## Differential Geometry

Hersh Singh

October 4, 2014

### Chapter 1

### **Basics**

**Definition 1.** A topological space M is called Haursdoff if any two points in it can be separated by open sets. Explicitly, for any two points  $p, q \in M$ , there must be open neighbourhoods of U, U' of p, q respectively such that  $U \cap U' = \emptyset$ .

#### 1.1 Manifold

M is an n dimensional topological manifold if it

- (i) is *Haursdoff*,
- (ii) has a countable basis for topology, that is, it is second countable,
- (iii) is locally homeomorphic to  $\mathbb{R}^n$ .

A basis for a topological space M is just an a family of open sets  $\{U_{\alpha} | \alpha \in I\}$  such that  $M = \bigcup_{\alpha \in I} U_{\alpha}$ .

The dimension of an n-dimensional manifold is unique.

#### **Coordinate Charts**

One of main motivations of a manifold is to find a space which looks locally euclidean. This section makes that explicit.

A pair  $(U, \varphi)$  where  $U \in M$  is open and  $\varphi : U \to \mathbb{R}^n$  is a homeomorphism is called a *coordinate chart*. By definition of a manifold, any point  $p \in M$  is contained in at least one such open set  $U \in M$ .

Given a pair  $(U, \varphi)$ , we call U the coordinate domain or coordinate neighbourhood of each of its points. The set U is called a coordinate ball or a coordinate cube if the image  $\varphi(U)$  is an open ball or an open cube in  $\mathbb{R}^n$ , respectively.

#### Topological Properties of Manifolds

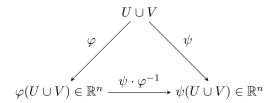
#### **Smooth Structures**

We can intuitively guess what a smooth map on the euclidean space means. A map  $F: V \to U$ , where  $U \in \mathbb{R}^n$ ,  $V \in \mathbb{R}^m$  are open subsets, is said to be

smooth if all its partial derivatives are continuous in all orders. If in addition F is bijective and has a smooth inverse map, then it's called a diffeomorphism.

Now, here's how we extend concept of smoothness to manifolds.

We have M, a topological n-manifold.



We already know how to talk about the smoothness of the map  $\psi \cdot \varphi^{-1}$ , since its just a map on the euclidean space. Lets call this the *transition map* from  $\varphi$  to  $\psi$ . Two charts  $(U, \varphi)$  and  $(V, \psi)$  are said to be smoothly compatible if either  $U \cup V = \emptyset$  or the if transition map  $\psi \cdot \varphi^{-1}$  is a diffeomorphism.

We define an atlas for M as a collection of charts whose domains cover M. An atlas  $\mathcal{A}$  is called a *smooth atlas* if any two charts in  $\mathcal{A}$  are smoothly compatible with each other.

more here

#### 1.2 Tangent Space

# Appendix A

## References

1. Introduction to Differential Topology by Brocker and Janich