Proof 1:

Let us Assume that f is associative.

If condition 1/2/3 (below) holds we can deduce that:

We can notice that , thus:

Since f is associative we can also deduce that

If condition 1/2/3 holds we can deduce that:

Condition 1:

For any MS input (x,y):

res(

Condition 2:

For any MS input (x,y) & For **any** constant :

And

Condition 3:

For any MS input (x,y) & For **any** constant :

And

Proof 2:

We will prove that for a group S of binary numbers.

Only If for any |B| (number of different bits between the binary numbers in S), the following holds:

Case 1:

* Meaning in S there are unique binary numbers, where |B| represents the number of bits these numbers differ one another.
* Thus, will be consisted with exactly |B| meta-stable bits, in the indices where the original binary numbers differ.
* From here we can deduce by the definition of res() that |.
* Meaning | , since the indices of the different bits in S & the MS bits in are identical we can deduce that

Case 2:

* Let’s assume that .
* Meaning in S there is a smaller number of unique binary numbers than , where |B| represents the number of bits these numbers differ one another.
* But in there are |B| MS bits.
* From here we can deduce by the definition of res() that |.
* We received a contradiction.

Case 3:

* Not feasible.

Conclusions:

* We can conclude from condition 1 & proof 2 that for an MS function as defined in the beginning , if **for** **any** MS input Where |B| is the number of different bits within the results of , than is associative.
* Instead of testing options we need to tests options.
* Testing this property is very simple:
  + Is
* The Diamond Operator holds only condition 2.

Examples of functions that holds this conjecture:

* Robust function (definition below & proof below).
* Constant functions.
* ID function.

**Robust Function**

For any input we shall receive .

For any input we shall receive .

We shall define a robust function as following:

If differ from in exactly 1 bit than differ exactly in 1 bit as well.

**Conjecture:**

For f that is robust and associative will be associative as well.

Proof 3 (conjecture):

* We will prove that f holds condition 1 and deduce that is associative.
* For a given input (x,y) let us assume:
* Where |B| is the number of MS bits in x concatenated with y.
* This is a reasonable assumption, because we are assuming not a general set (x,y) but a MS input, thus its resolution will contain binary numbers.
* Since f is robust, then we can deduce (see proof 4 below) that:
* Since f is robust we can deduce that each binary number in is different in exactly |B| bits, hence condition 1 holds.
* Therefor is associative.

**Conjecture:**

If f is robust and the set (x,y) has |B| MS bits than we can say that than:

Proof 4 (conjecture):

* We will prove in induction.
* For n = 1..
  + the concatenated set (x,y) has exactly 1 MS bit, hence the set :
  + By the definition of a robust function:
* Induction Assumption:
  + For n = k the concatenated set (x,y) has exactly k MS bits, hence the set :
  + By the induction assumption.
* Induction proof
  + For n = k+1 we will prove that for the concatenated set (x,y) that has exactly k + 1 MS bits, the following holds:
  + Lets assume that the k+1 MS bit is 0/1 than we will receive that in the set there exist only k MS bits and the following holds:
  + It doesn’t matter if the 1 extra MS bit is in x or y.
* Therefor we proved that for a robust f with |B| MS bits in its input