Laser Beam Parameters

Hersh Samdani (EP22B027)

Aim:

To determine the various parameters of a laser beam such as the divergence of the beam (θ_0), the size of the spot and the location of the radius of the beam.

Theory:

Power distribution within the beam can be studied by various methods, such as photographic, scanning, etc. In this experiment,we measure power past a knife edge which is slowly inserted in the beam. We base this on the assumption that the laser is oscillating in the TEM_{00} mode so that the spatial distribution of the beam is Gaussian. If P_0 is the total power of the beam and its spot size is $2\omega_0$, then the irradiance distribution $I(x, y)$ is given by:

$$I(x, y) = \frac{2P_0}{\pi\omega_0} \exp\left[-\frac{2(x^2 + y^2)}{{w_0}^2}\right]$$

The power P transmitted past a knife-edge blocking off all points for which $x \leq a$ is given by:

$$P = \frac{P_0}{2}\mathbf{erfc}(\frac{a_0\sqrt{2}}{\omega_0})$$

Where a is depth of knife-edge in the beam and $\mathbf{erfc}(x)$ is the complementary error function.

Observations:

10 cm		20 cm		30 cm	
x	V	x	V	x	V
8.12 mm	0.505	8.28 mm	0.505	8.16 mm	0.505
8.34 mm	0.500	8.41 mm	0.500	8.39 mm	0.500
8.43 mm	0.495	8.63 mm	0.480	8.61 mm	0.480
8.56 mm	0.480	8.74 mm	0.460	8.72 mm	0.460
8.64 mm	0.465	8.82 mm	0.440	8.80 mm	0.440
8.70 mm	0.450	8.88 mm	0.420	8.87 mm	0.420
8.75 mm	0.435	8.94 mm	0.400	8.93 mm	0.400
8.80 mm	0.420	9.01 mm	0.380	8.99 mm	0.380
8.85 mm	0.405	9.06 mm	0.360	9.04 mm	0.360
8.90 mm	0.390	9.12 mm	0.340	9.10 mm	0.340
8.94 mm	0.375	9.18 mm	0.320	9.17 mm	0.320
8.98 mm	0.360	9.27 mm	0.300	9.25 mm	0.300
9.03 mm	0.345	9.36 mm	0.280		
9.08 mm	0.330	9.53 mm	0.270		
9.12 mm	0.315				
9.17 mm	0.300				
9.22 mm	0.285				
9.30 mm	0.270				
9.38 mm	0.266				

Table 1: Measurements of x and V at 10 cm, 20 cm, and 30 cm

Calculations:

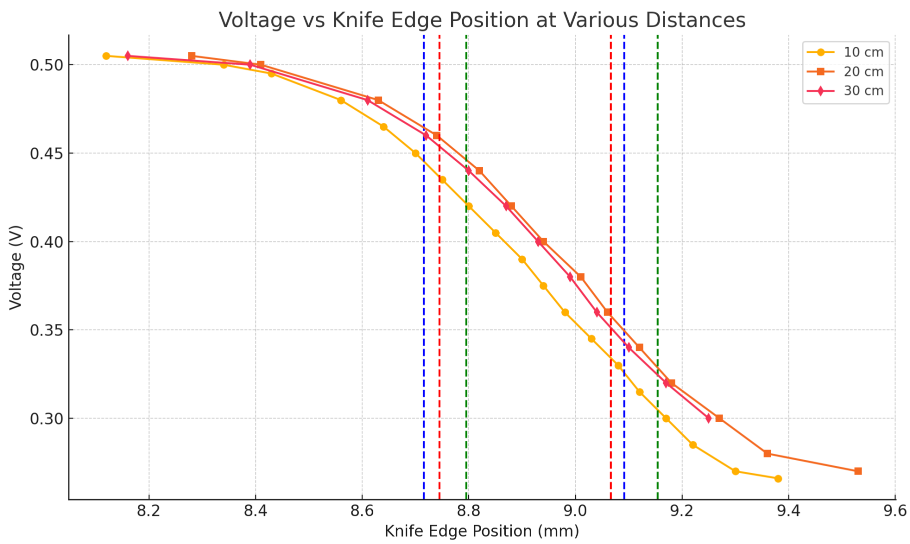

Figure 1: Plots of Voltage/Power vs knife edge position for $x = 10, 20, 30$cm

The positions are:

- 10cm:
  - 25%: 9.09mm
  - 75%: 8.72mm
- 20cm:
  - 25%: 9.15mm
  - 75%: 8.80mm
- 30cm:
  - 25%: 9.07mm
  - 75%: 8.75mm

Since

$$\omega' = \frac{x_{25\%} - x_{75\%}}{2 * 0.6745}$$

- 10cm:
  - ω' : 0.278mm
  - Spot size $2\omega_1 = 1.113$mm
- 20cm:

- ω' : 0.266mm
- Spot size $2\omega_2 = 1.064$mm

- 30cm:
  - ω' : 0.238mm
  - Spot size $2\omega_3 = 0.953$mm

$$\theta_0 = \frac{1}{\sqrt{2}\,D}\sqrt{\left|\,w_3^2 - 2\,w_2^2 + w_1^2\,\right|}, \quad D = 10 \text{ cm.}$$

$$\theta_0 = \frac{1}{\sqrt{2}\times 10}\sqrt{\left|\,0.04762^2 - 2(0.05319)^2 + 0.05569^2\,\right|} = 0.0012045 \text{ rad.}$$

$$\omega_0 = \frac{\lambda}{\pi\,\theta_0}, \quad \lambda = 633\times 10^{-7} \text{ cm.} = 633\text{nm}$$

$$\omega_0 = \frac{633\times 10^{-7}}{\pi \times 0.0012045} = 0.01673 \text{ cm} = 0.1673 \text{ mm.}$$

$$z_0 = \frac{\sqrt{w_1^2 - \omega_0^2}}{\theta_0},$$

$$z_0 = \frac{\sqrt{0.05569^2 - 0.01673^2}}{0.0012045} = 44.08 \text{ cm.}$$

Error Analysis

Starting from the exact beam-radius formula,

$$w^2(z) \;=\; w_0^2\left[1 + \frac{\lambda^2\,z^2}{\pi^2\,w_0^4}\right],$$

take logarithms:

$$\ln\!\big(w^2\big) = \ln\!\big(w_0^2\big) + \ln\!\Big(1 + \tfrac{\lambda^2 z^2}{\pi^2 w_0^4}\Big)\,.$$

Small–error propagation for w gives

$$\frac{\Delta w}{w} = \frac{1}{2}\,\frac{\Delta\big(w^2\big)}{w^2}\,, \qquad \frac{\Delta w}{w_0} = \frac{\Delta\big(w_0^2\big)}{2\,w_0^2}\,.$$

To find the uncertainty in z, invert the relation

$$w^2 - w_0^2 = \frac{\lambda^2}{\pi^2\,w_0^4}\,z^2 \quad\Longrightarrow\quad z = \frac{\pi\,w_0^2}{\lambda}\,\sqrt{w^2 - w_0^2}\,,$$

$$\Delta z = \sqrt{\Big(\frac{\partial z}{\partial w}\,\Delta w\Big)^2 + \Big(\frac{\partial z}{\partial w_0}\,\Delta w_0\Big)^2}\,.$$

Since

$$\frac{\partial z}{\partial w} = \frac{\pi\,w_0^2}{\lambda}\,\frac{w}{\sqrt{w^2 - w_0^2}}\,, \qquad \frac{\partial z}{\partial w_0} = \frac{2\pi\,w_0}{\lambda}\,\sqrt{w^2 - w_0^2} - \frac{\pi\,w_0^2}{\lambda}\,\frac{w_0}{\sqrt{w^2 - w_0^2}}\,,$$

We obtain:

$$\frac{\Delta z}{z} = \sqrt{\Big(\frac{2\,\lambda\,z}{\pi\,w_0^2}\,\frac{\Delta w}{z}\Big)^2 + \Big(-\frac{4\,\lambda\,z^2}{\pi\,w_0^3}\,\frac{\Delta w_0}{z}\Big)^2}\,.$$

$$\frac{\Delta z}{z} = \sqrt{\Big(\frac{2\,(633\,\mathrm{nm})\,(10\,\mathrm{cm})}{\pi\,(0.1673\,\mathrm{mm})^2}\,\frac{\Delta w}{10\,\mathrm{cm}}\Big)^2 + \Big(-\frac{4\,(633\,\mathrm{nm})\,(10\,\mathrm{cm})^2}{\pi\,(0.1673\,\mathrm{mm})^3}\,\frac{\Delta w_0}{0.1673\,\mathrm{mm}}\Big)^2}.$$

Assuming least counts as:

$$\Delta w = 0.01\,\mathrm{mm} = 0.001\,\mathrm{cm}, \qquad \Delta w_0 = 0.01\,\mathrm{mm} = 0.001\,\mathrm{cm}.$$

Plugging in,

$$\frac{\Delta z}{z} = \sqrt{\Big(\frac{2\,(633\times10^{-7})\,(0.001)}{\pi\,(0.01673)^2}\Big)^2 + \Big(\frac{4\,(633\times10^{-7})\,(10)\,(0.001)}{\pi\,(0.01673)^3}\Big)^2} \approx 0.172\,.$$

Hence

$$\Delta z = z \times 0.172 = 10\,\mathrm{cm} \times 0.172 = 1.72\,\mathrm{cm}.$$

Therefore, our final beam-waist location is

$$\boxed{z_0 = 44.08 \pm 1.72 \text{ cm}}.$$

Results:

- $z_0 = 44.08 \pm 1.72$ cm
- $\omega_0 = 0.1673$ mm.
- $\theta_0 = 1.2045$ mrad

Precautions and Discussion

- Don't look directly into the laser.
- Don't touch the knife edge.
- We need to take readings on the sharp side of the knife edge as on the other side we get diffused reading, the laser cut-off is not sharp.
- The laser is never exactly in the TEM_{00} mode so our beam parameter values will not be exactly right.