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Topics: Conditional Probability, Total Probability Theorem, Bayes' rule, Multiplication Rule, Independence, reliability, and counting

Homework and Exam Grading "Philosophy"

Answering a question is not just about getting the right answer, but also about **communicating** how you got there. This means that you should carefully define your model and notation, and provide a step-by-step explanation on how you got to your answer. Communicating clearly also means your homework should be **neat**. Take pride in your work it will be appreciated, and you will get practice thinking clearly and communicating your approach and/or ideas. To encourage you to be neat, if you homework or exam solutions are sloppy you **may NOT get full credit** even if your answer is correct.

Q. 1 What is the probability that at least one of a pair of fair six sided dice lands on 6, given that the sum of the dice is 9.

Q. 2 (**Hat Problem**) Alice and Bob play a game in which a hat is placed on their head. The hats can be one of two possible colors, say black or white at random. Each player can only see the color of the other player's hat, but not his own. The aim of the game is to guess the color of your own hat. To win the game they both have to guess their own hat color correctly.

1. Suppose both players guess the color of their hat independently . What is the probability that they win.
2. Although the players cannot communicate during the game, they can agree on a strategy at the start of the game. Suppose the two players agree on guessing based on the assumption that they have the same color hat. What is the probability that they win.

Q. 3 We have two coins; one is fair and the second two headed. We pick one of the coins at random , we toss it twice and head shows both times. Find the probability that the coin picked is fair.

Q. 4 (**Base rate fallacy**) Suppose α percent of a certain population suffer from a serious disease. A person suspected of the disease is given two conditionally independent tests. Each test makes a correct diagnosis 90 percent of the time. i.e., if person has the disease the test is positive with probability 0.9 and if the person does not have the disease the test results are negative with probability 0.9.

1. For $\alpha = 0.1$, find the probability that the person has the disease given that both tests are positive.
2. Explain why the porbability is less even though the tests make a correct diagnosis 90 percent of the time.
3. Suppose $\alpha = 10$, what do you think will happen to the probability that the person has the disease given that both tests are positive.
4. Are the two tests independent? Explain.

Q. 5 At the end of each day Professor Plum puts her glasses in her drawer with probability .90, leaves them on the table with probability .06, leaves them in her briefcase with probability 0.03, and she actually leaves them at the office with probability 0.01. The next morning she has no recollection of where she left the glasses. She looks for them, but each time she looks in a place the glasses are actually located, she misses finding them with probability 0.1, whether or not she already looked in the same place. (After all, she doesnt have her glasses on and she is in a hurry.)

1. Given that Professor Plum didnt find the glasses in her drawer after looking one time, what is the conditional probability the glasses are on the table?

2. Given that she didnt find the glasses after looking for them in the drawer and on the table once each, what is the conditional probability they are in the briefcase?

3. Given that she failed to find the glasses after looking in the drawer twice, on the table twice, and in the briefcase once, what is the conditional probability she left the glasses at the office?

Note : Assume conditional independence of each search i.e., if A is the event that the glasses were not found after the first drawer search and B is the event that the glasses were not found after the second drawer search, then we have $P(A \cap B|C) = P(A|C)P(B|C)$, if $P(C) > 0$.

Q. 6 (**Probability and Combinatorial Analysis**) A committee consisting of three electrical engineers and three mechanical engineers is to be formed at random from a group of seven electrical engineers and five mechanical engineers. Find the probability of the following events:

1. one particular electrical engineer must be on the committee.

2. two particular mechanical engineers cannot be together on the committee.

Q. 7 (**Independent vs. mutually exclusive**)This problem is aimed to help you refine your basic understanding of independence.

1. Suppose that an event E is independent of itself. Show that either $P(E) = 0$ or $P(E) = 1$.

2. Events A and B have probabilities $P(A) = 0.3$ and $P(B) = 0.4$. What is $P(A \cup B)$ if A and B are independent? What is $P(A \cup B)$ if A and B are mutually exclusive?

3. Now suppose that $P(A) = 0.6$ and $P(B) = 0.8$. In this case, could the events A and B be independent? Could they be mutually exclusive?

4. If A and B are independent, show that A^c and B are also independent.

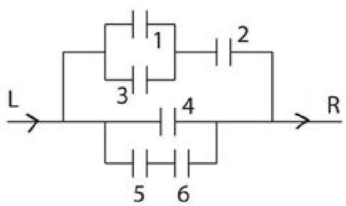
Q. 8 Suppose Bob has n keys, of which one will open his office door.

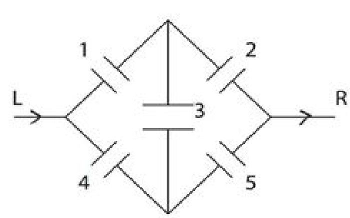
1. Suppose Bob tries the keys at random, discarding those that do not work, what is the probability that he will open the door on his kth try?

2. What is the probability that he will open the door if he does not discard previously tried keys?

Q. 9 In the two networks shown in figure below, assume that the probability of each relay being closed is p and that each relay is open or closed independently of any other relay. In each case find the probability that the current flows from L to R.

(1) (2)





(Hint : There are many ways of solving second network problem, one of them is to condition on Relay 3 and then use Total Probability Theorem.)