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Topics Covered:Introduction, Set Theory, Probability Axioms, countable and uncountable sets

Homework and Exam Grading "Philosophy"

Answering a question is not just about getting the right answer, but also about **communicating** how you got there. This means that you should carefully define your model and notation, and provide a step-by-step explanation on how you got to your answer. Communicating clearly also means your homework should be **neat**. Take pride in your work it will be appreciated, and you will get practice thinking clearly and communicating your approach and/or ideas. To encourage you to be neat, if you homework or exam solutions are sloppy you **may NOT get full credit** even if your answer is correct.

Occasionally I will give additional problems on a homework which will not be graded. These are intended as extra practice which in my experience many of you will ask for anyway.

Q. 1 Write down the sample space Ω for the following experiments :

1. A six sided die is rolled and the outcome is observed.
2. The die is rolled sequentially until the first six appears.

Q. 2 A power cell consists of two subcells, each of which can provide voltage from 0 to 5 volts, regardless of what the other subcell provides. The power cell is functional if and only if the sum of the two voltages of the subcells is at least 6 volts. An experiment consists of measuring and recording the voltages of the two subcells.

1. Suggest a sample space Ω for the experiment.
2. Using your proposed sample space, what is the event that the power cell is not functional but needs less than one additional volt to become functional.

Q. 3 We will often use notation like $A := \{(x,y) : 0 \leq x,y \leq 1\}$ which means A is set of all pairs (x,y) such that $0 \leq x \leq 1$ and $0 \leq y \leq 1$. Note that the values are assumed to be real valued unless otherwise specified. To develop a better understanding of the notations do sketch the following subsets of the $x-y$ plane.

1. $A_z := \{(x,y) : x^2+y^2 \leq z^2\}$ for $z=5$
2. $B_z := \{(x,y) : (x,y) \in \mathbb{Z}, x>0, y>0$ and $x+y \geq z\}$ for $z=5$
3. $C := \{(x,y) : 0 \leq y \leq x \leq 1\}$
4. $A_5 \cap B_3$
5. $D_z := \{(x,y) : x \leq z\}$ for $z=3$
6. $B_z \cup D_z$ for $z=3$

Q. 4 The following group of students are in a class: 24 male students of age over 18 , 16 female students of age over 18, 12 male students of age under 18 and 8 female students of age under 18. One student is choosen at random and the following events are defined $A =$ Student is male, $B =$ Student is female , C= Student is of age over 18 and $D =$ Student is of age under 18. Evaluate the following :

1. $P(A \cup C)$.
2. $P(B^c \cap D^c)$.

Q. 5 Two events A and B have the following probabilities: $P(A) = 0.6, P(B) = 0.5$ and $P(A \cap B) = 0.3$. Calculate the following

1. $P(A \cup B)$.
2. $P(A^c \cup B^c)$.

Q. 6 Consider rolling a six-sided die. Let A be the set of outcomes which are divisible by three. Let B be the set of outcomes which are prime numbers.

1. Show that $(A^c \cup B^c)^c \cup (A^c \cup B)^c = A$. (Hint : Using De Morgan's Law)
2. Calculate and compare the sets on both sides of the previous equation.

Q. 7 For any three events A ,B and C. Show that

1. $P(A \cap B) \geq P(A) + P(B) - 1$.
2. $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$

Q. 8 In class we informally defined a set as being *countable* if the elements of the set could be enumerated say as $a_1, a_2, \ldots,$. In other words, you can find an exhaustive way to list them. This problem is intended to help you refine your understanding of "countable" sets. You must provide clear succinct, preferably mathematical arguments.

1. Show that the positive rational numbers are countable – here we stick to positive rationals for simplicity. Recall that rational numbers are numbers that can be represented at the ratio of two integers, say i/j where $i, j \in \{1, 2, 3, \ldots\}$. Hint: Find a way to visualize the rationals as a 2 dimensional array, then think about how you can linearly enumerate the elements in that array.
2. Suppose A and B are countable. Argue that the union $A \cup B$ is also countable.
3. Consider the set A containing all infinite sequences of 0s and 1s, e.g., $\mathbf{a} = 10101010101....$ is an element in A. Argue that A is **not** countable. Hint: You can argue this by contradiction. Suppose you had an enumeration of all elements (sequences) in A, i.e., $\mathbf{a}_1, \mathbf{a}_2, \ldots$. Construct a sequence $\mathbf{a}^*$ which is not in the list. For example, $\mathbf{a}^*$ may differ from $\mathbf{a}_i$ in the ith digit for each $i = 1, 2, \ldots$ This is called a **diagonal argument.**
4. Based on the claim made in the previous question argue that the real numbers in $[0, 1]$ are uncountable.

Q. 9 My goal in this class is to serve as your guide in learning the fundamental of probability and statistics and applications to engineering, but also to bring what you will learn what you will learn in my class into your daily life. To that end this last problem requires that you watch the following video http://www.gapminder.org/videos/the-joy-of-stats/ and answer the following questions.

1. According to the program what is the origin of the term "statistics" ?
2. The program discusses correlation. Describe in your own words what they mean by correlation ?
3. The program explains a new paradigm for translation from one language to another. Explain in just a few sentences the previous versus the new approach.
4. The program also explains a new paradigm that involves using simulation to generate (even more) data. Explain how this kind of simulated data can/is be used.