M368K
Homework #1

Burden and Faires.

Section 7.1 (#1a, 4c, 12). Section 7.2 (#6[1]ae, 19[2]). Section 7.3 (#2a, 4a, 10[3]ac, 14).

[1] Indicate if the matrix is convergent.

[2] For simplicity, assume all eigenvalues/vectors of A are real. Verify result for the eigenvalues of $A = \begin{bmatrix} -1 & 4 \\ 1 & 2 \end{bmatrix}$ using the ∞-norm.

[3] Would Jacobi converge to the exact solution for arbitrary $x^{(0)}$? What about Gauss-Seidel?

Programming mini-project.

Consider the linear system of equations

$$\begin{array}{rcrcrcrcr} 4x_1 & + & x_2 & - & x_3 & + & x_4 & = & -2 \\ x_1 & + & 4x_2 & - & x_3 & - & x_4 & = & -1 \\ -x_1 & - & x_2 & + & 5x_3 & + & x_4 & = & 0 \\ x_1 & - & x_2 & + & x_3 & + & 3x_4 & = & 1. \end{array} \tag{1}$$

(a) Download the C++ program file `program1.cpp`, function file `jacobi.cpp` and library files `matrix.cpp` and `matrix.h` from the course webpage. These files implement the Jacobi method; read the program file for instructions on how to compile and run.

(b) Use the Jacobi method to solve (1) to within a tolerance of $||r^{(k)}||_\infty < 10^{-3}$, beginning from an initial guess $x^{(0)} = 0$, where $r^{(k)} = b - Ax^{(k)}$. Report the approximate solution $x^{(k)}$ and number of iterations k required.

(c) Write a function `gauss_seidel.cpp` to implement the Gauss-Seidel method. Repeat part (b) using this method with $x^{(0)} = 0$ and briefly compare results. Which method converges faster for this system? Do you get the same solution and convergence speed for different choices of $x^{(0)}$?

Turn in: responses to (b), (c) and a copy of `gauss_seidel.cpp`.