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Trabalho de CDI II - Integrais multiplas

Questão 1

a-) $\iint_R \frac{4y}{x^3+2}\,dx\,dy$ onde $R=\{(x,y);\ 1\leq x\leq 2 \text{ e } 0\leq y\leq 2x\}$

$$\int_1^2\int_0^{2x} \frac{4y}{x^3+2}\,dy\,dx = \int_1^2 \frac{\cancel{4}y^2}{\cancel{2}(x^3+2)}\Big|_0^{2x} dx = \int_1^2 \frac{2(2x)^2}{x^3+2}\,dx = \int_1^2 \frac{8x^2}{x^3+2}\,dx =$$

$$8\int_1^2 \frac{x^2}{x^3+2}\,dx = 8\int_3^{10} \frac{\cancel{x^2}}{u}\,\frac{du}{3x^2} = \frac{8}{3}\int_3^{10}\frac{1}{u}\,du = \frac{8}{3}\left(\ln(u)\Big|_3^{10}\right)$$

$$= \frac{8}{3}\left(\ln(10)-\ln(3)\right)$$

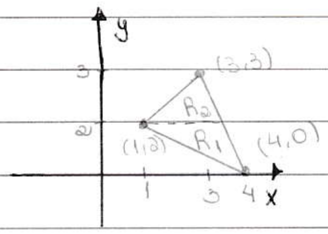
substituição

$u=x^3+2 \to du = 3x^2dx$

$u \to 2^3+2,\ u\to 10$

$u\to 1^3+2,\ u\to 3$

b-) $\iint_R x^2+y^2\,dxdy$ onde R é o interior do triângulo cujos vértices $(1,2),(3,3),(4,0)$



equação de reta:

$(3,3),(4,0)$	$(3,3),(1,2)$
$m = \frac{y_1-y_0}{x_1-x_0}$ $\quad y-y_0 = m(x-x_0)$	$m = \frac{3-2}{3-1}$ $\quad y-2 = \frac{1}{2}(x-1)$
$m = \frac{3-0}{3-4}$ $\quad y-0 = -3(x-4)$	$m = \frac{1}{2}$ $\quad y-2 = \frac{1}{2}x - \frac{1}{2}$
$y = -3x+12$	$y = \frac{1}{2}x + \frac{3}{2}$
$\hookrightarrow x = -\frac{y}{3} + \frac{12}{3}$	$\hookrightarrow 2y-3 = x$
$m = \frac{3}{-1} = -3$	

$(4,0)\ (1,2)$

$m = \frac{2-0}{1-4}$

$m = \frac{2}{-3} = -\frac{2}{3}$

$y-0 = -\frac{2}{3}(x-4)$

$y = -\frac{2}{3}x + \frac{8}{3}$

$\to -\frac{3y}{2} + 4 = y$

$R_1 \begin{cases} 0 \leq y \leq 2 \\ -\frac{3y}{2}+4 \leq x \leq -\frac{y}{3}+\frac{12}{3} \end{cases}$

$R_2 \begin{cases} 2 \leq y \leq 3 \\ 2y-3 \leq x \leq -\frac{y}{3}+\frac{12}{3} \end{cases}$

$$\iint_R x^2+y^2\,dxdy = \underbrace{\int_0^2\int_{-\frac{3y}{2}+4}^{-\frac{y}{3}+\frac{12}{3}} x^2+y^2\,dxdy}_{I_1} + \underbrace{\int_2^3\int_{2y-3}^{-\frac{y}{3}+\frac{12}{3}} x^2+y^2\,dxdy}_{I_2}$$

$\hookrightarrow$ continuação ...

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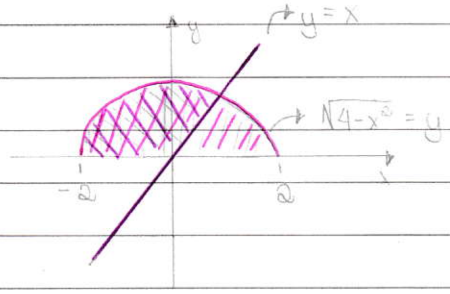
c-) $\iint_R \cos(x^2+y^2)\,dx\,dy$ onde R é a região onde $0 \leq y \leq \sqrt{4-x^2}$ e que esta acima da reta $y=x$



$y \geq 0$
$y \leq \sqrt{4-x^2}$

$y = \sqrt{4-x^2}$
$y^2 = (\sqrt{4-x^2})^2$
$y^2 = 4-x^2$
$y^2+x^2 = 4$
circunferência com $r=2$

transformando para coordenadas polares

- $\cos(x^2+y^2) = \cos(r^2)$
- $y^2+x^2=4 \rightarrow r^2=4 \rightarrow r=\pm 2 \rightarrow 0 \leq r \leq 2$
- $\frac{\pi}{4} \leq \theta \leq \pi$

substituição $u=r^2 \rightarrow du = 2r\,dr$
$r=2 \rightarrow u=r^2 \rightarrow u=4$ / $u=0$

$$\iint_R \cos(x^2+y^2)\,dx\,dy = \int_{\frac{\pi}{4}}^{\pi}\int_0^2 \cos(r^2)\,r\,dr\,d\theta = \int_{\frac{\pi}{4}}^{\pi}\int_0^4 \cos(u)\,\cancel{r}\,\frac{du}{2\cancel{r}}\,d\theta =$$

$$= \int_{\frac{\pi}{4}}^{\pi}\frac{1}{2}\int_0^4 \cos u\,du\,d\theta = \int_{\frac{\pi}{4}}^{\pi}\frac{1}{2}\left(\operatorname{sen} u\,\Big|_0^4\right)d\theta = \int_{\frac{\pi}{4}}^{\pi}\frac{\operatorname{sen}(4)}{2}\,d\theta = \frac{1}{2}\int_{\frac{\pi}{4}}^{\pi}\operatorname{sen}(4)\,d\theta$$

$$= \frac{1}{2}\left(\operatorname{sen}(4)\theta\,\Big|_{\frac{\pi}{4}}^{\pi}\right) = \frac{1}{2}\left(\operatorname{sen}(4)\pi - \frac{\operatorname{sen}(4)\pi}{4}\right) = +\frac{1}{2}\left(\frac{4\operatorname{sen}(4)\pi - \operatorname{sen}(4)\pi}{4}\right) =$$

$$= \frac{1}{2}\cdot\frac{3\operatorname{sen}(4)\pi}{4} = \frac{3\pi}{8}\operatorname{sen}(4)$$

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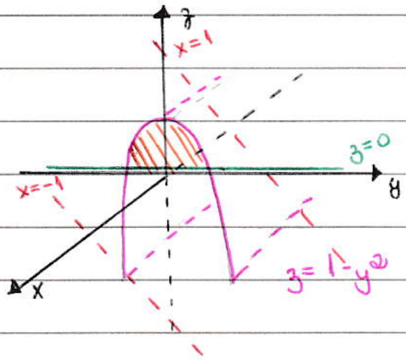
d-) $\iiint_S x^2 e^y \, dx\,dy\,dx$ onde S é o sólido delimitado pelas superfícies $z = 1 - y^2$, $z = 0$, $x = -1$, $x = 1$



superfície superior: $z = 1 - y^2$
superfície inferior: $z = 0$
$x = 1$, $x = -1$
$y = 1$, $y = -1$

$$\iiint_S x^2 e^y \, dx\,dy\,dx = \int_{-1}^{1}\int_{-1}^{1}\int_{0}^{1-y^2} x^2 e^y \, dz\,dy\,dx = \int_{-1}^{1}\int_{-1}^{1} x^2 e^y z \Big|_0^{1-y^2} dy\,dx =$$

$$= \int_{-1}^{1}\int_{-1}^{1} x^2 e^y (1-y^2)\, dy\,dx$$

↳ como tanto $-1 \leq x \leq 1$ e $-1 \leq y \leq 1$, podemos inverter a ordem da integração

$$\int_{-1}^{1}\int_{-1}^{1} x^2 e^y (1-y^2)\, dx\,dy = \int_{-1}^{1} e^y (1-y^2) \int_{-1}^{1} x^2 \, dx\,dy =$$

$$= \int_{-1}^{1} e^y (1-y^2) \left(\frac{x^3}{3}\right)\Big|_{-1}^{+1} dy = \int_{-1}^{1} e^y (1-y^2) \left(\frac{1}{3} + \frac{1}{3}\right) dy = \int_{-1}^{1} e^y (1-y^2) \frac{2}{3}\, dy$$

$$= \frac{2}{3}\int_{-1}^{1} e^y (1-y^2)\, dy = \frac{2}{3}\left[(1-y^2)(e^y)\Big|_{-1}^{1} - \int_{-1}^{1} (-2y)(e^y)\, dy \right]$$

integração por partes
$u = (1-y^2)$ ↳ $\int uv' = uv - \int u'v$
$v' = e^y$
$u' = -2y$
$v = e^y$

↳ continuação

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$$= \frac{2}{3}\left[(1-y^2)(e^y) - \int -2e^y y\, dy\right]_{-1}^{1}$$

$-2\int e^y y\, dy$ integração por parte

$-2((ye^y) - \int e^y dy)$ $u = y$ $v' = e^y$

$-2(ye^y - e^y)$ $u' = 1$ $v = e^y$

$$= \frac{2}{3}\left[e^y(1-y^2) - \left(-2(ye^y - e^y)\right)\right]_{-1}^{1} = \frac{2}{3}\left[e^y(1-y^2) + 2(ye^y - e^y)\right]_{-1}^{1} =$$

$$= \frac{2}{3}\left[e^y - e^y y^2 + 2ye^y - 2e^y\right]_{-1}^{1} = \frac{2}{3}\left[-e^y - e^y y^2 + 2ye^y\right]_{-1}^{+1} =$$

$$= \frac{2}{3}\left[\cancel{-e} \cancel{-e} + \cancel{2e} - (-e^{-1} - e^{-1}(-1)^2 + 2(-1)e^{-1})\right] =$$

$$= \frac{2}{3}\left[+e^{-1} + e^{-1}(-1)^2 - 2(-1)e^{-1}\right] = \frac{2}{3}\left[\frac{1}{e} + \frac{1}{e} + \frac{2}{e}\right] = \frac{2}{3} \cdot \frac{4}{e} = \frac{8}{3e}$$ //

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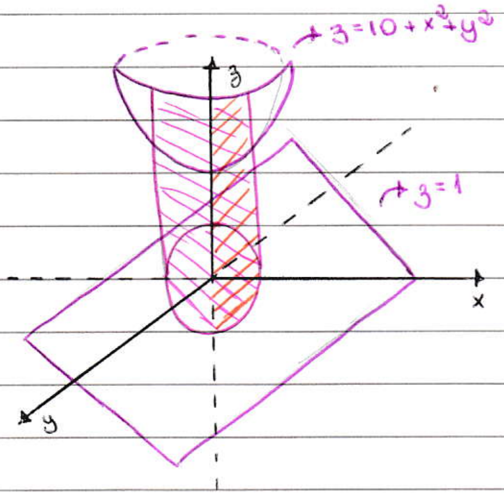
e-) o volume do sólido $S = \{(x,y,z);\ 1 \leq z \leq 10 + x^2 + y^2,\ x^2+y^2 \leq 4 \text{ e } x \geq 0\}$



superfície sup: $z = 10 + x^2 + y^2$
superfície inf: $z = 1$

$x^2 + y^2 = 4$
$y^2 = 4 - x^2$
$y = \pm\sqrt{4 - x^2}$

$0 \leq x \leq 2$

$$\int_0^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_1^{10+x^2+y^2} 1 \, dz\, dy\, dx$$ → transformar para coordenadas cilíndricas

- $10 + x^2 + y^2 \rightarrow 10 + r^2$
- $x^2 + y^2 = 4 \rightarrow r^2 = 4 \rightarrow r = \pm 2$ $0 \leq r \leq 2$
- $0 \leq \theta \leq \pi$

$$\rightarrow \int_0^\pi \int_0^2 \int_1^{10+r^2} r \, dz\, dr\, d\theta = \int_0^\pi \int_0^2 rz \Big|_1^{10+r^2} dr\, d\theta =$$

$$= \int_0^\pi \int_0^2 r(10 + r^2) - r \, dr\, d\theta = \int_0^\pi \int_0^2 10r + r^3 - r \, dr\, d\theta =$$

$$\int_0^\pi \int_0^2 r^3 + 9r \, dr\, d\theta = \int_0^\pi \frac{r^4}{4} + \frac{9r^2}{2} \Big|_0^2 d\theta = \int_0^\pi \frac{2^4}{4} + \frac{9 \cdot 2^2}{2} \, d\theta =$$

$$\int_0^\pi \frac{16}{4} + 18 \, d\theta = \int_0^\pi 4 + 18 \, d\theta = \int_0^\pi +14 \, d\theta = 22\theta \Big|_0^\pi = 22\pi$$

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questão 2

em coordenadas cilíndricas e esféricas

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{2-x^2-y^2}} xy \, dz \, dy \, dx$$

- $\sqrt{x^2+y^2} \leq z \rightarrow r \leq z$ e $z \leq \sqrt{2-x^2-y^2} \rightarrow z = \sqrt{2-r^2}$
- $0 \leq y \rightarrow 0 \leq r.\operatorname{sen}\theta$ e $y \leq \sqrt{1-x^2} \rightarrow y^2+x^2 \leq 1$

$0 \leq \theta \leq 2\pi$ $\quad$ $0 \leq R \leq 1$

$$\int_0^{2\pi} \int_0^1 \int_r^{\sqrt{2-r^2}} (r\cos\theta)(r\operatorname{sen}\theta) r \, dz \, dr \, d\theta = \int_0^{2\pi} \int_0^1 \int_r^{\sqrt{2-r^2}} r^3 \cos\theta \operatorname{sen}\theta \, dz \, dr \, d\theta$$

$$\int_0^{2\pi} \int_0^1 r^3 \cos\theta \operatorname{sen}\theta \, z \Big|_r^{\sqrt{2-r^2}} dr \, d\theta = \int_0^{2\pi} \int_0^1 \cancel{r^3} \cancel{\cos}\theta \cancel{\operatorname{sen}}\theta (\sqrt{2-r^2}) - \cancel{r^3} r . \cancel{\cos}\theta \cancel{\operatorname{sen}}\theta \, dr \, d\theta$$

$$\int_0^{2\pi} \int_0^1 \sqrt{2-r^2} - r \, dr \, d\theta \int_0^{2\pi} \frac{2\sqrt[3]{(1-r^2)^2}}{3} - \frac{r^2}{2} \Big|_0^1 d\theta = \int_0^{2\pi} \frac{2\sqrt[3]{(2-1)^2}}{3} - \frac{1}{2}^2 - 0 \, d\theta$$

$$\int_0^{2\pi} \frac{2}{3} - \frac{1}{2} \, d\theta = \int_0^{2\pi} \frac{1}{6} \, d\theta = \frac{1}{6} . 2\pi = \frac{\pi}{3} \,//$$