSR** with respect to the **Predicted** values…

$$\text{SSR} = \sum_{i=1}^{n=3} (\text{Observed}_i - \text{Predicted}_i)^2 \quad \text{Predicted}_i = \text{green squiggle}_i = \text{blue} + \text{orange} + b_3$$

$$\frac{d\ SSR}{d\ b_3} = \frac{d\ SSR}{d\ Predicted} \times \boxed{\frac{d\ Predicted}{d\ b_3}}$$

...and the derivative of the **Predicted** values with respect to $\boldsymbol{b_3}$.

$$= \sum_{i=1}^{n=3} -2 \times (\text{Observed}_i - \text{Predicted}_i) \boxed{\times\ 1}$$

$$\text{SSR} = \sum_{i=1}^{n=3} (\text{Observed}_i - \text{Predicted}_i)^2 \quad \text{Predicted}_i = \text{green squiggle}_i = \text{blue} + \text{orange} + b_3$$

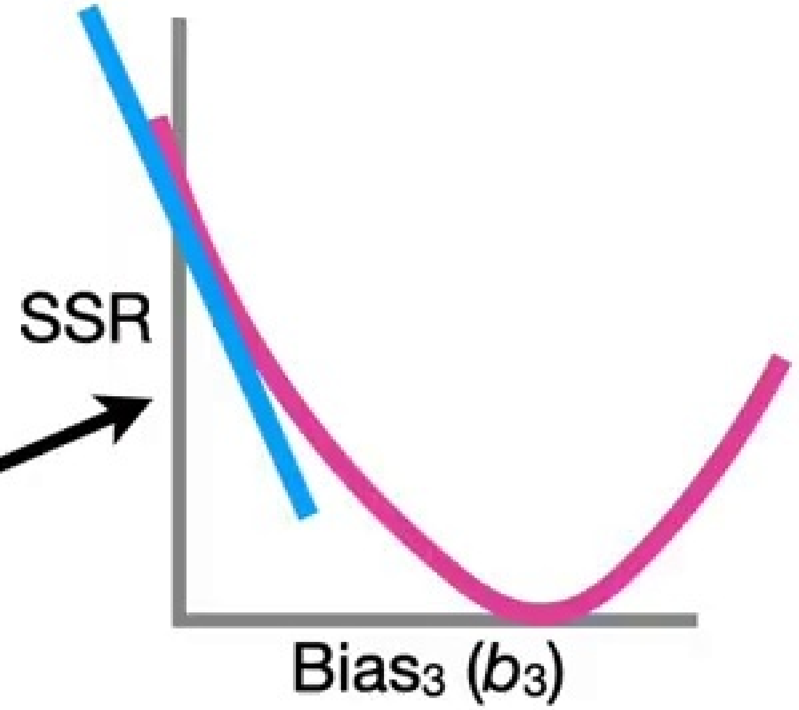
$$\frac{d\ SSR}{d\ b_3} = \frac{d\ SSR}{d\ Predicted} \times \frac{d\ Predicted}{d\ b_3}$$

$$= \sum_{i=1}^{n=3} -2 \times (\text{Observed}_i - \text{Predicted}_i) \times \mathbf{1}$$

And at long last, we have the derivative of the **SSR** with respect to b_3.

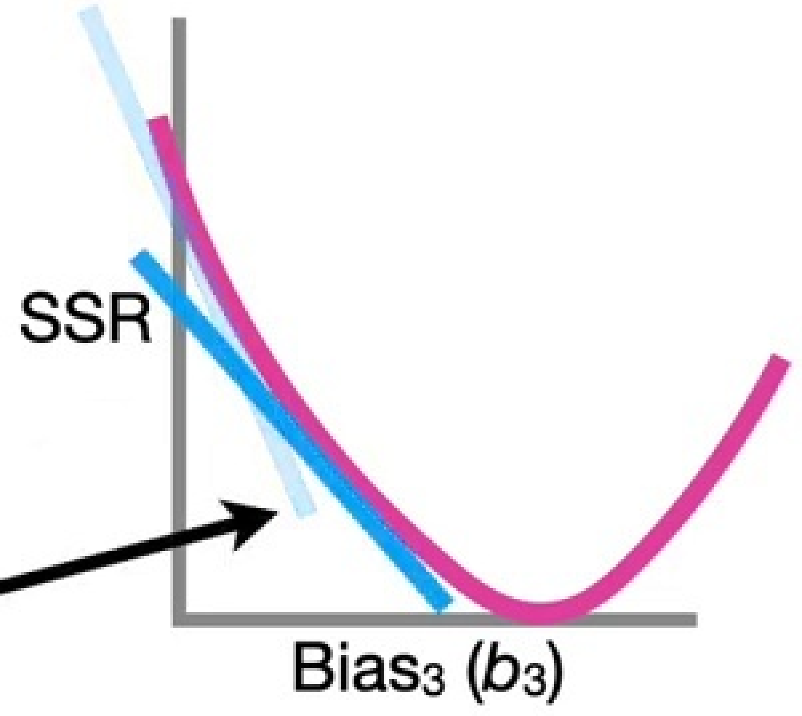
$$\frac{d\ SSR}{d\ b_3} = \frac{d\ SSR}{d\ Predicted} \times \frac{d\ Predicted}{d\ b_3}$$

$$= \sum_{i=1}^{n=3} -2 \times (\text{Observed}_i - \text{Predicted}_i) \times \mathbf{1}$$



And that means we can plug this derivative into **Gradient Descent** to find the optimal value for b_3.

$$\frac{d\ SSR}{d\ b_3} = \frac{d\ SSR}{d\ Predicted} \times \frac{d\ Predicted}{d\ b_3}$$

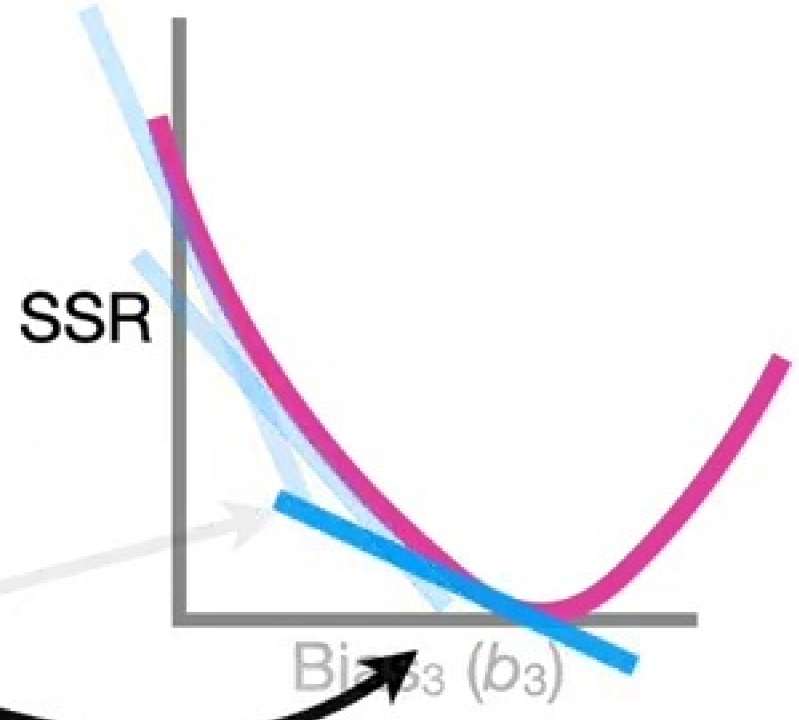


$$= \sum_{i=1}^{n=3} -2 \times (\text{Observed}_i - \text{Predicted}_i) \times \mathbf{1}$$

And that means we can plug this derivative into **Gradient Descent** to find the optimal value for b_3.

$$\frac{d\ SSR}{d\ b_3} = \frac{d\ SSR}{d\ Predicted} \times \frac{d\ Predicted}{d\ b_3}$$

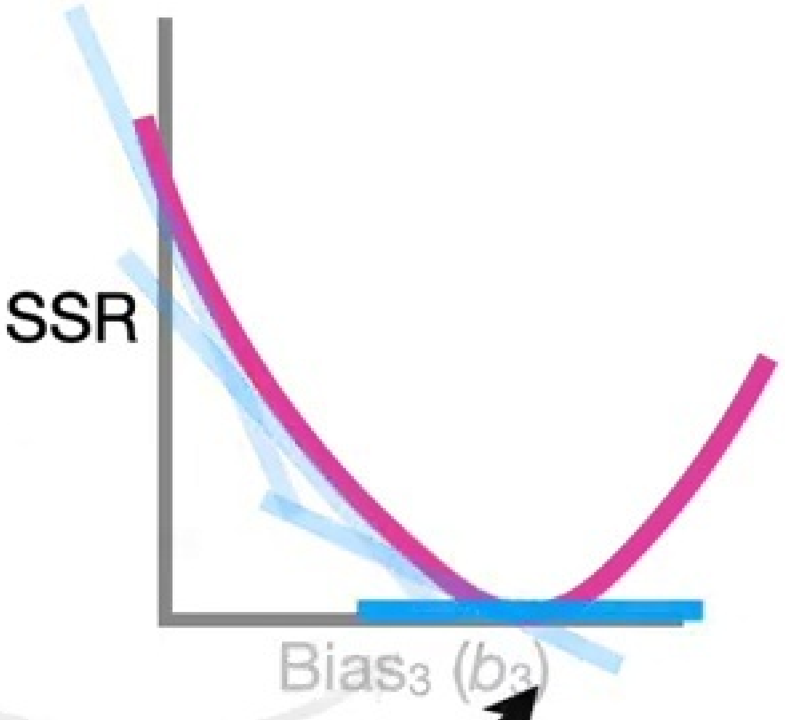
$$\boxed{= \sum_{i=1}^{n=3} -2 \times (\text{Observed}_i - \text{Predicted}_i) \times \mathbf{1}}$$



And that means we can plug this derivative into **Gradient Descent** to find the optimal value for $\boldsymbol{b_3}$.

$$\frac{d\ SSR}{d\ b_3} = \frac{d\ SSR}{d\ Predicted} \times \frac{d\ Predicted}{d\ b_3}$$

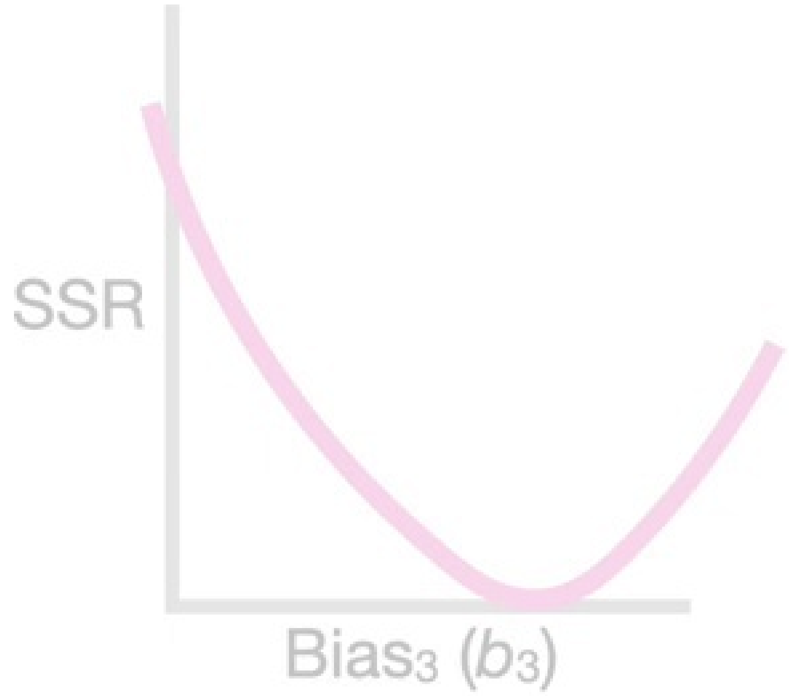
$$= \sum_{i=1}^{n=3} -2 \times (\text{Observed}_i - \text{Predicted}_i) \times \mathbf{1}$$



And that means we can plug this derivative into **Gradient Descent** to find the optimal value for b_3.

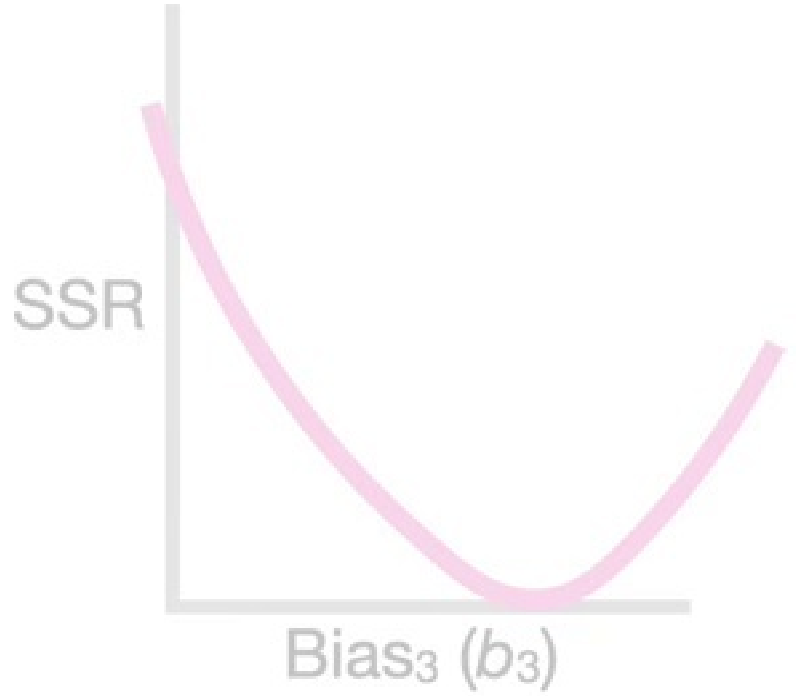
$$\frac{d\,SSR}{d\,b_3} = \sum_{i=1}^{n=3} -2 \times (\text{Observed}_i - \text{Predicted}_i) \times \mathbf{1}$$

Anyway, first, we expand the summation.

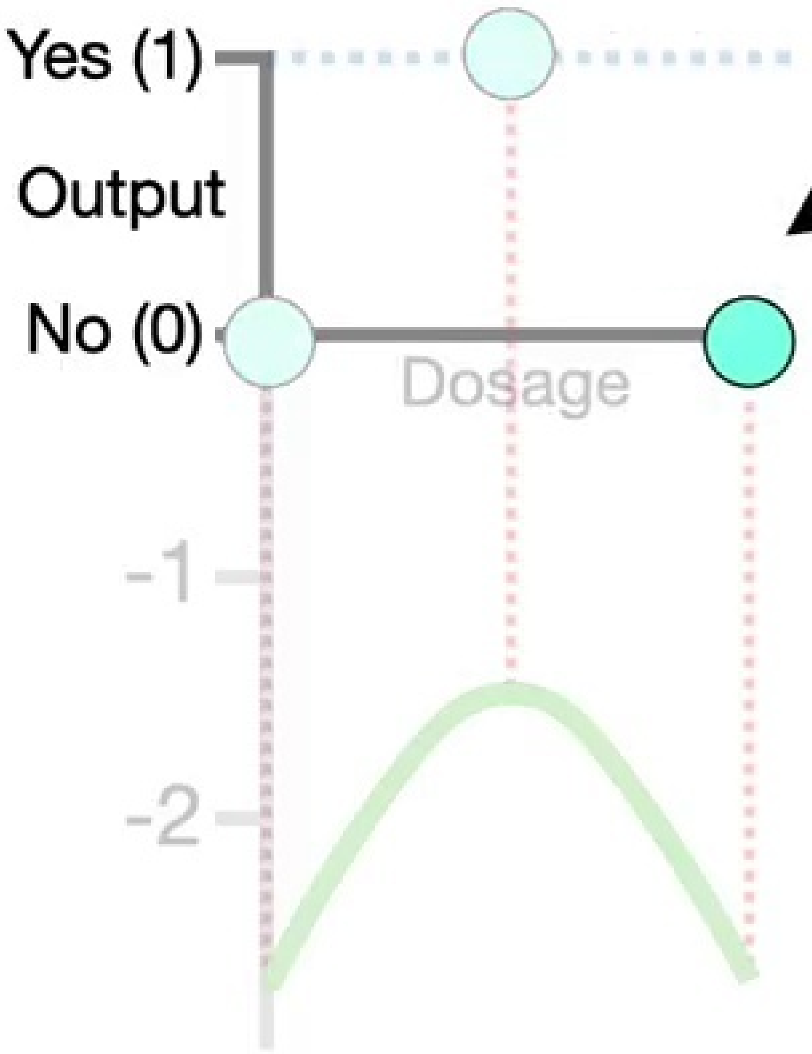


$$\frac{d\ SSR}{d\ b_3} = -2 \times (\text{Observed}_1 - \text{Predicted}_1) \times 1 + -2 \times (\text{Observed}_2 - \text{Predicted}_2) \times 1 + -2 \times (\text{Observed}_3 - \text{Predicted}_3) \times 1$$

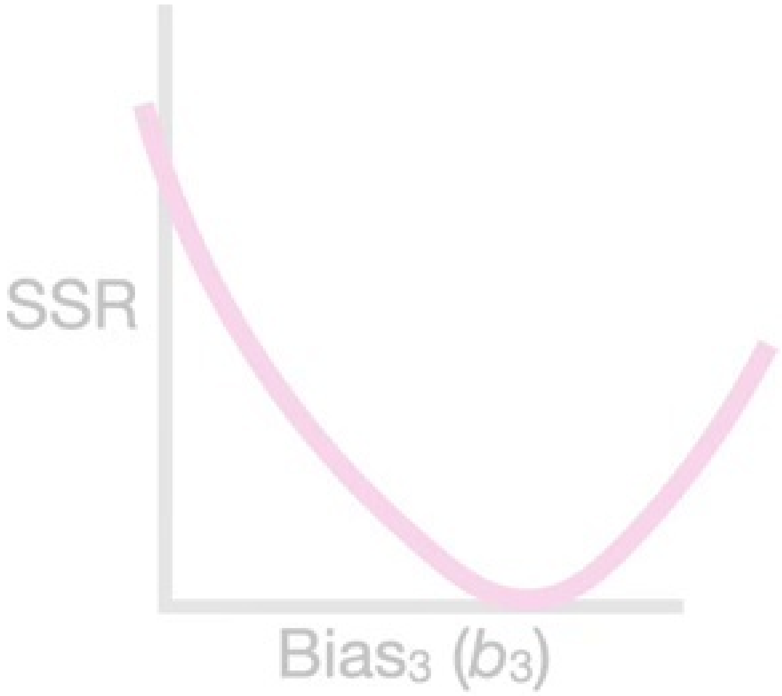
Anyway, first, we expand the summation.



$$\frac{d\ SSR}{d\ b_3} = -2 \times (0 - \text{Predicted}_1) \times 1$$
$$+ -2 \times (1 - \text{Predicted}_2) \times 1$$
$$+ -2 \times (0 - \text{Predicted}_3) \times 1$$

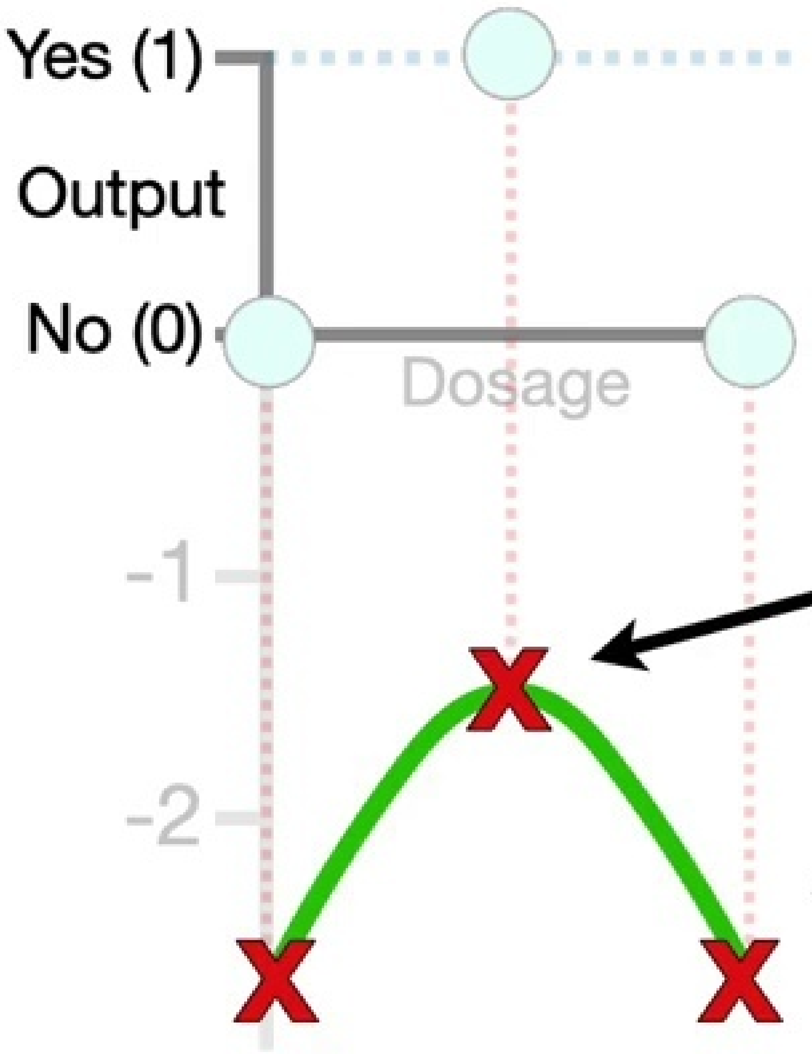


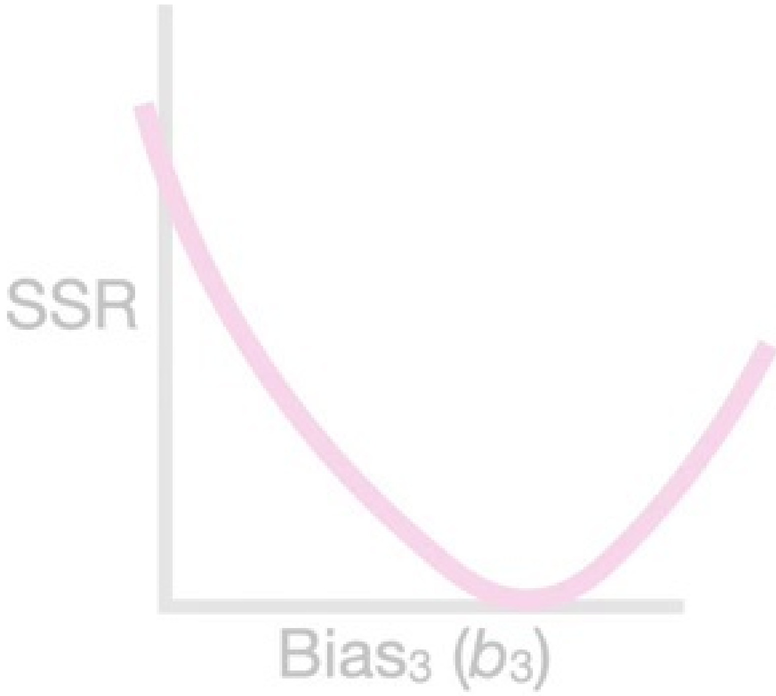
Then we plug in the **Observed** values….



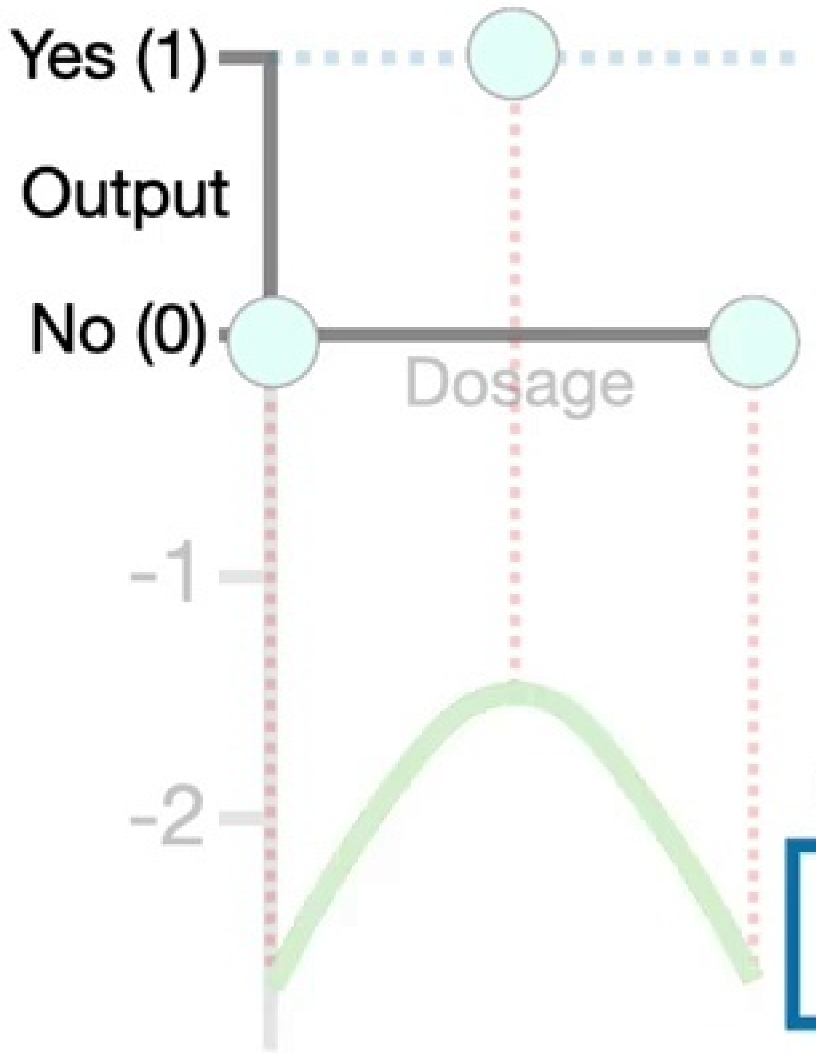
$$\frac{d\ SSR}{d\ b_3} = -2 \times (0 - \text{Predicted}_1) \times 1 + -2 \times (1 - \text{Predicted}_2) \times 1 + -2 \times (0 - \text{Predicted}_3) \times 1$$

...and the values **Predicted** by the **green squiggle**.

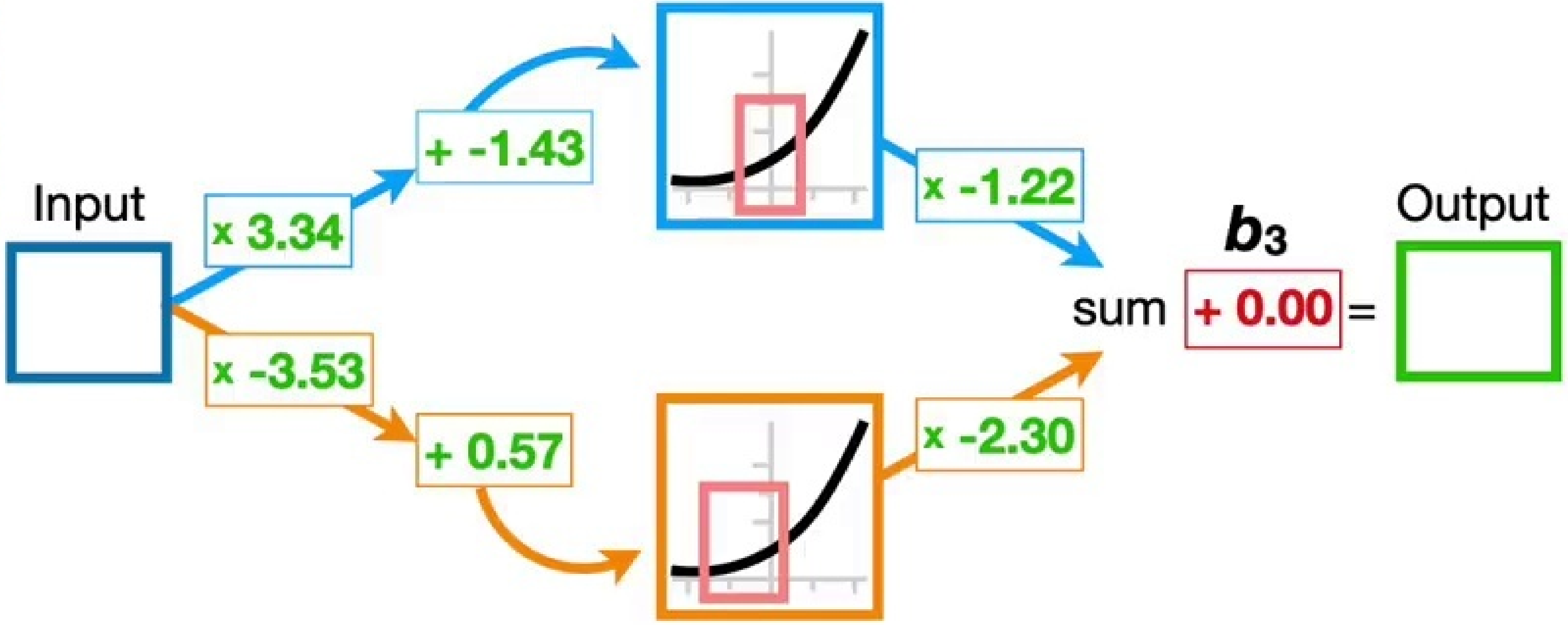




$$\frac{d\,SSR}{d\,b_3} = -2 \times (0 - \text{Predicted}_1) \times 1 + -2 \times (1 - \text{Predicted}_2) \times 1 + -2 \times (0 - \text{Predicted}_3) \times 1$$

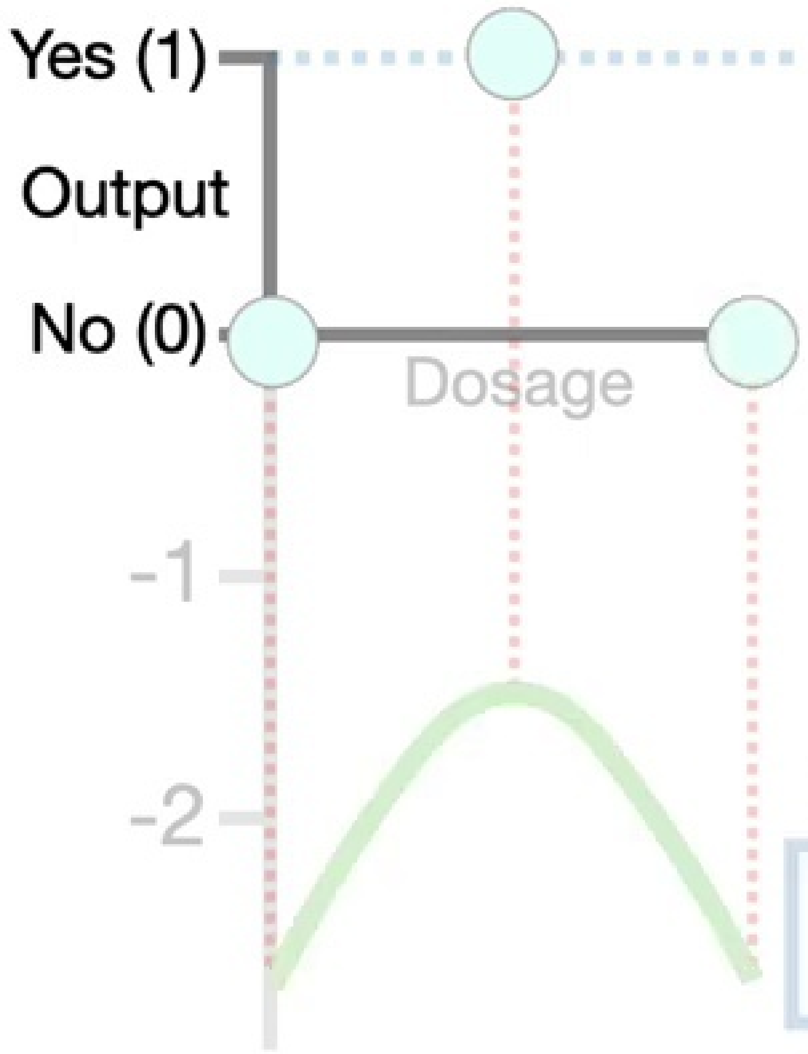


Remember, we get the **Predicted** values on the **green squiggle** by running the **Dosages** through the **Neural Network**.



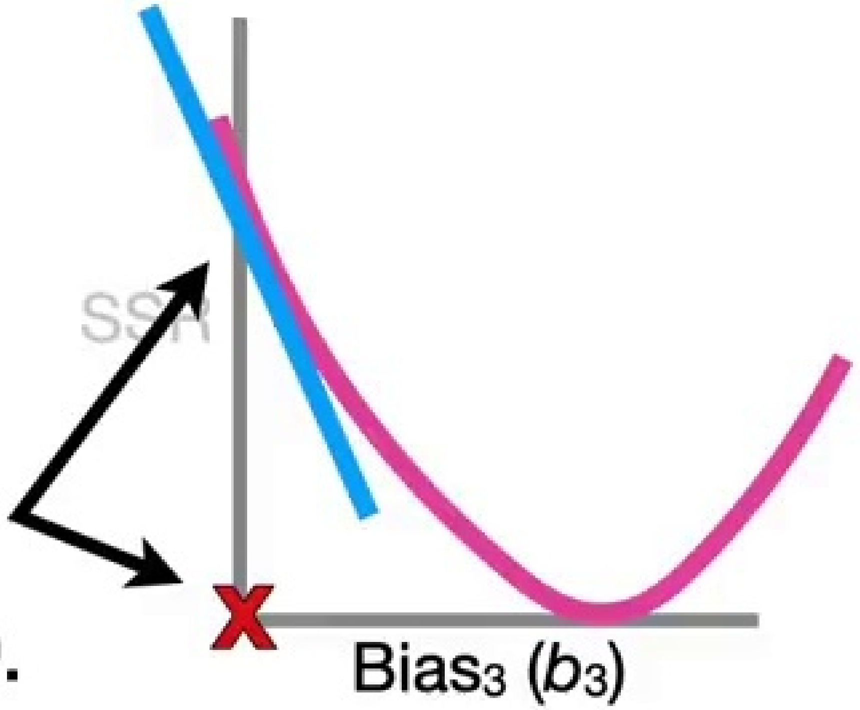
$$\frac{d\,SSR}{d\,b_3} = -2 \times (0 - -2.6) \times \mathbf{1} + -2 \times (1 - -1.6) \times \mathbf{1} + -2 \times (0 - -2.61) \times \mathbf{1} \boxed{= -15.7}$$

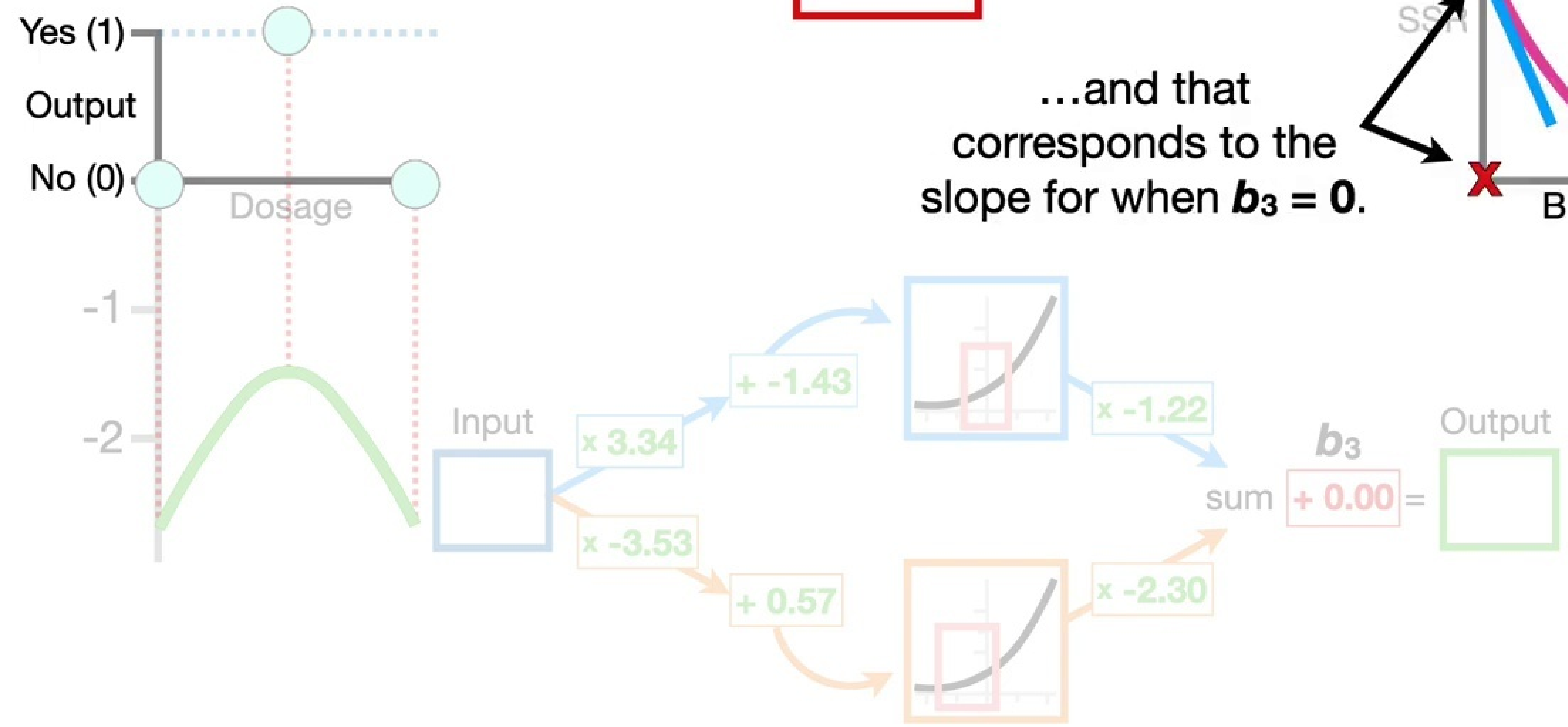
Now we just do the math and get **-15.7**…



$$\frac{d\,SSR}{d\,b_3} = -2 \times (0 - -2.6) \times \mathbf{1} + -2 \times (1 - -1.6) \times \mathbf{1} + -2 \times (0 - -2.61) \times \mathbf{1} \boxed{= -15.7}$$

...and that corresponds to the slope for when $\boldsymbol{b_3} = \mathbf{0}$.

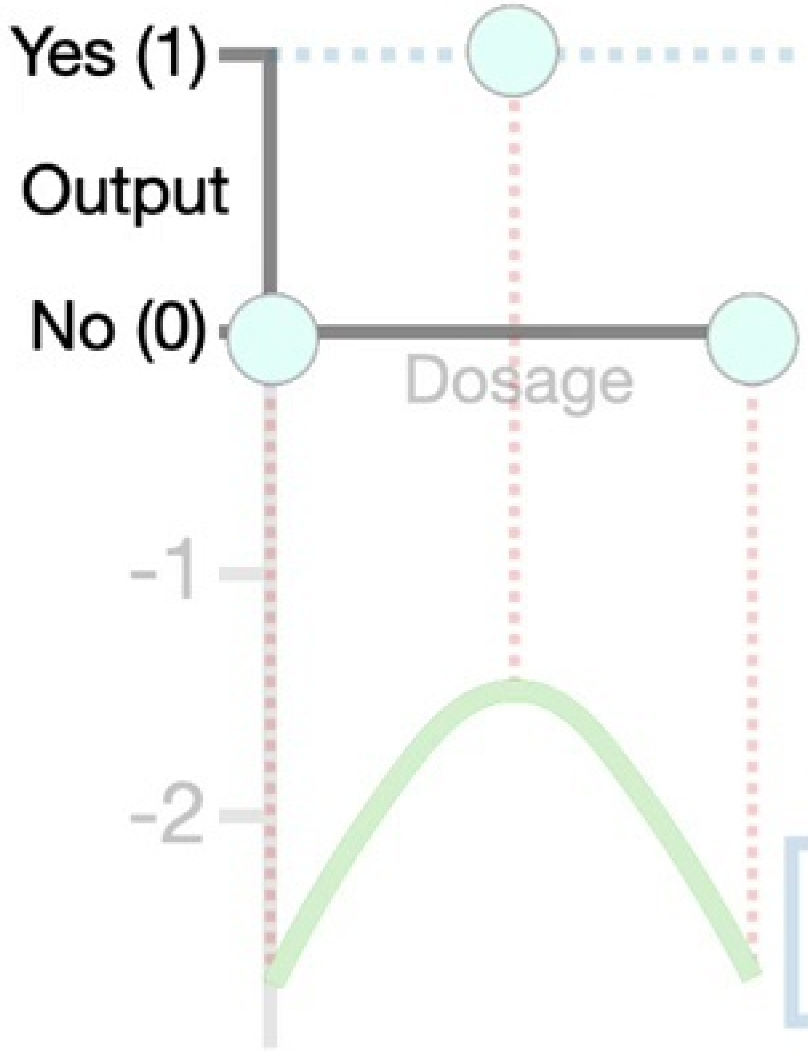


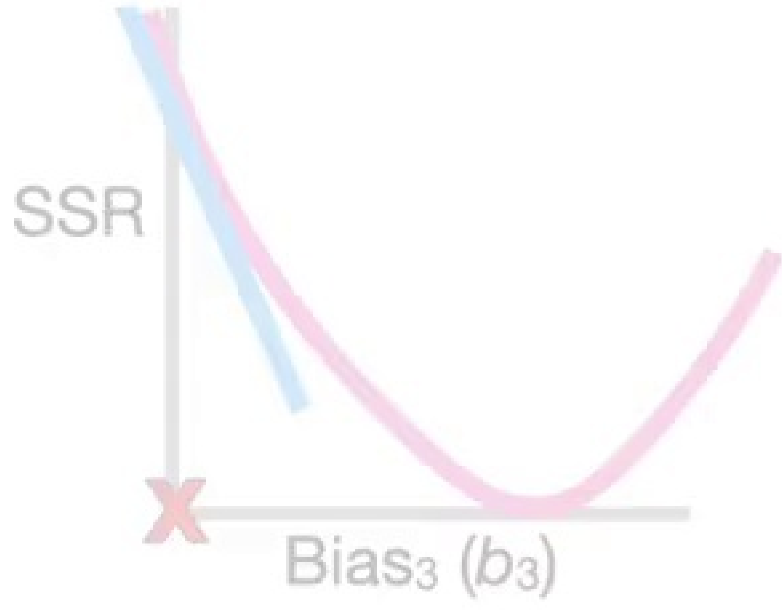


Now we plug the slope into the **Gradient Descent** equation for **Step Size**…

$$\frac{d\,SSR}{d\,b_3} = -2 \times (0 - -2.6) \times 1 + -2 \times (1 - -1.6) \times 1 + -2 \times (0 - -2.61) \times 1 = \boxed{-15.7}$$

Step Size = $\boxed{\textbf{Slope}}$ × Learning Rate

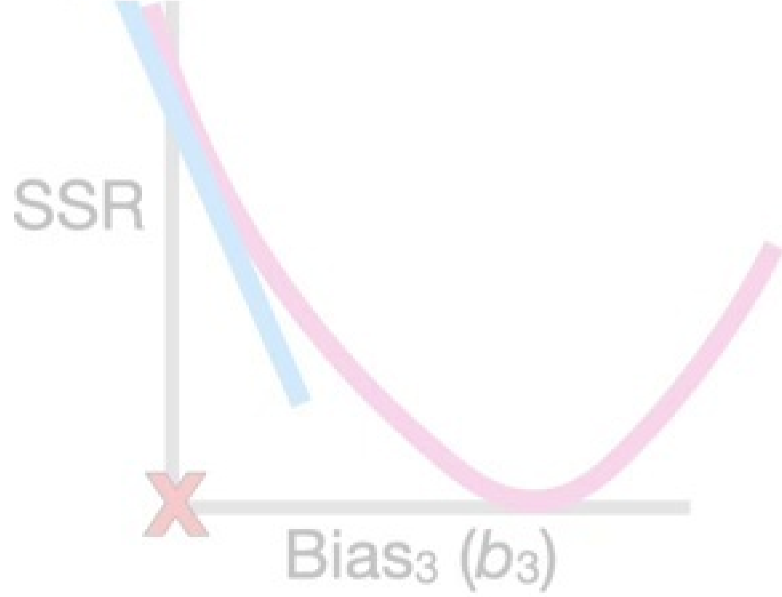


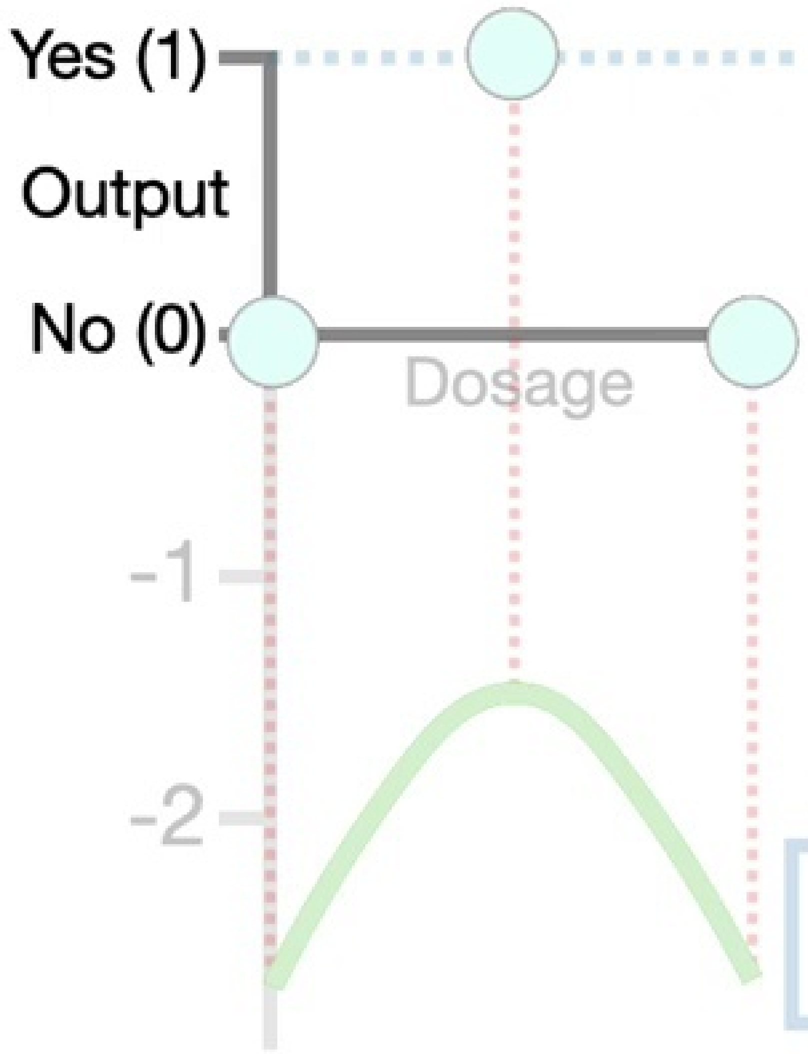


Now we plug the slope into the **Gradient Descent** equation for **Step Size**…

$$\frac{d\,SSR}{d\,b_3} = -2 \times (0 - -2.6) \times 1 + -2 \times (1 - -1.6) \times 1 + -2 \times (0 - -2.61) \times 1 = \boxed{-15.7}$$

$$\textbf{Step Size} = \boxed{-15.7} \times \textbf{Learning Rate}$$

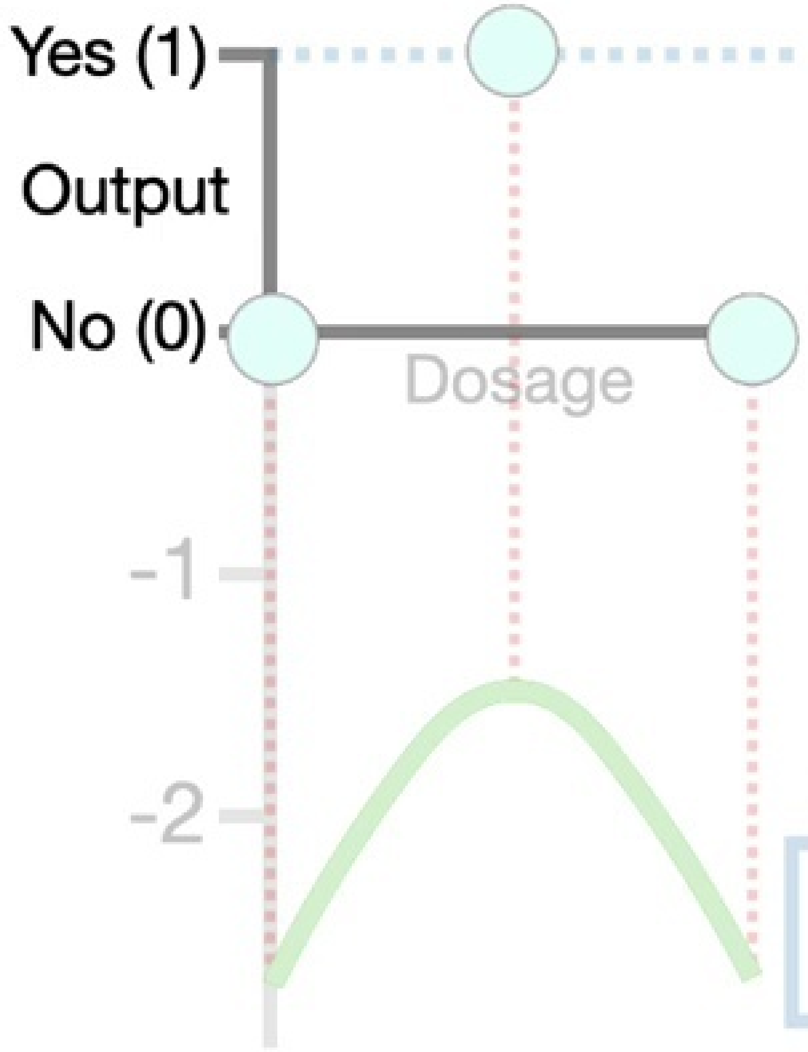


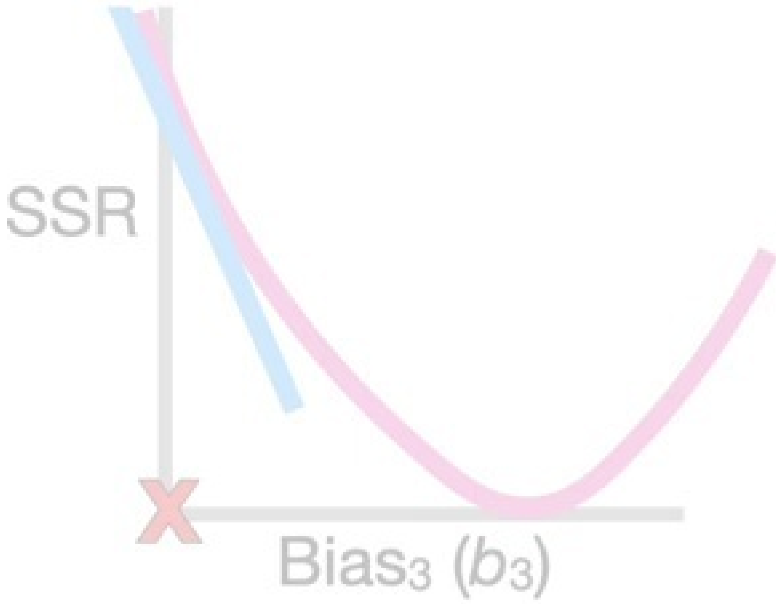


$$\frac{d\,SSR}{d\,b_3} = -2 \times (0 - -2.6) \times 1 + -2 \times (1 - -1.6) \times 1 + -2 \times (0 - -2.61) \times 1 = -15.7$$

…and in this example, we'll set the **Learning Rate** to **0.1**…

Step Size = -15.7 $\boxed{\times\ 0.1}$

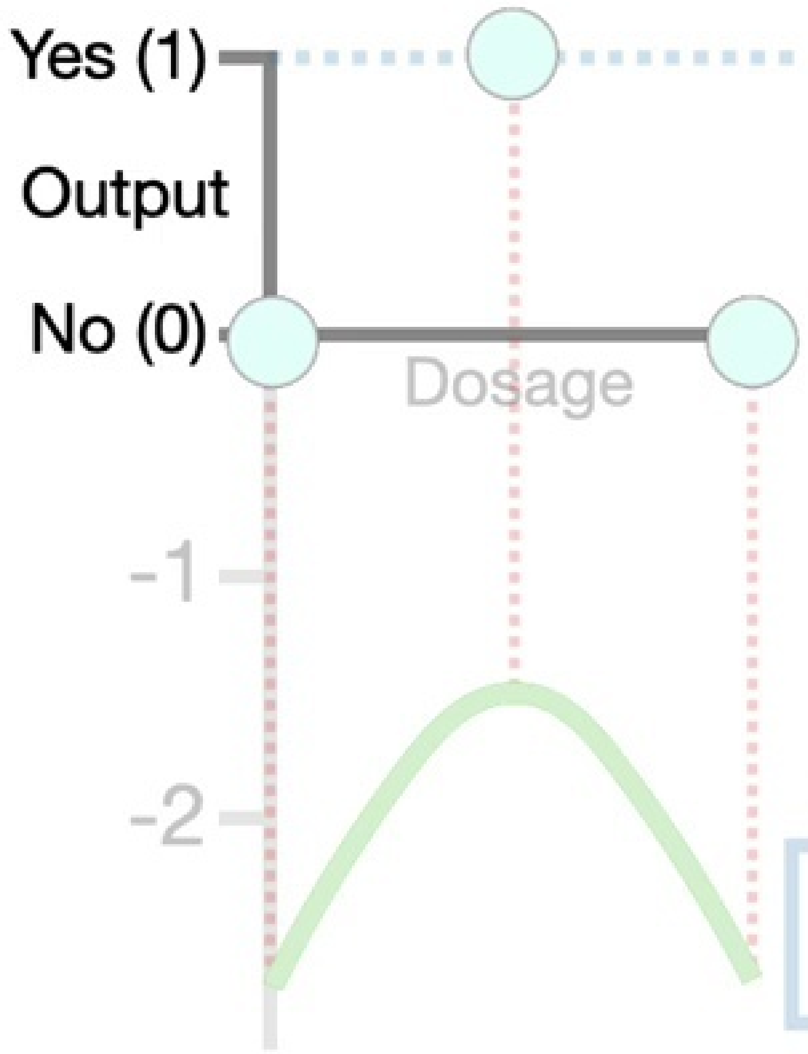


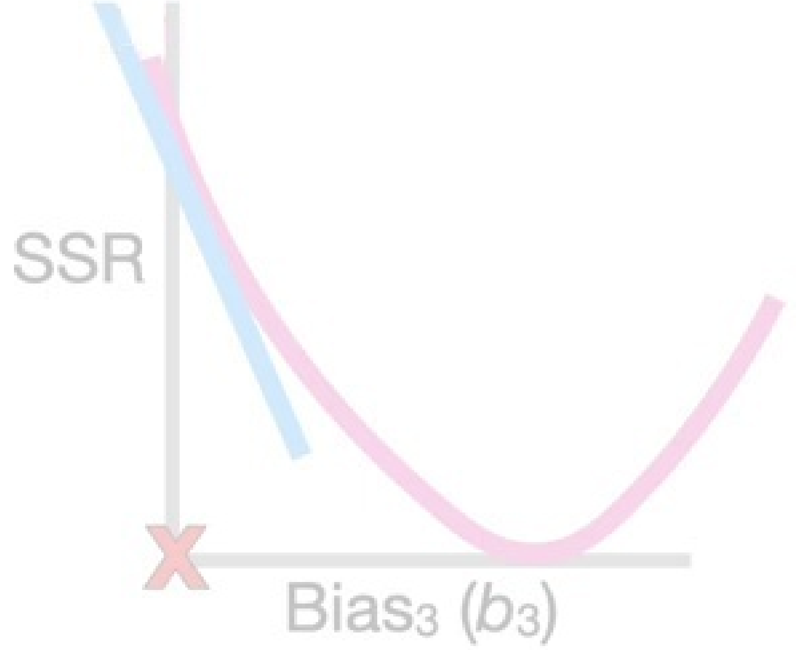


$$\frac{d\ SSR}{d\ b_3} = -2 \times (0 - -2.6) \times 1 + -2 \times (1 - -1.6) \times 1 + -2 \times (0 - -2.61) \times 1 = -15.7$$

…and that means the **Step Size** is **-1.57**.

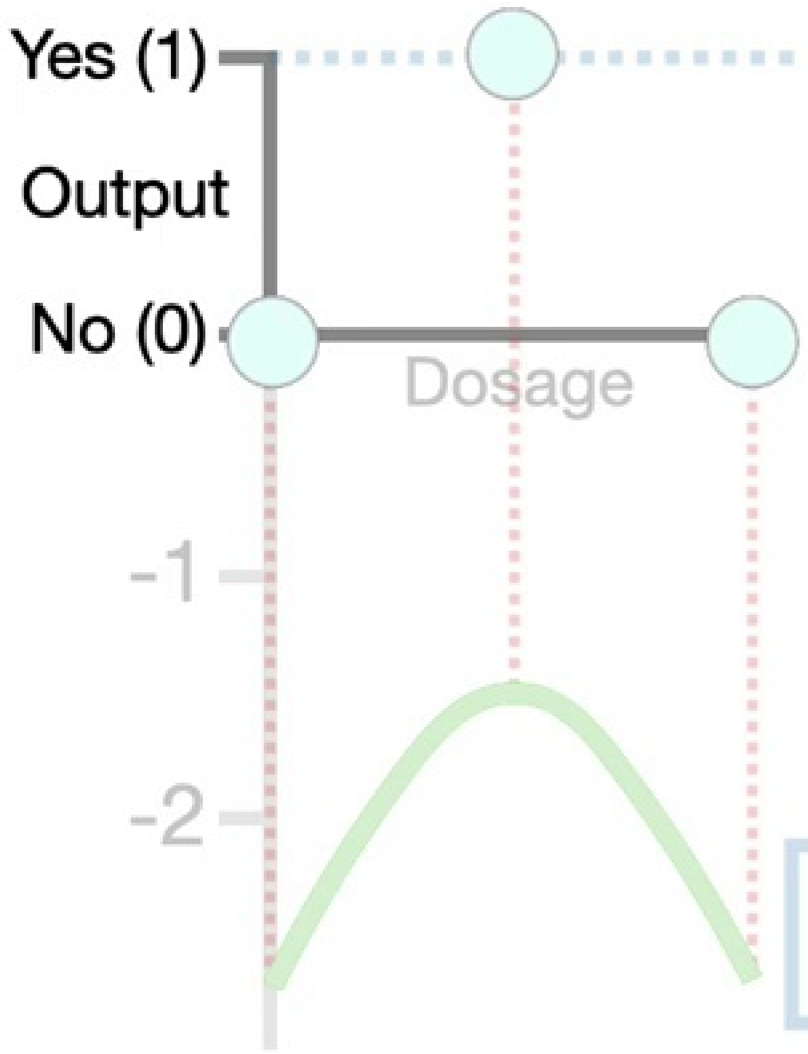
Step Size = $-15.7 \times 0.1 \boxed{= -1.57}$





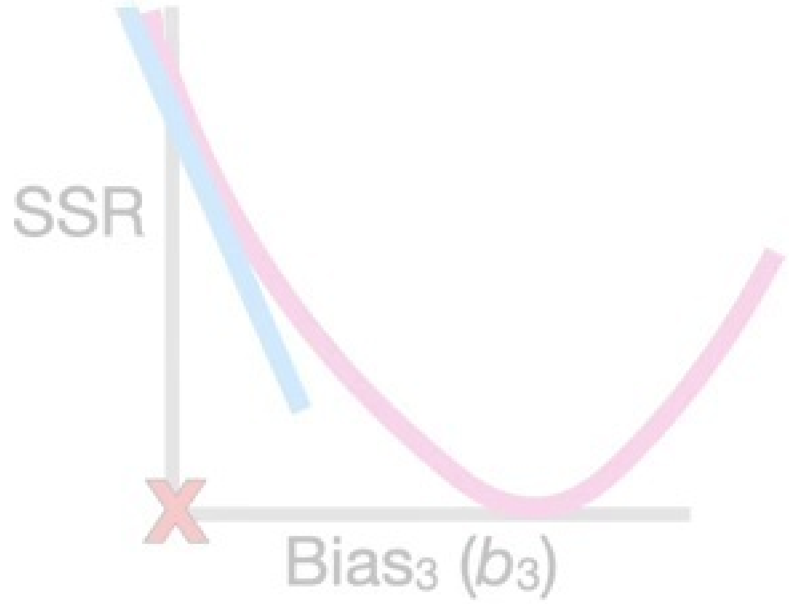
$$\frac{d\,SSR}{d\,b_3} = -2 \times (0 - -2.6) \times 1 + -2 \times (1 - -1.6) \times 1 + -2 \times (0 - -2.61) \times 1 = -15.7$$

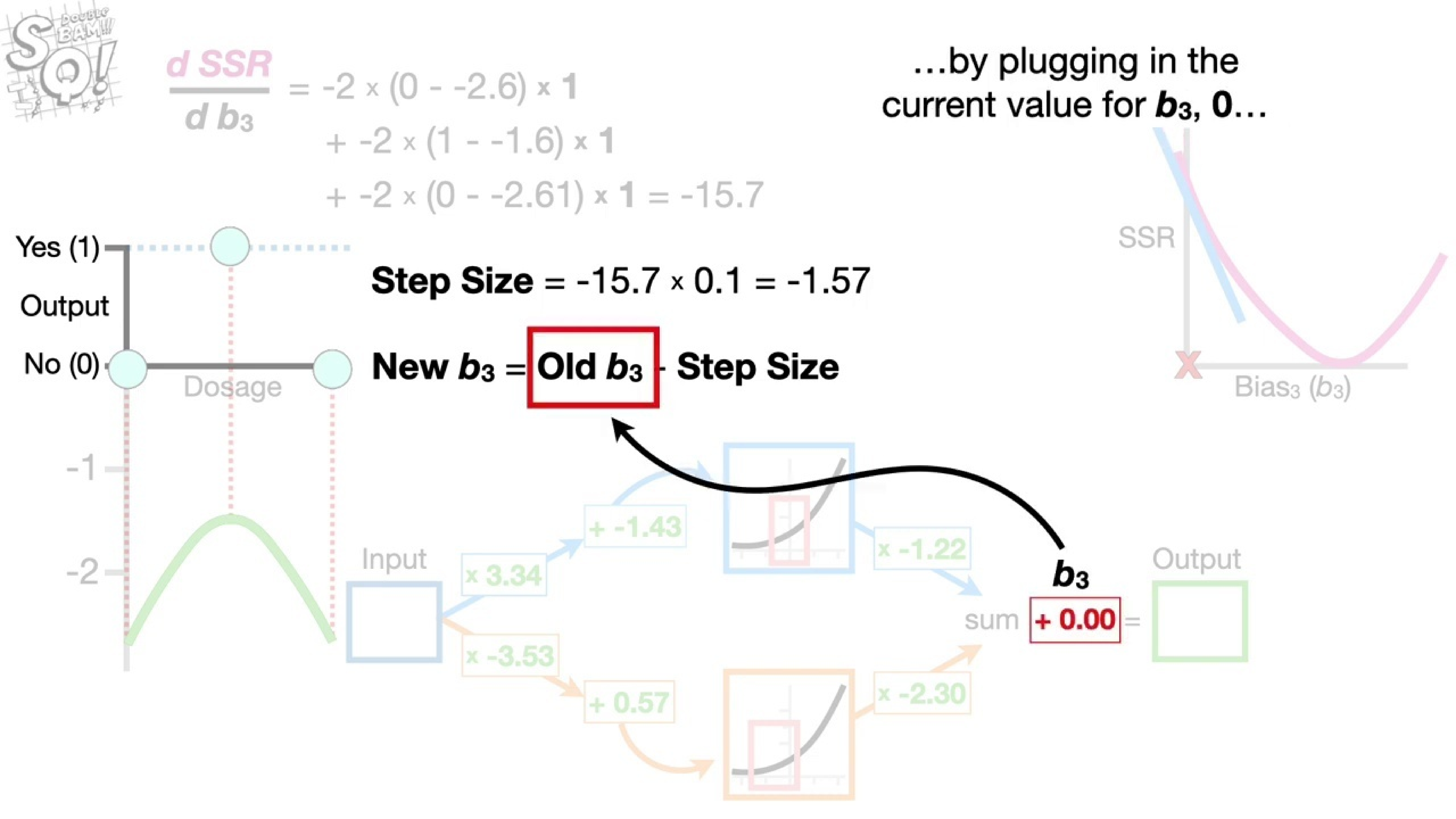
Now we use the **Step Size** to calculate the new value for b_3…



Step Size = $-15.7 \times 0.1 = -1.57$

New b_3 = Old b_3 - Step Size

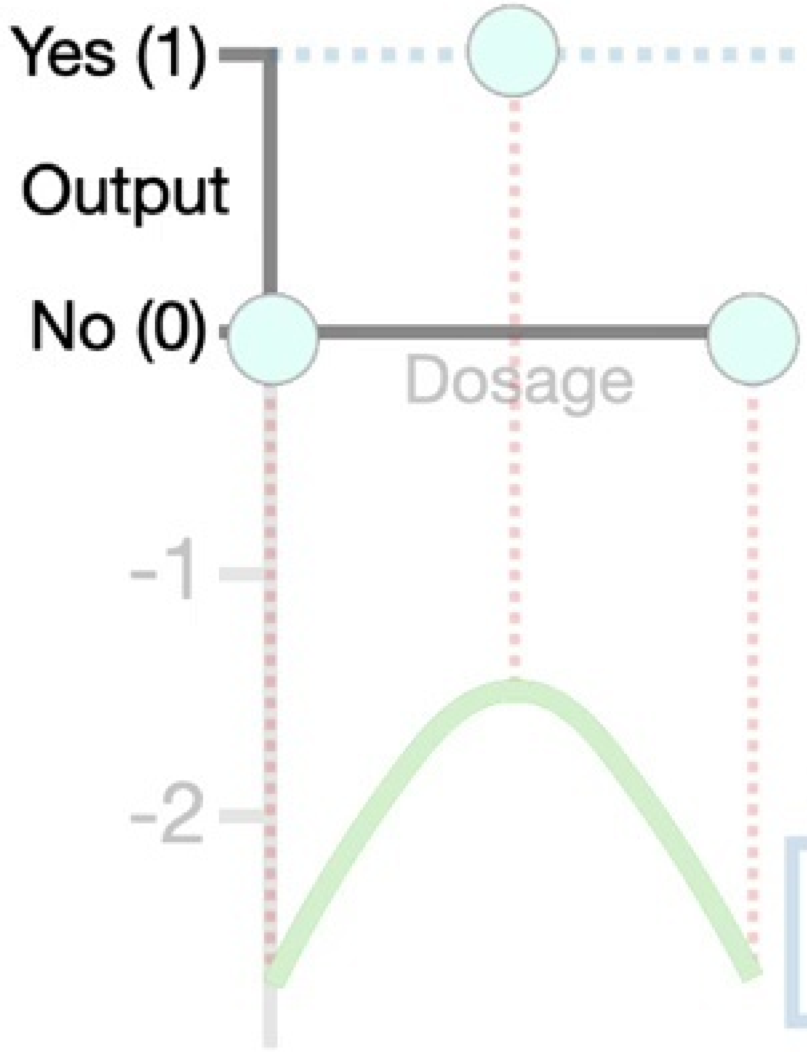




DOUBLE BAM!!!
$\frac{d\ SSR}{d\ b_3} = -2 \times (0 - -2.6) \times 1$
$+ -2 \times (1 - -1.6) \times 1$
$+ -2 \times (0 - -2.61) \times 1 = -15.7$
…by plugging in the current value for b_3, 0…
Step Size = -15.7 x 0.1 = -1.57
New b_3 = Old b_3 - Step Size
Yes (1)
Output
No (0)
Dosage
-1
-2
SSR
Bias$_3$ (b_3)
Input
x 3.34
+ -1.43
x -1.22
x -3.53
+ 0.57
x -2.30
b_3
sum
+ 0.00
=
Output

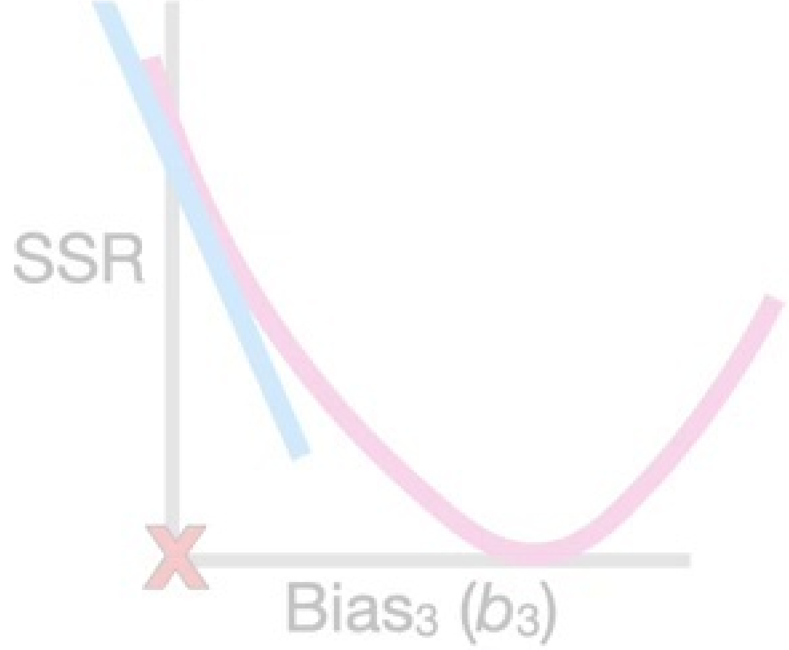
$$\frac{d\ SSR}{d\ b_3} = -2 \times (0 - -2.6) \times 1 + -2 \times (1 - -1.6) \times 1 + -2 \times (0 - -2.61) \times 1 = -15.7$$

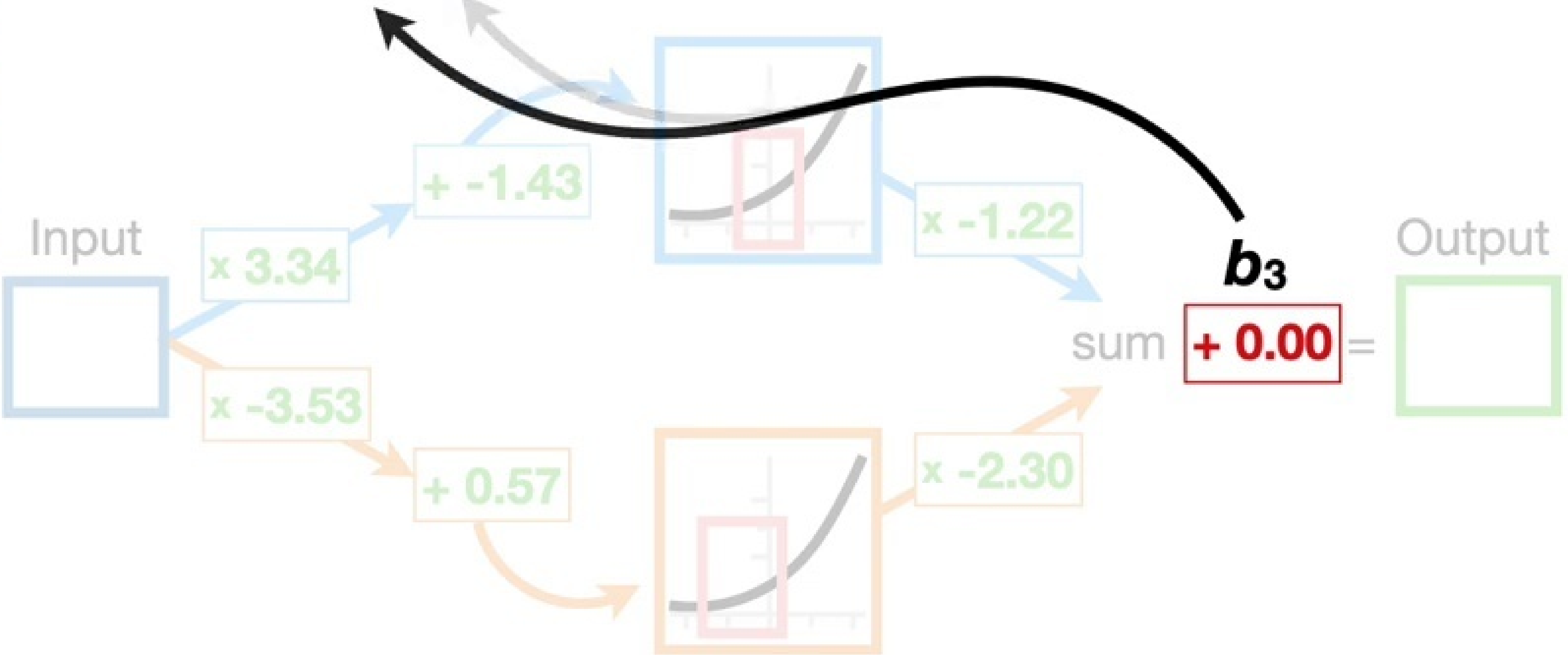
…by plugging in the current value for b_3, **0**…



Step Size = -15.7 × 0.1 = -1.57

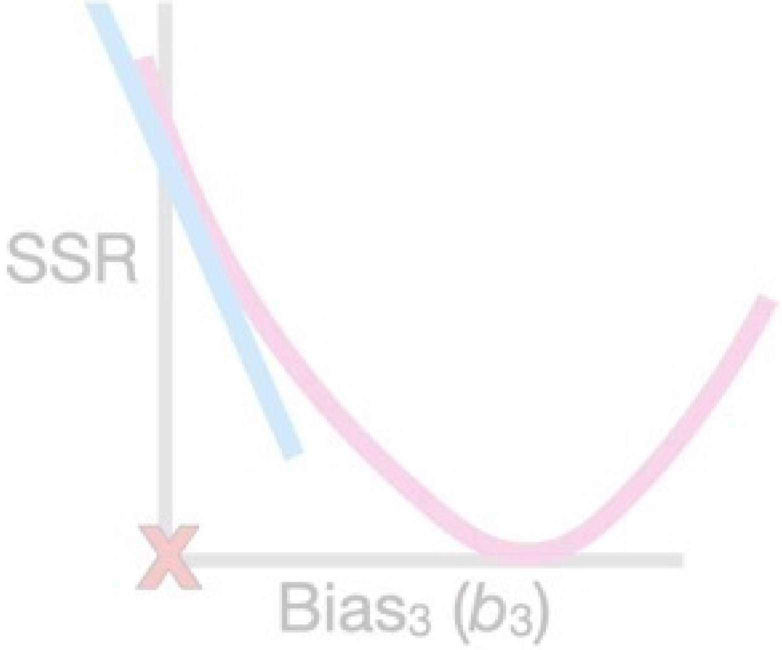
New b_3 = $\boxed{0}$ - **Step Size**

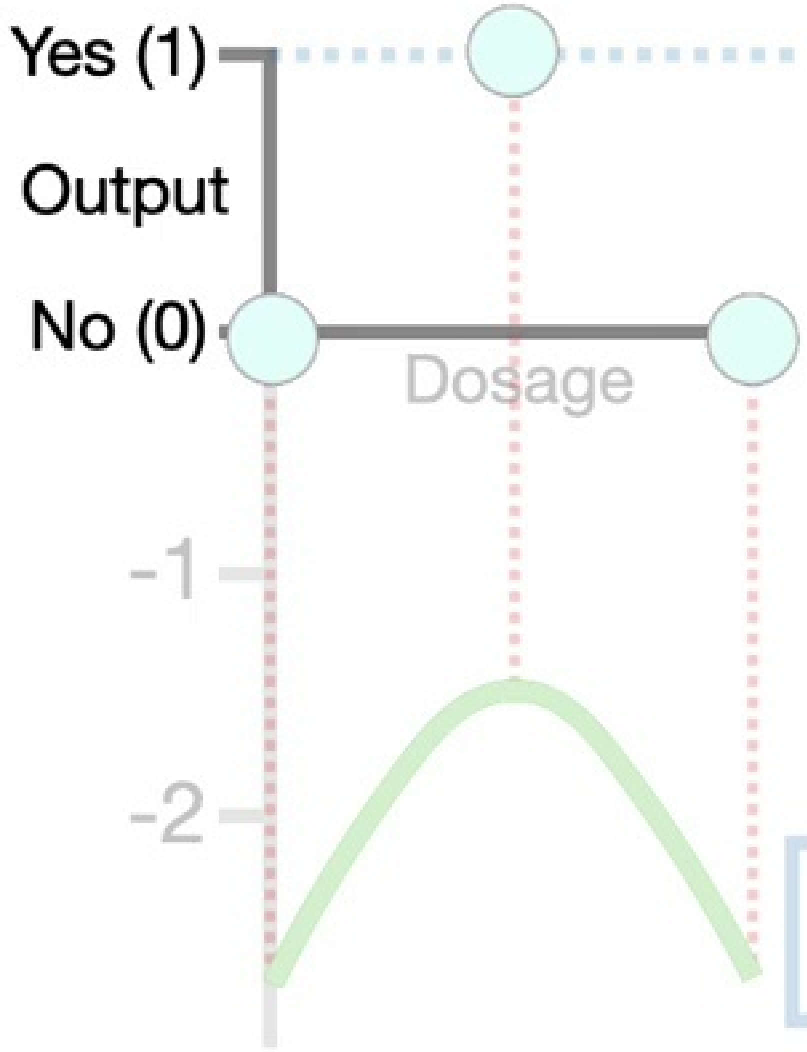




$$\frac{d\ SSR}{d\ b_3} = -2 \times (0 - -2.6) \times 1 + -2 \times (1 - -1.6) \times 1 + -2 \times (0 - -2.61) \times 1 = -15.7$$

…and the **Step Size**, **-1.57**.



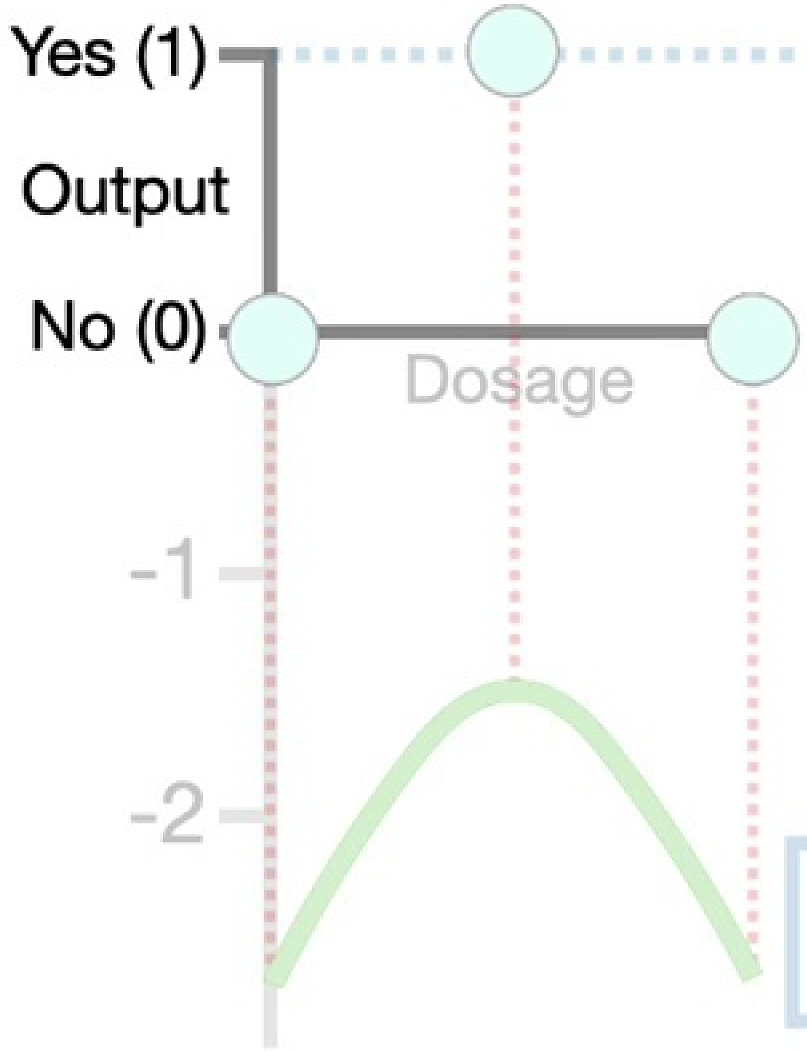


Step Size = $-15.7 \times 0.1 \boxed{= -1.57}$

New b_3 = **0** - $\boxed{\textbf{Step Size}}$

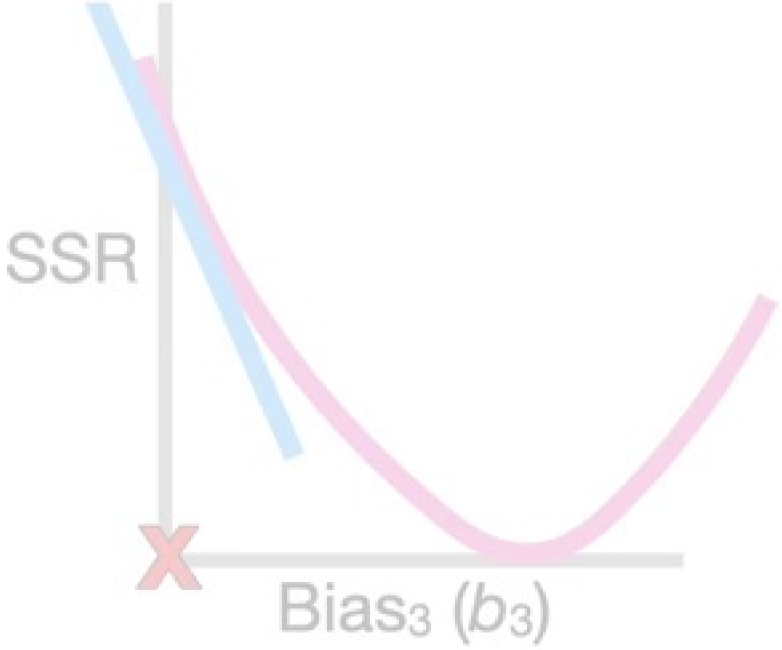
$$\frac{d\,SSR}{d\,b_3} = -2 \times (0 - -2.6) \times 1 + -2 \times (1 - -1.6) \times 1 + -2 \times (0 - -2.61) \times 1 = -15.7$$

...and the **Step Size**, **-1.57**.



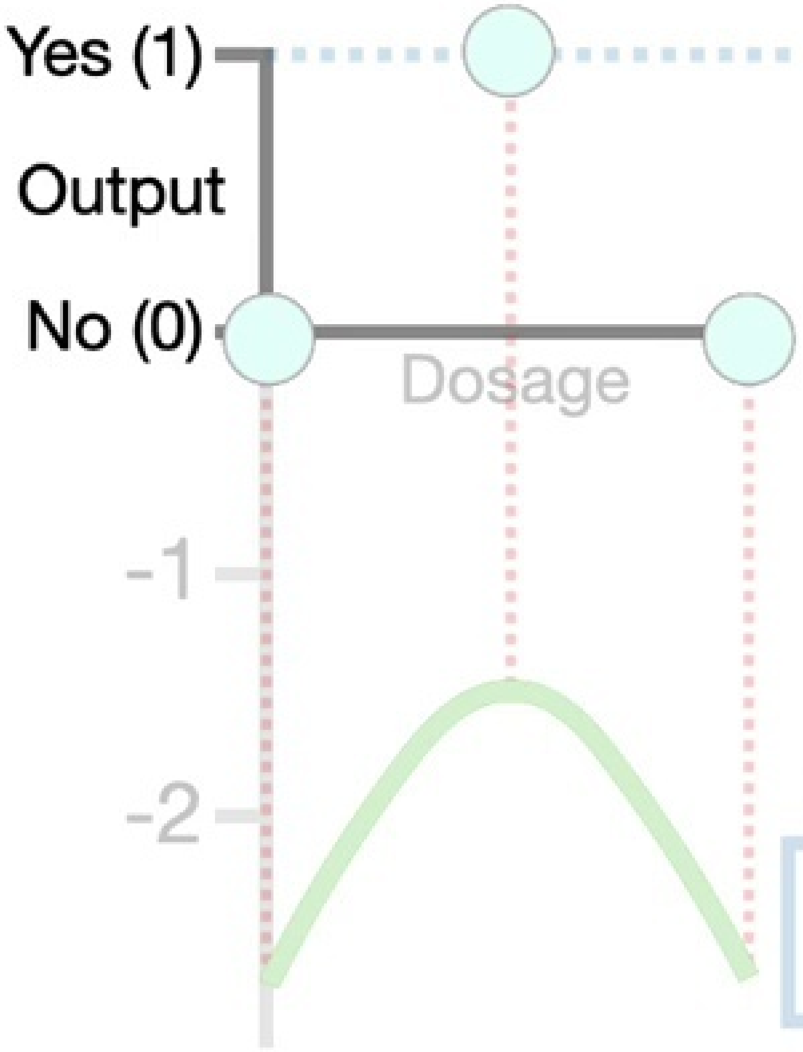
Step Size = $-15.7 \times 0.1 \boxed{= -1.57}$

New $\boldsymbol{b_3}$ = $\color{red}{\mathbf{0}}$ - $\boxed{(-1.57)}$



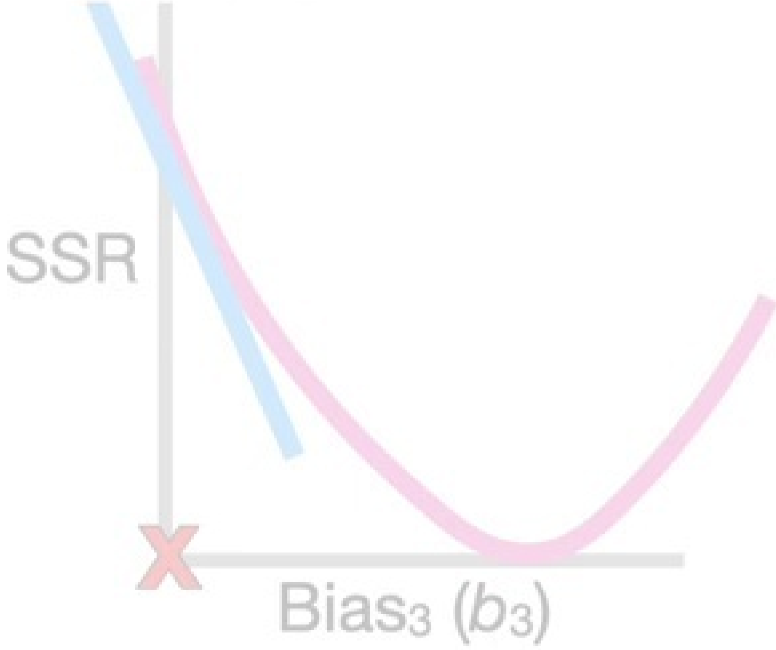
$$\frac{d\ SSR}{d\ b_3} = -2 \times (0 - -2.6) \times \mathbf{1} + -2 \times (1 - -1.6) \times \mathbf{1} + -2 \times (0 - -2.61) \times \mathbf{1} = -15.7$$

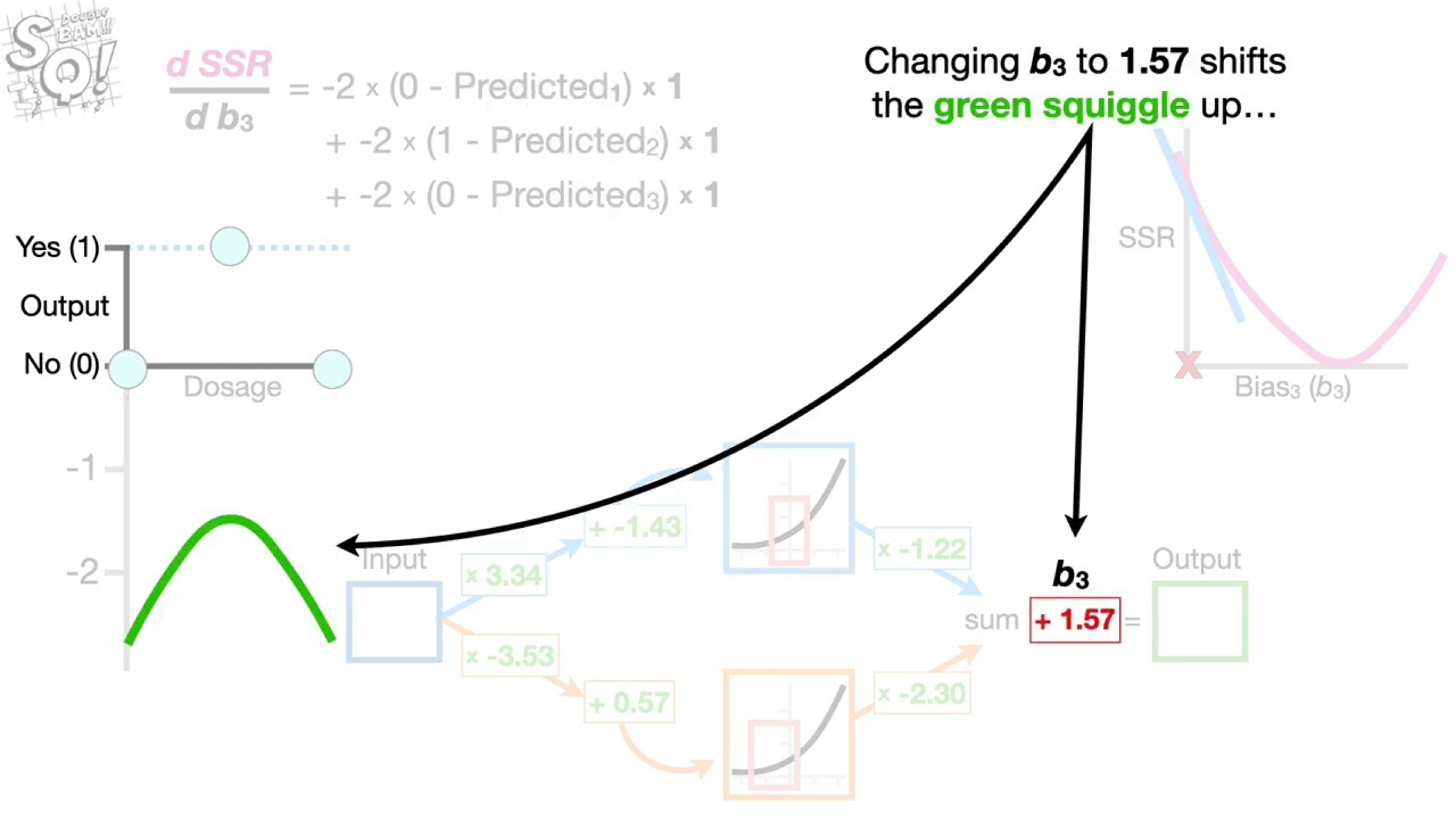
Step Size = -15.7 × 0.1 = -1.57



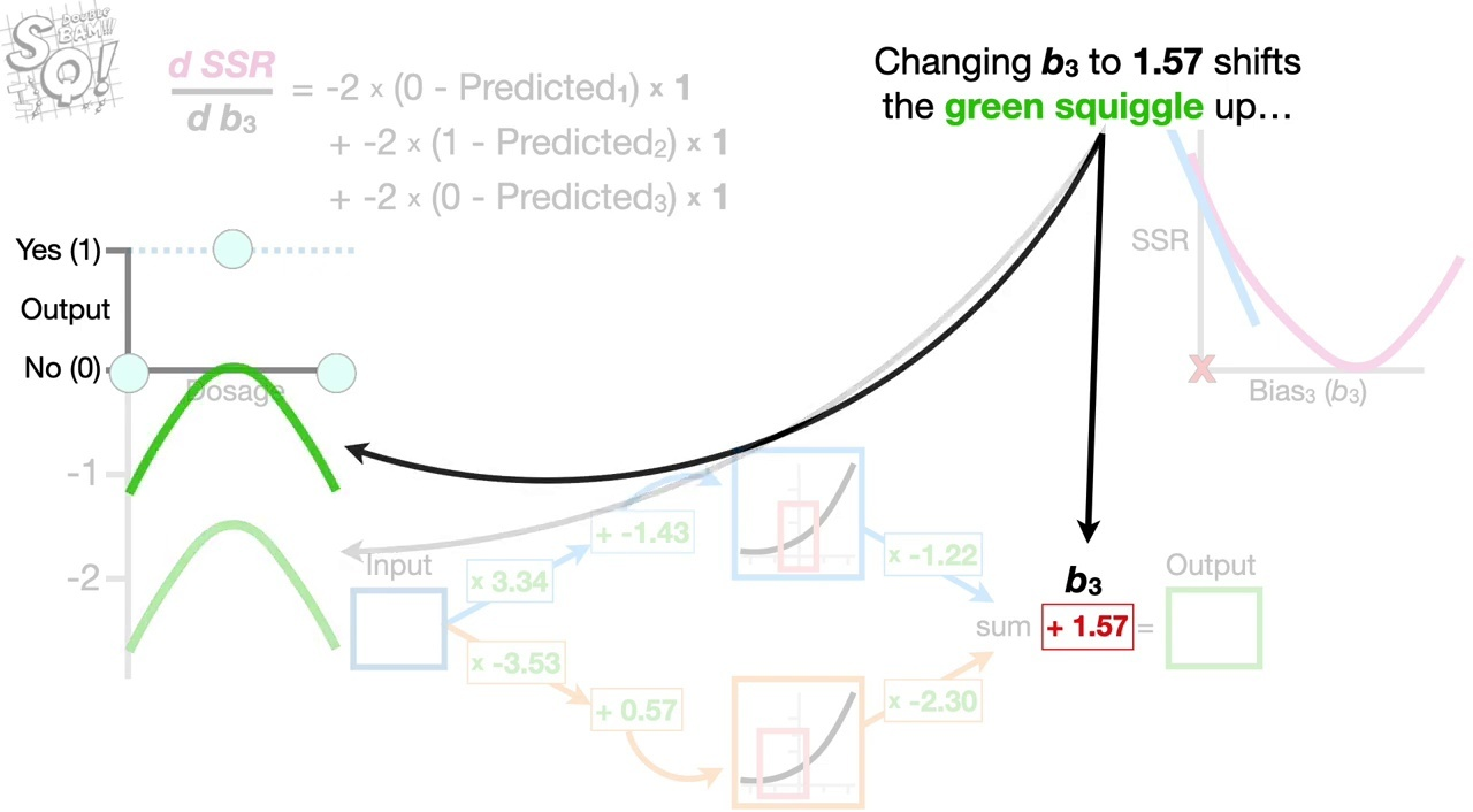
New b_3 = **0** - (-1.57) $\boxed{= 1.57}$

And the new value for b_3 is **1.57**.

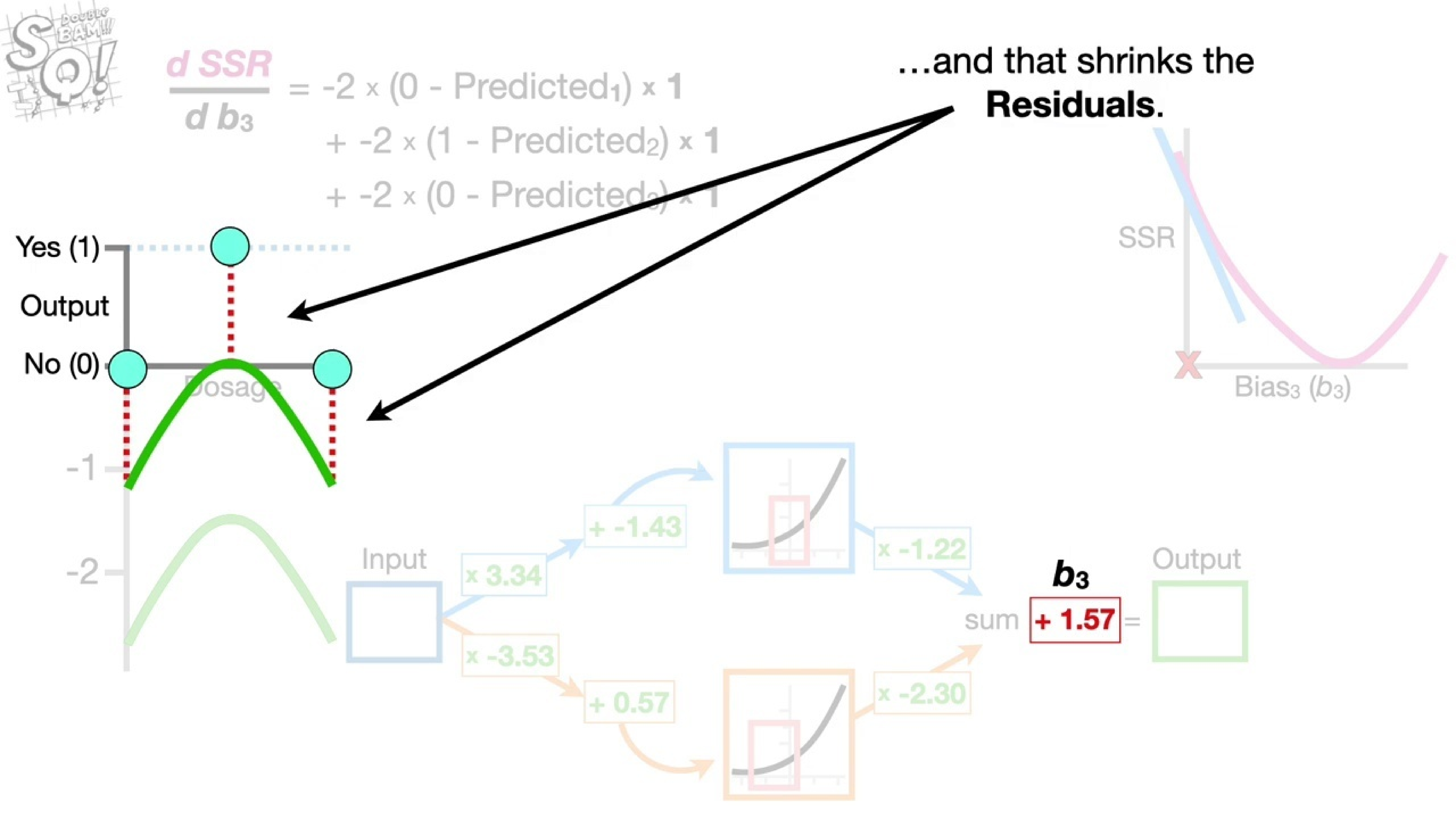




Changing b_3 to 1.57 shifts the green squiggle up…
$\frac{d\,SSR}{d\,b_3} = -2 \times (0 - \text{Predicted}_1) \times 1 + -2 \times (1 - \text{Predicted}_2) \times 1 + -2 \times (0 - \text{Predicted}_3) \times 1$
Yes (1)
Output
No (0)
Dosage
-1
-2
Input
x 3.34
+ -1.43
x -1.22
x -3.53
+ 0.57
x -2.30
b_3
sum
+ 1.57
=
Output
SSR
Bias$_3$ (b_3)



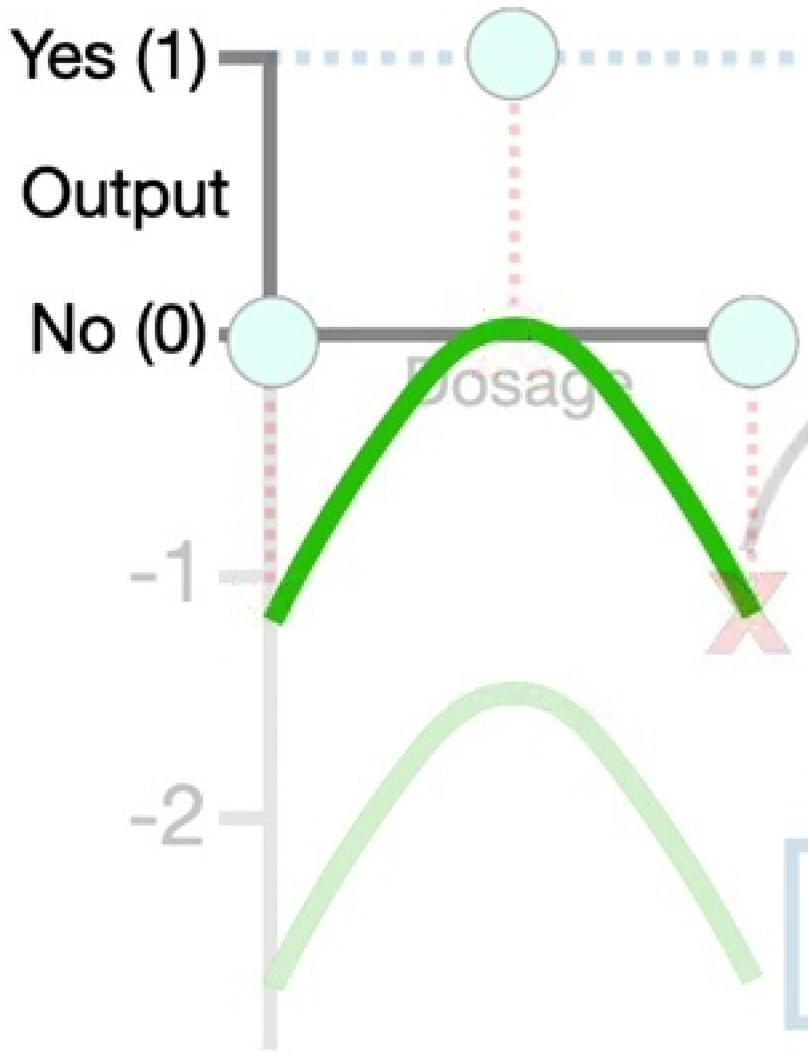
$$\frac{d\,SSR}{d\,b_3} = -2 \times (0 - \text{Predicted}_1) \times 1 + -2 \times (1 - \text{Predicted}_2) \times 1 + -2 \times (0 - \text{Predicted}_3) \times 1$$
Changing b_3 to 1.57 shifts the green squiggle up…
Yes (1)
Output
No (0)
Dosage
-1
-2
SSR
Bias$_3$ (b_3)
Input
x 3.34
+ -1.43
x -1.22
x -3.53
+ 0.57
x -2.30
b_3
sum + 1.57 =
Output

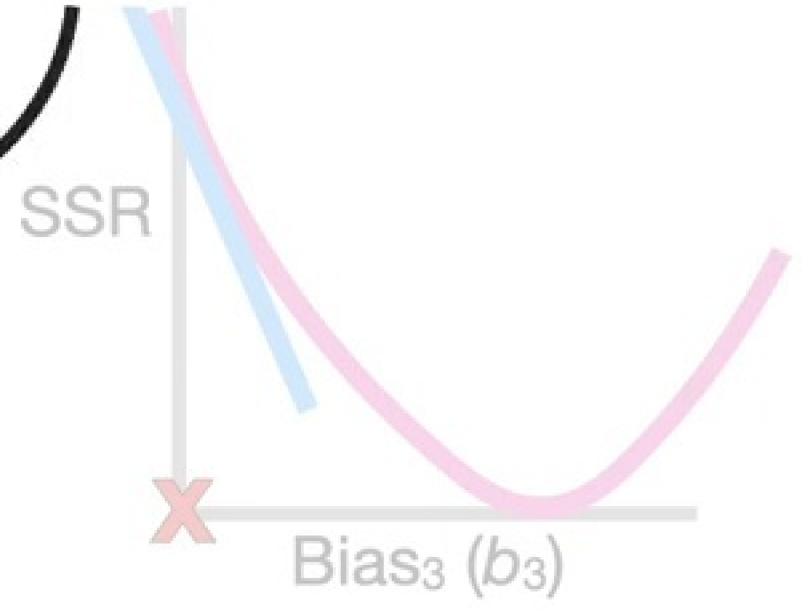


d SSR / d b3 = -2 x (0 - Predicted1) x 1
+ -2 x (1 - Predicted2) x 1
+ -2 x (0 - Predicted3) x 1
...and that shrinks the
Residuals.
Yes (1)
Output
No (0)
Dosage
-1
-2
SSR
Bias3 (b3)
Input
x 3.34
+ -1.43
x -1.22
x -3.53
+ 0.57
x -2.30
b3
sum
+ 1.57
=
Output

Now, plugging in the new **Predicted** values and doing the math gives us **-6.26**…

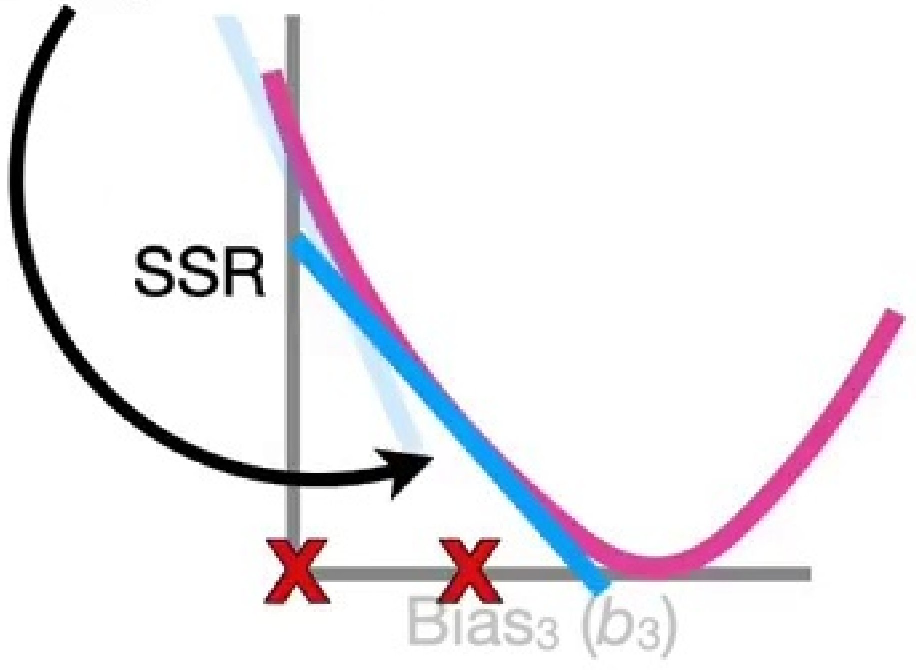
$$\frac{d\ SSR}{d\ b_3} = -2 \times (0 - -1.03) \times 1 + -2 \times (1 - -0.03) \times 1 + -2 \times (0 - -1.04) \times 1 \ \boxed{= -6.26}$$

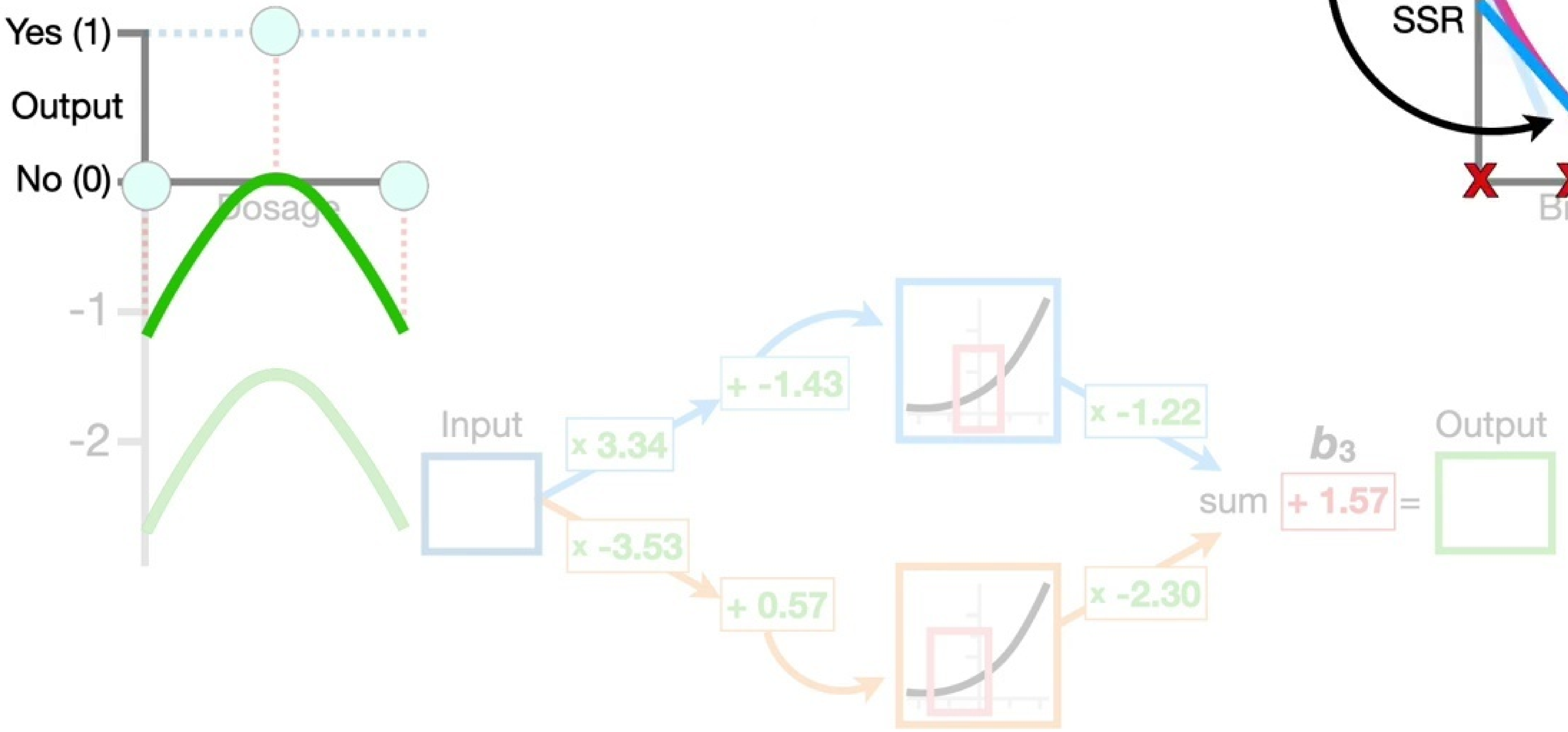




$$\frac{d\ SSR}{d\ b_3} = -2 \times (0 - -1.03) \times \mathbf{1} + -2 \times (1 - -0.03) \times \mathbf{1} + -2 \times (0 - -1.04) \times \mathbf{1} = -6.26$$

…which corresponds to the slope when $\boldsymbol{b_3} = \mathbf{1.57}$.

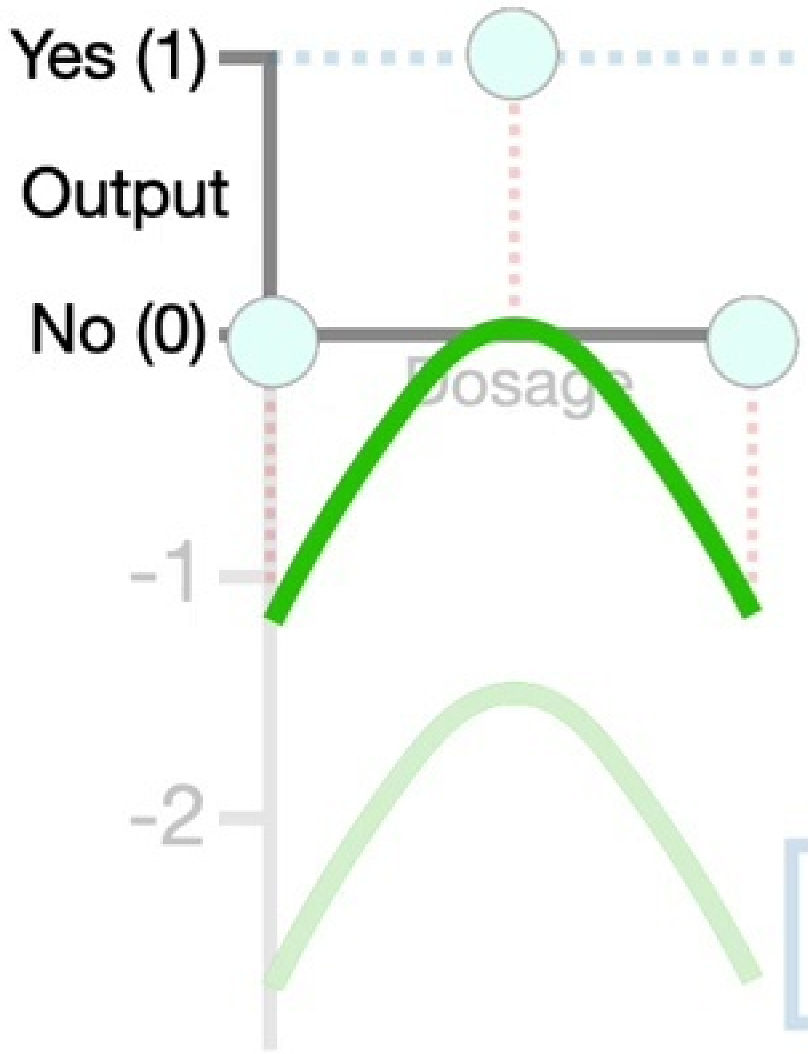


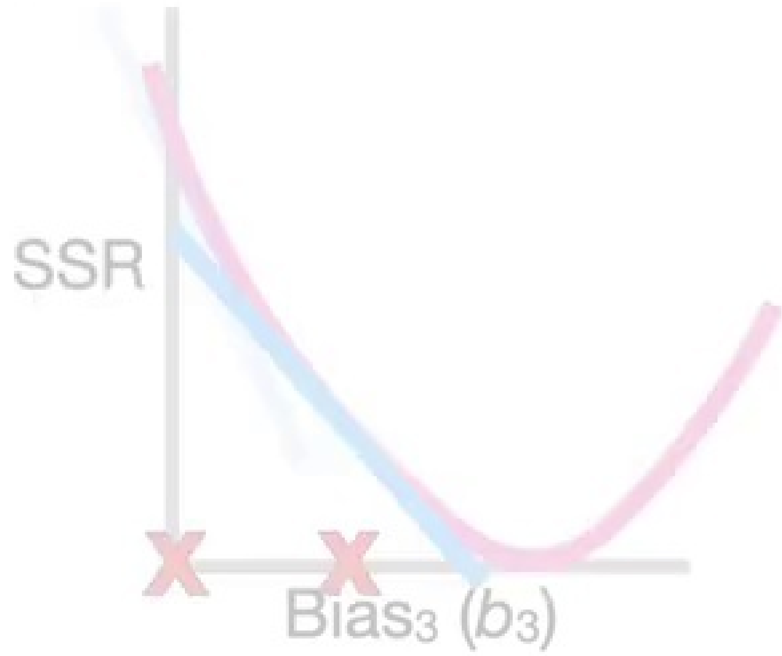


$$\frac{d\ SSR}{d\ b_3} = -2 \times (0 - -1.03) \times 1 + -2 \times (1 - -0.03) \times 1 + -2 \times (0 - -1.04) \times 1 = -6.26$$

Then we calculate the **Step Size**…

Step Size = Slope × Learning Rate

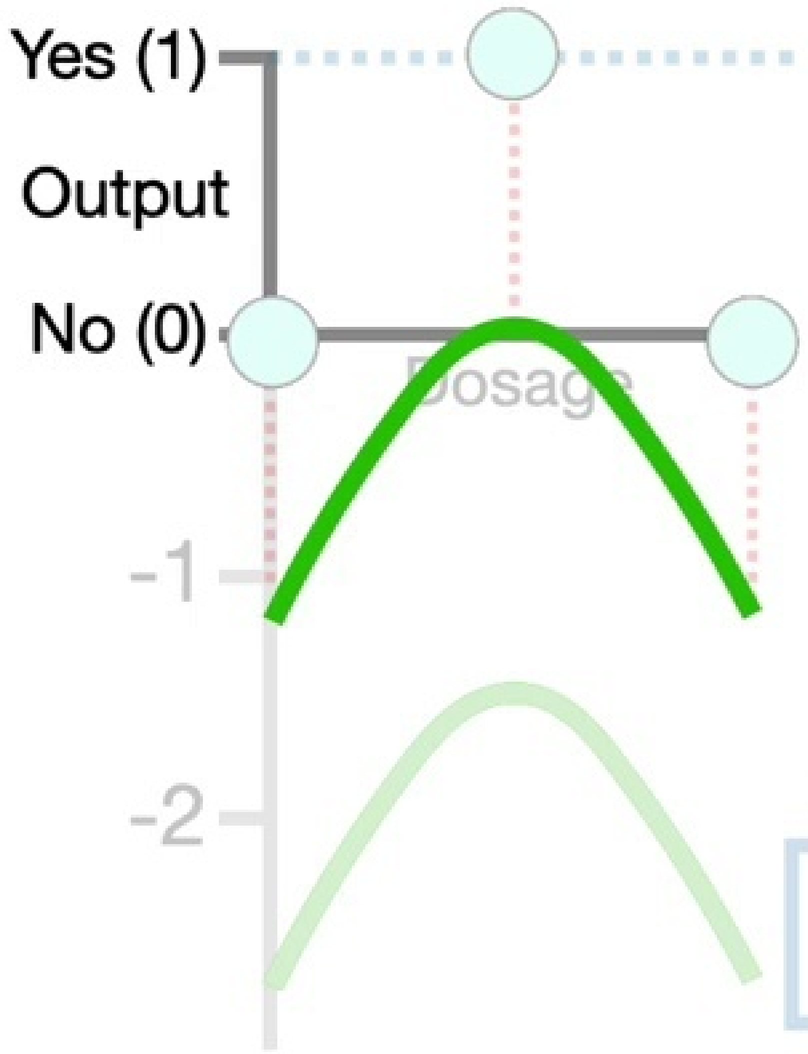


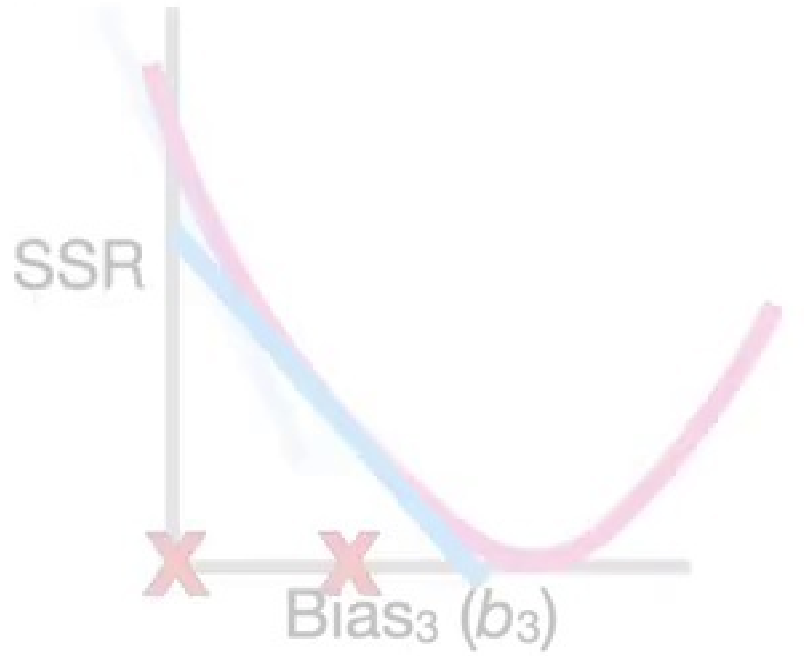


Then we calculate the **Step Size**…

$$\frac{d\ SSR}{d\ b_3} = -2 \times (0 - -1.03) \times 1 + -2 \times (1 - -0.03) \times 1 + -2 \times (0 - -1.04) \times 1 = \boxed{-6.26}$$

Step Size = $\boxed{\textbf{Slope}}$ × **Learning Rate**

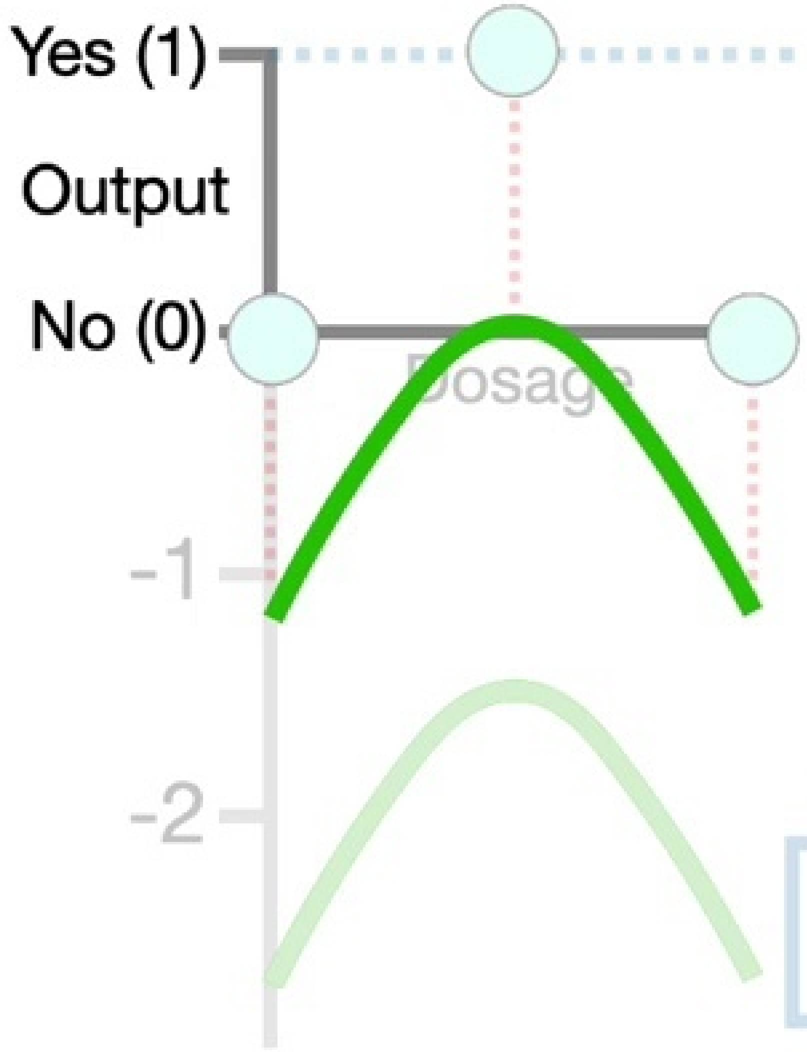


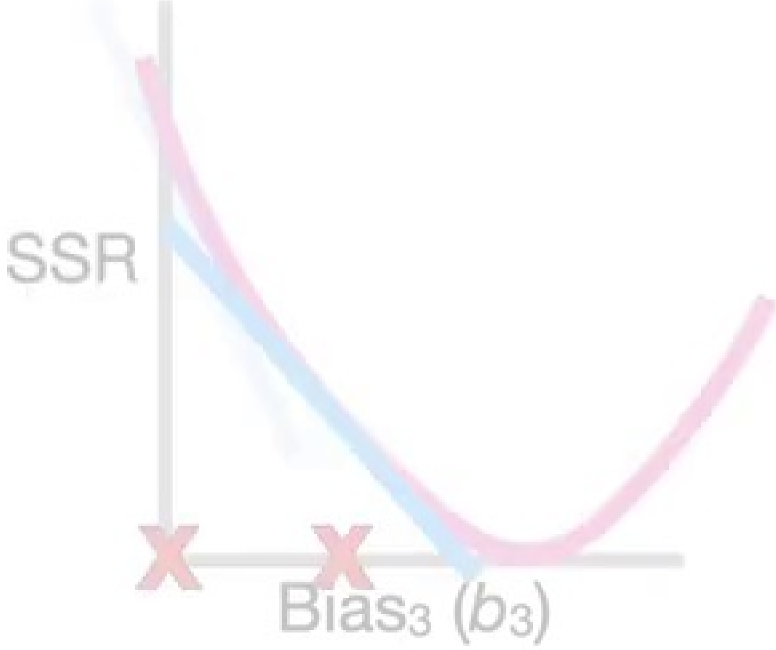


$$\frac{d\ SSR}{d\ b_3} = -2 \times (0 - -1.03) \times 1 + -2 \times (1 - -0.03) \times 1 + -2 \times (0 - -1.04) \times 1 = \boxed{-6.26}$$

Then we calculate the **Step Size**…

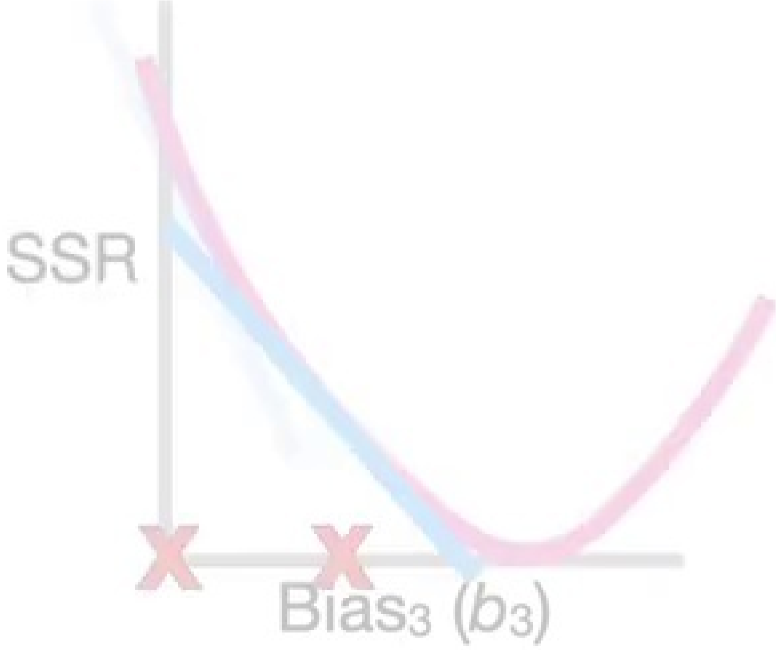
Step Size = $\boxed{-6.26}$ × **Learning Rate**



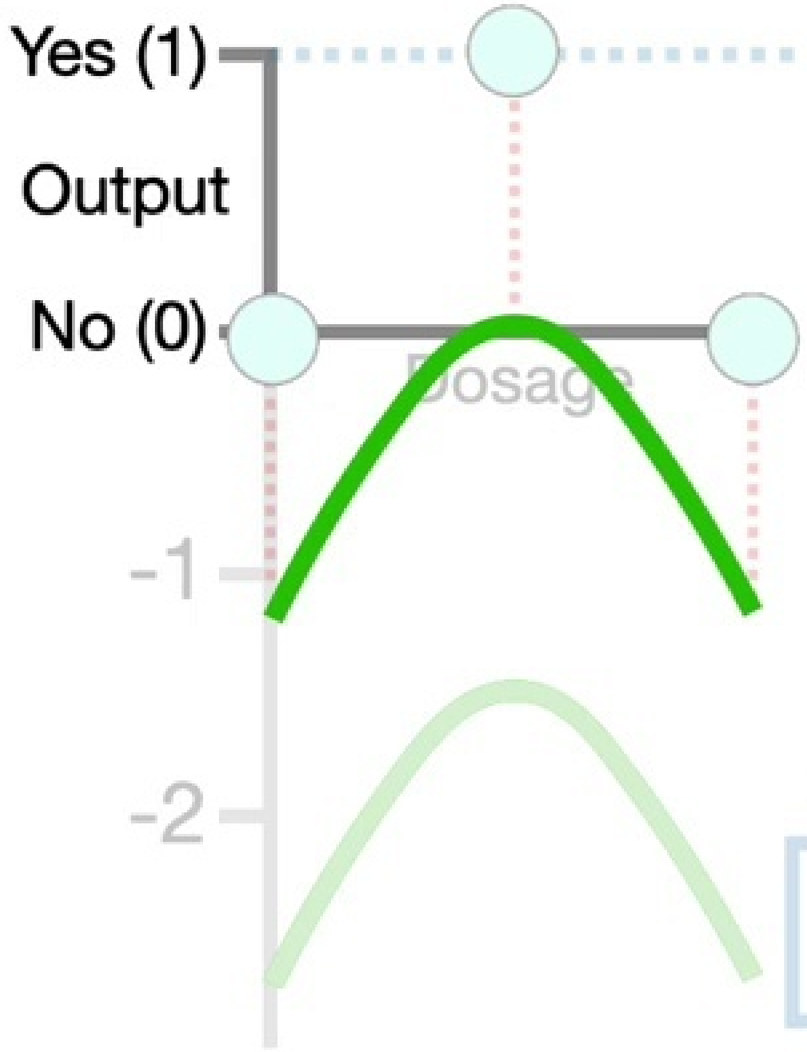


$$\frac{d\,SSR}{d\,b_3} = -2 \times (0 - -1.03) \times 1 + -2 \times (1 - -0.03) \times 1 + -2 \times (0 - -1.04) \times 1 = -6.26$$

Then we calculate the **Step Size**…

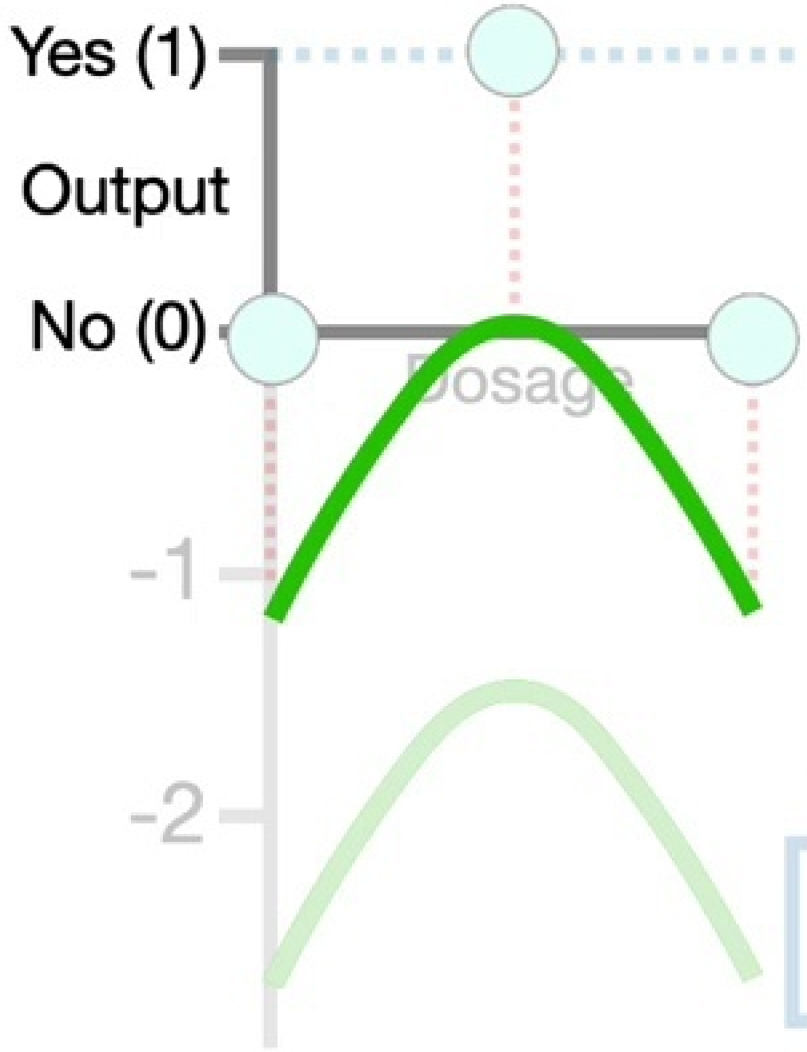


Step Size = -6.26×0.1

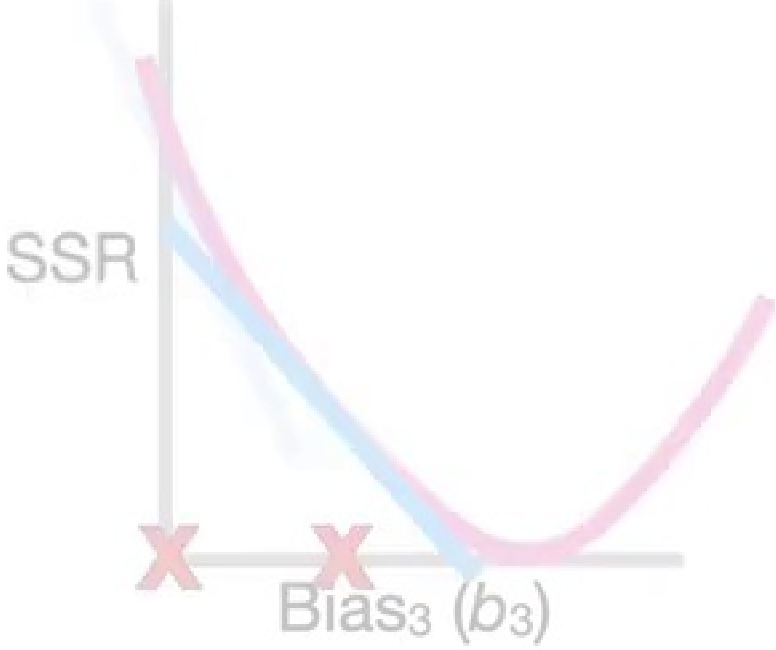


$$\frac{d\,SSR}{d\,b_3} = -2 \times (0 - -1.03) \times 1 + -2 \times (1 - -0.03) \times 1 + -2 \times (0 - -1.04) \times 1 = -6.26$$

Then we calculate the **Step Size**…

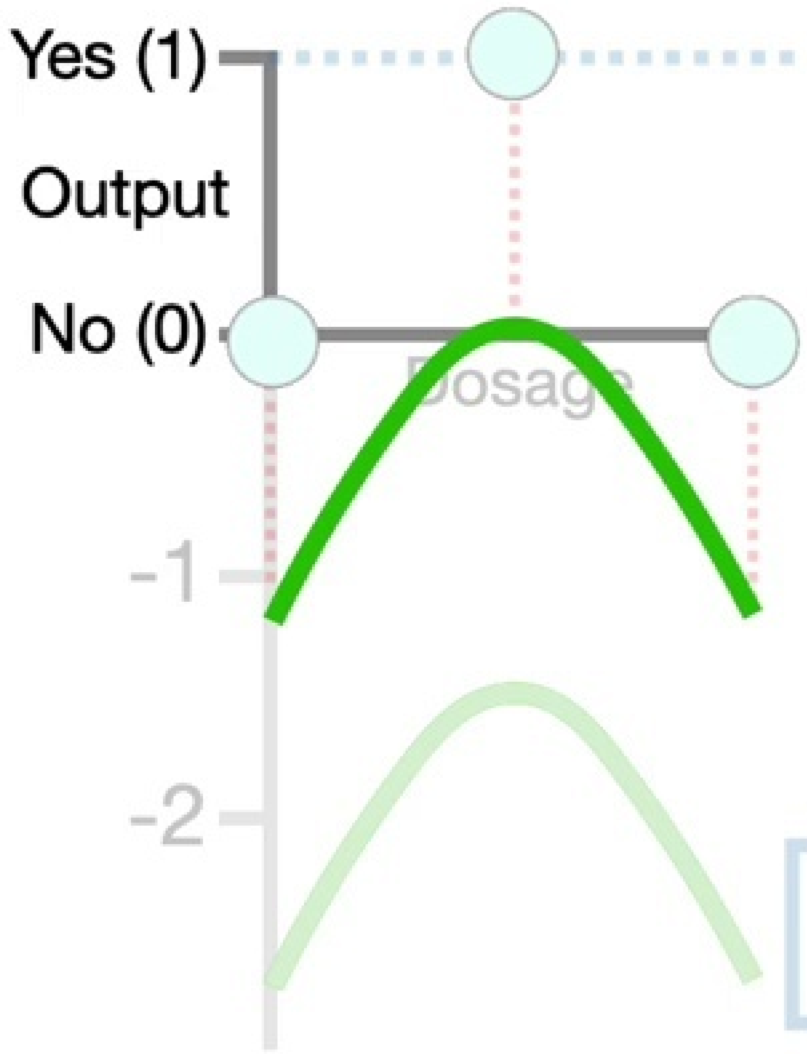


Step Size = $-6.26 \times 0.1 = -0.626$



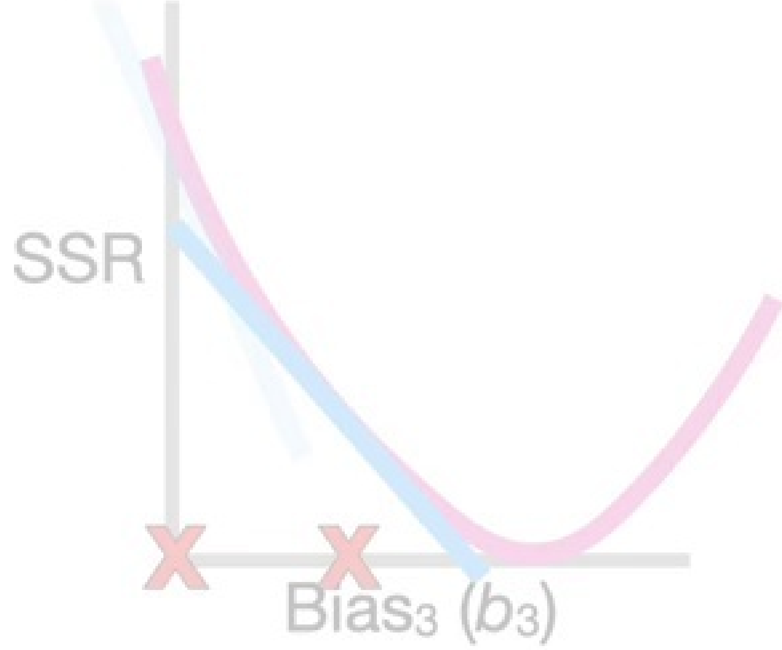
$$\frac{d\ SSR}{d\ b_3} = -2 \times (0 - -1.03) \times 1 + -2 \times (1 - -0.03) \times 1 + -2 \times (0 - -1.04) \times 1 = -6.26$$

...and the new value for $\boldsymbol{b_3}$...



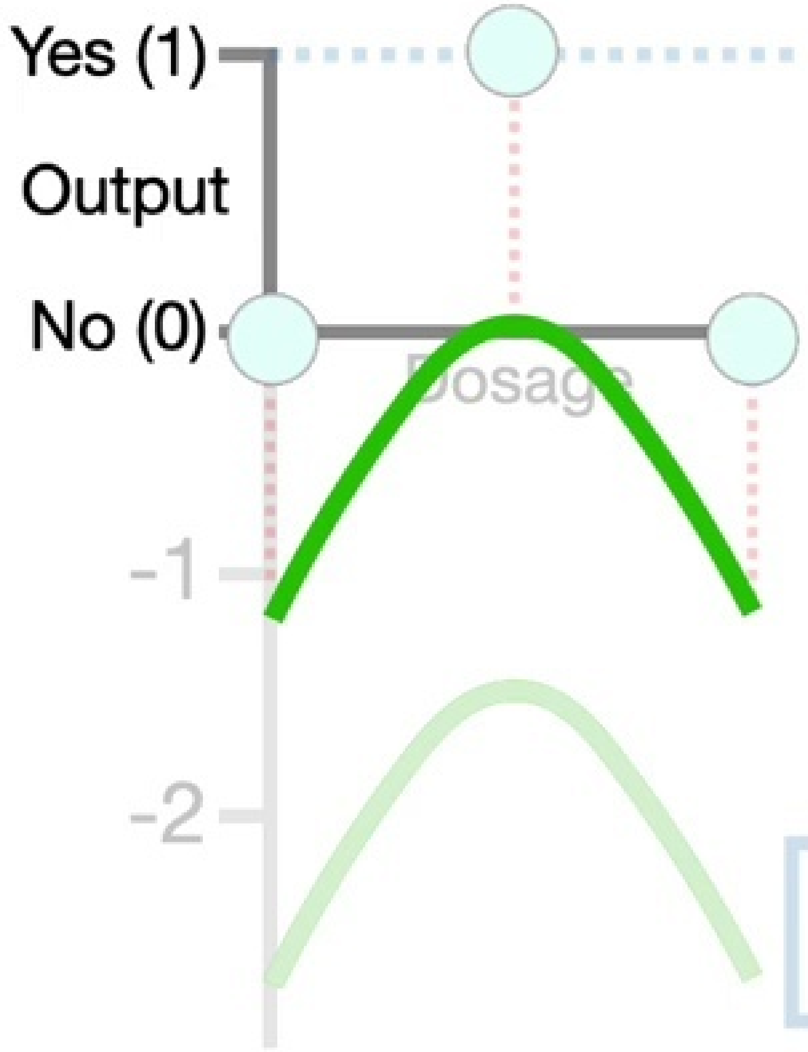
Step Size = $-6.26 \times 0.1 = -0.626$

New b_3 = Old b_3 - Step Size



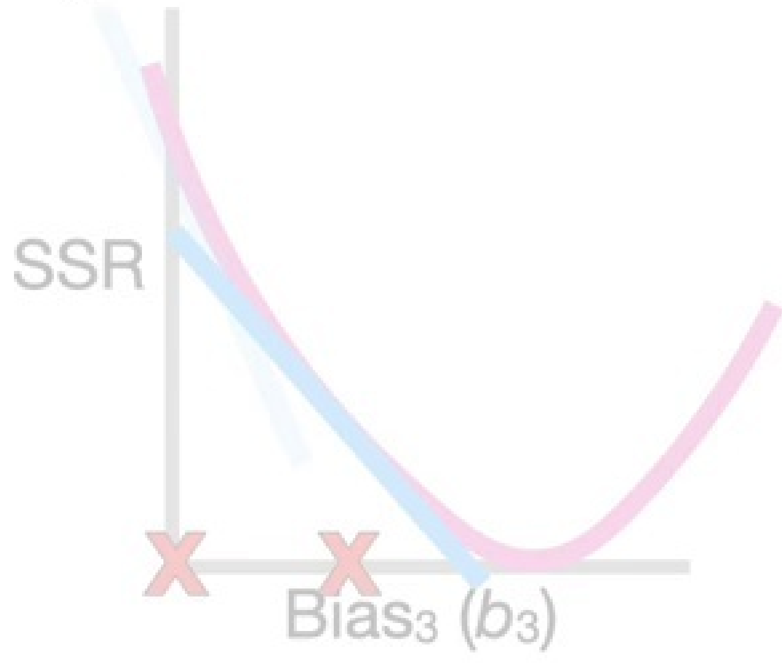
$$\frac{d\,SSR}{d\,b_3} = -2 \times (0 - -1.03) \times 1 + -2 \times (1 - -0.03) \times 1 + -2 \times (0 - -1.04) \times 1 = -6.26$$

…and the new value for b_3…



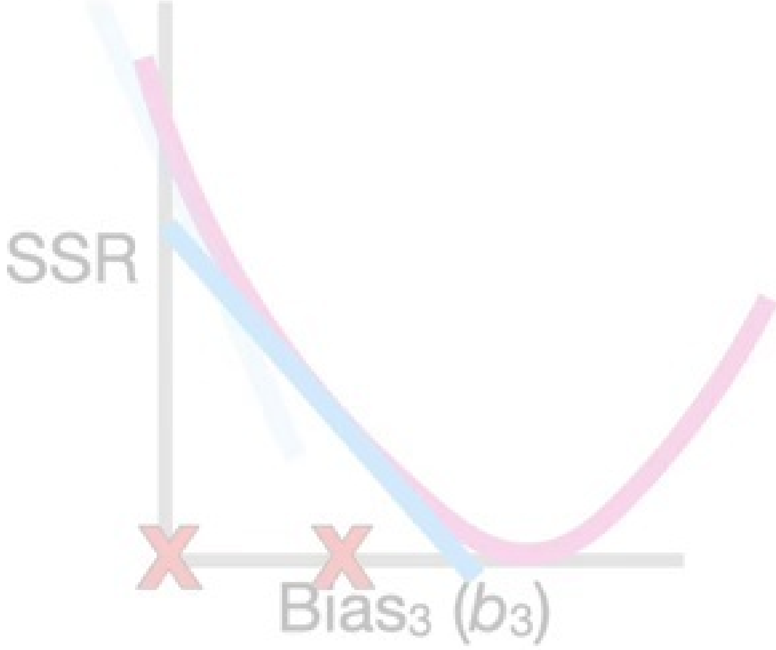
Step Size = -6.26 × 0.1 = -0.626

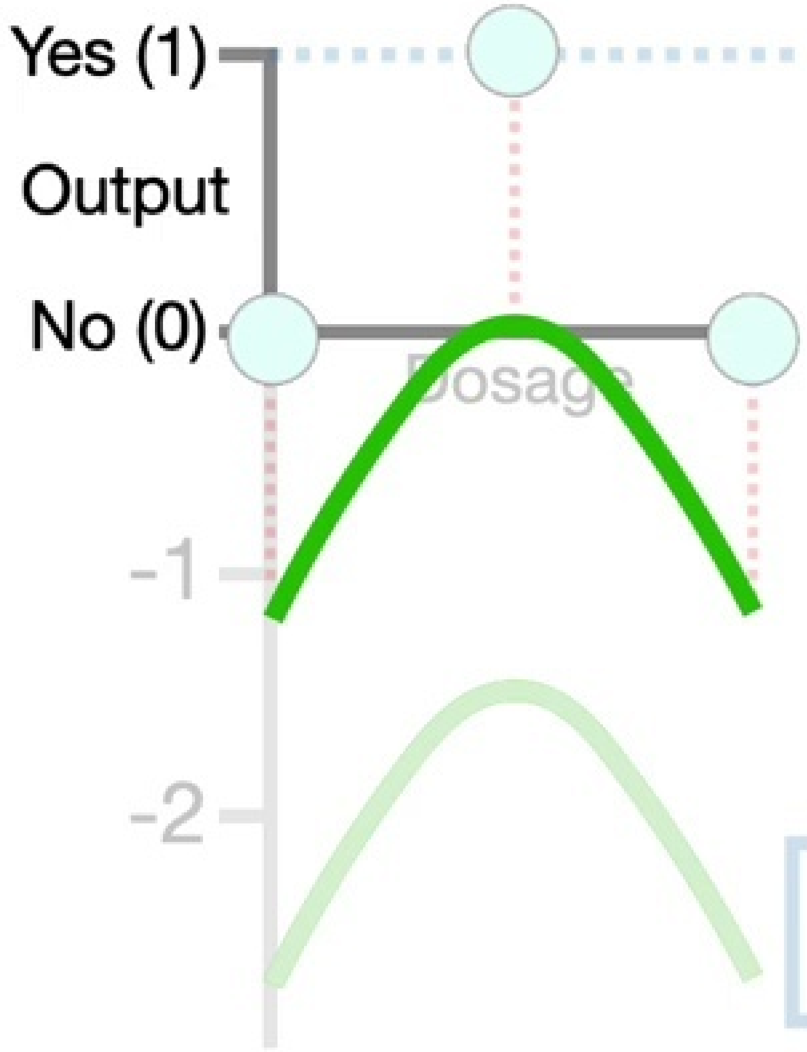
$$\textbf{New } b_3 = \boxed{\textbf{Old } b_3} - \textbf{Step Size}$$



$$\frac{d\ SSR}{d\ b_3} = -2 \times (0 - -1.03) \times 1 + -2 \times (1 - -0.03) \times 1 + -2 \times (0 - -1.04) \times 1 = -6.26$$

…and the new value for b_3…



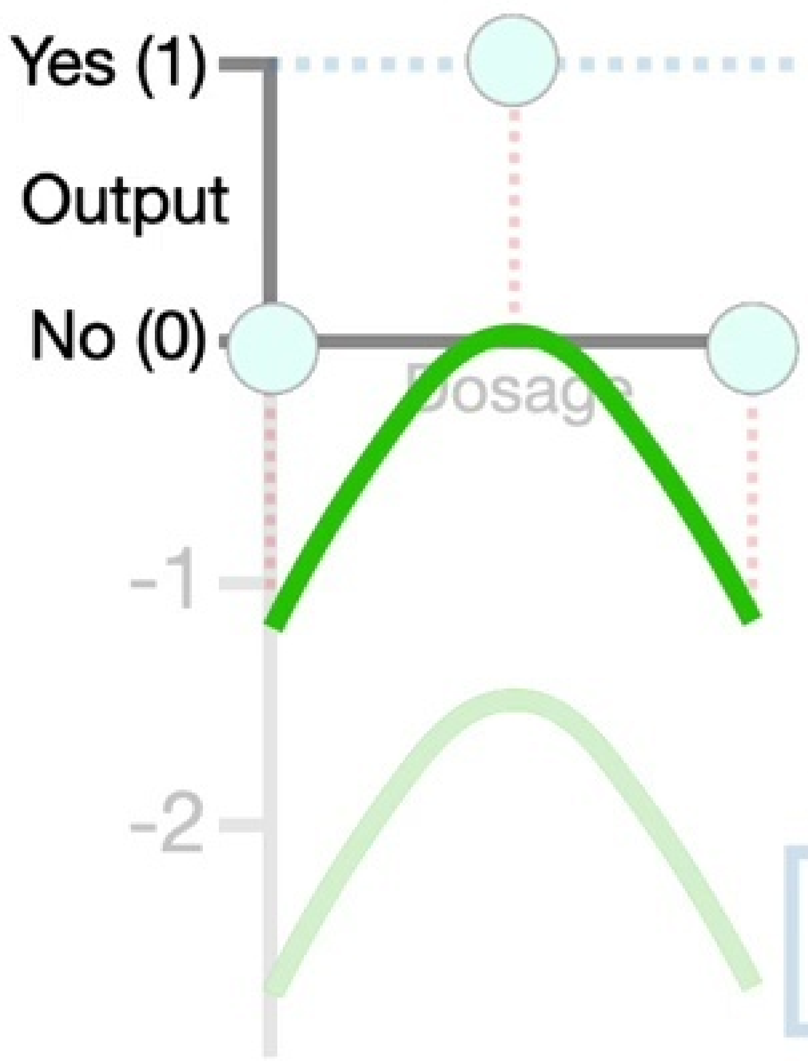


Step Size = -6.26 × 0.1 = -0.626

New b_3 = $\boxed{1.57}$ - Step Size

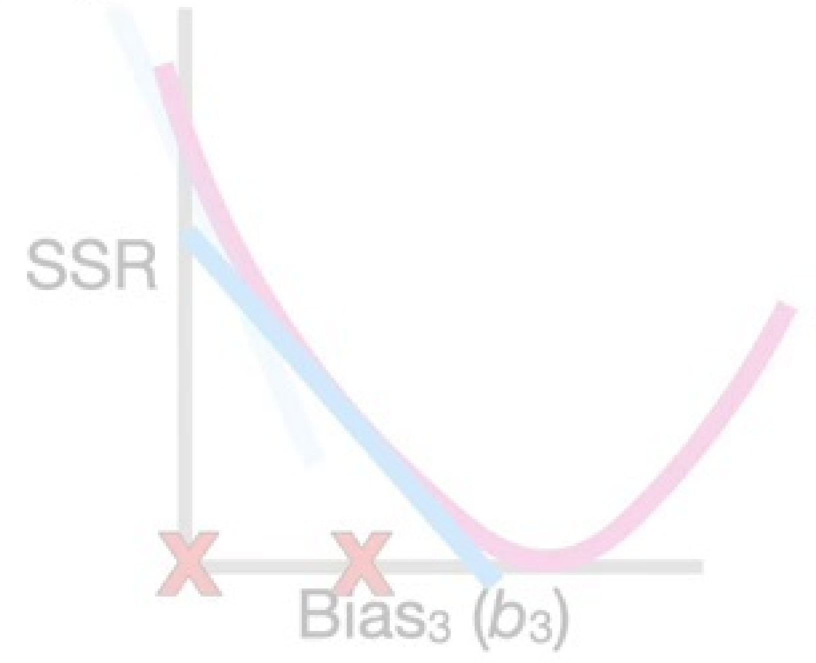
$$\frac{d\,SSR}{d\,b_3} = -2 \times (0 - -1.03) \times 1 + -2 \times (1 - -0.03) \times 1 + -2 \times (0 - -1.04) \times 1 = -6.26$$

…and the new value for $\boldsymbol{b_3}$…



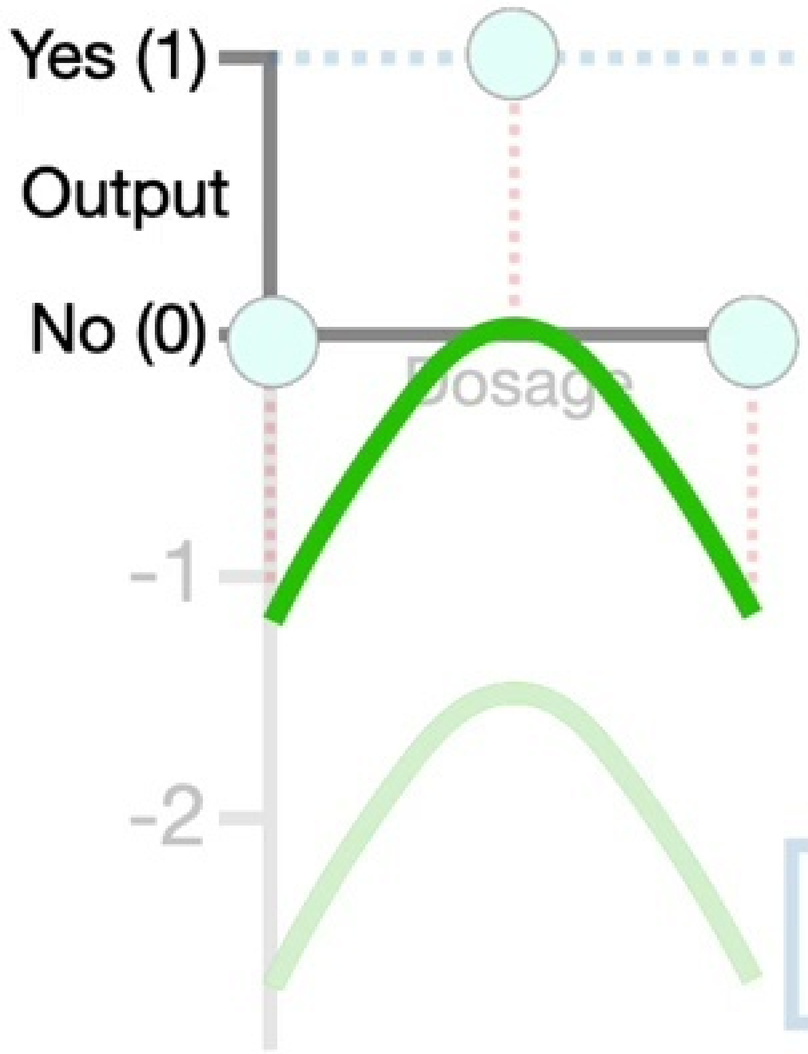
Step Size = $-6.26 \times 0.1 \boxed{= -0.626}$

New b_3 = 1.57 $\boxed{\text{- Step Size}}$



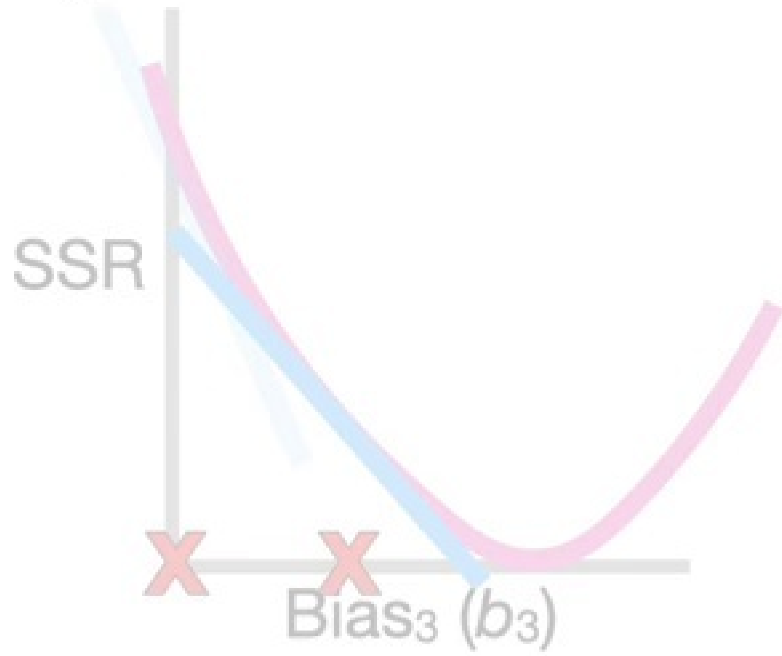
$$\frac{d\ SSR}{d\ b_3} = -2 \times (0 - -1.03) \times 1 + -2 \times (1 - -0.03) \times 1 + -2 \times (0 - -1.04) \times 1 = -6.26$$

…and the new value for $\boldsymbol{b_3}$…



Step Size = -6.26 × 0.1 $\boxed{= -0.626}$

New $\boldsymbol{b_3}$ **= 1.57** $\boxed{- (-0.626)}$

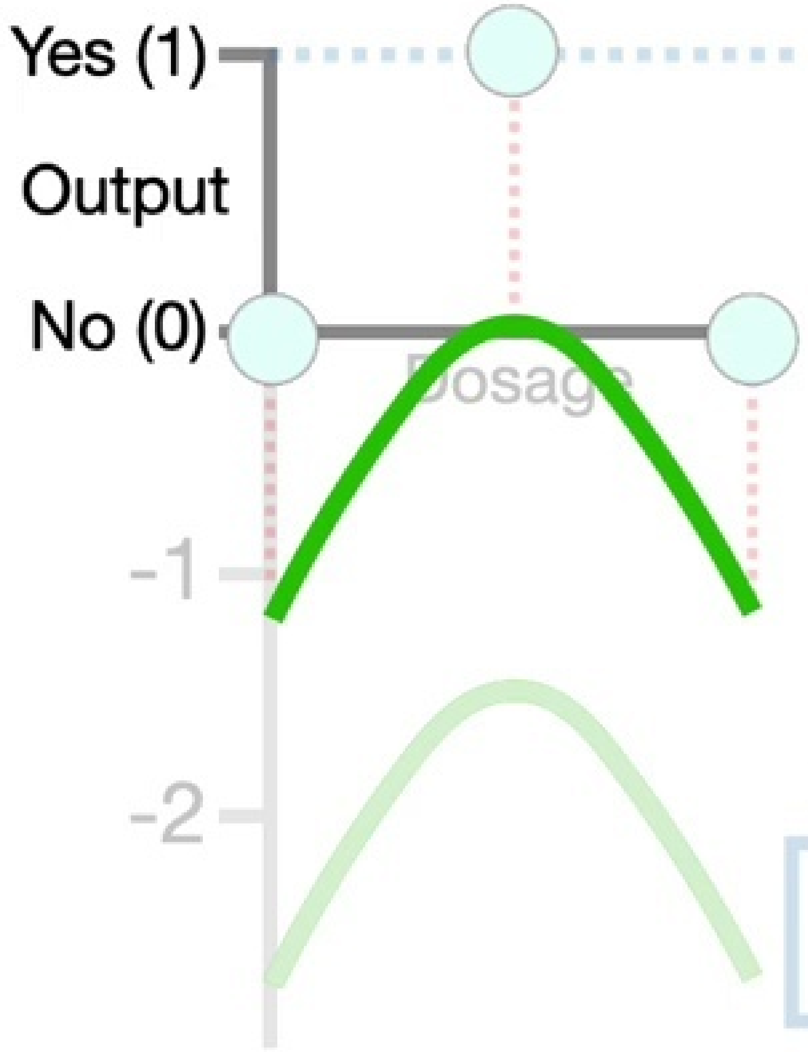


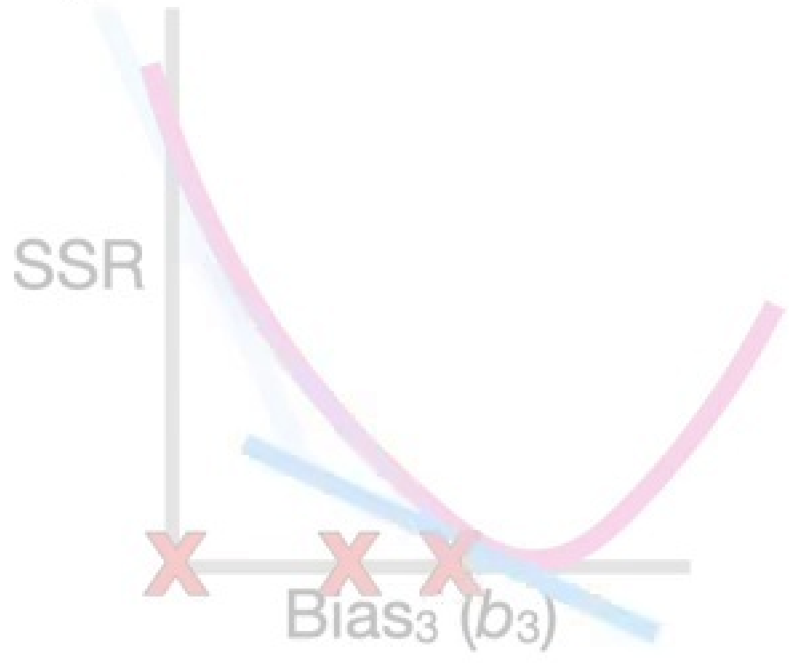
...and the new value for $\boldsymbol{b_3}$...

$$\frac{d\ SSR}{d\ b_3} = -2 \times (0 - -1.03) \times 1 + -2 \times (1 - -0.03) \times 1 + -2 \times (0 - -1.04) \times 1 = -6.26$$

Step Size = -6.26 × 0.1 = -0.626

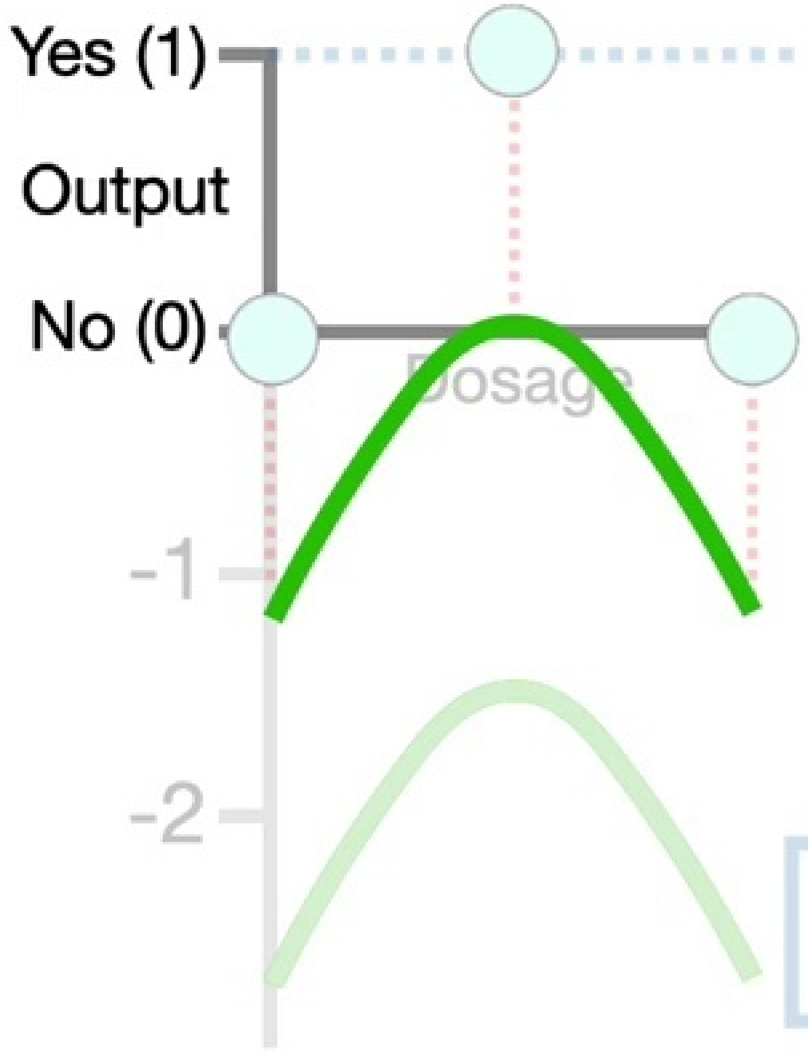
New b_3 = 1.57 - (-0.626) = 2.19

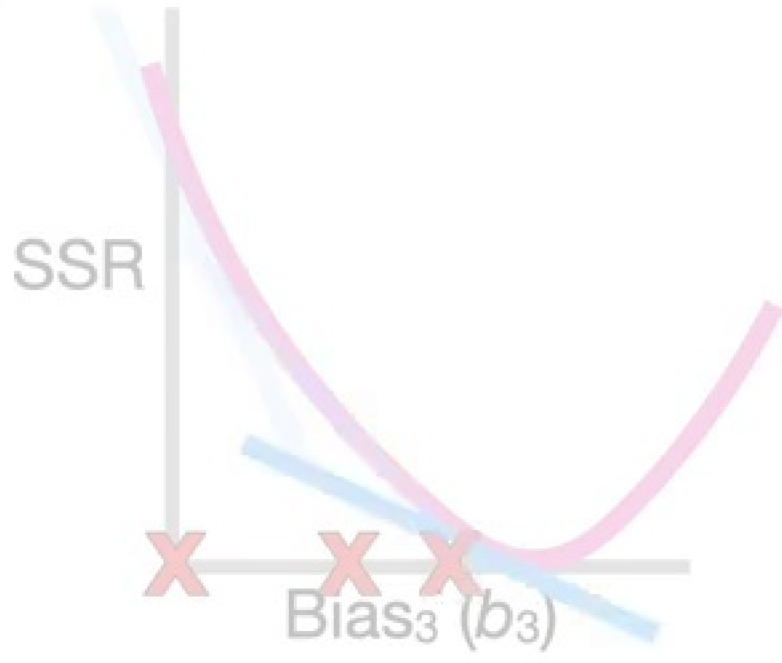




$$\frac{d\,SSR}{d\,b_3} = -2 \times (0 - -1.03) \times 1 + -2 \times (1 - -0.03) \times 1 + -2 \times (0 - -1.04) \times 1 = -6.26$$

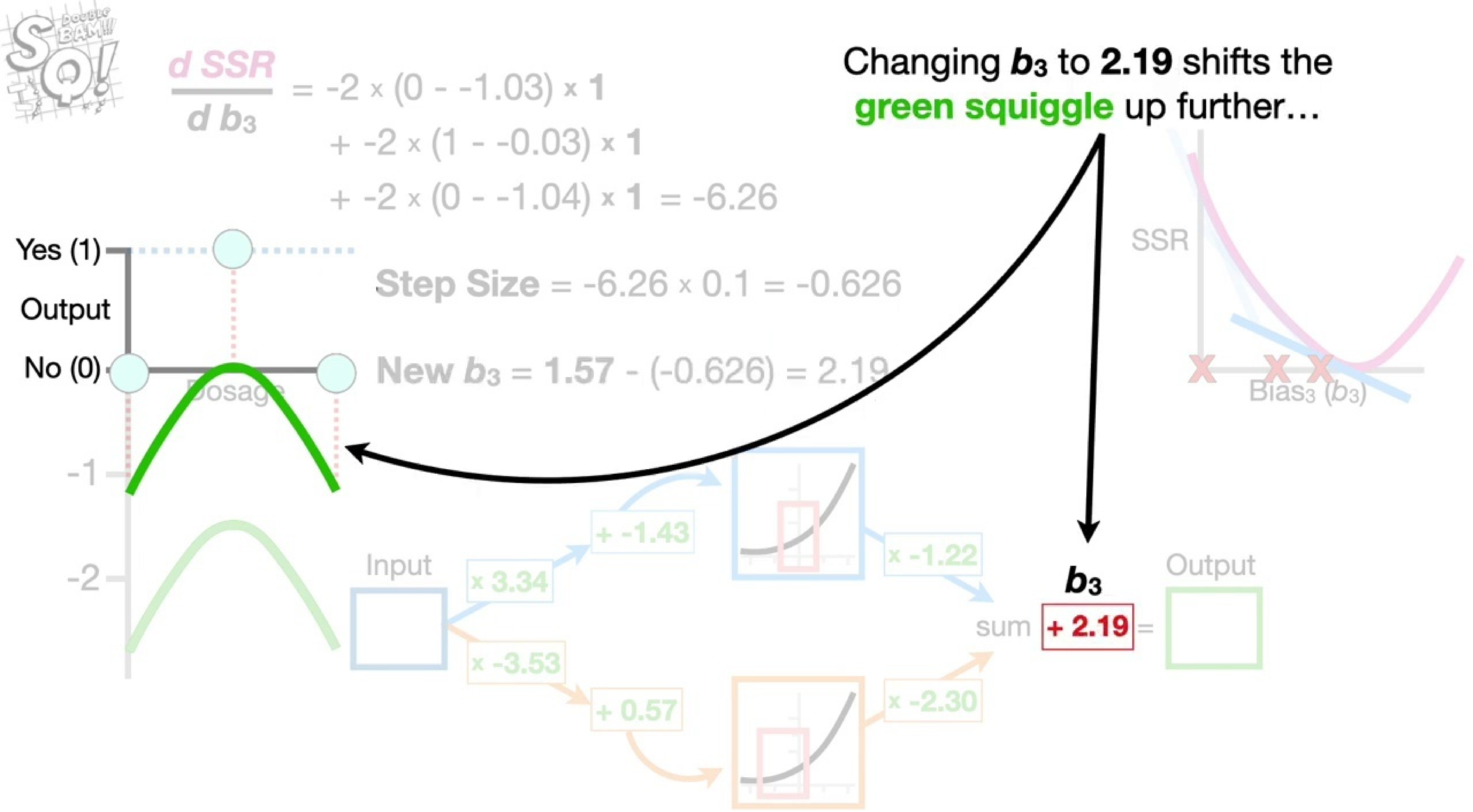
…which is **2.19.**

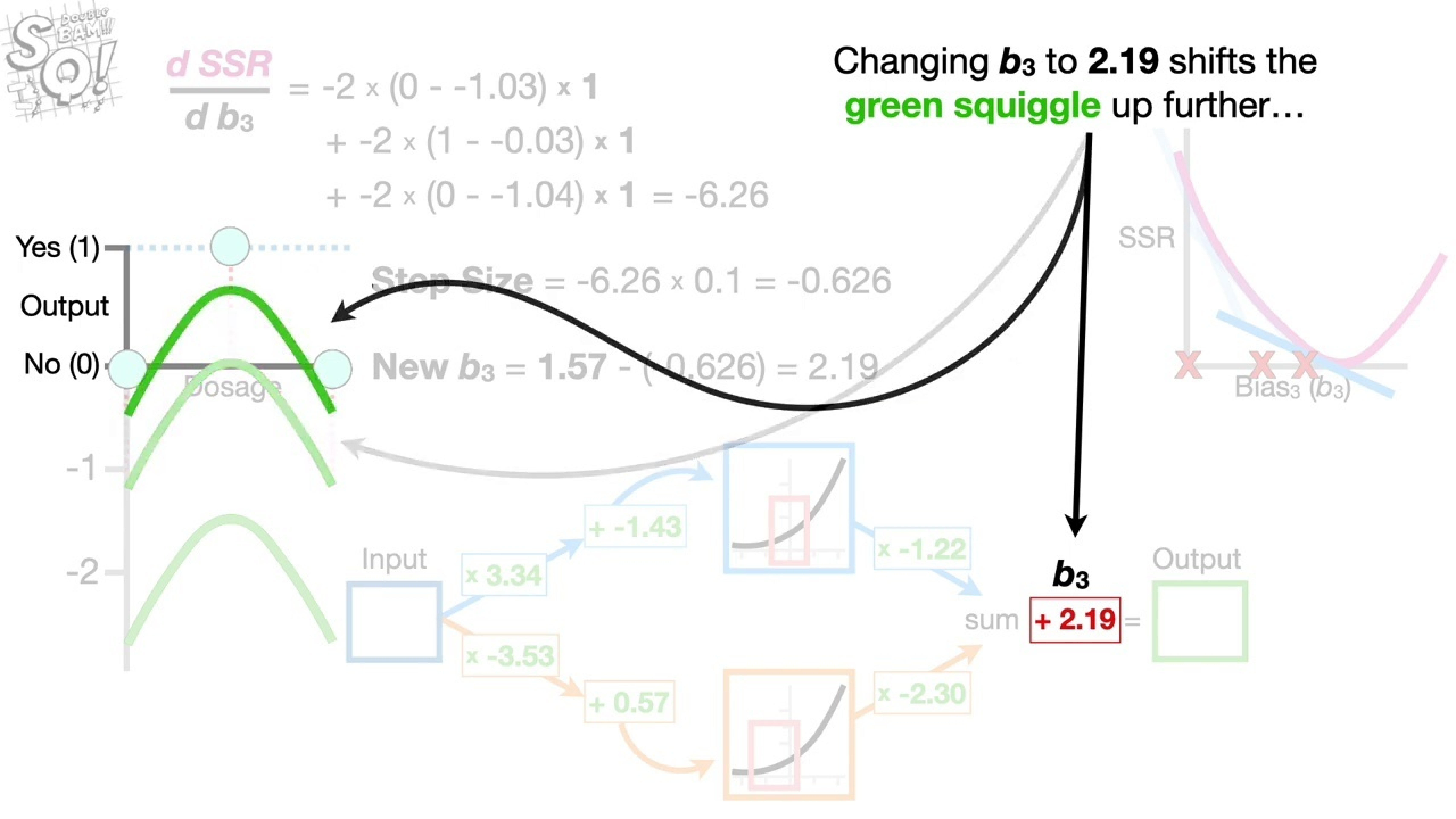


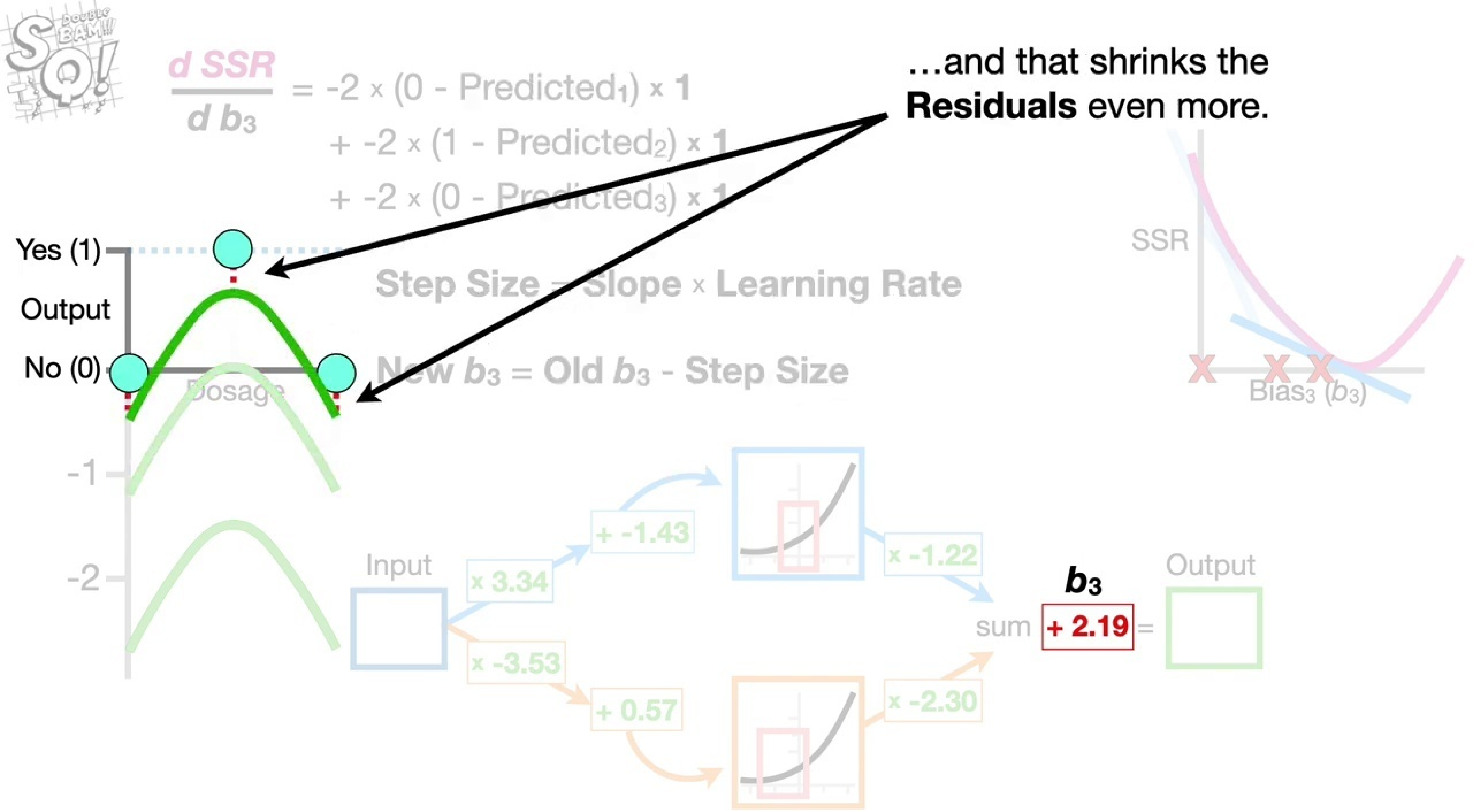


Step Size = -6.26 × 0.1 = -0.626

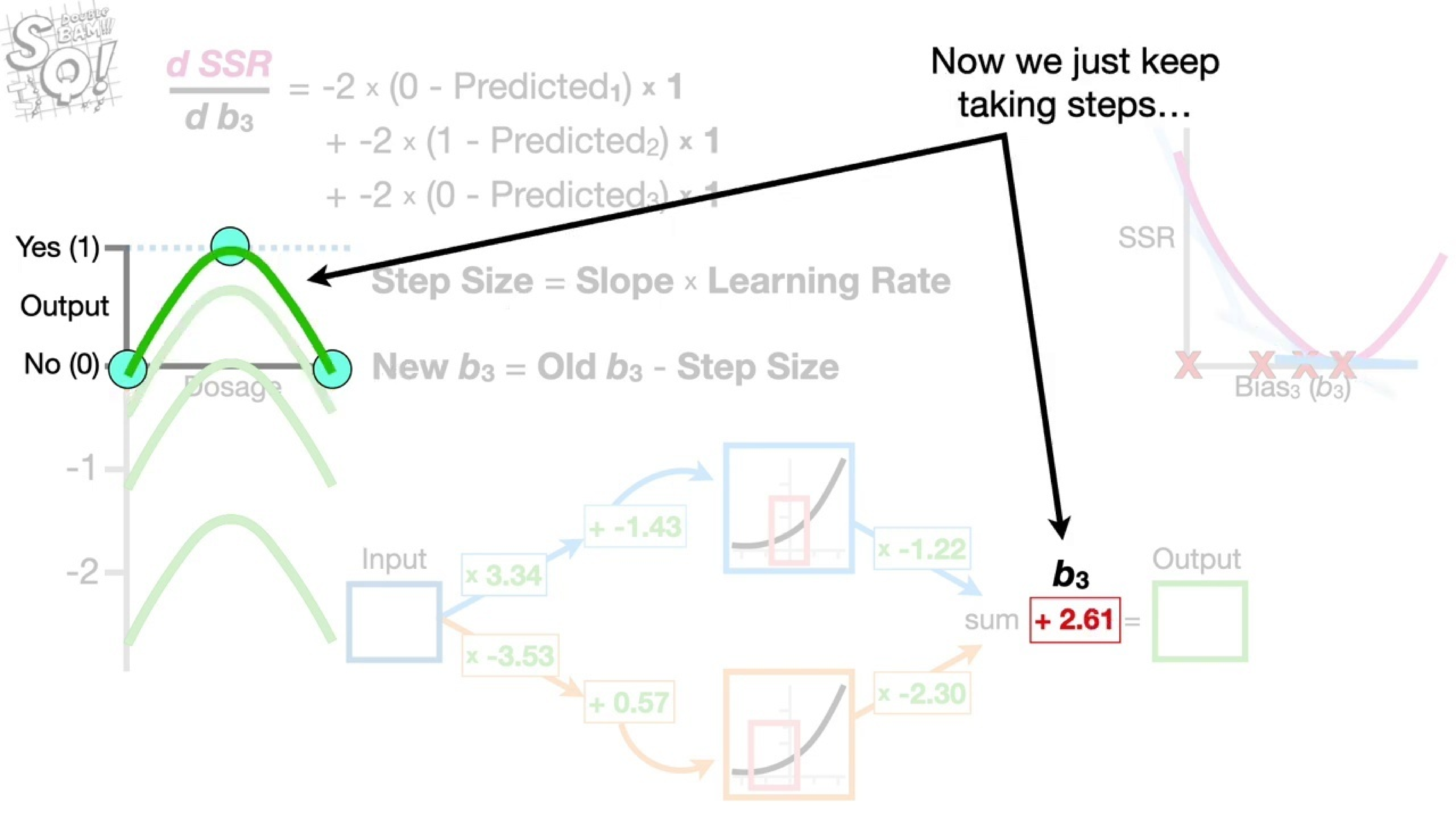
New b_3 = 1.57 - (-0.626) $\boxed{= 2.19}$


Changing b_3 to 2.19 shifts the green squiggle up further…
$\frac{d\ SSR}{d\ b_3}$ = -2 x (0 - -1.03) x 1
+ -2 x (1 - -0.03) x 1
+ -2 x (0 - -1.04) x 1 = -6.26
Step Size = -6.26 x 0.1 = -0.626
New b_3 = 1.57 - (-0.626) = 2.19
Yes (1)
Output
No (0)
Dosage
-1
-2
SSR
Bias3 (b_3)
Input
x 3.34
+ -1.43
x -1.22
x -3.53
+ 0.57
x -2.30
b_3
sum + 2.19 =
Output


Changing b_3 to 2.19 shifts the green squiggle up further…
$\frac{d\ SSR}{d\ b_3}$ = -2 x (0 - -1.03) x 1
+ -2 x (1 - -0.03) x 1
+ -2 x (0 - -1.04) x 1 = -6.26
Step Size = -6.26 x 0.1 = -0.626
New b_3 = 1.57 - (-0.626) = 2.19
Yes (1)
Output
No (0)
Dosage
-1
-2
SSR
Bias3 (b_3)
Input
x 3.34
+ -1.43
x -1.22
x -3.53
+ 0.57
x -2.30
b_3
sum + 2.19 =
Output



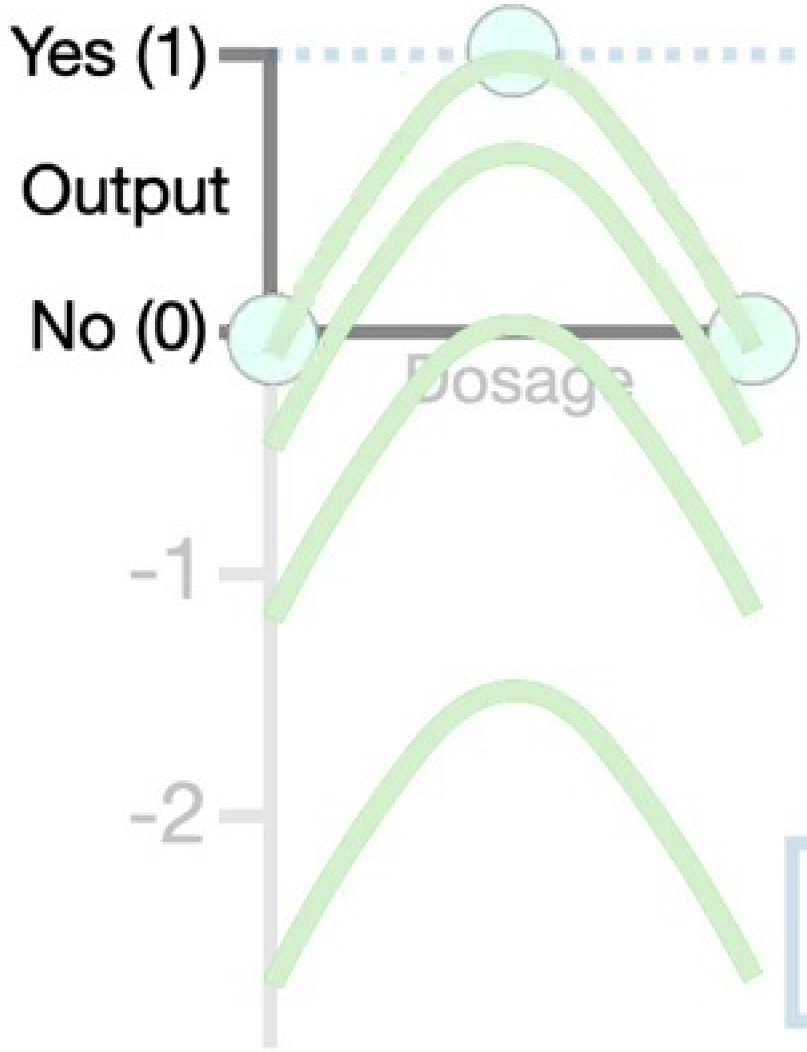
...and that shrinks the **Residuals** even more.
$\frac{d\,SSR}{d\,b_3} = -2 \times (0 - \text{Predicted}_1) \times 1$
$+ -2 \times (1 - \text{Predicted}_2) \times 1$
$+ -2 \times (0 - \text{Predicted}_3) \times 1$
Step Size = Slope × Learning Rate
New b_3 = Old b_3 - Step Size
Yes (1)
Output
No (0)
Dosage
-1
-2
SSR
Bias$_3$ (b_3)
Input
x 3.34
+ -1.43
x -1.22
x -3.53
+ 0.57
x -2.30
b_3
sum
+ 2.19
=
Output



Now we just keep taking steps…
$\frac{d\,SSR}{d\,b_3} = -2 \times (0 - \text{Predicted}_1) \times 1 + -2 \times (1 - \text{Predicted}_2) \times 1 + -2 \times (0 - \text{Predicted}_3) \times 1$
Step Size = Slope x Learning Rate
New b_3 = Old b_3 - Step Size
Yes (1)
Output
No (0)
Dosage
-1
-2
SSR
Bias$_3$ (b_3)
Input
x 3.34
+ -1.43
x -1.22
x -3.53
+ 0.57
x -2.30
b_3
sum
+ 2.61 =
Output

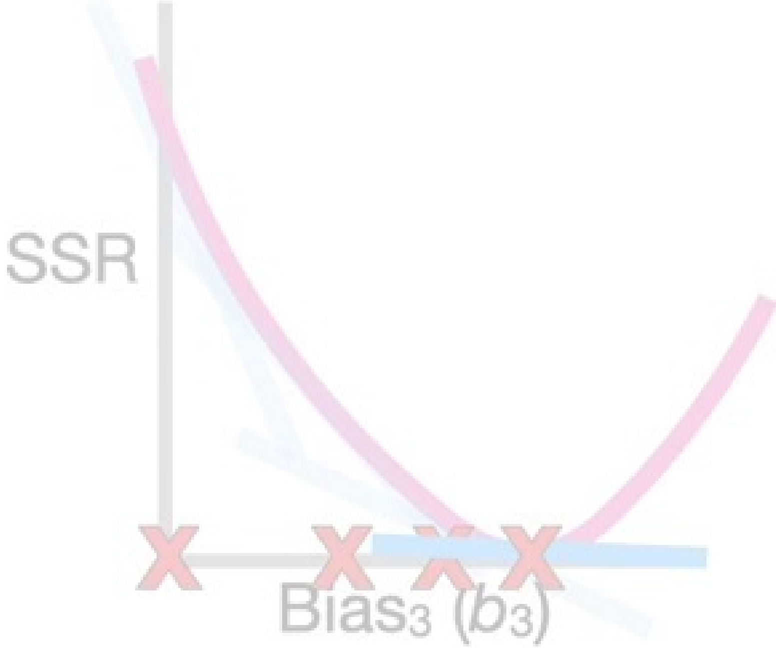
$$\frac{d\,SSR}{d\,b_3} = -2 \times (0 - 0.01) + -2 \times (1 - 1.01) + -2 \times (0 - 0) = \text{close to } 0$$

…until the **Step Size** is close to **0**.



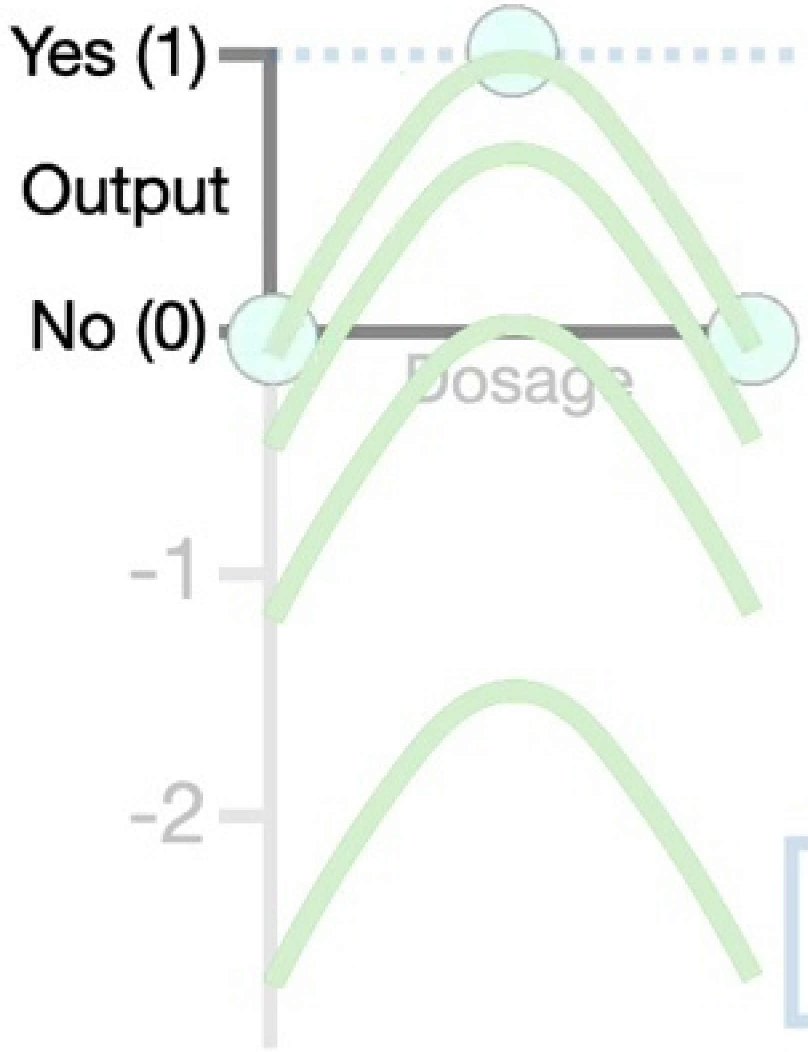
Step Size = close to 0 × 0.1 = close to 0

New b_3 = Old b_3 - Step Size



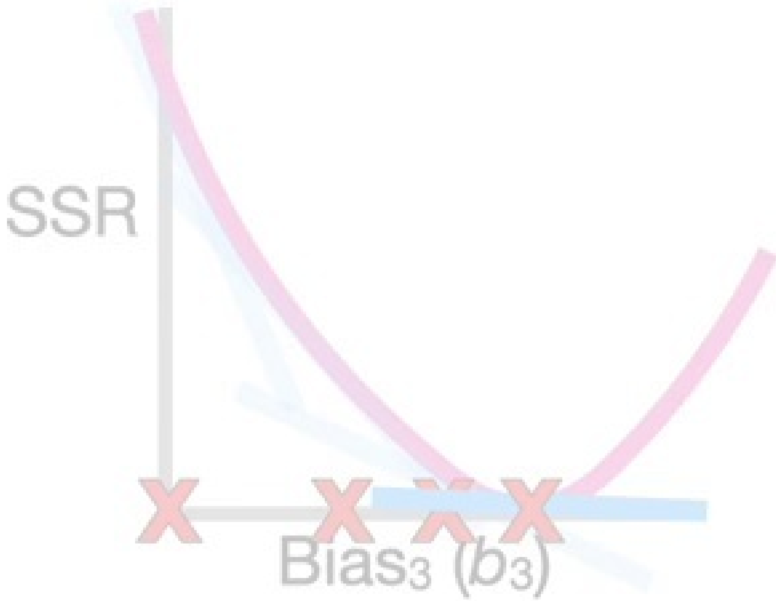
$$\frac{d\ SSR}{d\ b_3} = -2 \times (0 - 0.01) + -2 \times (1 - 1.01) + -2 \times (0 - 0) = \text{close to } 0$$

And because the **Step Size** is close to **0** when $\boldsymbol{b_3}$ **= 2.61**…



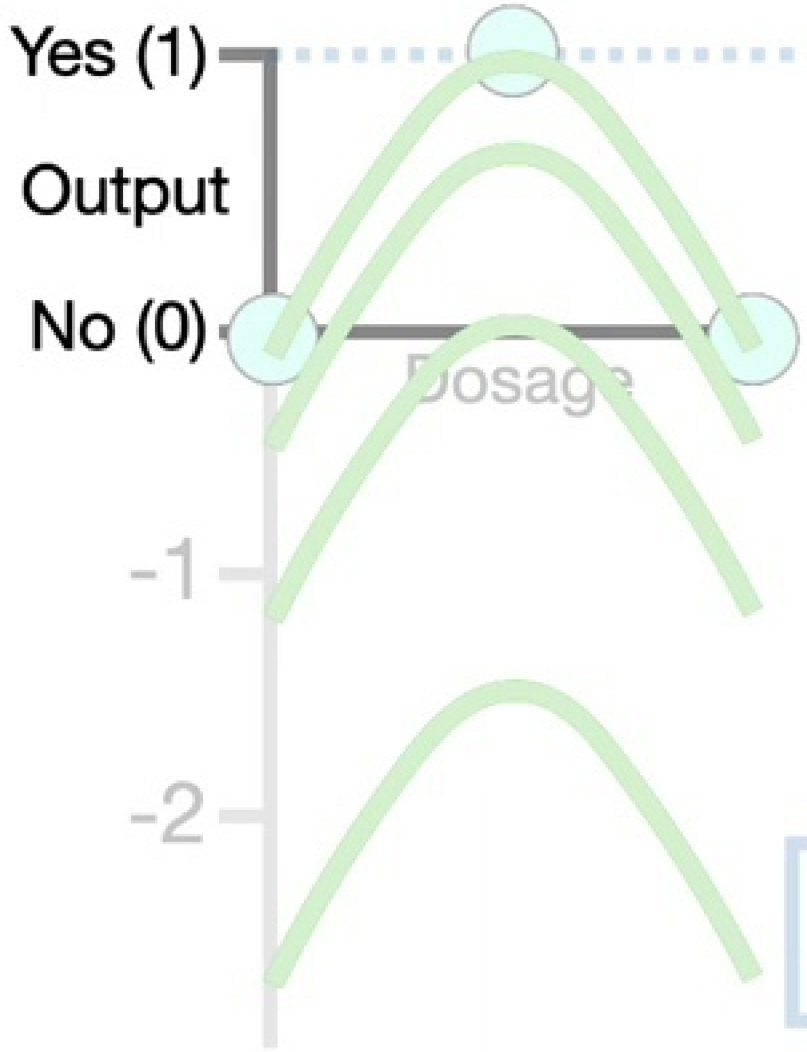
Step Size = close to 0 × 0.1 $\boxed{= \text{close to } 0}$

New $\boldsymbol{b_3}$ **= Old** $\boldsymbol{b_3}$ **- Step Size**



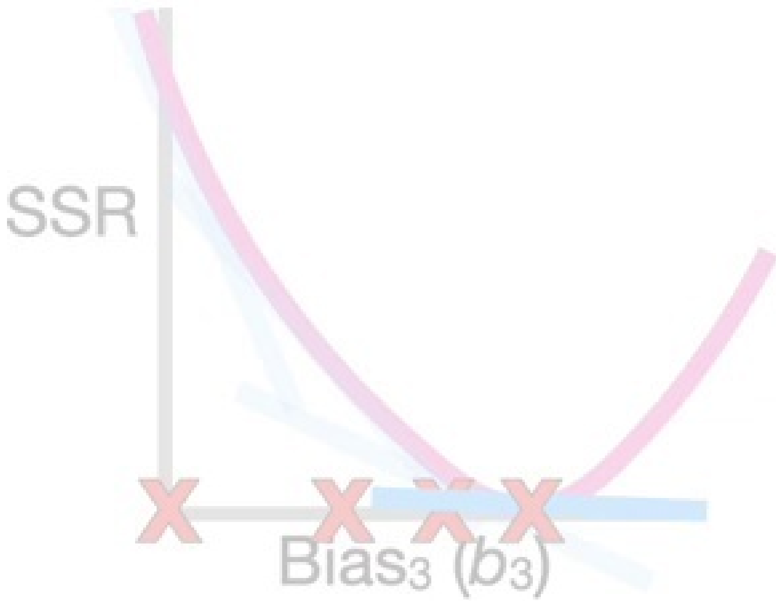
$$\frac{d\ SSR}{d\ b_3} = -2 \times (0 - 0.01) + -2 \times (1 - 1.01) + -2 \times (0 - 0) = \text{close to } 0$$

…we decide that **2.61** is the optimal value for ***b*₃**.

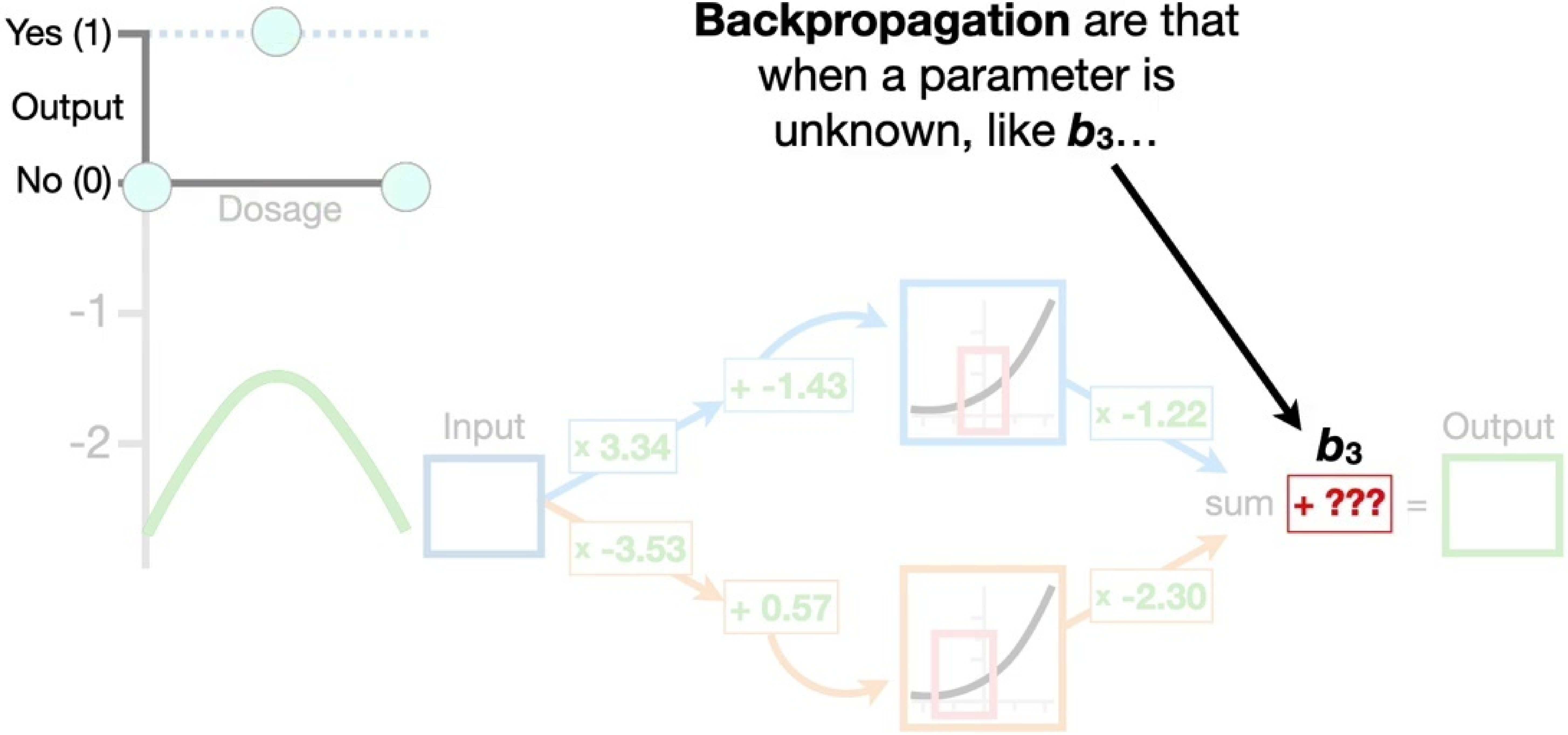


Step Size = close to 0 × 0.1 $\boxed{= \text{close to } 0}$

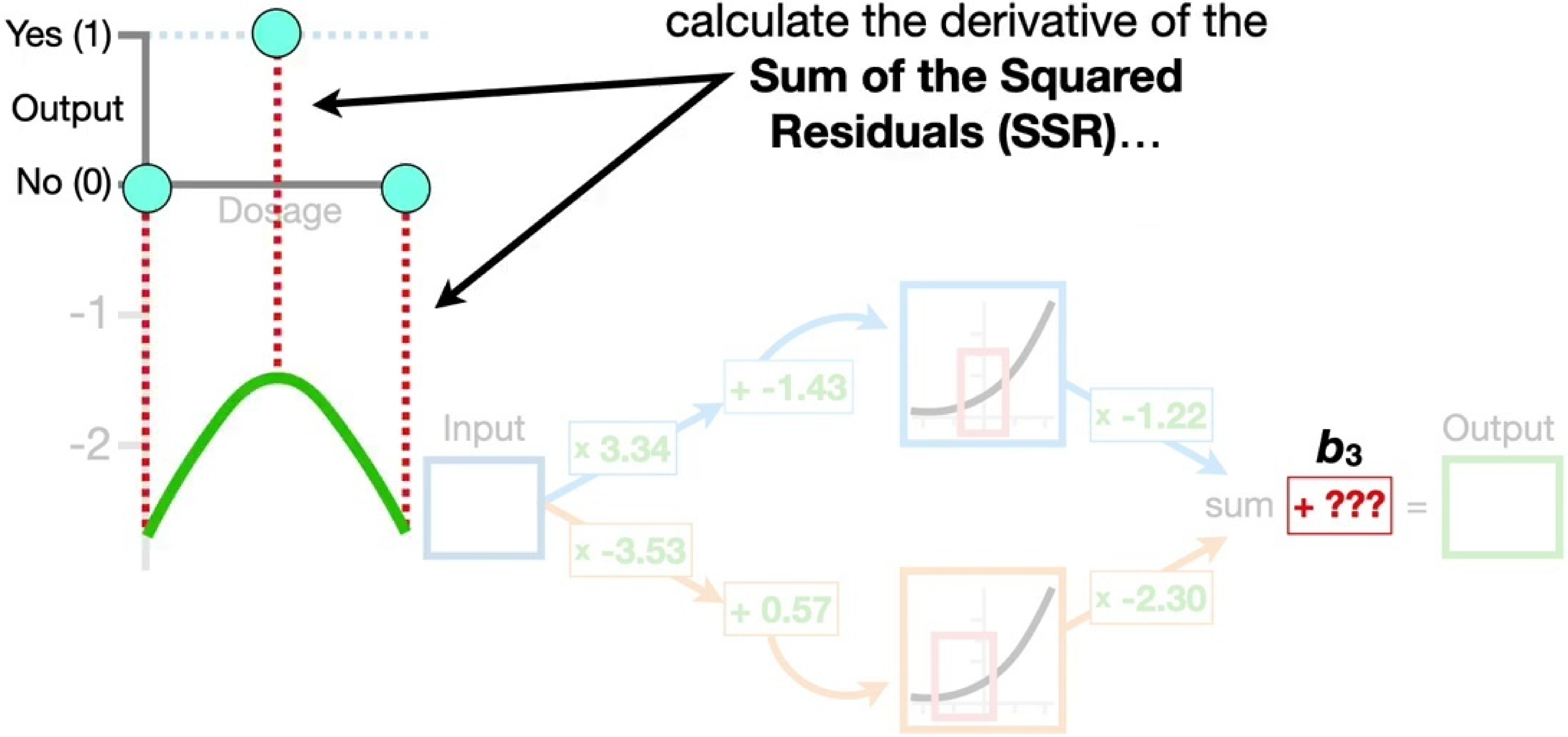
New b_3 = Old b_3 - Step Size



So the **Main Ideas** for **Backpropagation** are that when a parameter is unknown, like b_3…

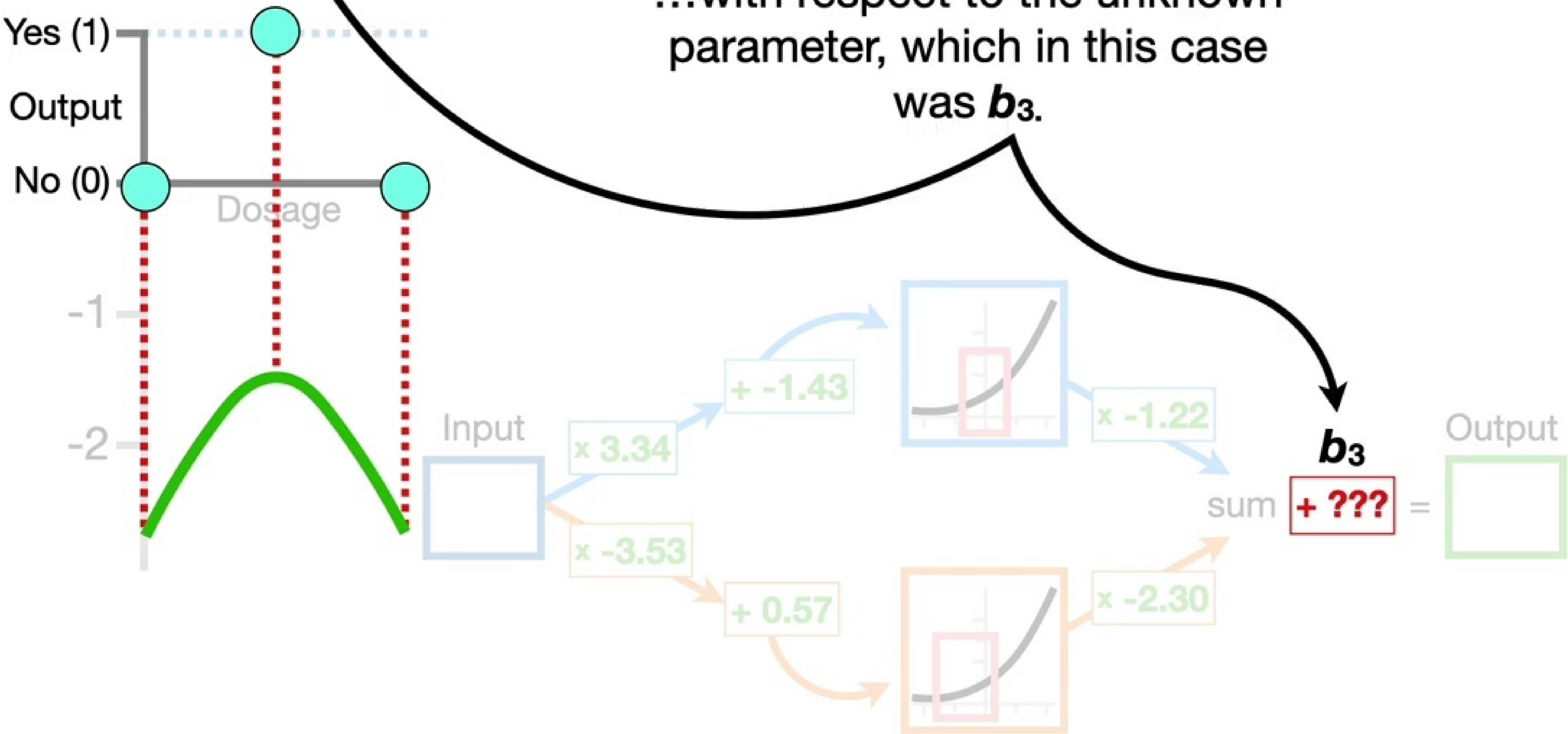


…we use **The Chain Rule** to calculate the derivative of the **Sum of the Squared Residuals (SSR)…**



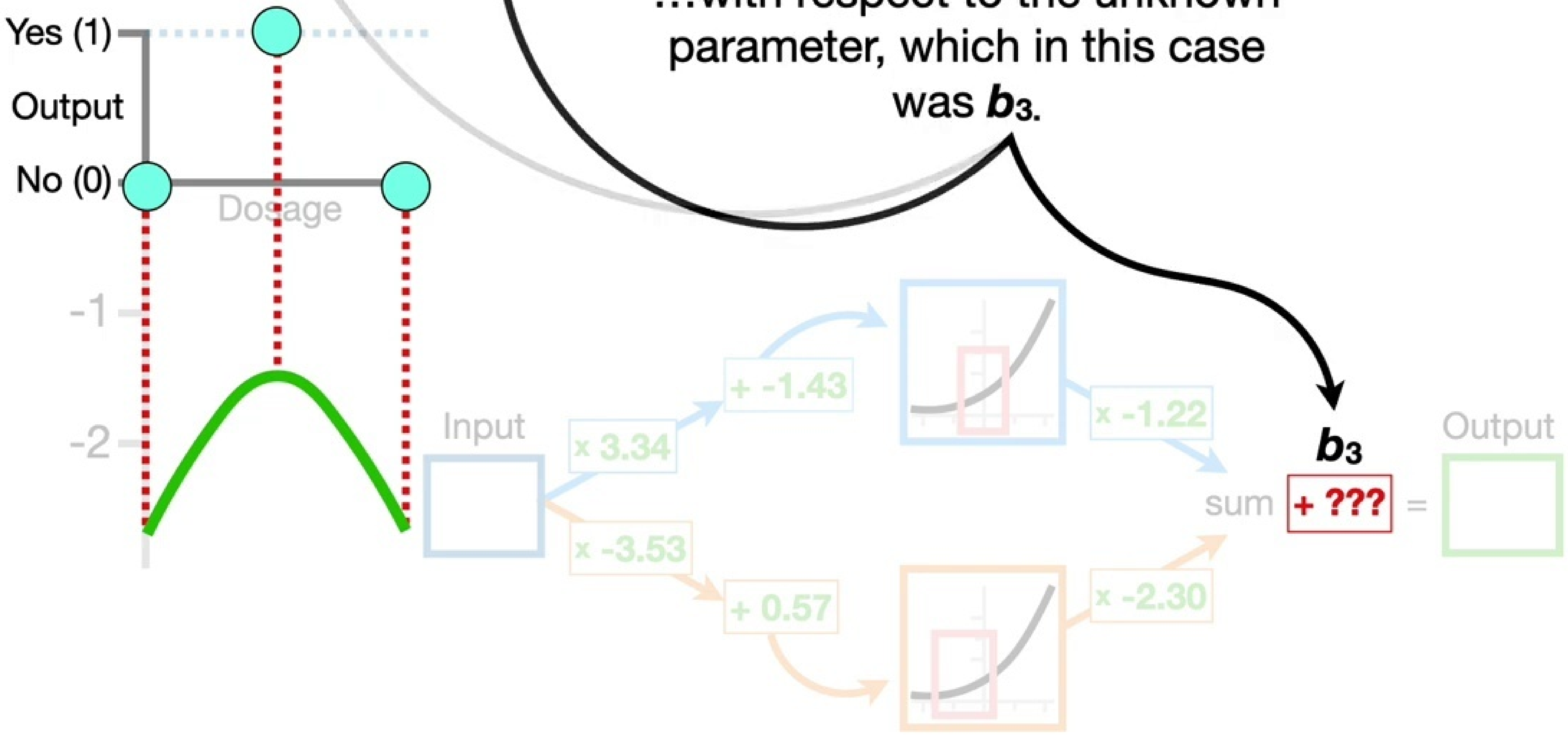
$$\frac{d\ SSR}{d\ b_3}$$

...with respect to the unknown parameter, which in this case was b_3.



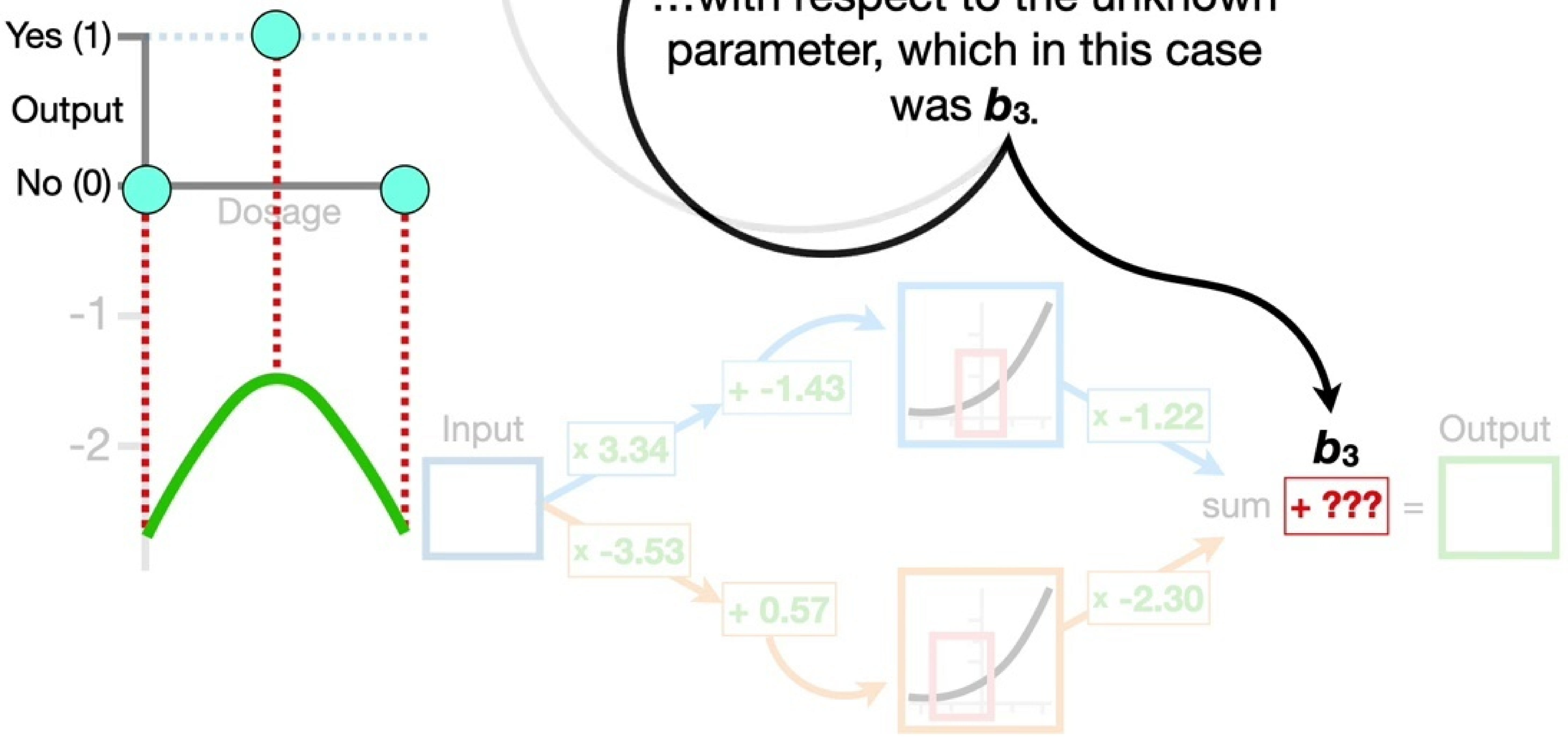
$$\frac{d\ SSR}{d\ b_3} = \frac{d\ SSR}{d\ Predicted}$$

…with respect to the unknown parameter, which in this case was b_3.



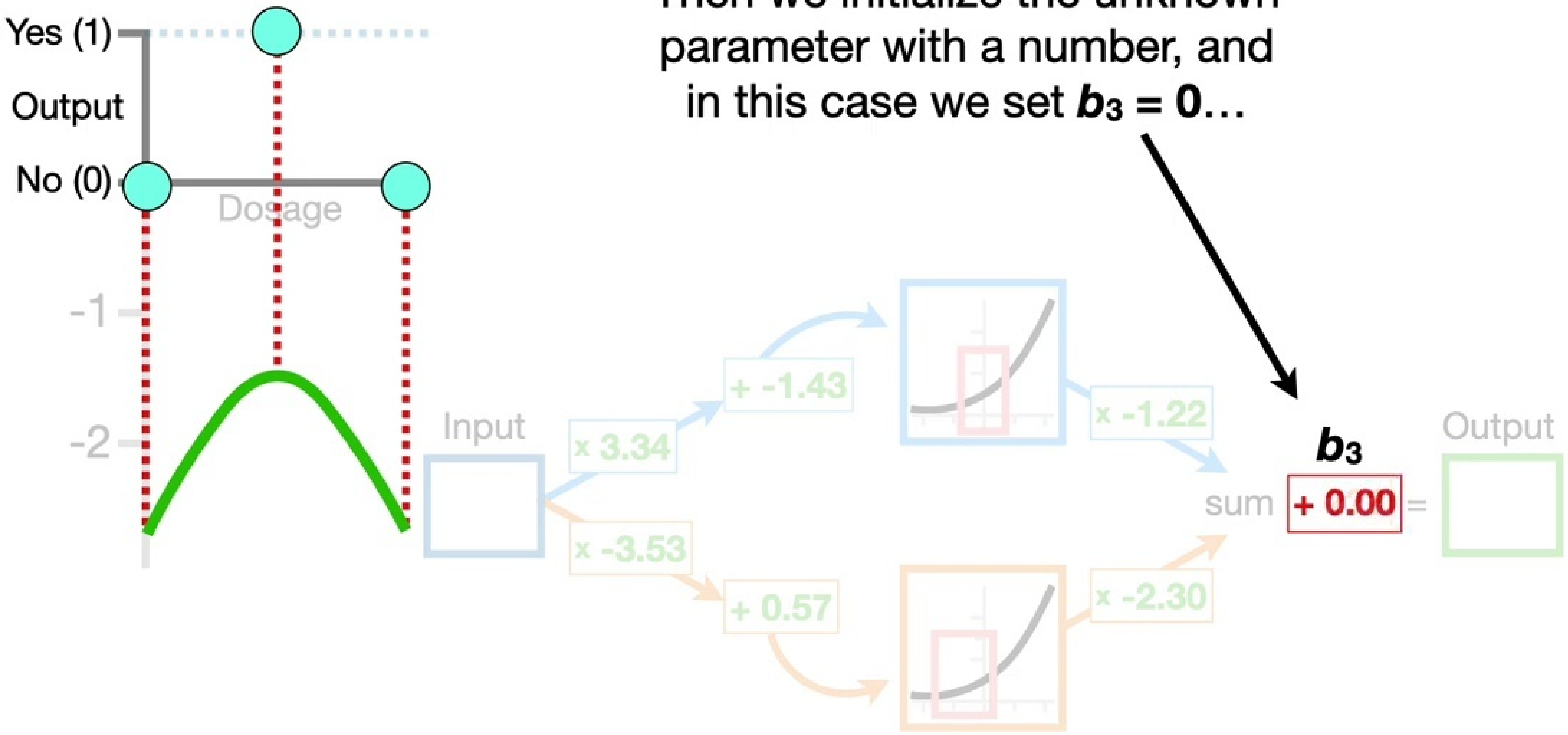
$$\frac{d\ SSR}{d\ b_3} = \frac{d\ SSR}{d\ Predicted} \times \frac{d\ Predicted}{d\ b_3}$$

…with respect to the unknown parameter, which in this case was b_3.



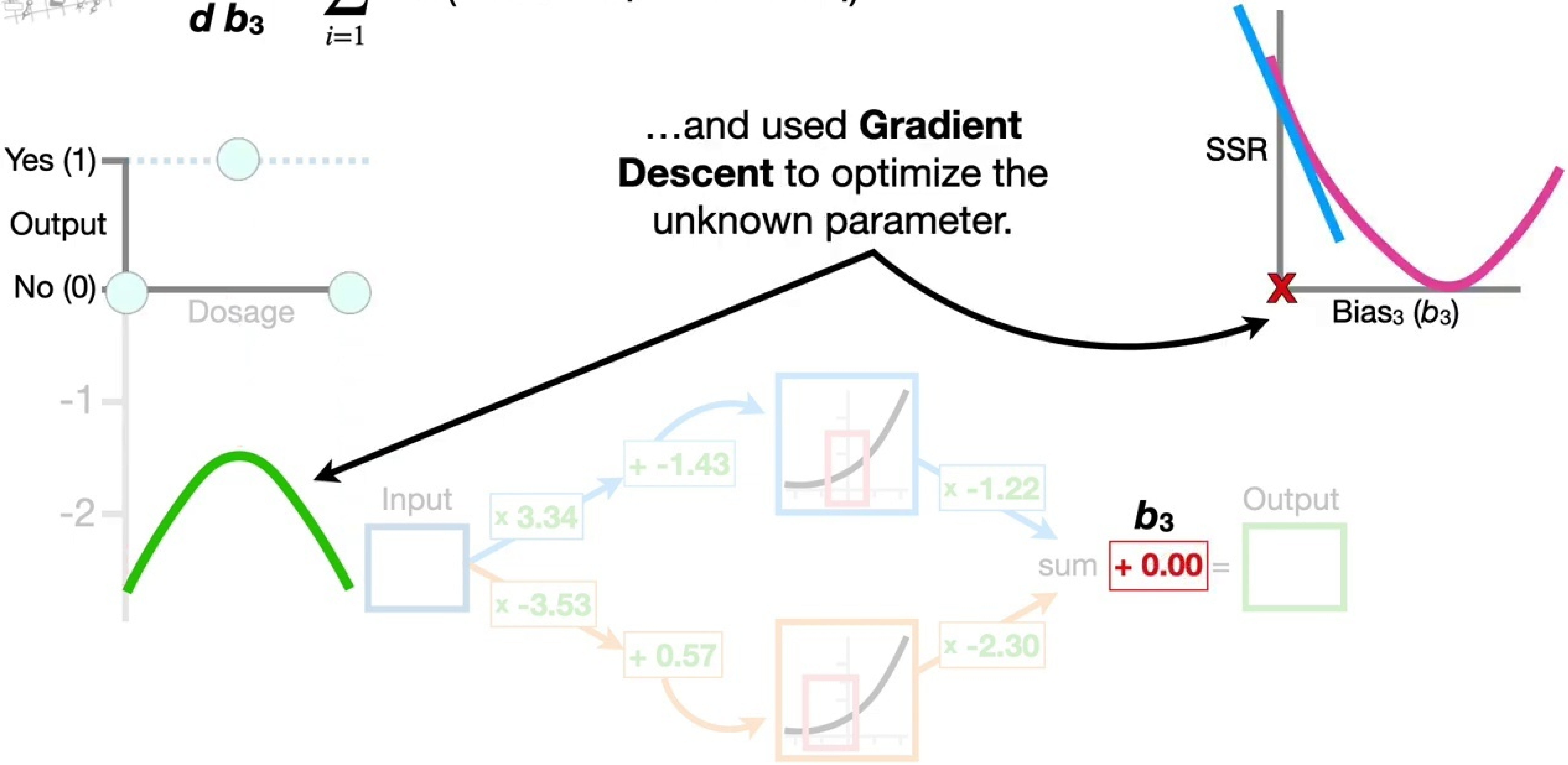
$$\frac{d\ SSR}{d\ b_3} = \sum_{i=1}^{n=3} -2 \times (\text{Observed}_i - \text{Predicted}_i) \times 1$$

Then we initialize the unknown parameter with a number, and in this case we set $b_3 = 0$…



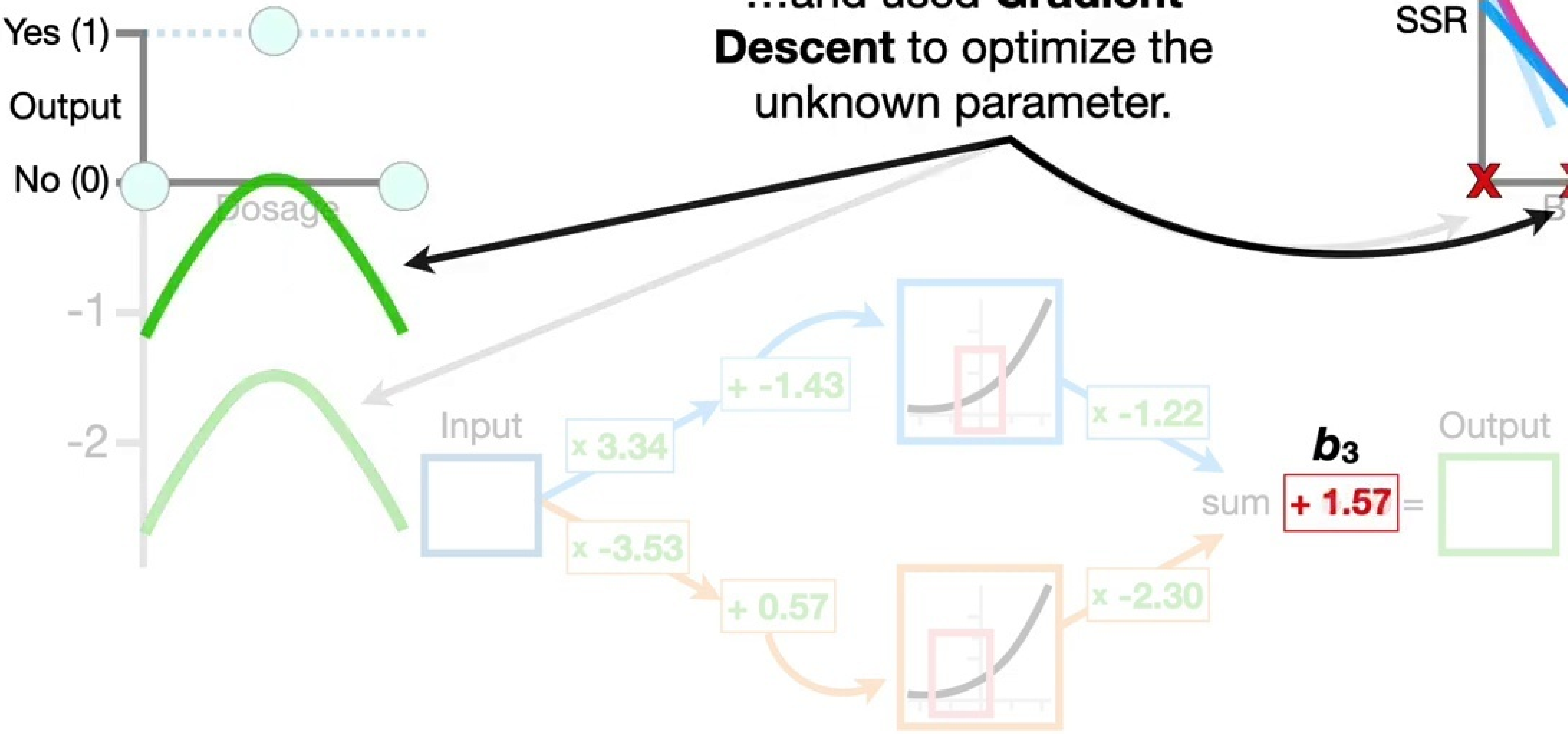
$$\frac{d\ SSR}{d\ b_3} = \sum_{i=1}^{n=3} -2 \times (\text{Observed}_i - \text{Predicted}_i) \times \mathbf{1}$$

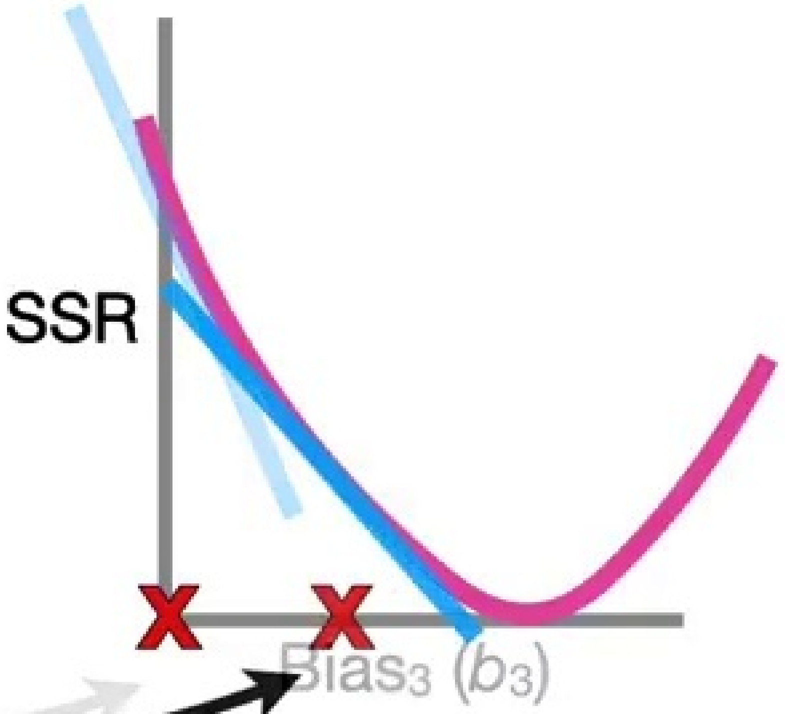
...and used **Gradient Descent** to optimize the unknown parameter.



$$\frac{d\ SSR}{d\ b_3} = \sum_{i=1}^{n=3} -2 \times (\text{Observed}_i - \text{Predicted}_i) \times \mathbf{1}$$

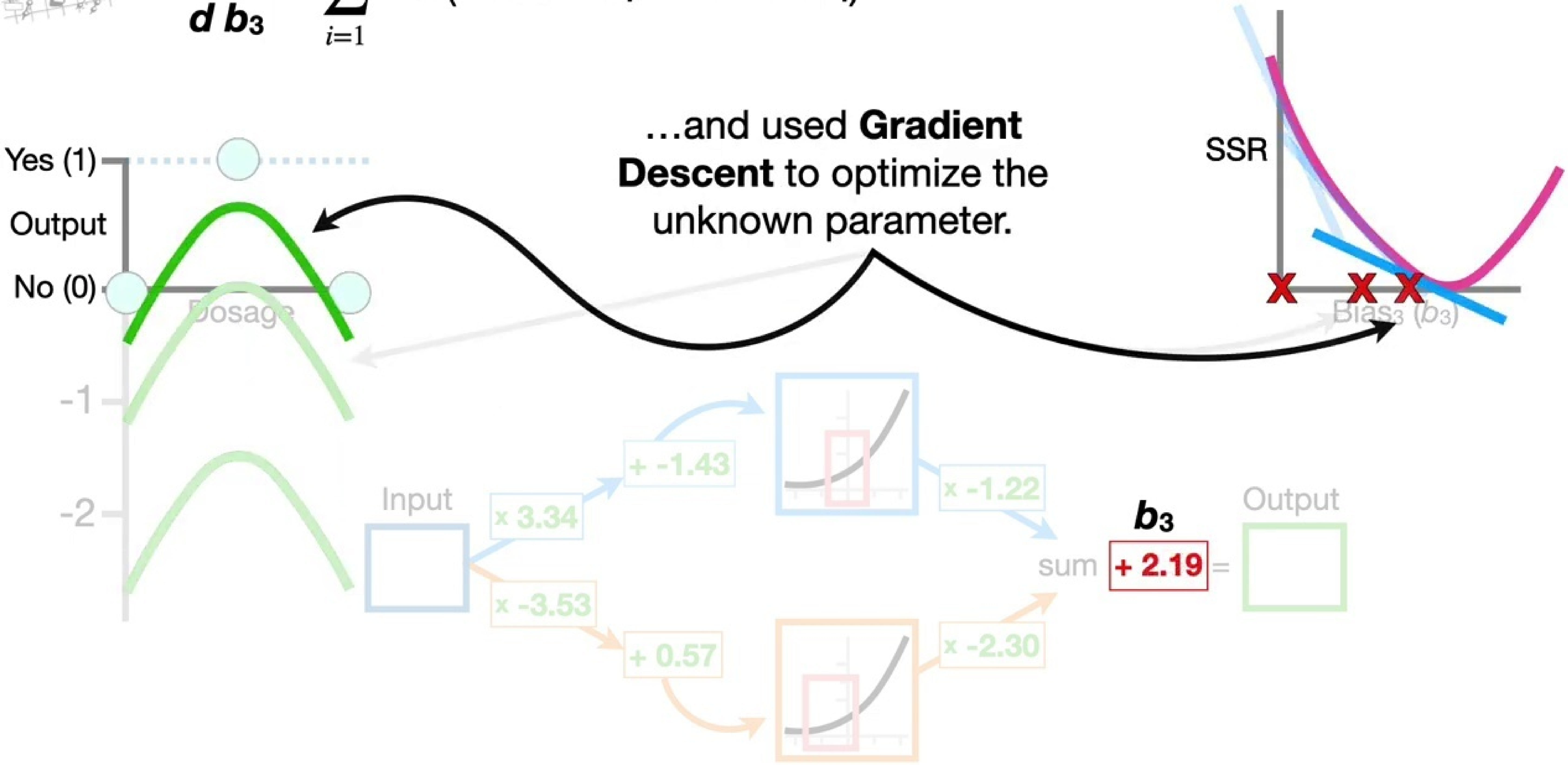
...and used **Gradient Descent** to optimize the unknown parameter.





$$\frac{d\ SSR}{d\ b_3} = \sum_{i=1}^{n=3} -2 \times (\text{Observed}_i - \text{Predicted}_i) \times 1$$

…and used **Gradient Descent** to optimize the unknown parameter.



$$\frac{d\ SSR}{d\ b_3} = \sum_{i=1}^{n=3} -2 \times (\text{Observed}_i - \text{Predicted}_i) \times \mathbf{1}$$

...and used **Gradient Descent** to optimize the unknown parameter.