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Topics:Multiple continuous, RVs, expectations, conditional expectation, total probability, derived distributions

Q. 1 Random variables X and Y are described by the joint PDF

$$f_{X,Y}(x,y) = ax \quad \text{for } 2 \leq x \leq 4, \quad 0 \leq y \leq x$$

and zero otherwise.

1. Evaluate the constant a.
2. Determine the marginal PDF $f_Y(y)$
3. Determine the expected value for $\frac{1}{X}$ given that $Y = 3$

Q. 2 Suppose X and Y are described by the joint PDF

$$f_{X,Y}(x,y) = \begin{cases} 5x^2/2 & -1 \leq x \leq 1,\ 0 \leq y \leq x^2 \\ 0 & \text{otherwise} \end{cases}$$

Let A be the event $\{Y \leq 1/3\}$.

1. What is the conditional joint density $f_{X,Y|A}(x,y)$?
2. What are $f_{Y|A}(y)$ and $f_{X|A}(x)$?
3. What are $E[Y|A]$ and $E[X|A]$?

Q. 3 Suppose X has pdf:

$$f_X(x) = \begin{cases} \frac{1}{2}x & \text{if } 0 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

X denotes the length of a stick. Now suppose the stick is randomly broken and let Y denote the length of the remaining stick.

1. Find the joint PDF of Y and X.
2. Find the marginal PDF of Y.
3. Find $E[Y]$.

Q. 4 A customer entering a store is served by clerk i with probability p_i, $i = 1, \ldots, n$. The time taken by clerk i to serve a customer is an exponentially distributed random variable with parameter α_i.

1. Find the pdf of T, the time taken to service a customer.
2. Find $E[T]$ and $\text{Var}(T)$. You should find expressions in terms of p_i and α_i, $i = 1, \ldots, n$
3. Suppose $T > 5$. Find an expression for the probability that clerk i served the customer. Hint: You will need to use a version of Bayes Rule.

Q. 5 Suppose $X = e^Y$ where $Y \sim \text{Normal}(\mu, \sigma^2)$, i.e.,

$$f_Y(y) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(y-\mu)^2}{2\sigma^2}} \text{for} - \infty < y < \infty,$$

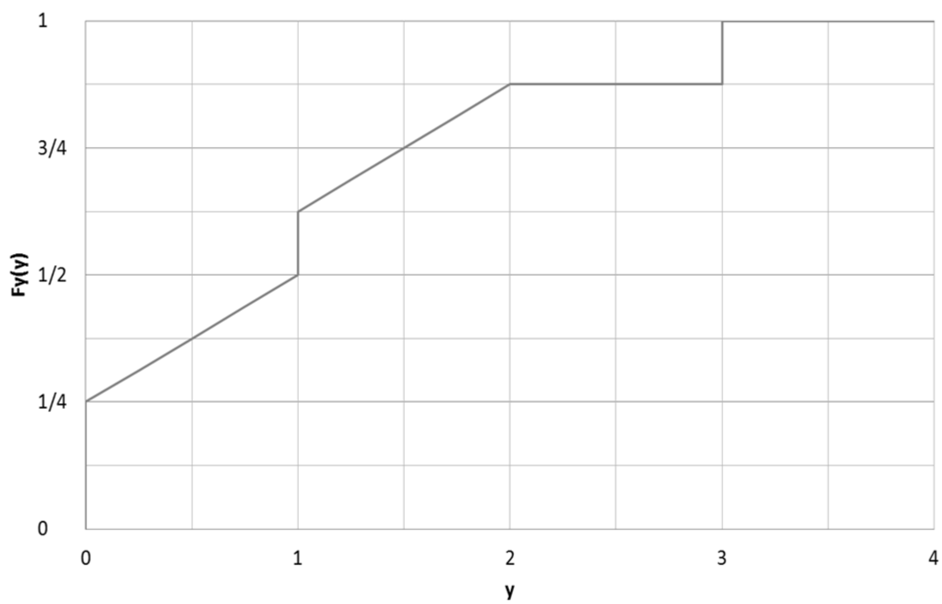
then X is said to have a log normal distribution with parameters μ, σ^2 , denoted $X \sim Lognormal(\mu, \sigma^2)$. Determine the PDF of X.

Q. 6 A Mixed RV is a "mixture" of a discrete and a continuous RV. For example, suppose X is discrete with CDF $F_X(x)$ and Y is continuous with CDF $F_Y(y)$. Define:

$$Z = \begin{cases} X & w.p.\ \alpha \\ Y & w.p.\ (1-\alpha) \end{cases} \quad \text{where } \alpha \in [0,1].$$

Z is said to be a Mixed RV.

1. Suppose $X \sim \text{Bernoulli}(p)$, $Y \sim \exp(\lambda)$, and $\alpha = \frac{1}{2}$. Find the CDF of Z.
2. Suppose Z has CDF $F_Z(z)$:



find F_X discrete, F_Y continuous, and α for Z.

Q. 7 Jane goes to the bank to make a withdrawal, and is equally likely to find 0 or 1 customers ahead of her. The service time of the customer ahead, if present, is exponentially distributed with parameter λ What is the CDF of Jane's waiting time?