Measurement of Dielectric Constant

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Aim:

To determine the dielectric constant of a non-magnetic low loss dielectric (solid) material using the microwave bench with the von Hippel or shorted waveguide method.

Apparatus:

- Rectangular Wave Guide
- Two samples of same material but with different thickness
- Microwave source
- Voltmeter.

Without Sample		Sample 1		Sample 2	
Distance (mm)	Voltage (mV)	Distance (mm)	Voltage (mV)	Distance (mm)	Voltage (mV)
7.5	44	7.5	71	7.5	63
7.7	68	7.6	79	7.6	57
7.8	68	7.7	71	7.7	45
7.9	64	7.8	61	7.8	33
8.0	58	7.9	52	7.9	20
8.1	53	8.0	43	8.0	4
8.3	48	8.1	37	**8.1**	**0**
8.5	40	8.2	30	8.2	8
8.7	31	8.3	22	8.3	46
8.9	20	8.4	15	8.4	98
9.0	14	8.5	6	8.5	106
9.1	**11**	**8.6**	**1**	8.6	111
9.2	15	8.7	5	8.7	106
9.3	32	8.8	28	8.8	102
9.4	56	8.9	77	8.9	99
9.5	67	9.0	98	9.0	94
9.6	74	9.1	106	9.1	90
9.7	73	9.2	104	9.2	82
9.8	69	9.3	98	9.3	75
9.9	64	9.4	90	9.4	67

Table 1: Voltage vs. Distance (7.5–9.9 mm) for three cases.

Without Sample		Sample 1		Sample 2	
Distance (mm)	Voltage (mV)	Distance (mm)	Voltage (mV)	Distance (mm)	Voltage (mV)
10.0	58	9.5	81	9.5	55
10.2	49	9.6	73	9.6	41
10.4	41	9.7	62	9.7	31
10.6	31	9.8	53	9.8	18
10.8	20	9.9	45	9.9	5
10.9	**10**	10.0	35	**10.0**	**0**
11.0	10	10.1	29	10.1	10
11.1	17	10.2	21	10.2	52
11.2	35	10.3	15	10.3	101
11.3	57	10.4	8	10.4	124
11.4	72	**10.5**	**1**	10.5	119
11.5	77	10.6	7	10.6	110
11.6	77	10.7	33	10.7	105
11.7	72	10.8	72	10.8	101
11.9	59	10.9	101	10.9	97
12.1	49	11.0	112	11.0	91
12.2	44	11.1	111	11.1	85
12.4	34	11.2	105	11.2	76
12.7	17	11.3	96	11.3	66
12.8	12	11.4	83	11.4	53
12.9	**9**	11.5	71	11.5	41
13.0	19	11.6	62	11.6	28
		11.7	53	11.7	18
		11.8	45	11.8	7
		11.9	36	**11.9**	**0**
		12.0	29	12.0	14
		12.1	21		
		12.3	6		
		12.4	**1**		
		12.5	7		
		12.6	37		

Table 2: Voltage vs. Distance (10.0–13.0 mm) for three cases.

Calculation:

Given (Without Sample):

Minima positions:

$$d_1 = 9.1\,\text{cm}, \quad d_2 = 10.0\,\text{cm}, \quad d_3 = 11.9\,\text{cm}$$

$$\Delta d = 11.9 - 10.0 = 1.9\,\text{cm}$$

$$\beta_{g0} = \frac{\pi}{\Delta d} = \frac{\pi}{1.9} = 1.65347\,\text{rad/cm}$$

Sample 1

Thickness:

$$t = 0.815\,\text{cm}$$

Minima positions:

$$8.60,\ 10.50,\ 12.40\,\text{cm}$$

$$x = 9.1 - 8.60 = 0.5\,\text{cm}$$

From the equation:

$$\frac{\tan(\beta_{g0}(t+x))}{\beta_{g0}t} = \frac{\tan(X)}{X}$$

Substituting:

$$\frac{\tan(1.65347 \cdot (0.815 + 0.5))}{1.65347 \cdot 0.815} = \frac{\tan(X)}{X}$$

Solving numerically:

$$X = 2.00471$$

$$X = \beta_{gs}t \Rightarrow \beta_{gs} = \frac{X}{t} = \frac{2.00471}{0.815} = 2.4597\,\text{rad/cm}$$

$$\beta_0^2 = \beta_{g0}^2 + \left(\frac{2\pi}{\lambda_c}\right)^2 = (1.65347)^2 + \left(\frac{6.28}{4.46}\right)^2 = 4.71864\,\text{rad}^2/\text{cm}^2$$

Dielectric constant:

$$\varepsilon_r - 1 = \frac{\beta_{gs}^2 - \beta_{g0}^2}{\beta_0^2} = \frac{2.4597^2 - 1.65347^2}{4.71864} = 0.1641$$

$$\Rightarrow \varepsilon_r = 1.1641$$

Sample 2

Thickness:

$$t = 1.516\,\text{cm}$$

Minima positions:

$$8.10,\ 10.00,\ 11.90\,\text{cm}$$

$$x = 9.1 - 8.10 = 1.0\,\text{cm}$$

Using:

$$\frac{\tan(1.65347 \cdot (1.516 + 1.0))}{1.65347 \cdot 1.516} = \frac{\tan(X)}{X}$$

Solving:

$$X = 4.37283 \Rightarrow \beta_{gs} = \frac{X}{t} = \frac{4.37283}{1.516} = 2.8844\,\text{rad/cm}$$

$$\varepsilon_r - 1 = \frac{2.8844^2 - 1.65347^2}{4.71864} = 0.2508 \Rightarrow \varepsilon_r = 1.2508$$

Error Analysis

Sample 1

$$\Delta x = 0.01\,\text{cm}, \quad \Delta t = 0.001\,\text{cm}$$

$$\frac{\Delta\beta_{gs}}{\beta_{gs}} = \frac{\Delta x}{x} + \frac{\Delta t}{t} = \frac{0.01}{0.5} + \frac{0.001}{0.815} = 0.02012 + 0.00123 = 0.02135$$

$$\frac{\Delta\varepsilon_r}{\varepsilon_r} = 2 \cdot \frac{\Delta\beta_{gs}}{\beta_{gs}} = 0.04270 \Rightarrow \Delta\varepsilon_r = 0.04270 \cdot 1.1641 = 0.0497$$

Sample 2

$$\Delta x = 0.01\,\text{cm}, \quad \Delta t = 0.001\,\text{cm}$$

$$\frac{\Delta\beta_{gs}}{\beta_{gs}} = \frac{0.01}{1.0} + \frac{0.001}{1.516} = 0.01000 + 0.000659 = 0.01065$$

$$\frac{\Delta\varepsilon_r}{\varepsilon_r} = 2 \cdot 0.01065 = 0.02130 \Rightarrow \Delta\varepsilon_r = 0.02130 \cdot 1.2508 = 0.0267$$

Results:

- The dielectric constant of sample 1 is 1.1641 ± 0.0497
- The dielectric constant of sample 2 is 1.2508 ± 0.0267

Discussion and Sources of Error:

- The probe should be moved only in one direction only, because moving it the other way could cause hysteresis error.
- There should be no gap between the sample and the metal plate that is below the sample.
- Dielectric properties are frequency-dependent. This could be explained by the fact that at low frequencies, the molecules in the material have enough time to align with the field, leading to a higher dielectric constant while the opposite occurs at high frequencies.
- Dielectric constants are also temperature dependent; increasing temperature decreases the constant because thermal motion makes it harder for the dipoles to align.