

## MATH 368K: Numerical Methods for Applications

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### Study guide for Exam #2

#### Eigenvalues/vectors and matrix factorizations (Sec 9.1)

- Defn of eigenvalues and eigenvectors; properties
- Defn of similar, diagonalizable and orthogonal matrices
- Thm on similar matrices
- Thm on diagonalizable matrices
- Spectral thm for symmetric matrices (Schur's thm)
- Gersgorin's circle theorem

#### Power, deflation methods for eigenvalues/vectors of arbitrary matrices (9.2)

- General and symmetric power methods for dominant eigenvalue/vector
- Convergence of general and symmetric methods
- Wielandt deflation thm; deflation of a matrix
- Inverse power method for refining a specific eigenvalue/vector
- Convergence of inverse method
- Combined method for finding all eigenvalues/vectors

#### Similarity methods for eigenvalues/vectors of symmetric matrices (9.3,9.4)

- Spectral thm for symmetric matrices:  $D = Q^T A Q$
- Idea of similarity methods for approximating  $D$
- Householder algo for general  $\rightarrow$  tridiagonal reduction
- QR algo for tridiagonal  $\rightarrow$  diagonal reduction
- Convergence of QR algo

#### Fixed-point methods for systems of nonlinear eqns (10.1)

- Relation between root-finding and fixed-point problems
- Solvability thm for fixed-point problems
- Standard fixed-point algo for fixed-point problem
- Convergence of fixed-point algo; rate of convergence
- Gauss-Seidel version of fixed-point algo

#### Newton-type methods for systems of nonlinear eqns (10.2,10.3)

- Newton algo for root-finding problem
- Props of Newton: idea of algo, rate of convergence
- Broyden algo for root-finding problem
- Props of Broyden: idea of algo, advantages, rate of convergence
- Sherman-Morrison formula

#### Steepest descent method for systems of nonlinear eqns (10.4)

- Relation between root-finding and minimization problems
- Steepest descent algo for minimization problem
- Idea of algo; basic step size selection via search
- Step size improvement via quadratic interpolation
- Rate of convergence; zig-zag issue

#### Continuation or homotopy methods for systems of nonlinear eqns (10.5)

- Relation between root-finding and path-following problems
  - IVP/ODE formulation of path-following problem
  - Fwd Euler and RK4 continuation algos
  - Convergence of Fwd Euler and RK4 continuation
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