M368K
Homework #5

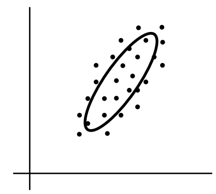
Burden and Faires.

Section 9.1 (#2a, 4b). Section 9.2 (#6c, 7c). Section 9.3 (#2a, 6a).

Programming mini-project.

In statistics, the distribution of a set of points $x^{(k)} \in \mathbb{R}^n$ $(k = 1, \ldots, m)$ can be described in terms of a mean vector $\mu \in \mathbb{R}^n$ and a symmetric, positive-definite covariance matrix $A \in \mathbb{R}^{n \times n}$ defined as

$$\mu_i = \frac{1}{m}\sum_{k=1}^{m} x_i^{(k)}, \quad A_{ij} = \frac{1}{m}\sum_{k=1}^{m}(x_i^{(k)} - \mu_i)(x_j^{(k)} - \mu_j).$$



The vector μ and the eigenpairs (λ, v) of A define the standard-deviation ellipsoid for the data: μ defines the center, and each v defines an axis of the ellipsoid with corresponding radius $\sqrt{\lambda}$. Here we find the center and maximum radius for a set of data $x^{(k)} \in \mathbb{R}^4$ $(k = 1, \ldots, 100)$ using different power methods.

(a) Download the C++ program file `program5.cpp`, the data file `program5.dat` and the function file `genpower.cpp` from the course webpage. Complete the program so that it computes μ and A for the given data. Report μ and A.

(b) Use the general power method to approximate the most dominant eigenvalue of the matrix A to within a tolerance of $||\lambda^{-1}Ax - x||_\infty < 10^{-4}$. Report the initial vector you used, number of iterations, approximate eigenvalue and approximate unit eigenvector. Do you get the same results for different choices of the initial vector? What is the maximum radius of the ellipsoid?

(c) Write a function `sympower.cpp` to implement the symmetric power method. Repeat part (b) using the same initial vectors, but with the tolerance condition $||\lambda^{-1}Ax - x||_2 < 10^{-4}$. Notice that the symmetric method is based on the 2-norm whereas the general method is based on the ∞-norm. Do you get the same eigenvalue and eigenvector (up to scaling) as in part (b)?

Turn in: responses to (a), (b), (c) and completed versions of `program5.cpp` and `sympower.cpp`.