

Sigma Model Lattice VMC

The 1D σ model Hamiltonian is given by:

$$\hat{H} = \eta g^2 \sum_{i=0}^N \mathbf{L}_i^2 + \frac{\eta}{g^2} \sum_{i=0}^N \hat{n}_i \cdot \hat{n}_{i+1}$$

Where \hat{n}_i is parameterized by two angles, ρ_i and ϕ_i . We define our ansatz as:

$$\psi(\{\rho_i, \phi_i\}) = e^{-\text{NN}(\{\rho_i, \phi_i\})}$$

We define the energy in the usual VMC way:

$$\mathcal{E} = \left\langle \frac{H\psi}{\psi} \right\rangle_{\psi^2}$$

And the gradient in the usual way:

$$\frac{\partial \mathcal{E}}{\partial \theta} = 2 \left\langle \frac{\partial \ln \psi}{\partial \theta} \frac{H\psi}{\psi} \right\rangle - \left\langle \frac{\partial \ln \psi}{\partial \theta} \right\rangle \left\langle \frac{H\psi}{\psi} \right\rangle$$