# M368K
# Homework #13

Burden and Faires.

Section 12.3 (#2[1]). Section 12.4 (#1[2]).

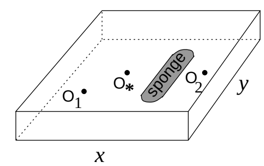
[1]Use the central-difference method with $\Delta x = \frac{1}{8}$ and $\Delta t = \frac{1}{10}$ instead of the given spacings. Explicitly write the discrete equations. Compute up to $t = \frac{3}{10}$; compare approx and exact solns at each node at final time.

[2]Consider the simpler eqns $u_{xx} + 4u_{yy} = 3$ in $D$, $u(x, 0.5) = 2x$ and $u(0, y) = 0$ on $S_1$ and $u_x \cos\theta_1 + 4u_y \cos\theta_2 = (y - x)\sqrt{2}/2$ on $S_2$. Find the finite-element basis functions, arrays and approx soln; report approx soln at each node.

Programming mini-project.

A study is being conducted on how a porous, sponge-like material can be used to attenuate sound waves in a liquid. In an experimental tank, a loud ping is emitted at location $O_*$ and propagates outward in all directions. The sound wave is reflected off the tank walls in some directions, passes through a sponge barrier in others, and is measured by sensors at locations $O_1$ and $O_2$. Considering a 2D model, the amplitude $u(x, y, t)$ of the sound wave is described by

$$\begin{array}{lll}
u_{tt} + su_t = \alpha\Delta u + \eta, & a \le x \le b, & c \le y \le d, \quad 0 \le t \le T \\
u(a, y, t) = 0, \quad u(b, y, t) = 0, & c \le y \le d, & 0 \le t \le T \\
u(x, c, t) = 0, \quad u(x, d, t) = 0, & a \le x \le b, & 0 \le t \le T \\
u(x, y, 0) = 0, & a \le x \le b, & c \le y \le d \\
u_t(x, y, 0) = 0, & a \le x \le b, & c \le y \le d
\end{array}$$




where $\Delta u = u_{xx} + u_{yy}$ is the Laplacian of $u$, $\alpha$ is a sound speed coefficient, $\eta$ is a source function which generates the localized ping, and $s$ is a dissipation coefficient which represents the sponge. Zero boundary and initial conditions are assumed for simplicity. For concreteness, we take $[a, b] = [c, d] = [-3, 3]$, $\alpha \equiv 0.3$, $\eta = 10e^{-30(x-x_*)^2 - 30(y-y_*)^2 - 30(t-t_*)^2}$ and $s = 10\cos^2(\pi w)$ if $0 \le w \le 0.5$ and $s = 0$ if $w > 0.5$, where $w = \sqrt{(x - x_s)^2 + 0.25(y - y_s)^2}$. The ping happens at $(x_*, y_*, t_*) = (-1, 0, 0.5)$, the sponge is centered at $(x_s, y_s) = (1.5, 0)$, the sensor $O_1$ is at $(x, y) = (-1, -1.5)$ and $O_2$ is at $(x, y) = (2, 0)$.

(a) Download the C++ files `program13.cpp` and `ctrdiff2D.cpp` from the course webpage. Complete the file `ctrdiff2D.cpp` so that it implements the central-difference method for initial-boundary-value problems of the type $u_{tt} + su_t = Pu_{xx} + Qu_{yy} + pu_x + qu_y + ru + \eta$ on rectangular domains.

(b) To identify reasonable grid spacings for this problem, we consider the longest time interval of interest $[0, T] = [0, 7]$ and perform some preliminary runs. Using $\Delta x = \Delta y = \frac{1}{20}$, solve the IBVP using $\Delta t = \frac{1}{10}, \frac{1}{15}, \frac{1}{20}, \frac{1}{25}$ and $\frac{1}{30}$ and record the value of $u$ at $(x, y, t) = (0, 2, 7)$ for each time grid. [As a check, the fourth grid should give $u = 0.03558$.] Was a stability threshold crossed? Does $u$ approach a definite value? Repeat using $\Delta x = \Delta y = \frac{1}{25}$ and the same sequence of $\Delta t$. [The final value of $u$ should not change very much, indicating that either of the final spacings is reasonable.]

(c) Using the spacings $\Delta x = \Delta y = \frac{1}{20}$ and $\Delta t = \frac{1}{30}$, solve the IBVP on $[0, T] = [0, 3.5]$, $[0, 5]$ and $[0, 7]$. Use the supplementary file `matlab13.m` to produce contour maps of the sound wave amplitude at each final time. If the objective is to measure the outgoing wave at each sensor, but not any reflections (echos), then would the interval $[0, 7]$ be appropriate? For the interval $[0, 7]$, compute the maximum sound amplitude $I(x, y) = \max_{0 \le t \le T} |u(x, y, t)|$ at the sensor locations $O_1$ and $O_2$. How effective is the sponge barrier at absorbing the wave, specifically, what is the ratio $I_{O_2}/I_{O_1}$?

Turn in: versions of `ctrdiff2D.cpp` and `program13.cpp` used for this problem and responses to (b),(c).