M368K
Homework #9

Burden and Faires.

Section 11.1 (#2a[1],9). Section 11.2 (#4a[2]).

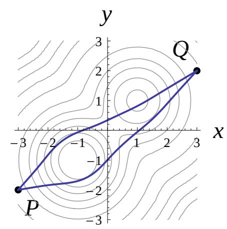
[1,2]Find approx soln using one step of Secant-Euler method discussed in class with $N = 2$, $t_0 = \frac{\beta-\alpha}{b-a}$, $t_1 = 1.1t_0$. Compare approx soln associated with t_2 to exact soln at each node.

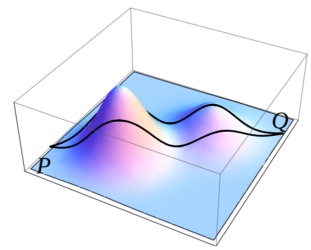
Programming mini-project.

A cartographer is surveying a region of hills described by a graph $z(x, y)$, and he is interested in finding the shortest path from P to Q over the terrain; such a path is called a geodesic. If we describe the path by a curve $y(x)$ in a contour map of the region, then any candidate for the shortest path must satisfy the Euler-Lagrange boundary-value problem

$$y'' = \frac{(py' - q)[u + 2vy' + wy'y']}{1 + p^2 + q^2}, \quad -3 \le x \le 3$$

$$y(-3) = -2, \quad y(3) = 2$$





where $p = z_x$, $q = z_y$, $u = z_{xx}$, $v = z_{xy}$ and $w = z_{yy}$ are functions of x and y. Here we use a shooting method to find two candidates for the shortest path from P to Q with coordinates $(-3, -2)$ and $(3, 2)$ in the map.

(a) Download the C++ files `program9.cpp` and `shootRK4.cpp` from the course webpage. These files implement the Newton-RK4 shooting method for general, two-point boundary-value problems $y'' = f(x, y, y')$, $a \le x \le b$, $y(a) = \alpha$ and $y(b) = \beta$; read the files for instructions on how to use.

(b) Consider the case when the terrain is planar, described by a graph $z = Ax + By + C$. Show analytically that the only solution of the BVP is a straight line in the map, and hence on the terrain.

(c) The terrain shown in the figure is described by the graph $z = e^{-(x+1)^2-(y+1)^2} + 0.5e^{-(x-1)^2-(y-1)^2}$. For this case, use the Newton-RK4 code with $N = 20$ to find two solutions of the BVP as illustrated above; for each you will need to make an appropriate initial guess of the shooting slope t. For each solution curve (x_j, y_j), $j = 0, \ldots, N$ report the first five points and the converged value of the slope t. Also, compute and report the approximate total path length from P to Q by summing the distances between successive points $(x_j, y_j, z(x_j, y_j))$ on the terrain. Which of the two paths illustrated above (upper or lower) is the shortest?

Turn in: modified version of `program9.cpp` used to solve this problem and responses to (b), (c).