Falling Magnet

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Aim:

To determine the damping coefficient of the falling magnet through different conducting materials due to generated eddy currents.

Apparatus:

- Magnets of different masses/dimensions
- Cylindrical pipes of conducting material (Aluminium and Copper)
- 2 coils of copper wire
- Oscilloscope and probes

Theory:

When the magnet falls through the conducting tube, it experiences a repulsive force generated by the eddy currents. This can be understood using Faraday's law. The repulsive force is modeled to be proportional to the velocity. Hence, the equation of motion governing the experiment is:

$$m\frac{dv}{dt} = mg - kv \tag{1}$$

We integrate this differential equation using the method of separation of variables:

$$\frac{k\,dv}{mg - kv} = \frac{k}{m}\,dt \tag{2}$$

Integrating both sides:

$$-\ln(mg - kv)\Big|_{v_1}^{v_2} = \frac{kt}{m} \tag{3}$$

Solving for $v(t)$ gives:

$$v(t) = \left(v_0 - \frac{mg}{k}\right)e^{-\frac{kt}{m}} + \frac{mg}{k} \tag{4}$$

- The first term, $\left(v_0 - \frac{mg}{k}\right)e^{-\frac{kt}{m}}$, represents the **damping term**.
- The second term, $\frac{mg}{k}$, represents the **terminal velocity**.

From this, it is evident that the damping term becomes negligible compared to the terminal velocity within a fraction of a second after the magnet is released. Therefore, if the experiment is observed after giving sufficient time (or distance from the top), the damping term effectively vanishes.

For all practical purposes in the experiment, $v(t)$ becomes constant and equal to the terminal velocity:

$$v(t) \approx \frac{mg}{k} \tag{5}$$

Setup:

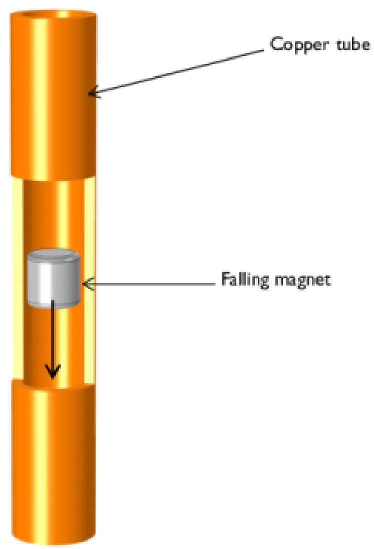

Figure 1: Cross section of the experimental setup.

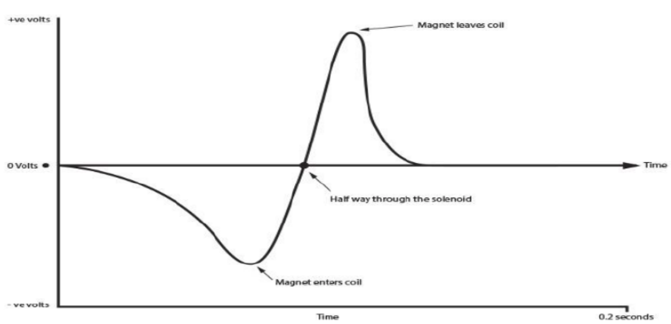

Figure 2: Expected EMF profile as the magnet passes through the coil.

Copper Measurements

Distance (cm)	t(20x10) (s)	t(20x20) (s)	t(20x30) (s)
40	1.8	2.46	2.00
45	2.04	2.78	2.27
50	2.22	3.10	2.48
55	2.44	3.38	2.70
60	2.68	3.72	3.00
65	2.90	4.00	3.24
70	3.12	4.30	3.46
75	3.36	4.64	3.72

Aluminium Measurements

Distance (cm)	t(20x10) (s)	t(20x20) (s)	t(20x30) (s)
40	1.98	2.62	2.52
45	2.22	2.98	2.86
50	2.44	3.26	3.16
55	2.68	3.62	3.50
60	2.90	3.84	3.82
65	3.14	4.20	4.16
70	3.44	4.60	4.48

Calculations:

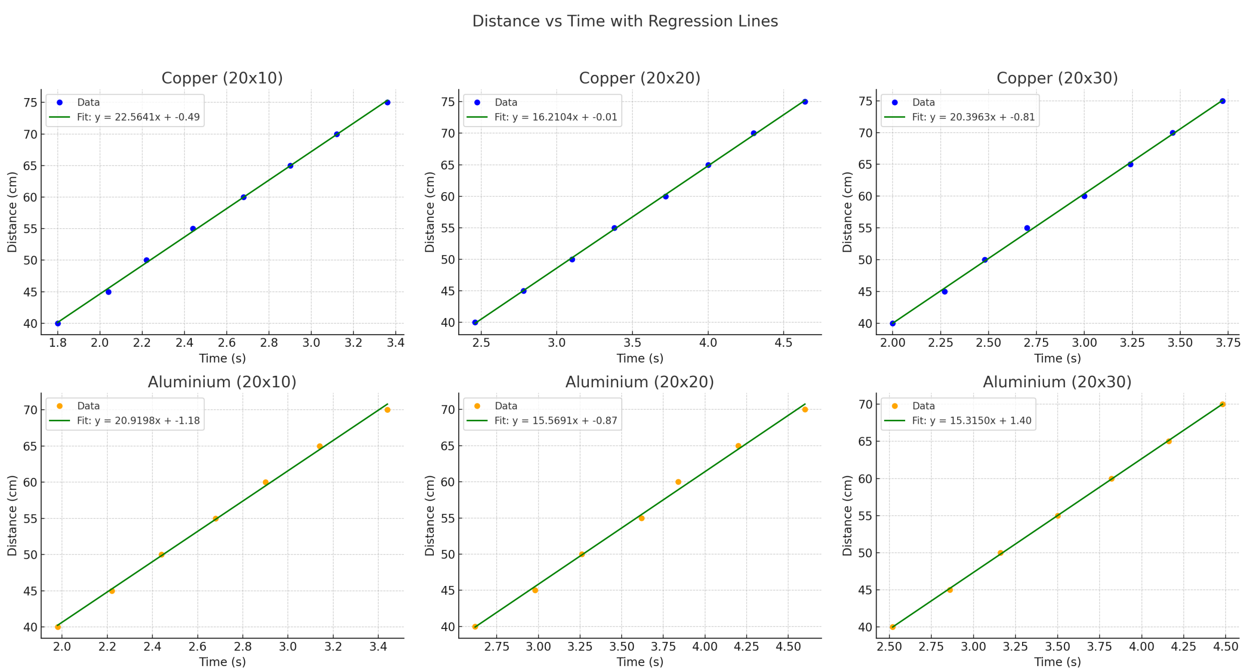

Figure 3: Enter Caption

Terminal Velocities and Damping Coefficients

Pipe Configuration	Avg. Velocity (cm/s)
Copper (20×10)	22.56
Copper (20×20)	16.21
Copper (20×30)	20.39
Aluminium (20×10)	20.92
Aluminium (20×20)	15.57
Aluminium (20×30)	15.32

Table 1: Average terminal velocities.

Copper Tube

$k_{M1} = \frac{(0.02184)(9.81)}{0.22560} = 0.950$ kg/s
$k_{M2} = \frac{(0.04707)(9.81)}{0.16210} = 2.849$ kg/s
$k_{M3} = \frac{(0.07023)(9.81)}{0.20400} = 3.377$ kg/s
Average k for Copper:
$k_{\text{avg, Copper}} = \frac{0.950+2.849+3.377}{3} = \boxed{2.392 \text{ kg/s}}$

Aluminium Tube

$k_{M1} = \frac{(0.02184)(9.81)}{0.20920} = 1.024$ kg/s
$k_{M2} = \frac{(0.04707)(9.81)}{0.15570} = 2.966$ kg/s
$k_{M3} = \frac{(0.07023)(9.81)}{0.15320} = 4.497$ kg/s
Average k for Aluminium:
$k_{\text{avg, Aluminium}} = \frac{1.024+2.966+4.497}{3} = \boxed{2.829 \text{ kg/s}}$

Error Analysis:

Relative error formula:

$$\frac{dK}{K} = \frac{dm}{m} + \frac{ds}{s} + \frac{dt}{t}$$

Copper Tube Relative Errors:
$\frac{dK}{K}(M1) = 0.00964$
$\frac{dK}{K}(M2) = 0.00728$
$\frac{dK}{K}(M3) = 0.00861$
Average Relative Error (Copper):
$\frac{dK}{K}_{\text{avg}} = \frac{0.00964+0.00728+0.00861}{3} = \boxed{0.00851}$
Aluminium Tube Relative Errors:
$\frac{dK}{K}(M1) = 0.00910$
$\frac{dK}{K}(M2) = 0.00707$
$\frac{dK}{K}(M3) = 0.00692$
Average Relative Error (Aluminium):
$\frac{dK}{K}_{\text{avg}} = \frac{0.00910+0.00707+0.00692}{3} = \boxed{0.00769}$

Results:

- $k_{copper} = 2.392 \pm 0.85\%$
- $k_{aluminium} = 2.829 \pm 0.77\%$

Sources of Error and Discussion:

- We definitely face some error due to external influence on the falling magnet. For example, a metal clamp might have damped the motion of the magnet.
- We also have some ambiguity in measuring the length that the magnet has fallen since there is no clear reference marked.
- The magnet also probably does not reach its full critical velocity by the time it passes through the first coil.

- We observed that the damping constant is influenced to a large extent by the mass and shape/dimensions of the magnet