M368K
Homework #7

Burden and Faires.

Section 10.1 (#6[1]ab, 10[2]c). Section 10.2 (#2b, 7[3]b, 10[4]).

[1]Put system in fixed-pt form by isolating $x_{1,2}$ in $\text{eqn}_{1,2}$ in simplest way. For (b), find $x^{(2)}$ given $x^{(0)} = (1/4, 1/4)$.

[2]Put system in fixed-pt form by isolating $x_{1,2,3}$ in $\text{eqn}_{1,2,3}$ in simplest way. Find $x^{(2)}$ given $x^{(0)} = (5, 2, 0)$.

[3]Find $x^{(2)}$ given $x^{(0)} = (2, 2)$.

[4]Show $x^{(1)}$ is exact solution for any $x^{(0)}$.

Programming mini-project.

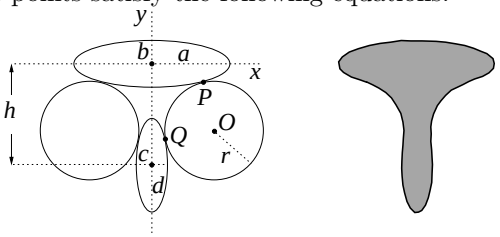
A machine component to be cut from a sheet of plastic is designed using arcs of ellipses and circles as shown. For given values of a, b, c, d, h and r, the design is completed by finding the locations of points $O = (z_1, z_2)$, $P = (z_3, z_4)$ and $Q = (z_5, z_6)$. These points satisfy the following equations.

P on ellipse: $z_3^2/a^2 + z_4^2/b^2 = 1$.

Q on ellipse: $z_5^2/c^2 + (z_6 + h)^2/d^2 = 1$.

P on circle: $(z_3 - z_1)^2 + (z_4 - z_2)^2 = r^2$.

Q on circle: $(z_5 - z_1)^2 + (z_6 - z_2)^2 = r^2$.



Ellipse and circle have same slope at P: $z_3(z_4 - z_2)b^2 = z_4(z_3 - z_1)a^2$.

Ellipse and circle have same slope at Q: $z_5(z_6 - z_2)d^2 = (z_6 + h)(z_5 - z_1)c^2$.

Here we solve for O, P and Q using the Newton method. For concreteness, we assume $a = 3$, $b = 1.5$, $c = 1$, $d = 2$, $h = 3.5$ and $r = 2.5$ [inches].

(a) Download the C++ files `program7.cpp`, `newton.cpp` and `gauss_elim.cpp` from the course webpage. These files implement the Newton method for systems of equations; read the files for instructions on how to use.

(b) Write the above equations in the form $F(x) = 0$, where $x = (z_1, \ldots, z_6)$, and find an expression for the Jacobian matrix $\frac{\partial F}{\partial x}(x)$. Use the Newton method to solve the equations; iterate until $||x^{(k)} - x^{(k-1)}||_\infty < 10^{-6}$. Use the diagram and the values of the constants to make a rough, but reasonable choice of the initial guess $x^{(0)}$. Briefly explain the initial guess you used and report the approximate solution and number of iterations required. [Note: a properly implemented problem should only take about 5-7 iterations.]

Turn in: modified version of `program7.cpp` appropriate for this problem and response to (b).