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Research

Analysis of Tandem Solar Cell Efficiencies Under AM1.5G Spectrum Using a Rapid Flux Calculation Method

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We report the use of a rapid flux calculation method using incomplete Riemann zeta functions as a replacement for the Bose–Einstein integral in detailed balance calculations to study the efficiency of tandem solar cell stacks under the terrestrial AM1.5G spectrum and under maximum concentration. The maximum limiting efficiency for unconstrained and constrained tandem stacks of up to eight solar cells, under the AM1.5G spectrum and maximum concentration, are presented. The results found agree well with previously published results with one exception highlighting the precautions necessary when calculating for devices under the AM1.5G spectrum. The band gap sensitivities of two tandem solar cell stack arrangements of current interest were also assessed. In the case of a three solar cell tandem stack the results show a large design space and illustrate that the constrained case is more sensitive to band gap variations. Finally, the effect of a non-optimum uppermost band gap in a series constrained five solar cell tandem stack was investigated. The results indicate that a significant re-design is only required when the uppermost band gap is greater than the optimum value with a relatively small effect on the limiting efficiency. It is concluded that this rapid flux calculation method is a powerful tool for the analysis of tandem solar cells and is particularly useful for the design of devices where optimum band gaps may not be available. Copyright © 2007 John Wiley & Sons, Ltd.

KEY WORDS: limiting efficiency; detailed balance; Tandem solar cells

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INTRODUCTION

The limiting efficiency of a solar cell serves a dual purpose in photovoltaics research, providing a guide as to the ultimate achievable efficiency, as well as being a testbed for novel concepts for achieving efficiency enhancements. Traditionally tandem solar cell stacks have been used in space applications, meaning that the majority of analyses of limiting efficiency has focused on the tandem stacks under the AM0, or maximally concentrated blackbody spectra. Recent results[1] have shown the ability to fabricate three solar cell tandem stacks with terrestrial efficiencies in excess of 40%, under 240x concentrated light. In addition to these results, investigations of five solar cell stacks, and the potential for even greater numbers of solar cells in tandem stacks,[2,3] mean a relatively quick and stable method for determining optimum band gaps is an essential design tool.

Since first being used by Shockley and Queisser in 1961,[4] the use of the detailed balance principle has been the standard manner of calculating the limiting efficiency of a solar cell. This principle states that the fundamental limit of the total current that can be extracted from a solar cell is the difference between

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the total absorbed incident light and the light emitted by the solar cell. The extension of this methodology, to stacks of solar cells, naturally followed for the case of an unconstrained stack[5–7] and a constrained (current matched) stack.[8] The method has also been modified and applied to assessing the impact of the Impurity Photovoltaic (IPV)[9,10] effect, as well as the intermediate band solar cell (IBSC).[11,12] In this approach, the solar cell is assumed to act as a blackbody emitter, with the emission from the solar cell corresponding to the lost photo-generated carriers or dark current. The flux is found, in the standard manner for a black-body, by using the Bose–Einstein (BE) integral.

The use of the BE integral for the flux calculation is straightforward for the case of a single band gap, but when analyzing solar cell stacks several of these calculations are required at each point, and the rapidity and stability of the BE integral is compromised. In cases where the bias (chemical potential) of the solar cell is close to the band gap, the calculation is particularly troublesome, as the calculation is being performed near a singularity. Since the BE integral has no closed form solution, one must always numerically calculate the flux.[13,14] This can be time consuming and can be prone to error, due to the limited accuracy of the numerical routine used and the higher order interpolation needed to fit the spectrum.[15]

In this paper, we report the use of incomplete Riemann zeta integrals (IRZIs) to estimate the photon flux for a blackbody. This allows the analysis of the limiting efficiency of tandem solar cell stacks with up to eight solar cells, under a terrestrial spectrum, namely AM1.5G. The improved stability and speed of this approach allow us to easily characterize the limiting efficiency over a large range of band gap values. This proves beneficial for two key reasons: firstly, as will be shown, it circumvents a potential source of error in obtaining the maximum limiting efficiency and optimum band arrangement. Secondly, it allows the determination of the band gap sensitivity of a solar cell stack, that is, how is the limiting efficiency affected by deviations of the band gaps away from the optimum values. These results are of particular interest due to an increasing interest in using tandem solar cell stacks for the realization of very high efficiency terrestrial devices.

After outlining the model used, the maximum limiting efficiencies for tandem stacks of up to eight solar cells are presented for the AM1.5G spectrum and maximum concentration cases. Following this, a key issue in finding maximum limiting efficiencies under the AM1.5G spectrum is highlighted. Finally the band gap sensitivities of two key tandem solar cell arrangements are investigated. First, the case of a three solar cell stack, with the lowest band gap fixed at 0.7 eV, is looked at. Second, the case of a five solar cell stack in the constrained configuration is also investigated. The top band gap is varied around its optimum value and the effect on the efficiency and the optimum values for the remaining band gaps in the stack is assessed. The band gap sensitivity calculations were done for 1x concentration under the AM1.5G spectrum; the trends identified should be applicable, however, to all concentration cases.

MODEL USED

Rapid flux calculation framework

The BE integral, used for the determination of flux from a blackbody in typical detailed balance calculations, has the general form

$$\Phi(E_A, E_B, T, \mu) = \frac{2\pi}{h^3c^2}\int_{E_A}^{E_B} \frac{E^P \mathrm{d}E}{\exp\left(\frac{E-\mu}{kT}\right) - 1} \quad (1)$$

for $\mu < E_A$ and $0 < E_A < E_B$ and zero otherwise, where E_A and E_B are the lower and upper energy limits of emission, respectively μ is the emitter chemical potential, T is the emitter temperature, c is the speed of light, h is Planck's constant and k is Boltzmann's constant. For the case of particle flux P takes the value of 2, whilst for the case of energy flux P is 3. By first substituting the dummy variable $y = (E - \mu)/kT$ into Equation (1), and then substituting $x = 1/y$ into the result, Equation (1) can be restated in terms of transformed IRZIs (E_A, E_B, T and μ are omitted for brevity)

$$\Phi = \sum_{p=1}^{P+1} a_{p-1}\int_{x_B}^{x_A} \frac{\mathrm{d}x}{x^{p+1}[\exp(1/x) - 1]} \quad (2)$$

where $x_A = \frac{kT}{E_A-\mu}$ and $x_B = \frac{kT}{E_B-\mu}$. This form of the flux has the advantage of avoiding the evaluation of improper integrals and allowing the construction of look up tables (LUTs) for rapid calculations. A domain, D_x, of x values is found for which Equation (2) is stable and computable. Outside of D_x, limiting expressions are found both for $x > \min(D_x)$ and $x > \max(D_x)$, allowing the following expression for the flux over an

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arbitrary domain, to be defined

$$\Phi = \sum_{p=1}^{P+1} a_{p-1} \left(\int_{x_N^p}^{x_A} \frac{\mathrm{d}x}{x^{p+1}[\exp(1/x) - 1]} + \sum_{k=n}^{N-1} \int_{x_k^{(p)}}^{x_{k+1}^{(p)}} \frac{\mathrm{d}x}{x^{p+1}[\exp(1/x) - 1]} + \int_{x_B}^{x_n^{(p)}} \frac{\mathrm{d}x}{x^{p+1}[\exp(1/x) - 1]} \right) \quad (3)$$

where the second term is determined from two LUTs, the first for $\min(D_x) < x < \max(D_x)$ and the second containing the computed values of the IRZI between successive entries of x in the first LUT. The first and third terms represent the computed remainders, that is, the small end parts where x lies between values of x, listed in the first LUT. For the cases of $x < \min(D_x)$ and/or $x > \max(D_x)$, the first and third terms are respectively approximated by the following:

$$\Phi^{\text{high}} = \sum_{p=1}^{P+1} a_{p-1} \int_{\max(D_x)}^{x_A} \frac{\mathrm{d}x}{x^p} \quad (4)$$

$$\Phi^{\text{low}} = \sum_{p=1}^{P+1} a_{p-1} \int_{x_B}^{\min(D_x)} \frac{\mathrm{d}x}{x^{p+1} \exp(1/x)} \quad (5)$$

both of these expressions are analytic, aiding precision and speed further. The process of consulting LUTs and finding the portions of x not covered by the LUTs provides not only a much greater speed, but also better precision, as the continued use of higher order interpolation polynomials is avoided.[15] The stability and rapidity provided by this approach makes it ideal for use in the calculation of tandem solar cell efficiencies over large band gap design spaces. For a more detailed discussion of the method outlined, a previous report[16] can be reviewed. As outlined in the following sections, this allows one not only to find the maximum limiting efficiency for a large number of band gaps, but also to gauge the band gap sensitivity of the tandem solar cell stacks.

Solar cell specifications

Tandem solar cells consist of a series of solar cells stacked in the direction of the incident light, with the top most solar cell having the highest band gap and the bottom most solar cell the lowest band gap, as indicated in Figure 1. Each solar cell targets a portion of the incident spectrum for absorption and hence conversion to electrical power. It is assumed that the absorption profile for the stack is step-wise, with no overlap of absorption between any of the solar cells in the stack. In other words, each solar cell absorbs light greater than its own band gap but does not absorb photons with energy greater than the solar cell above it in the stack. It is assumed that each solar cell absorbs all incident photons with energy greater than the cell's band gap and is transparent to photons with energy below the band gap. Each photon produces exactly one electron hole pair upon absorption.

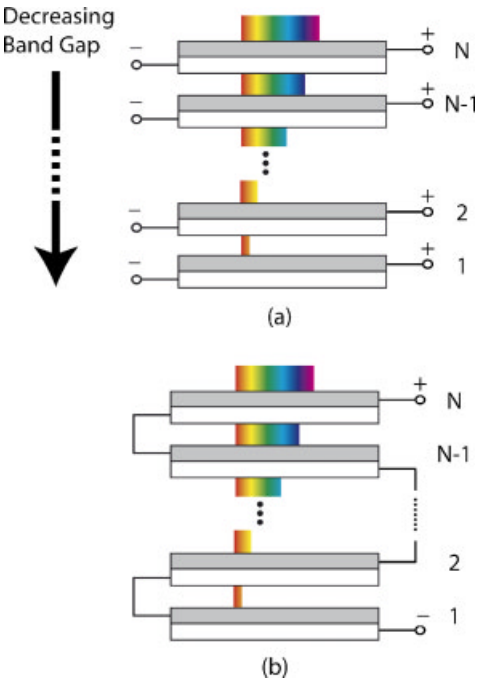

Figure 1. Tandem solar cell stack in the (a) unconstrained and (b) constrained cases

Figure 1a shows the unconstrained tandem solar cell stack, where each solar cell is connected to a separate circuit and is free to operate at its maximum power point. Figure 1b shows the constrained case, where the solar cells are connected in series, with any mismatch in the output current of each cell needing to be minimized for optimum operation. The constrained tandem stack finds preference in practice, despite the current matching restriction, since only two electrodes for the whole stack are required, rather than for each solar cell.

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RESULTS AND DISCUSSION

Limiting efficiency calculations

In the calculations for an unconstrained tandem stack an LUT based approach was used. Firstly, the photocurrent available between band gap values was calculated and the results stored as a LUT, in order to minimize re-calculations for each band gap combination. Secondly, the limiting efficiency of solar cell absorbing between two band gap values was calculated, using the IRZI method, and also maintained as a LUT. Efficiencies of band gap combinations were calculated by summation of the LUT entries for the efficiency of two consecutive values in the band gap combination. In all the calculations presented the band gap step size was 0.01 eV. A comparison was made between the results found using the IRZI method and the standard BE integral approach. For tandem stacks of up to five solar cells, no difference in the limiting efficiency of optimum band gaps was found.

The constrained case calculations are more complex, as each set of band gaps requires the output currents to be matched, in order to find the optimum bias, and hence current (I_{opt}), for the stack. By starting off with a combination of band gaps that give matched short-circuit currents, we should be close to the final solution, since we would expect each cell to have an $I_{opt} \simeq I_{SC}$. For each combination of band gaps the solar cell with the lowest I_{SC} was found and used as the bias control cell for the stack. The bias was varied across this control cell and the biases for the other cells corresponding to current matching were found. The maximum power point of the constrained stack was found for each band gap combination by sweeping the bias of the control cell from 0 to V_{OC}.

The AM1.5G spectrum data were taken from ASTM G173-03, with linear interpolation and trapezoidal integration used for calculations of photocurrent. Table I summarizes the results, showing the limiting efficiencies for each case calculated, for AM1.5G under 1x concentration and maximum concentration, where the sun is assumed to be a blackbody emitter at $T =$ 6000 K. The optimum band gap configurations have also been included. It should be noted that the efficiency limits found in this work are consistently higher than reported previously,[7,8] by a few tenths of a percent absolute. This was attributed to the updated AM1.5G spectrum used in these calculations.

All of the optimum band gap combinations returned for up to six solar cells were the same as previously published,[7,8] with the exception being the case of 3 solar cells in the unconstrained stack under AM1.5G spectrum. In this study the optimum band gaps were found to be 0.93, 1.40, and 2.05, in comparison to the previous combination of roughly 0.70, 1.16, and 1.85.[7,8] The influence of the spectrum was discounted as the source of disagreement between this and previous work, after calculations using the latest AM1.5G spectrum and those using an earlier AM1.5G spectrum showed no difference in the optimum band gap combinations.

Calculation method influence on results

A possible explanation for the difference in reported optimum band gap combinations for the three solar cell unconstrained case,[7,8] can be best understood by looking at the maximum efficiency surfaces for 1x maximum concentration (blackbody) and AM1.5G spectrum. The 1x blackbody spectrum was calculated using the IRZI method, with the sun being a blackbody at 6000 K. Each surface was calculated for an unconstrained two solar cell stack and are displayed in Figure 2.

The efficiency surface for a blackbody spectrum is smooth, with no discernible features, but this contrasts strikingly with the presence of two 'humps' in the AM1.5G surface. This is not surprising, since the AM1.5G spectrum is piecewise continuous, with a number of wavelength bands with very low photon flux, due to absorption in the earth's upper atmosphere. These humps appear to be the source of the disagreement in optimum band gap arrangement.

Typically, when calculating the limiting efficiency an optimizing routine is employed to search for the optimum result, in order to reduce the number of calculations required. These methods do have hidden perils, however, in that the routines can return the nearest maximum whether it is the global maximum or not,[17] when the function being optimized has multiple maxima. So while it is straightforward (if computationally intensive) to find the maximum for the blackbody spectrum case, for the AM1.5G this is not the case. The routine can return values for the optimum band gap combination that reside in the secondary hump, and hence are not the values for the global maximum.

It can be seen in Figure 2 that the limiting efficiency for the AM1.5G spectrum is higher than for the blackbody case. This seems counter-intuitive, but it must be kept in mind that the AM1.5G spectrum differs

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Table I. Summary of results for constrained and unconstrained solar cell stacks under the AM1.5G spectrum for 1x concentration and under maximum concentration. The band gap values shown correspond to the optimum combinations for the 1x AM1.5G case.

E_1	E_2	E_3	E_4	E_5	E_6	E_7	E_8	η (%)	E_1	E_2	E_3	E_4	E_5	E_6	E_7	E_8	η (%)
Unconstrained AM1.5G Spectrum									*Constrained AM1.5G spectrum*								
1.34	—	—	—	—	—	—	—	33.68	1.34	—	—	—	—	—	—	—	33.68
0.94	1.73	—	—	—	—	—	—	46.06	0.94	1.60	—	—	—	—	—	—	45.71
0.93	1.40	2.05	—	—	—	—	—	51.94	0.94	1.37	1.90	—	—	—	—	—	51.58
0.70	1.13	1.64	2.23	—	—	—	—	55.91	0.71	1.11	1.49	2.00	—	—	—	—	55.31
0.69	0.98	1.38	1.81	2.40	—	—	—	58.37	0.70	1.01	1.33	1.67	2.14	—	—	—	57.61
0.69	0.94	1.19	1.53	1.92	2.45	—	—	59.93	0.69	0.96	1.20	1.47	1.79	2.24	—	—	59.41
0.69	0.94	1.15	1.41	1.72	2.11	2.57	—	61.36	0.69	0.93	1.14	1.37	1.60	1.90	2.33	—	60.78
0.51	0.70	0.94	1.15	1.41	1.73	2.11	2.57	62.34	0.51	0.75	0.98	1.18	1.40	1.63	1.92	2.35	61.42
Unconstrained maximum concentration									*Constrained maximum concentration*								
1.11	—	—	—	—	—	—	—	40.74	1.11	—	—	—	—	—	—	—	40.74
0.77	1.70	—	—	—	—	—	—	55.80	0.76	1.54	—	—	—	—	—	—	55.47
0.62	1.26	2.10	—	—	—	—	—	63.75	0.60	1.14	1.82	—	—	—	—	—	63.15
0.52	1.03	1.61	2.41	—	—	—	—	68.67	0.49	0.93	1.38	2.01	—	—	—	—	67.85
0.45	0.88	1.34	1.88	2.65	—	—	—	72.00	0.44	0.81	1.17	1.58	2.18	—	—	—	71.02
0.40	0.78	1.17	1.60	2.12	2.87	—	—	74.41	0.38	0.71	1.01	1.33	1.72	2.30	—	—	73.33
0.36	0.70	1.04	1.40	1.81	2.32	3.06	—	76.22	0.37	0.66	0.92	1.18	1.48	1.85	2.42	—	75.09
0.33	0.64	0.94	1.25	1.59	1.98	2.47	3.20	77.63	0.30	0.60	0.83	1.06	1.29	1.57	1.96	2.50	76.19

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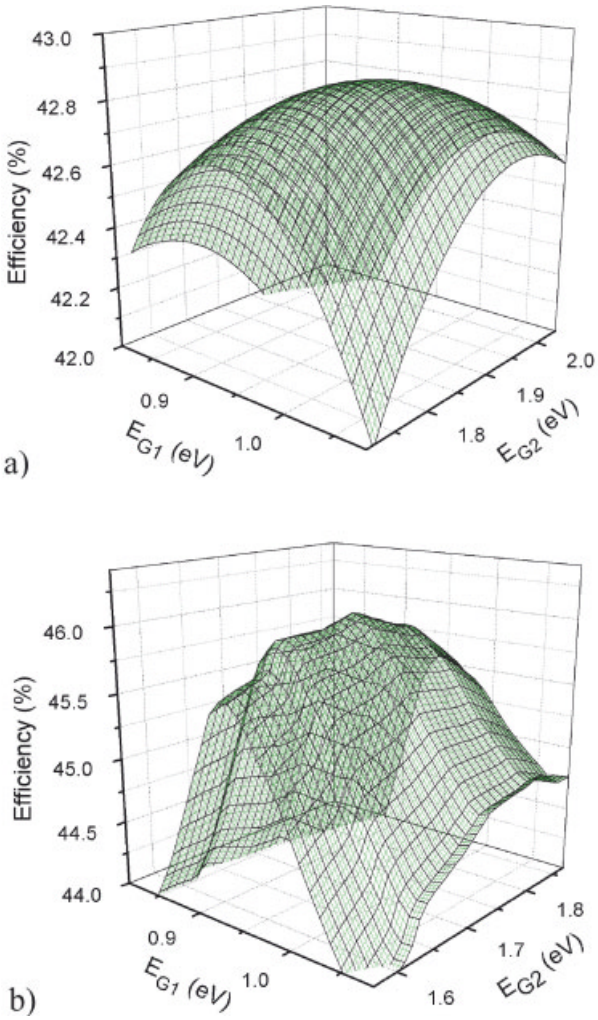

Figure 2. Efficiency contour for an unconstrained two solar cell tandem stack under (a) 1x concentration blackbody spectrum and (b) 1x AM1.5G spectrum

from a blackbody spectrum in two important ways. First, it has a limited band width, unlike the blackbody spectrum, and it has bands of low photon counts. So, for instance, the blackbody spectrum contains more light at the blue end of the spectrum, this high energy light is not converted efficiently by a solar cell. This means that a more optimal band gap combination can be found for the AM1.5G spectrum, where the number of high energy photons that are in effect 'wasteful' for the energy conversion process, is minimized. In addition, the bands of low photon numbers in the AM1.5G spectrum mean that the band width for a particular solar cell in the stack can be narrowed, improving efficiency further. The nett effect of these two features is seen in Figure 2 where for 1x concentration the AM1.5G spectrum gives a larger limiting efficiency. As such, there will be some concentration where the behavior will cross over, and the blackbody spectrum will give a higher limiting efficiency.

Band gap sensitivity

The calculation of the limiting efficiency over a large range of band gap combinations allows the analysis of an important feature of tandem solar cell stacks—the band gap sensitivity. Ideally, one can select materials with optimum band gaps, as well as optical and electrical properties. In practice, however, the choices are somewhat restricted due to issues such as lattice mismatch and vastly different material growth conditions. It is, therefore, important to know the sensitivity of the efficiency of a solar cell stack to variations in the band gap of any of the solar cells. As mentioned in the section Introduction, the case of 1x concentration was examined in order to determine general behavior trends, which can be applied to the analysis of specific devices.

Three solar cell tandem

One way to obtain insightful results is to vary two of the band gaps whilst keeping the remaining band gaps fixed. As an example of this, Figure 3a shows a contour surface plot of the limiting efficiency of a three band gap stack under 1x AM1.5G spectrum, with the lowest band gap fixed at 0.7 eV. As can be seen, the peak in limiting efficiency is broad and two humped. The results imply that, not only is the unconstrained tandem arrangement relatively insensitive to changes in the two upper band gaps, but it may be that if it is not possible to, say, get E_{G2} at the optimum value, then it may be advantageous to try to obtain a value significantly away from the global optimum. So, for instance, it may be advantageous to have $E_{G2} \approx 1.1$ eV, the band gap of silicon, rather than at 1.3 eV, even though the latter value is closer to the global optimum. By comparison, in the constrained case, as shown in Figure 3b, the stack has, as expected, a greater band gap sensitivity, due to the current matching condition, necessary for optimum operation. It does, however, retain the feature of a secondary efficiency hump, though in this case the drop in limiting efficiency, when E_{G2} and E_{G3} deviate from the global maximum values, is greater.

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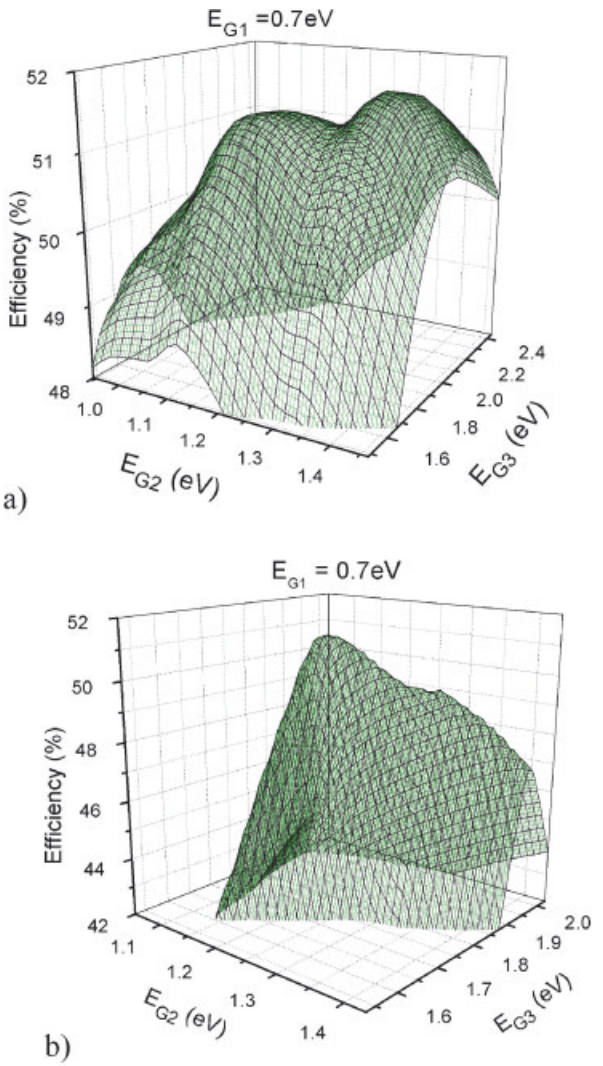

Figure 3. Efficiency contour plots for (a) an unconstrained and (b) a constrained three solar cell tandem stack under 1x AM1.5G spectrum with the lowest band gap fixed at 0.7 eV

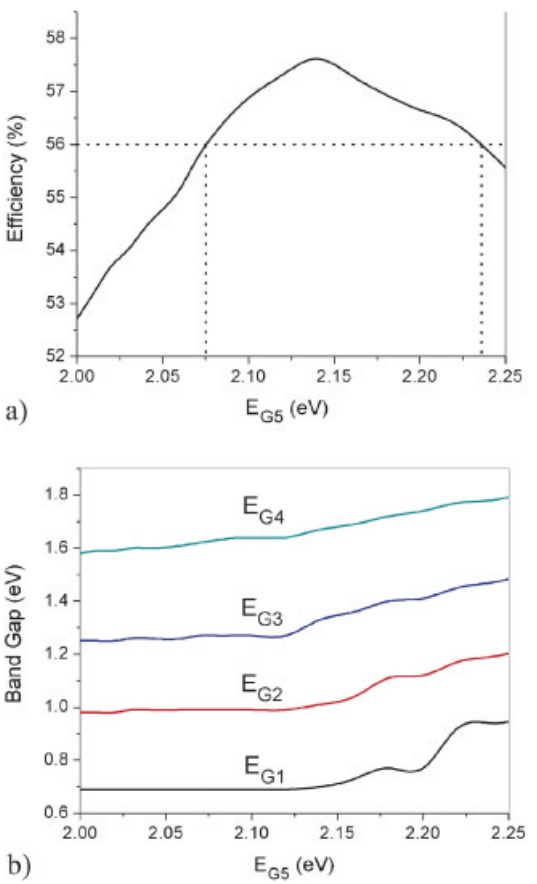

Figure 4. Results from optimization of a five solar cell series connected tandem arrangement under 1x AM1.5G spectrum, (a) the optimal efficiency as the value of the uppermost band gap is varied and (b) the optimal values of the the four lower band gaps as the upper most band gap is varied

Five solar cell tandem

As a final example of the insights made possible by the larger design space of the band gaps, the effect of changing the uppermost band gap in a series constrained five stack tandem solar cell arrangement was investigated. The lower four band gaps were optimized for each value of the uppermost band gap, which was varied between 2.00 eV and 2.25 eV. This is centered around the absolute maximum for the unconstrained five solar cell case ($E_{G5} = 2.14$ eV). The results are shown in Figure 4a, which indicates that the stack arrangement is robust to substantial changes in E_{G5}. As indicated by the dashed lines, the efficiency is $\geq$56% for E_{G5} ranging from roughly 2.08 eV to 2.22 eV. As shown in Figure 4b, the values for the lower four band gaps slowly vary up to around the global optimum arrangement, after which there is much greater variation.

This result is of importance in relation to recent efforts to implement five stack tandem solar cells under high concentration levels. In particular, the uppermost solar cell in the stack requires a band gap greater than 2.0 eV, a condition that has proven to be problematic.[3,18,19] The results presented in Figure 4 show that, in the case of 1x concentration AM1.5G spectrum, when the uppermost band gap is below its optimum of 2.14 eV, no adjustment is necessary to the band gaps of the lower solar cells. This is not, however, the case when the uppermost band gap is above 2.14 eV, meaning the band gaps of all of the

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solar cells in the stack need adjustment. Assuming this general result extends to any concentration, it suggests the inability to fabricate devices with a top band gap greater than 2.0 eV may not exclude the possibility of very high efficiency devices, and that radical re-designs of the tandem stacks may be unnecessary.

CONCLUSION

Analysis of the limiting efficiency of terrestrial tandem solar cell stacks was performed, using a rapid flux calculation method based on IRZIs. Limiting efficiencies were presented, along with the optimum band gap arrangements for stacks of up to eight solar cells under 1x AM1.5G spectrum and under maximum concentration. The rapidity of the IRZI method alleviates the need for optimizer routines, since efficiency surfaces over a large band gap range can be easily calculated. The variation of the optimum band gap arrangement for the unconstrained three solar cell stack with previously reported values was ascribed to the multiple maxima nature of efficiency under the AM1.5G spectrum that leads to the potential of optimizer routines to identify a local maximum as the global.

The stability afforded by the IRZI method outlined also allows the analysis of the band gap sensitivity of tandem solar cell stacks, in both the constrained and unconstrained cases. It was shown that the band gap sensitivity of the constrained tandem solar cell stack is greater than that of the unconstrained stack. This is not surprising as the constrained case means that the output current of the solar cells, comprising the tandem stack, needs to be matched. This means that any change in one of the band gaps can reduce the efficiency of each other cell, and hence the effect on the overall efficiency is large.

The case of a series constrained 5 tandem solar cell stack under 1x AM1.5G spectrum was analyzed using the IRZI approach, to assess the effect of changes in the value of the uppermost band gap on the limiting efficiency of the tandem stack. It was found that the limiting efficiency remains above 56% for a relatively large range of uppermost band gap values. It was also found that the optimum values of the lower four band gaps remain relatively unchanged for sub-optimum values of the uppermost band gap, but a significant re-design of the stack is required when the uppermost band gap exceeds the optimum value. If this result is extended to higher concentrations, it suggests that very high efficiency devices are possible, with little re-design of the tandem stacks necessary, and limiting efficiencies close to those for an optimum band gap arrangement.

The examples reported in this paper highlight the insights that can be obtained due to the large design space offered by the rapidity and stability of the flux calculations using the IRZI method. It is concluded that the IRZI method provides a valuable tool for the design of terrestrial tandem solar cell stacks, especially for stacks with a large number of solar cells.

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