Sensitivity

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Sensitivity analysis

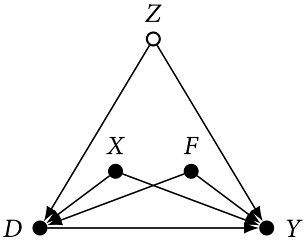
Sensitivity to

- → Collider bias: nope
- → Mediator-outcome confounding: week 7
- → Confounding…
- → Exclusion violation restrictions? [link] (Conley et al., 2012; van Kippersluis & Rietveld, 2018)

The state of the art

- → Omitted variable bias in coefficients
- → Omitted variable bias in R^2
- → Plots, plots, plots

Unmeasured confounding



Problem:

- → We know about X but we wonder if there's an unmeasured Z too

Cinelli and Hazlett (2020) example:

- → What is the effect of direct harm D due to government-organized or perpetrated violence on Y peace preferences in Darfur 2003-4?
- → X demographics including age, occupation, household size, voting experience
- → F being female

- → Z? village centrality, asset types, village accessibility…

See Hazlett (2020) for study details

Example

Observed variables

peace index (Y):

- → min; 0, max: 1,
- → mean: 0.32, sd: 0.35,
- → 0: 40%, 1: 9%

directlyharmed (D):

- → 0: 59%, 1: 41%

female (F):

- → 0: 54%, 1: 46%

	Model 1
(Intercept)	1.082 (0.315)***
directlyharmed	0.097 (0.023)***
age	−0.002 (0.001)*
farmer_dar	−0.040 (0.029)
herder_dar	0.014 (0.032)
pastvoted	−0.048 (0.024)*
hhsize_darfur	0.001 (0.002)
female	−0.232 (0.024)***
R^2	0.512
Num. obs.	1276

*** $p < 0.001$; ** $p < 0.01$; * $p < 0.05$

Omitted variables

What we want to estimate

$$Y = D\hat{\tau} + X\hat{\beta} + Z\hat{\gamma} + \hat{\epsilon}$$

What we estimate

$$Y = D\hat{\tau}_{\text{res}} + X\hat{\beta}_{\text{res}} + \hat{\epsilon}_{\text{res}}$$

The difference (bias) in estimates

$$\text{bias} = \hat{\tau}_{\text{res}} - \hat{\tau}$$

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What is $\hat{\tau}_{\text{res}}$ as a function of $\hat{\tau}$?

Omitted variables formula

Cinelli and Hazlett use $A^{\perp B}$ to mean the *residuals* from a regression of A on B

→ what's left of A after the 'effect' of B has been removed

Reminder: the regression coefficient of B predicting A in a *bivariate* regression is

$$\beta_B = \frac{\text{cov}(B, A)}{\text{var}(B)}$$

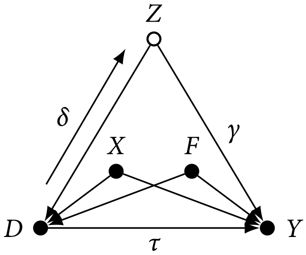
and from the Frisch-Lowell-Waugh theorem, the regression coefficient of B predicting A *controlling for* X is

$$\beta_B = \frac{\text{cov}(B^{\perp X}, A^{\perp X})}{\text{var}(B^{\perp X})}$$

Omitted variables formula

Then

$$
\begin{aligned}
\hat{\tau}_{\text{res}} &= \frac{\operatorname{cov}(D^{\perp X}, Y^{\perp X})}{\operatorname{var}(D^{\perp X})} \\
&= \frac{\operatorname{cov}(D^{\perp X}, D^{\perp X} + Z^{\perp X})}{\operatorname{var}(D^{\perp X})} \\
&= \hat{\tau} + \hat{\gamma}\frac{\operatorname{cov}(D^{\perp X}, Z^{\perp X})}{\operatorname{var}(D^{\perp X})} \\
&= \hat{\tau} + \hat{\gamma}\hat{\delta}
\end{aligned}
$$



So

$$
\text{bias} = \hat{\gamma}\hat{\delta}
$$

→ δ is a measure of 'imbalance'

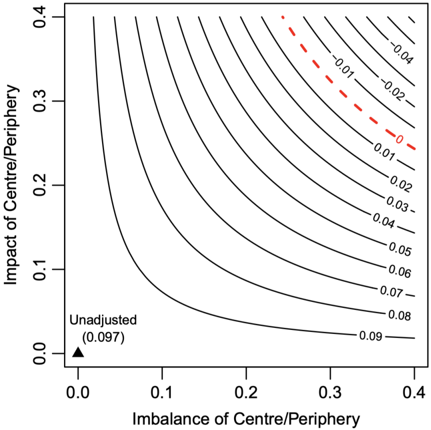
→ γ is a measure of the (not necessarily causal) 'impact' of Z on Y

Bias is imbalance times impact

> *the only way in which Z's relationship to D enters the bias is captured by its 'linear imbalance', parameterized by $\hat{\delta}$. In other words, the linear regression of Z on D and X need not reflect the correct expected value of Z—rather it serves to capture the aspects of the relationship between Z and D that affects the bias.*
>
> *(Cinelli & Hazlett, 2020)*

OVB PLOT

Plotting $\hat{\tau}_{\text{res}} - \hat{\gamma}\hat{\delta}$



Limitations

- → Hard to interpret if Z is not binary
- → What about *multiple* Z?
- → How much 'robustness' is enough? Compared to what?
- → Can get sensitivity for more than the coefficient's point estimate?

Suggestion: Switch from coefficients to (partial) R^2s

Reminder: R^2 is symmetrical in bivariate regressions:

$$R^2_{Z\sim D} = \frac{\text{var}(\hat{Z})}{\text{var}(Z)} = \frac{1 - \text{var}(Z^{\perp D})}{\text{var}(Z)} = \text{cor}(Z, \hat{Z})^2 = \text{cor}(Z, D)^2$$

and so is *partial* R^2

$$R^2_{Z\sim D|X} = \frac{1 - \text{var}(Z^{\perp D,X})}{\text{var}(Z^{\perp X})} = \text{cor}(Z^{\perp X}, D^{\perp X})^2 = R^2_{D\sim Z|X}$$

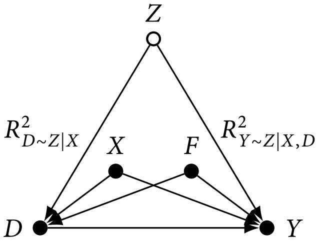
Advantages

The relevant quantities are now:

- How much variance in Y is explained by Z controlling for everything else: $R^2_{Y\sim Z|X,D}$
- How much variance in D is explained by Z controlling for everything else: $R^2_{D\sim Z|X}$

There can be *lots* of Zs working in concert

- Their scales are irrelevant now we work in variance explained, a.k.a. 'explanatory power'



Sensitivity statistics

Functions of these two quantities:

The *robustness value*

$$RV_q = R^2_{Y\sim Z|X,D} = R^2_{D\sim Z|X}$$

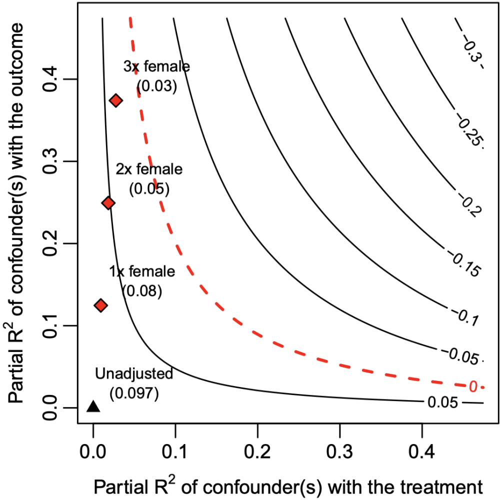
describes how strong Z has to be to reduce the $\hat{\tau}$ by a factor of q

- → e.g. RV_1 is the strength of confounding that would reduce it to zero

Comparative robustness measures, e.g. relative bounds

- → the *maximum effect* that a Z not more than k times as strong as, say 'Female' would have on $\hat{\tau}$
- → These results are upper bounds on the effects of multiple Zs

Graphical reporting



Interpretation:

- → For multivariate Z contours are upper bounds
- → RV_q is a summary of the *diagonal* of this plot
- → $\hat{\tau}$ when Z is k times as powerful as 'Female' are shown as diamonds

Tabular reporting

Table 1. Proposed minimal reporting on sensitivity to unobserved confounders†

Treatment	*Outcome, PeaceIndex:*					
	Estimate	*Standard error*	*t-value*	$R^2_{Y\sim D\|\mathbf{X}}$ *(%)*	*RV (%)*	$RV_{\alpha=0.05}$ *(%)*
DirectHarm	0.097	0.023	4.18	2.2	13.9	7.6

†df = 783; bound (Z as strong as Female), $R^2_{Y\sim Z|D,\mathbf{X}} = 12\%$, $R^2_{D\sim Z|\mathbf{X}} = 1\%$.

- → Unmeasured confounders with equal effect on D and Y would have to explain 13.9% to reduce $\hat{\tau} = 0.097$ to zero
- → or 7.6% to make $\hat{\tau}$ not statistically distinguishable from zero (at the 5% level)
- → if confounders explained 100% of the residual variance of the Y, they would need to explain at least 2.2% of the residual variance of the D to reduce $\hat{\tau}$ to zero
- → the footnote shows the two relevant quantities for a Z like 'Female'. Notice that 1% < 2.2%, so even in the worst case scenario above, $\hat{\tau}$ would not be reduced to zero

Aside: functional form

Sensitivity to misspecification comes for free!

> Finally, note that the set of confounders $\mathbf{Z}$ is arbitrary; thus it accommodates non-linear confounders as well as misspecification of the functional form of the observed covariates $\mathbf{X}$. To illustrate the point, let $Y = \hat{\tau}D + \hat{\beta}X + \hat{\gamma}_1 Z + \hat{\gamma}_2 Z^2 + \hat{\gamma}_3 ZX + \hat{\gamma}_4 X^2 + \hat{\varepsilon}_{\text{full}}$, and imagine that the researcher did not measure Z and did not consider that X could also enter the equation with a squared term. Now just call $\mathbf{Z} = (Z_1 = Z, Z_2 = Z^2, Z_3 = ZX, Z_4 = X^2)$ and all the previous arguments follow.

Sensitivity analysis

What thresholds are reasonable for sensitivity testing?

→ Terrible question: this question makes (almost) no sense

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So what do I do with all this information then?

> This transition from a qualitative to a quantitative discussion about unobserved confounding can often be enlightening. As put by Rosenbaum (2017), page 171, it may ‘provide grounds for caution that are not rooted in timidity, or grounds for boldness that are not rooted in arrogance’. A sensitivity analysis raises the bar for the sceptic of a causal estimate—not just any criticism can invalidate the research conclusions. The hypothesized unobserved confounder now must meet certain standards of strength; otherwise, it cannot logically account for all the observed association. Likewise, it also raises the bar for defending a causal interpretation of an estimate—proponents must articulate how confounders with certain strengths can be ruled out.

References

Cinelli, C. & Hazlett, C. (2020). ‘Making sense of sensitivity: Extending omitted variable bias’. *Journal of the Royal Statistical Society: Series B (Statistical Methodology)*, *82*(1), 39–67.

Conley, T. G., Hansen, C. B. & Rossi, P. E. (2012). ‘Plausibly exogenous’. *The Review of Economics and Statistics*, *94*(1), 260–272.

Hazlett, C. (2020). ‘Angry or weary? how violence impacts attitudes toward peace among darfurian refugees’. *Journal of Conflict Resolution*, *64*(5), 844–870.

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