Fermionic Ansatz

We have a system of N fermions, $N^{\uparrow}$ being spin up, and $N^{\downarrow}$ being spin down:

$$\underbrace{x_1, \ldots, x_{N^{\uparrow}}}_{\text{Up spin}}, \underbrace{x_{N^{\uparrow}+1}, \ldots, x_N}_{\text{Down spins}}$$

We can construct $\Phi^{\uparrow}$ and $\Phi^{\downarrow}$:

$$\Phi^{\uparrow}(x_1, \ldots x_N) = \frac{1}{\sqrt{N!}} \begin{vmatrix} \phi_1(x_1, \overbrace{\underbrace{x_2, \ldots x_{N^{\uparrow}}}_{\text{Symmetric}}, \ldots x_N}^{\text{Remaining coordinates}}) & \phi_1(x_2, \underbrace{x_1, x_3, \ldots x_{N^{\uparrow}}}_{\text{Symmetric}}, \ldots x_N) & \cdots \\ \phi_2(x_1, \underbrace{x_2, \ldots x_{N^{\uparrow}}}_{\text{Symmetric}}, \ldots x_N) & \phi_2(x_2, \underbrace{x_1, x_3, \ldots, x_{N^{\uparrow}}}_{\text{Symmetric}}, \ldots x_N) & \\ \vdots & & \ddots \end{vmatrix}$$

Where $\Phi^{\downarrow}$ is constructed in the analogous way, using $N^{\downarrow}$ neural network functions, $\phi_{N^{\uparrow}+1}, \ldots, \phi_N$.

Our ansatz is

$$\Psi(x_1, \ldots x_N) = \underbrace{\Phi^{\uparrow}(x_1, \ldots x_N)}_{\text{Anti. } \uparrow} \underbrace{\Phi^{\downarrow}(x_1, \ldots x_N)}_{\text{Anti. } \downarrow} \underbrace{f(x_1, \ldots x_N)}_{\text{Symmetric}} e^{-\sum x_i^2}$$

Delta Function Sampling

Energy Computation

$$\begin{aligned} E &= \frac{\int dx\, \psi^2 \left(-\frac{1}{2m}\frac{1}{\psi}\frac{d^2\psi}{dx^2} + \frac{1}{2}m\omega^2 x^2\right)}{\int dx\, \psi^2} + \frac{\int dx\, N_{\uparrow} N_{\downarrow} g \psi^2(x) \frac{\psi^2(x')}{\psi^2(x)} d(x_\mu)}{\int dx\, \psi^2(x)} \\ &\to \langle \underbrace{-\frac{1}{2m}\frac{1}{\psi}\frac{d^2\psi}{dx^2} + \frac{1}{2}m\omega^2 x^2}_{\text{E w/ no delta}} + \underbrace{N_{\uparrow} N_{\downarrow} g \frac{\psi^2(x')}{\psi^2(x)} d(x_\mu)}_{\text{E w/ delta}} \rangle \end{aligned}$$

Where x_μ is the coordinate of the first down-spin fermion.

Gradient Computation

$$\begin{aligned} \frac{\partial E}{\partial \theta} &= \frac{2\int dx\, \psi^2(x) \left(\frac{1}{\psi(x)} H\psi - E\right) \frac{1}{\psi}\partial_\theta \psi(x)}{\int dx\, \psi^2(x)} \\ &= \frac{\int dx\, \psi^2(x) \left(\frac{2}{\psi}\left(-\frac{1}{2}\frac{d^2\psi}{dx^2} + \frac{1}{2}m\omega^2 x^2 \psi + g\delta(x_1 - x_\mu)\psi - E\right) \frac{1}{\psi}\partial_\theta \psi\right)}{\int dx\, \psi^2(x)} \\ &= \frac{\int dx\, \psi^2(x) \left(\frac{2}{\psi(x)}\left(-\frac{1}{2}\frac{1}{\psi}\frac{d^2\psi}{dx^2} + \frac{1}{2}m\omega^2 x^2\right) - E\right)\partial_\theta \psi}{\int dx\, \psi^2(x)} + \frac{\int dx\, \psi^2 g\delta(x_1 - x_\mu) \frac{2}{\psi}\partial_\theta \psi}{\int dx\, \psi^2(x)} \end{aligned}$$

$$= \frac{\int dx\, \psi^2(x) \left(\frac{2}{\psi(x)}\left(-\frac{1}{2}\frac{1}{\psi}\frac{d^2\psi}{dx^2} + \frac{1}{2}m\omega^2 x^2\right) - E\right)\partial_\theta \psi}{\int dx\, \psi^2(x)} + \frac{\int dx\, \psi^2 N_\uparrow N_\downarrow \frac{\psi^2(x')}{\psi^2(x)}\frac{2g}{\psi(x')}\partial_\theta \psi(x')\, d(x_\mu)}{\int dx\, \psi^2(x)}$$

$$\to \langle \left(-\frac{1}{2m}\frac{1}{\psi}\frac{d^2\psi}{dx^2} + \frac{1}{2}m\omega^2 x^2 - E\right)\frac{2}{\psi}\partial_\theta \psi + \left(N_\uparrow N_\downarrow g \frac{\psi^2(x')}{\psi^2(x)} d(x_\mu)\right)\frac{2\partial_\theta \psi(x')}{\psi(x')}\rangle$$