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Topics: Continuous Random Variables, CDF (discrete and continuous), Normal Random Variables, Joint PDF

Q. 1 Let Z be a continuous random variable with probability density function

$$f_Z(z) = \begin{cases} c(1+z^2), & \text{if} -2 < z < 1 \\ 0, & \text{otherwise} \end{cases}$$

1. For what value of c is this possible?
2. Find the cumulative distribution function of Z.

Q. 2 Let X and Y be Gaussian random variables, with $X \sim N(0,1)$ and $Y \sim N(1,4)$.

1. Find $P(X \leq 1.5)$ and $P(X \leq -1)$.
2. What is the distribution of $\frac{Y-1}{2}$.
3. Find $P(-2 \leq Y \leq 1)$.
4. Find $P(|Y|^2 < 1.5)$.

Q. 3 Suppose that the cumulative distribution function of X is given by

$$F_X(b) = \begin{cases} 0 & b < 0 \\ \frac{b}{4} & 0 \leq b < 1 \\ \frac{1}{2} + \frac{b-1}{4} & 1 \leq b < 2 \\ \frac{11}{12} & 2 \leq b < 3 \\ 1 & 3 \leq b \end{cases}$$

1. Find $P(X = i)$, for $i = 1, 2, 3$.
2. Find $P(\frac{1}{2} < X < \frac{2}{3})$.

Q. 4 A machine produces bolts the length of which obeys a normal distribution with mean 5 and standard deviation 0.2. A bolt is called defective if its length is not within a standard deviation of its mean.

1. What is the probability that a bolt produced by this machine is defective?
2. What is the probability that among ten bolts none will be defective?

Q. 5 A random variable X is uniformly distributed between 0 and 10. Find the probability that X lies between a standard deviation σ_X from its mean μ_X.

Q. 6 Two random variables X and Y have the joint PDF given by

$$f_{XY}(x,y) = \begin{cases} ke^{-(2x+3y)}, & \text{x} \geq 0, y \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

1. Find the value of the constant k that makes $f_{XY}(x,y)$ a true joint PDF.
2. Find the marginal PDFs of X and Y.
3. Find $P(X < Y < 2)$
4. Without actually performing the integration, obtain the integral that expresses the $P(X \leq Y^2)$ (That is, just give the exact limits of the integration.)
5. Find $E[X+Y]$.

Q. 7 Show that for a positive continuous Random Variable X, $E[X] = \int P(X > x)dx$