M368K
Homework #2

Burden and Faires.

Section 7.4 (#2a, 9[1]). Section 7.5 (#1d, 2c, 8[2]). Section 7.6 (#6a).

[1]Explain the proof on p.826 by indicating which of the following properties of determinants for $A, B \in \mathbb{R}^{n\times n}$ is used at each step: (i) $\det A = \Pi_{i=1}^{n} \lambda_i$, (ii) $\det(AB) = \det A \det B$, (iii) $\det A = \Pi_{i=1}^{n} A_{ii}$ if A is triangular, (iv) $\det(A^{-1}) = 1/\det A$ if A is invertible. Also, show why $\det(cA) = c^n \det A$ for any $c \in \mathbb{R}$. For simplicity, assume all eigenvalues of T_ω are real.

[2]Compute the relative error in x using $||\cdot||_\infty$ and verify the inequality in Equation (7.25).

Programming mini-project.

Consider a symmetric, positive-definite, tri-diagonal system $Ax = b$ of size $n = 100$ where for $i, j = 1, \ldots, n$ we have

$$A_{ij} = \left\{ \begin{array}{cl} -1 & \text{if } j = i-1 \\ 2 + (i/10) & \text{if } j = i \\ -1 & \text{if } j = i+1 \\ 0 & \text{otherwise} \end{array} \right], \qquad b_i = 1 + (i/20).$$

(a) Download the C++ program file `program2.cpp` and function file `conjgrad.cpp` from the course webpage. These files implement the Conjugate Gradient method; read the program file for instructions on how to compile and run. The files from the previous homework will also be needed.

(b) Use the Conjugate Gradient method with pre-conditioner $C^{-1} = D^{-1/2}$ to solve $Ax = b$ to within a tolerance of $||r^{(k)}||_\infty < 10^{-5}$, beginning from an initial guess $x^{(0)} = 0$. (You should build A and b with a loop.) Report the approximate solution components $x_1^{(k)}, \ldots, x_4^{(k)}$ and number of steps k required, and also the residual norm $||r^{(k)}||_\infty$.

(c) Repeat part (b) using the Jacobi, Gauss-Seidel and SOR ($\omega = 1.2$) methods and briefly compare results. Begin each method from a zero initial guess and use the same tolerance on the residual. (You should set `maxIter` to a sufficiently large value.) Try SOR with different ω values, say $\omega = 1.6$ and $\omega = 2.3$. Does the value of ω have a significant impact on convergence? Try Conjugate Gradient without pre-conditioning ($C^{-1} = I$). Does pre-conditioning make a noticeable difference?

Turn in: responses to (b) and (c), a copy of `sor.cpp`, and a copy of the loop for building A and b.