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The Evolution of Dynamical Cores for Global Atmospheric Models

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Abstract

The evolution of global atmospheric model dynamical cores from the first developments in the early 1960s to present day is reviewed. Numerical methods for atmospheric models are not straightforward because of the so-called pole problem. The early approaches include methods based on composite meshes, on quasi-homogeneous grids such as spherical geodesic and cubed sphere, on reduced grids, and on a latitude-longitude grid with short time steps near the pole, none of which were entirely successful. This resulted in the dominance of the spectral transform method after it was introduced. Semi-Lagrangian semi-implicit methods were developed which yielded significant computational savings and became dominant in Numerical Weather Prediction. The need for improved physical properties in climate modeling led to developments in shape preserving and conservative methods. Today the numerical methods development community is extremely active with emphasis placed on methods with desirable physical properties, especially conservation and shape preservation, while retaining the accuracy and efficiency gained in the past. Much of the development is based on quasi-uniform grids. Although the need for better physical properties is emphasized in this paper, another driving force is the need to develop schemes which are capable of running efficiently on computers with thousands of processors and distributed memory.

Test cases for dynamical core evaluation are also briefly reviewed. These range from well defined deterministic tests to longer term statistical tests with both idealized forcing and complete parameterization packages but simple geometries. Finally some aspects of coupling dynamical cores to parameterization suites are discussed.

1. Introduction

Global atmospheric models were first developed in the early 1960's. Since then they have evolved for application to the problems of deterministic weather forecasting, seasonal prediction, and climate simulation. These applications rely on basically the same models, but each emphasizes different aspects due to the relative importance of the time scales and processes involved in each application. In the early days the weather forecasting and climate simulation the models for the two applications were very distinct and the modelers made very different choices in the model formulation. NWP emphasized accurate prediction of the fluid flow by applying the highest resolution feasible and climate emphasized the parameterized forcing with conservation being essential for very long runs. For climate simulation relatively low resolution was feasible. With the inclusion of seasonal prediction and more emphasis on moist processes in NWP, the distinction between the two applications is blurring and there is a movement toward unified models applicable to all applications, e.g., Met Office Unified Model (Cullen and Davies 1991). Although a common model might be developed for the forecast and climate applications, they continue to be set up differently for the two applications. The most notable differences being the trend to extremely high resolution for fore-

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casting, e.g., triangular spectral truncation T511 or about 40 km grid at the ECMWF with plans to go to T799 or about 25 km soon, (WGNE, 2005), and to extremely long runs at low to modest resolution for climate simulation. Both applications now apply ensembles to establish measures of the uncertainty for climate and probability for forecasting.

Global models calculate the evolution of a modeled atmosphere or fluid, but are unable to resolve all processes involved, some of which occur on very small scales. Therefore models are conceptually divided into two components: a resolved dynamical fluid flow component and an unresolved, sub-grid-scale component which is parameterized as functions of the large-scale, resolved flow. The parameterized component consists of approximations to the radiative processes, planetary boundary layer, convective processes, and processes associated with stratiform clouds. The later three include phase change of water and precipitation processes. These parameterized components provide a forcing to the resolved fluid flow. We will later mention a few issues which have received less attention concerning coupling of the parameterized components to the resolved fluid flow component.

We define the dynamical core to be the resolved, fluid flow component of the model. In nonlinear fluid flow, enstrophy and energy cascade through the resolved scales and can accumulate at the truncation limit. In the continuous problem these would normally continue to cascade to smaller and smaller scales until dissipated by friction. However, such dissipative processes occur in nature at scales well below those that can be included in models. In the discrete approximation system energy can build up at the truncation limit. Thus we consider processes designed to prevent and control such buildup to be part of the dynamical core. These can involve terms explicitly added to the equations such as ∇^2 or ∇^4 diffusion of momentum and temperature, or damping that is implicitly designed into a numerical scheme which controls the accumulation. The dynamical core also includes a component that transports water in all forms (vapor, liquid, and ice) as well as chemical constituents when included in a model. The dynamical core determines the wind field that transports these quantities, often referred to as tracers. The core, in turn, is forced by the parameterizations which are strongly affected by these tracers.

Dynamical cores are often thought of, or are evaluated as dry dynamical processes. However, as alluded to above, one should also consider the pure transport component alone since it is a primary component of the fluid core. In fact the transport component is often developed and evaluated first in problems with specified wind fields. Then a dynamical core is built around the transport scheme and evaluated as a dry fluid problem. Finally the coupled dry core and moist transport are evaluated as a moist core. We will discuss some aspects of dynamical core evaluation later.

Development of numerical methods for global models has been an active area of research since the first global models were developed in the early 1960s, and continues with renewed interest today. Research falls into two areas, the first deals with numerical methods for calculating transport and for calculating hydrostatic, large scale fluid motion of the type occurring in the atmosphere, from the largest scales down to those below synoptic scales. Recently attention is also being turned toward non-hydrostatic cloud resolving models over the entire sphere such as Tomita and Satoh (2004). The numerical methods are often developed first in Cartesian geometry or more general curvilinear coordinates. In some instances they have been adopted from other areas of computational fluid dynamics, such as methods for calculating flows that develop shocks. In such cases consideration should be given to how they need to be modified to properly calculate atmospheric-like flows. The second area of research is in adapting these numerical methods to the spherical geometry of the earth, which presents unique problems, usually and vaguely referred to collectively as the pole problem. We will argue that, overall, the pole problem is primarily an economic problem. Of course these two areas are not completely independent.

This paper is not intended as a mathematical review of the properties of numerical methods. That would require textbooks which do exist. For example, for details about basic numerical methods used in the past in atmospheric models and the concepts involved see Haltiner and Williams (1980) and the less accessible, but still very valuable, GARP Publications Series No 17 (Mesinger and Arakawa 1976; Kasahara 1979) on Numerical Methods Used in Atmospheric Models. For a summary of numerical methods that have been used for global models and additional references see Williamson (1992) and Williamson and Laprise (2000). An excellent book by Durran (1999) covers the fundamentals of numerical methods for atmospheric problems and fluid flow in general. More recent reviews on specific types of schemes include Machenhauer et al. (2007) on finite-volume methods and other contributions to that volume.

This paper concentrates on the adaptations to spherical geometry, the pole problem, rather than the details of numerical methods. This primarily involves the horizontal aspects of the schemes and we do not discuss the vertical aspects although they are of course very important. We cannot go into all details but try to point the reader toward current research directions that they might follow if interested.

2. History of the pole problem

2.1 Early developments

The earliest atmospheric models were grid point, finite difference based. The thrust in atmospheric modeling was in basic numerical approximations for nonlinear fluid flow and dealt with issues such as accuracy, efficiency, conservation, and nonlinear instability. The earliest models involved transforming the equations to a map projection over a limited region of the sphere. Cartesian coordinates were then applied on the map projections. Applying these approaches which were developed for Cartesian coordinates on map projections to the entire sphere introduced an additional set of difficulties. Almost all the approaches applied to the sphere in the early days with varying degrees of success, have been resurrected in recent times and remain foci of current developments. Williamson (1979, 1992) provide extensive references and more details of these early developments than can be included here. Several of these early approaches which did not reach fruition then are now looking promising with recent work which combines them with more modern numerical methods.

Spherical curvilinear coordinates (latitude and longitude) present the most obvious coordinate system for the surface of the sphere. The most natural grid for spherical coordinates is equally spaced lines of constant latitude and longitude and therein lies the problem. Figure 1 illustrates such a grid. The meridians convergence approaching the pole making the longitudinal grid interval (measured in distance rather than degrees) approach zero at the pole, and the coordinate system becomes singular there. This convergence and singularity led to the term pole problem being applied to a rather vaguely defined problem. But this pole problem is really one of economics rather than a fundamental problem. The singularity itself can be dealt with in a variety of ways.

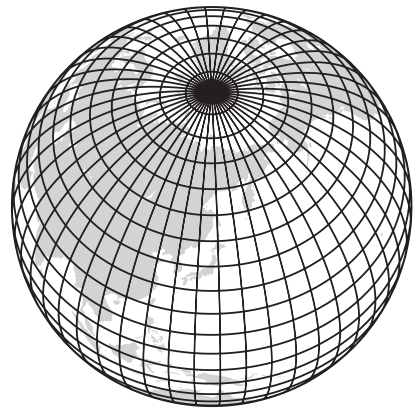

Fig. 1. A latitude-longitude grid consisting of equally spaced lines of constant latitude and longitude.

The earliest numerical weather prediction models were formulated with the equations transformed to a map projection. Conformal projections were chosen because they result in greater symmetry in the equations when written in terms of Cartesian coordinates on the map projection. Stereographic and Mercator projections were the most common, with domains at most hemispheric. It was natural to attempt to extend these projections to sphere, but no single conformal projection maps the entire sphere onto a finite section of the plane. Therefore Phillips (1957) proposed combining several projections to cover the sphere. This approach would later be referred to as a composite mesh, and more recently as overset grids. He used two polar stereographic projec-

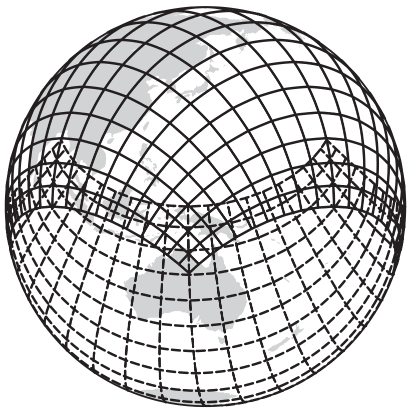

Fig. 2. A composite or overset grid consisting of uniform grids on a North Polar and a South Polar stereographic projection. The meshes on the projections are mapped back to the surface of the sphere to illustrate the overlap.

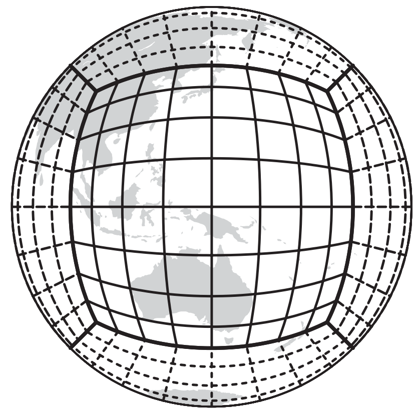

Fig. 3. A "cubed sphere" grid obtained by projecting a Cartesian coordinate system on each face of a cube onto the surface of the sphere.

tions for the Northern and Southern hemispheres, and a Mercator projection for the equatorial band, and applied second-order finite differences to uniform grids on each projection. Values needed for the approximations at points not included in the computational grid of one projection were obtained by interpolation within the grid of another projection. Figure 2 shows a composite grid consisting of a North Polar and a South Polar stereographic grid projected back to the surface of the sphere to illustrate the overlap. Note, this grid does not include the equatorial Mercator projection included by Phillips (1957). Although Phillips (1962) showed that with careful definition of the finite-difference scheme and interpolation procedures the composite mesh approach could give good results, his approach never gained popularity. Conservation aspects of composite meshes were revisited by Stoker and Isaacson (1975) with the addition of a conserving technique for the interpolations (Bayliss and Isaacson 1975; Sasaki 1976). Again composite meshes were not adopted for a complete baroclinic model, perhaps because of lingering concerns about conservation and noise on the part of practitioners of that time given the very large investment needed to develop a complete baroclinic model.

Sadourny (1972) developed a method to cover the sphere with several non-conformal projections which required no interpolations between meshes. It is based on a regular polyhedron circumscribed to the sphere. A coordinate system is derived for each face for a gnomonic or central projection. He tested this approach with a cube for the polyhedron in which case the sides of the polyhedral faces are coordinate lines and grid points are common to the two sides defining the edge. Such a system is illustrated in Fig. 3. Finite differences were developed at the boundaries from flux or conservation considerations so no interpolations were necessary to obtain information from adjacent faces. He encountered a diffculty with two-grid interval noise arising from the boundaries where it is difficult to maintain the accuracy of the interior scheme.

As mentioned above, the pole problem with spherical coordinates is primarily and economic one, not a technical problem. Explicit finite difference schemes have a restriction on the time step related to the wind speed and the grid interval. Essentially the time step must be small enough that the advection or wave propagation remains within the grid stencil used by the finite differences. The relation between the wind speed, grid length and time step is referred to as the Courant-Fredrich-Levy or CFL condition, after the mathematicians who first de-

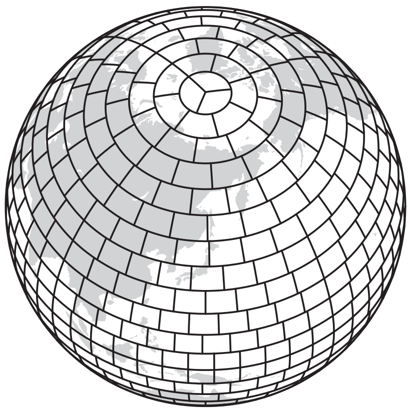

Fig. 4. A Kurihara or reduced grid in which the longitudinal grid interval measured in degrees is increased approaching the poles to keep the physical length as uniform as possible.

scribed it. Thus as the longitudinal grid interval goes to zero approaching the poles the time step must go to zero since the wind does not go to zero. Use of such a small time step makes a method extremely expensive. In order to make schemes based on latitude-longitude grids applied to spherical coordinates economical, two early attempts were to (1) shorten the time steps near the poles to satisfy longitudinal CFL restrictions arising from the decreasing longitudinal grid distance (Grimmer and Shaw 1967) and (2) to lengthen the longitudinal grid intervals over which the finite differences are taken near the poles again to create a less restrictive CFL condition (Gates and Regal 1962; Kurihara 1965). While the first approach gave satisfactory results, it still had an economic disadvantage in that more than half the computer time was spent integrating the two rows next to the pole which comprised only 2 percent of the earth's area. The grids resulting from the second approach were originally referred to as Kurihara grids, but more recently have been referred to as reduced grids. One is illustrated in Fig. 4. Such grids were tried by a number of investigators and resulted in differential phase errors which produced artificial meridional tilts to waves and spurious meridional transports. Shuman (1970) and Williamson and Browning (1973) showed that errors from the curvilinearity of the coordinate system are largest near the polar singularities, and arise from the variation in the unit vectors. For uniform accuracy in approximations dealing with vector components, points should be added in longitude near the poles, not subtracted, thus making the economic problem even worse. This is not an issue with scalar variables.

Uniform latitude-longitude grids were made economical by applying a spatial filter near the poles in longitude to remove the fastest moving, computationally unstable waves, which fortunately are also the shortest waves (Umscheid and Sankar-Rao 1971). This is a rather unsatisfying, engineering approach but still in use today for example in the optional finite volume core of the Community Atmosphere Model (CAM) (Lin 1997; Collins et al. 2004). Purser (1988) has shown that errors introduced by such filtering may be significant and that near the pole some additional information in latitude should be used to reduce them.

In the mid 1960s, Buckminster Fuller's geodesic domes (e.g., McHale 1962) inspired spherical grids which were constructed by covering the sphere with nearly uniform triangles. Such grids, illustrated in Fig. 5, are referred to as spherical geodesic or icosahedral. Sadourny et al. (1968) and Williamson (1968) obtained very good solutions of the barotropic vorticity equation for a test with a Rossby-Haurwitz wave, an exact solution to the equations. The Rossby wave showed none of the distortion that was seen in the reduced grid approaches described above. Additional tests with the barotropic primitive (shallow water) equations were also successful, however solutions with higher order schemes exhibited noise (Williamson 1971). Cullen (1974) integrated the shallow water equations on an icosahedral grid using finite element methods. His results were better than those obtained with second-order finite differences on a latitude-longitude grid with four times the number of grid points but his method was not pursued further.

2.2 *Dominance of spectral transform*

In the mid to late 1970s further development of grid-point schemes for spherical geometry was stifled by the success of the spectral transform method. Although grid-point models continued to be used, the level of research and

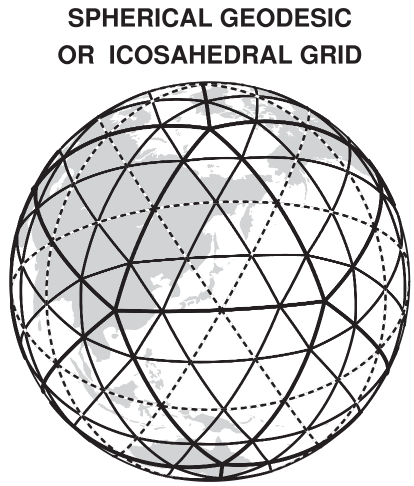

Fig. 5. A spherical geodesic or icosahedral grid obtained by subdividing the twenty triangles of an icosahedron into smaller triangles. The twenty icosahedral triangles are indicated by the thicker solid lines. Each of these triangles is divided into four smaller triangles indicated by the dashed lines combined with the thicker solid lines. These are further divided into four triangles indicated by the thiner solid lines.

development into explicit grid-point approximations dropped dramatically. Spectral transform became the method of choice for both NWP and climate models and dominated the field, although they were not universally adopted and a few notable examples of grid-point models continued to be applied.

The introduction of the spectral transform method by Eliasen et al. (1970) and Orszag (1970) made the spectral method cost effective. The spectral transform method represents fields by a series of spherical harmonics. Linear terms are calculated directly in spectral space while nonlinear terms are calculated from grid point values obtained by synthesizing the field from the spectral coefficients. The results of the nonlinear calculations are transformed back to spectral space. Machenhaur (1979) provides an excellent review of the method.

The spectral transform method calculates linear advection of a resolved field exactly except for time truncation, so there is no computational dispersion. With triangular spectral truncation it presents a natural filter for spherical geometry by providing an isotropic representation in spectral space even though the computationally adopted underlying Gaussian grid does not. Since it is based on an isotropic representation, short longitudinal structures near the pole are not present and therefore do not restrict the time step. Application of a quadratic unaliased transform grid provides a natural way to eliminate aliasing of quadratic terms and thus makes the method immune to nonlinear instability, although that problem had also been solved in grid point models by Arakawa (1966) type differences. Unlike grid-point schemes, the spectral transform method does not have a number of arbitrary parameters to define it and its application to global atmospheric models became amazingly standard following Bourke's (1974) implementation. More recently Swarztrauber (1996) compared the accuracy of nine spectral transform methods for solving the shallow water equations. They vary by being based on the shallow water equations written in different forms. Eight of the methods compute almost identical results with standard test cases. For practical purposes the ninth is also comparable to the others.

The spectral transform method became dominant at that time when the modeling issues consisted primarily of large scale, relative smooth dynamical motions. It provided a very elegant solution to sphere problem. It also has advantages at the relatively low resolution used for climate modeling at that time: linear advection is accurate to the truncation limit unlike grid-point schemes which in general damp short wave rather severely and in addition can have significant phase errors. Of course the spectral transform method could not properly capture the nonlinear interactions at scales near the truncation limit, but then no scheme does.

To indicate the continuing popularity of the spectral transform method until recently we compare its use to that of grid point methods in recent production models. The spectral transform method is the basis of 11 out of 14 recent operational global NWP systems, the remaining 3 are grid point based (WGNE 2005, Appendix E). Concerning models applied to climate simulation, in AMIP I, which was carried out from 1990 to 1996 (Gates 1995; Gates et al.

1999), 20 out of 32 models were spectral transform with truncations ranging from R15 to T63, 7 of those below T40 and 13 at or above T40. The grid point models were all based on latitude-longitude grids, with all but two on $4^\circ \times 5^\circ$ grids. (http://www-pcmdi.llnl.gov/projects/amip/AMIP1/modeling_groups.php). More recently, the proportion of spectral transform models in AMIP II (http://www-pcmdi.llnl.gov/projects/amip/amip2_workshop_proceedings.pdf) increased with 18 out of 25 models being spectral transform, only four with truncations below T40. The AMIP II grid point models remained at their AMIP I resolutions. (http://www-pcmdi.llnl.gov/projects/modeldoc/amip2/). Even more recently, the atmospheric model components of the coupled models used for the IPCC 4th Assessment Report calculations (http://www-pcmdi.llnl.gov/ipcc/about_ipcc.php) consisted of 10 spectral transform and 6 grid-point schemes, almost all at resolutions higher than those in AMIP II (http://www-pcmdi.llnl.gov/ipcc/model_documentation/ipcc_model_documentation.php).

With the widespread adoption of spectral transform models development of grid-point methods suitable for global modeling had a brief respite, although it did not stop completely. Development of spectral transform methods on the other hand continued, driven primarily by the quest for increased efficiency. During this period NWP groups pursued increased efficiency primarily via semi-Lagrangian approaches while climate modeling groups became concerned about physical aspects of the solution, such as conservation and positive definiteness of tracers such as water vapor.

3. Efficiency developments

3.1 *Efficiency gains for spectral methods*

a. Reduced grid

By the late 1970s as spectral transform models were becoming generally accepted, it was recognized that underlying latitude-longitude Gaussian transform grids are highly non-isotropic due to convergence of meridians towards the poles, and inconsistent with triangular spectral truncation which is isotropic. Machenhauer (1979) concluded that a certain reduction of points along latitude circles could be made in middle to higher latitudes without significantly changing the integration results. Hortal and Simmons (1991) reduced the number of longitudinal points at latitudes approaching the pole as in the earlier Kurihara grid with grid point schemes to create a quasi-homogeneous *reduced grid* for their spectral transform model. Their choice of longitude grid intervals was based on geometric arguments. Courtier and Naughton (1994) developed reduced grids based on properties of the associated Legendre functions, following the arguments of Jarraud and Simmons (1983). Williamson and Rosinski (2000) also defined reduced grids based on the expected loss of accuracy that would occur when certain basis functions were eliminated from the spectral representation poleward of some latitude. Juang (2004) further refined the spectral approach with the reduced grid. Reduced grids have up to 30% fewer points and thus offer substantial savings in the grid point computations without degrading the solutions.

The reduced grid has been successful primarily with spectral transform methods where an underlying reduced transform grid is consistent with the isotropic spectral representation. The spectral calculations are essentially unaffected so the problems seen earlier with grid point methods on the Kurihara grid do not arise. The reduced grid is also applicable to grid point schemes for transport of tracers, since the variation of the unit vectors near the pole discussed earlier is less of an issue. Such grids are also suitable for semi-Lagrangian methods which will be discussed shortly in which the trajectory is calculated using transformed coordinates near the poles and the vector velocity is advected rather than the components.

b. Fast transforms

One path that has been followed to develop more efficient spectral models is the quest for "fast" transforms. The cost of the spherical harmonic transforms in a global model increases with increasing resolution as $\mathrm{O}(K^3)$, where K is spectral truncation and the two-dimensional spherical grid has $\mathrm{O}(K^2)$ points. The cost of the FFTs on the other hand increase as $\mathrm{O}(K^2 \log K)$, a single FFT costing $\mathrm{O}(K \log K)$. The term "fast transform" in the spectral context implies an algorithm that scales as $K^2 \log K$ rather than as K^3. A fast pseudospectral method on the sphere was developed by Merilees (1973, 1974) and recently revived by Fornberg (1995). Merilees applied FFTs in both longitude and latitude directions to calculate the needed derivatives. However, because it was based on a latitude-longitude grid, longitudinal filters were again required at higher latitudes to avoid time-step restrictions and maintain stability. As was the case with grid point methods, these were never really satisfactory. The issue for a double Fourier transform model comes down to developing appropriate two dimensional isotropic filters.

Spots et al. (1998) applied the double Fourier series method with a harmonic filter which consisted of spherical harmonic analysis followed by a synthesis. They demonstrated that the stability and accuracy of their approach was identical to the traditional spectral method. They also showed that their spherical harmonic projection operator or spherical harmonic filter prevents nonlinear instability with the equivalent of quadratic unaliased truncation. In their approach fewer Legendre transforms are required than in the standard spectral transform method since they are needed to filter only the prognostic variables and not to compute spatial derivatives which are computed from the Fourier representation. Spots et al. (1998) also examine alternative fast Fourier filters. To make a semi-implicit semi-Lagrangian spectral transform model more efficient, Layton and Spotz (2003) developed a shallow water based on double Fourier series with a spherical harmonic projection. They adopted a quadratic projection grid to control aliasing.

Rather than a spherical harmonic filter as in Spots et al. (1998), Cheong (2000) applied a spectral filter consisting of a high-order diffusion operator applied to prognostic variables to prevent aliasing error or nonlinear instability. However, his approach also required a polar filter. Cheong (2006) extended his approach to a baroclinic hydrostatic model with sigma vertical coordinates. Jakob-Chien and Alpert (1997) published a fast multipole spherical harmonic filter. However the crossover truncation at which the filter approach becomes cheaper was still generally higher than production model resolutions. Suda (2005) implemented a fast spherical harmonic transform algorithm in a traditional spectral transform shallow water model and compared its performance to the traditional method. The fast transform version became faster at T170 resolution. In general, fast transforms have not made their way into production models. More work is needed to make them cost effective at current popular resolutions.

3.2 Efficiency gains via semi-Lagrangian methods

a. Semi-Lagrangian

The NWP goal of developing more efficient methods let to the development of semi-Lagrangian methods coupled to grid point methods as well as to spectral transform method for the global dynamical cores. Semi-Lagrangian approximations have become dominant in NWP applications. The semi-Lagrangian method can be thought of as a Lagrangian method with a remapping to a specified grid every time step. For transport the method uses the Lagrangian form of the equations which says that a transported quantity is constant along a trajectory. The application then calculates trajectories either backward in time or forward in time from a fixed set of grid points, backward being more common. In the backward case the backward end of the trajectory at the previous time step, commonly called the departure point, is in general not a grid point and the value there is determined by interpolation. That interpolated value is the forecast at the grid point end of the trajectory at the forecast time, referred to as the arrival point. In the case of foreword trajectories, the values at the departure point at the past time are the grid point values and the arrival points with the forecast are not grid points. A remapping is required to obtain the forecast at grid points. Because the method is based on the Lagrangian equations, the time step is not restricted as it is in an Eulerian method as long as the interpolation stencil incorporates the departure point. There is a weaker stability condition which essentially requires that trajectories do not cross. Therefore the method allows a long time step which can be chosen for accuracy rather than stability. The remaining terms in the model equations can be approximated in a variety of ways. For example horizontal derivatives associated with the pressure gradient can be approximated with grid point methods or with the spectral transform method. In addition, if terms associated with gravity waves are approximated in a semi-implicit manner, then the method becomes very economical for a complete model, not just for transport. Staniforth and Côté (1991) provide an excellent review of the development of semi-Lagrangian methods.

The first semi-Lagrangian models were three-time-level, much like centered differences. Later two-time-level implementations were developed in which values needed at the center of the trajectory are obtained by extrapolation from past values rather than being directly predicted. This reduces the number of time steps for a given forecast period by two, resulting in substantial savings.

The method is well suited to the sphere, but care is needed if global spherical coordinates are the basis of the model. Near the poles, if the trajectory is calculated using spherical coordinates, the trajectories become very inaccurate. Ritchie (1988) developed a method based on a straight line trajectory in three-dimensional Cartesian geometry that does not follow the surface of the sphere. He maps the departure point back to the surface of the sphere. McDonald and Bates (1989) applied a coordinate transform before calculating the trajectory. For the calculation associated with each grid point, they transform to a spherical coordinate system that is rotated with respect to the original system so that arrival point is on the equator of the rotated system.

As mentioned above, semi-Lagrangian transport can be coupled with both grid point and spectral methods. It offers an additional advantage with spectral methods. Since the transport is treated in a Lagrangian manner it is no longer a quadratic term as in the Eulerian formulation. A quadratic unaliased grid is no longer needed to prevent nonlinear instability. A linear grid can be used giving a 50% increase in resolution. Côté and Staniforth (1988) argued that in the semi-Lagrangian method the nonlinearities are hidden in the trajectory calculation and interpolations. They showed that the linear grid was stable in integrations with an unforced barotropic model. Williamson (1997) showed that the linear grid was also advantageous for long climate simulations.

Although semi-Lagrangian methods have many desirable properties, including coupling with semi-implicit approximations and solving the pole problem very well, they have their trade-offs as well. Stationary forcing such as that provided by mountains generally presents no stability problems with Eulerian schemes because with Eulerian methods forced, stationary problems are usually less restrictive than free, traveling waves. That is not necessarily the case for semi-implicit, semi-Lagrangian schemes as first noted by Coiffier et al. (1987). Forcing that is stationary in the Eulerian framework is Doppler shifted to higher frequencies in the Lagrangian framework, and may result in large truncation errors associated with semi-implicit schemes. The problem is illustrated in Williamson and Laprise (2000) who also provide a survey of variants and references to semi-Lagrangain semi-implicit algorithms to address this problem. These variants usually involve some un-centering of the time average semi-implicit operator along the trajectory which unfortunately introduces undesirable damping of other important signals. This mountain resonance problem is likely to occur with other long-time-step Lagrangian based schemes.

b. *Semi-Lagrangian models on latitude-longitude grids*

We indicated above the popularity of spectral transform models. Many of the operational NWP spectral forecast models adopted semi-Lagrangian approximations for their cores. Nevertheless, although spectral transform method dominated, several NWP centers active in semi-Lagrangian development continued to pursue grid point methods on latitude-longitude grids. These efforts have led to operational or production models. The new dynamical core of the Met Office's unified model (Davies et al. 2005) is based on a latitude-longitude grid, as were its previous cores. It uses semi-Lagrangian advection for all prognostic variables except density which has a flux form Eulerian treatment to achieve mass conservation. Conservative (Priestly 1993) and monotonic (Bermejo and Staniforth 1992) constraints are applied to the semi-Lagrangian transport of tracers. While the semi-Lagrangian scheme is accurate on the latitude-longitude grid, the iteration count needed by the elliptic solver arising from the semi-implicit component increases because of slower convergence near the poles. The number of iterations needed is reduced by apply a longitudinal filter based on a conservative horizontal diffusion operator. It eliminates two-grid-length waves and heavily damps short wavelengths. A design goal of the model was to include no explicit diffusion, in principal the required dissipation being provided by the monotonic advection schemes. However, selective smoothing was needed to overcome numerical difficulties.

Côté et al. (1998a) describe the Canadian unified global variable resolution model that evolved from the shallow water model of Côté et al. (1993). For global applications it is run on a uniform latitude-longitude grid. In its global configuration it is a finite element, two-time-level semi-implicit semi-Lagrangian formulation on a latitude-longitude grid. The semi-Lagrangian component applies the monotonic interpolants of Bermejo and Staniforth (1992) with Priestley's (1993) scheme to enforce conservation via a minimization procedure with local adjustments made where the interpolation procedure is most susceptible to introduce errors. It has recently been extended into a nonhydrostatic version (Yeh et al. 2002).

c. Net efficiency gains

Hortal (1999) argued that NWP obtained an efficiency increase of a "factor of 72 by algorithmic changes in forecast model." This was obtained primarily from semi-Lagrangian aspects: longer time step, elimination of every other time step by replacing three-time-level with two-time-level approximations, and application on a linear grid in place of a quadratic grid. A relatively small component of the gain arises from the adoption of a reduced grid. Williamson (2002a) estimated the gain that might be realized with climate models as more like a factor of 13, but such a gain could only be achieved by going to higher resolution than was commonly used at the time. The smaller potential gain for the climate application arises because of different characteristics of the two applications. At low climate model resolution, the time step cannot be increased at the same proportion that was allowed with high NWP model resolution. The time step in climate models becomes limited by the physical parameterizations, for both accuracy and stability reasons. Nonlinear parameterizations are often linearized about the current predicted state in order to solve them economically. The linearization is accurate only for a limited time. Time steps that are too long lead to inaccuracies and occasionally to instabilities. Most of these properties have not been studied formally. Much of the knowledge of these issues remains in the climate modeling lore and is not described in technical reports or journal papers. Another reason climate models have less of a savings with increased time step is that climate models devote a larger fraction of computer time to radiation which is usually done at a time step which is longer than the dynamical time step. It remains a fixed cost even if the dynamical time step is lengthened.

Semi-Lagrangian methods have not been widely adopted for climate models for several reasons. At the low resolutions typical of climate models of the past the damping from the interpolations of the method is noticeable. In addition, conservation properties for mass and water vapor were not as good with semi-Lagrangian as with Eulerian spectral transform models. Many climate modeling groups have apparently been willing to forgo the economical advantages of semi-Lagrangian dynamics in order to have more desirable physical properties in their solutions. However, Mizuta et al. (2006) have developed a quasi-conservative semi-Lagrangian model and applied it at very high resolution as well as at resolutions more common in climate models today.

3.3 Shape preservation and conservation

a. Semi-Lagrangian

Although climate modeling groups have not adopted the semi-Lagrangian approach for dynamical cores, they took interest in it for transport of water vapor and chemical constituents. As water vapor began to play a more dominant role in the parameterizations, dissatisfaction arose with the spectral transform method for transport of scalars such as water vapor and chemical constituents. The underlying spectral representation leads to regions of negative water vapor (Rasch and Williamson 1990), an unrealistic physical condition. Of course finite difference models are not automatically immune to this problem either. Various fixes were applied to address under and overshoots, but none were totally satisfactory. The degree of the problem and many of the attempts to address it are summarized in Williamson (1992). Shape preserving semi-Lagrangian methods were developed to avoid this problem as well as the more general problem of over- and under-shooting (Williamson and Rasch 1989). These methods have been adopted in spectral-transform climate models to replace Eulerian spectral-transform transport of moisture and chemistry while retaining the Eulerian spectral-transform dry dynamics (Hack et al. 1993; Feichter et al. 1996). They were adopted later by some NWP centers when they came essentially free with the semi-Lagrangian dynamics.

b. Conservation

The first applications with the traditional point-wise semi-Lagrangian transport were not inherently conservative. More recently conservative semi-Lagrangian schemes for tracer transport have been developed and coupled with spectral transform dynamical cores. Yabe et al. (2001) introduced a family of conservative semi-Lagrangian schemes using multi-moments based on the Constrained Interpolation Profile (CIP) philosophy introduced by Yabe and Aoki (1991) and the finite-volume concept, in which the cell integrated value is predicted via a volume remapping while the interface value is predicted via a point remapping. They named their scheme CIP-CSL for CIP-conservative semi-Lagrangian schemes. Xiao et al. (2002) developed a set of positive definite conservative schemes (CIP+CSLR) and an efficient multi-dimensional algorithm by using monotonic rational functions for interpolation. Based on Xiao et al. (2002), Peng et al. (2005) developed a 3D extension in spherical geometry with latitude-longitude grid by using dimensional splitting. To apply it to the sphere they introduce polar mixing and a divergence correction to the dimensional splitting. They applied it to water vapor transport component in a spectral transform dynamical core and concluded that the polar mixing avoids numerical difficulty in the polar regions and produces adequate numerical results for practical simulations.

Coupling a semi-Lagrangian transport scheme for tracers to an Eulerian spectral transform dynamical core is not entirely satisfactory. Jockel et al. (2001) argue that the continuity equations for atmospheric mass and for chemical constituents should be consistent with each other. This is not the case with semi-Lagrangian tracer transport coupled to spectral transform dynamical cores described above.

4. Continuing developments in grid point methods

The traditional semi-Lagrangian methods are not inherently conservative. This has led to them falling into disfavor. Because of their global nature, spectral-transform models are difficult to implement efficiently on computers consisting of thousands of processors. Although opportunities to use such large machine have not been common until recently, they are expected to dominate the market in the future. In addition spectral models suffer from “spectral ringing” in vicinity of steep gradients. For example west of the Andes, off the coast of South America, the flat ocean surface becomes a wavy surface in a spectral model. This drives spurious upward and downward vertical motions which can interact with the parameterizations in a deleterious way. This is most problematical just off the coast and modulates the parameterized stratus clouds inappropriately. For all these reasons, recent developments in global models have focused on local grid point, flux based schemes. These can provide the desired conservation and monotonicity properties and are easier to implement across thousands of processors. Once again, the issue comes down developing accurate methods with the desired physical properties, and adapting them to spherical geometry. Although to date many developments have only been applied to the shallow water equations, the natural starting point, many are now being extended to the baroclinic equations. In fact, some of the methods are already used in some models, or are expected to be used soon, replacing the spectral transform method in many instances.

4.1 Latitude-longitude grids

Tolstykh (2002) developed a semi-Lagrangain shallow water model with vorticity and divergence as prognostic variables and fourth-order compact finite differences. Lin (2004) has developed a finite volume dynamical core that has recently been implemented into several atmospheric models: namely those at NASA, GFDL and NCAR. Lin describes his transport component as flux-form semi-Lagrangian. It is stable for long time steps. Thus the convergence of the meridians does not pose an economic problem for the transport scheme. He also has optional monotonicity constraints resulting on oscillation free solutions. The gravity wave component, however, is treated explicitly limiting the overall dynamical time step and leading to the economic pole problem if nothing else is done. He includes a longitudinal filter to reduce this penalty. The tracer transport is done over a longer time step than that of the dry dynamics. That time step can be chosen from accuracy considerations, rather that the stability considerations limiting the dynamics time step. Like the Met Office model, the finite-volume dynamical core can also be run with no explicit diffusion. The monotonicity constraint provides a nonlinear diffusion which creates strong local mixing when monotonicity principles are violated. However, when the finite volume core is run with no explicit diffusion in the NCAR CAM, the kinetic energy spectrum shows an upturn at small wavenumbers, rather than a more straight k^{-3} that might be expected and desired. Lin has added a diffusion of divergence to produce a more realistic kinetic energy spectrum. The finite-volume core is an option in the CAM3 (Collins et al. 2004, 2006) and it is being used in the development version of CAM which will lead to a future release.

Because of their economical advantage and because they do not suffer as much from the pole problem, semi-Lagrangian methods are being developed on latitude-longitude grids with emphasis on addressing conservation concerns. With semi-Lagrangian methods the difference between spectral transform and grid point methods is in the approximations for the horizontal pressure gradient and in the elliptic problem introduced by the semi-implicit component. It has not been established whether spectral transform models still hold an advantage in the semi-Lagrangian environment.

Point-wise semi-Lagrangian methods do not guarantee integral conservation of the transported quantity since they predict the point value on trajectories. The semi-Lagrangian ideas have been extended to finite volume based conservative schemes by considering upstream cells rather than upstream grid points. The mass within the upstream cell provides the forecast of the mass in the arrival grid cell. To make the forecast, the mass in the upstream cells, which do not in general align with the grid cells, must be determined from the mass in the grid cells, just as with the pointwise versions the upstream point value was determined by interpolation between the regular grid cells. The process is referred to as remapping.

Machenhauer and Olk (1998) designed such a cell integrated semi-Lagrangian semi-implicit model which Nair and Machenhauer (2002) later generalized. They used a two-dimensional remapping algorithm and the cost was comparable to conventional point-wise semi-Lagrangian methods.

Nair et al. (2002) presented a conservative semi-Lagrangian advection scheme in which the remapping is based on a computationally efficient dimension splitting *cascade* method, introduced by Purser and Leslie (1991). The cascade scheme replaces a tensor product multi-dimensional interpolation method with a sequence of one-dimensional interpolations and thus is significantly cheaper. This approach is based on an extension of the cascade method to spherical geometry by Nair et al. (1999). However, near the poles their conservative cascade procedure breaks down and they convert to locally approximate scheme as in Nair and Machenhauer (2002). Zerroukat et al. (2004) generalized their Semi-Lagrangian Inherently Conservative and Efficient (SLICE) transport scheme (Zerroukat et al. 2002) to spherical geometry. It is based on a control volume approach with multiple sweeps of a one-dimensional conservative remapping algorithm. More recently they incorporated a monotonic and positive-definite filter into their scheme (Zerroukat et al. 2005).

4.2 *Composite mesh*

The composite mesh approach along the lines of Phillips (1957) has also been taken up again. Browning et al. (1989) developed a composite mesh finite-difference method on two overlapping stereographic coordinate systems such as in Fig. 2. They applied fourth- and sixth-order finite-difference approximations coupled with Lagrangian polynomial interpolation of comparable order. With several test cases for the shallow water equations, they compared it to the common spectral transform method based on scalar spherical harmonics and a spectral transform method based on vector spherical harmonics. They did not develop a baroclinic model however. Dudhia and Bresch (2002) created a global version of the PSU-NCAR Mesoscale Model (MM5) by using two stereographic grids, each centered on one of the two poles and covering that hemisphere. They used bilinear interpolation. Neither of these approaches strove to preserve conservation with the interpolations. The bilinear interpolation applied in the later is rather damping and may help prevent noise from forming. The mean precipitation in a long run of latter appeared to be noise free. However, there remains the concern that noise will develop in a baroclinic model with higher order interpolation and with strong parameterized forcings in the region of overlap.

A very different type of composite mesh has been proposed recently. Kageyama and Sato (2004) suggested a quasi-uniform composite mesh for spherical geometry without singular points which they named the "Yin-Yang" grid. A similar grid system was introduced independently by Purser (presented at the Workshop on the Solution of Partial Differential Equations on the Sphere, 20–23 July 2004, Yokohama, Japan). The Yin-Yang grid consists of two notched latitude-longitude grids which are normal to each other. Each of the two components is based on a low-latitude piece of a latitude-longitude grid on the sphere, with a gap in longitude. One component, the Yin grid illustrated in Fig. 6a, is oriented as the traditional latitude-longitude grid, the other, the Yang grid illustrated in Fig. 6b, is rotated 90 degrees to fill the gap in the first and to cover the polar regions left open in the first. The domain of the two grids, the Yin-Yang grid illustrated in Fig. 6c, looks much like the cover of a tennis ball or baseball, but with overlap at the seams. Since the two components are based on

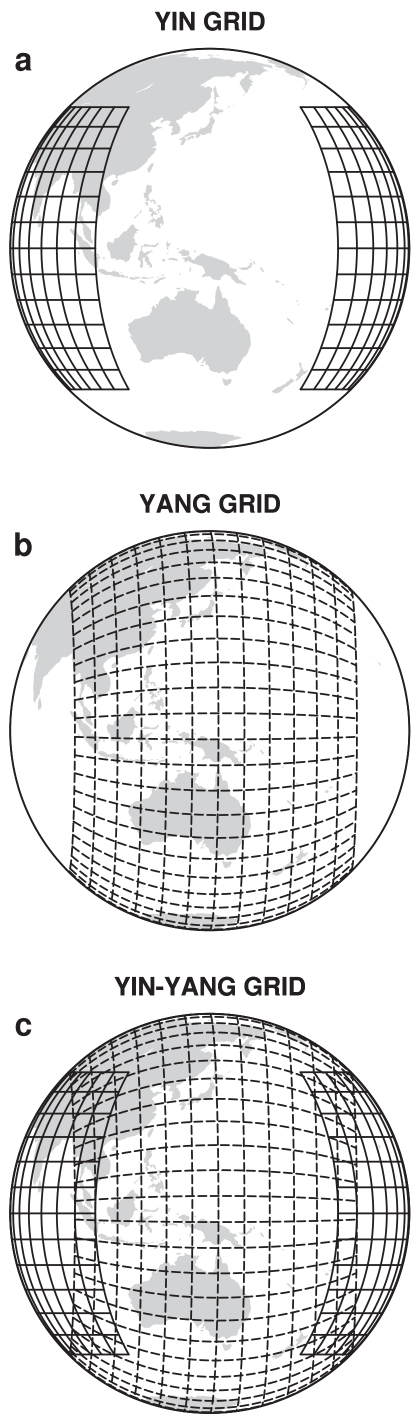

Fig. 6. (a) A Yin grid, a low-latitude latitude-longitude grid with a gap in longitude oriented as the traditional latitude-longitude grid. (b) A Yang grid, the Yin grid rotated 90 degrees to fill the gap in the Yin grid and to cover the polar regions left open in the Yin grid. The gap is on the back side. (c) A Yin-Yang grid, the combination of the Yin and Yang grids showing the overlap of the two grids.

normal spherical coordinates, each component is orthogonal and techniques developed for orthogonal curvilinear coordinates can be easily applied. The only difference is that the Coriolis term on the rotated component is not simply a function of the transformed latitude coordinate. Near the boundaries of the components, in the overlap regions interpolation is needed to fill the grid stencil of one component from predicted values from the other. Peng et al. (2006) developed a interpolation algorithm that guarantees that the fluxes on the boundaries of the two grid components are identical yielding local conservation. They applied this conservative constraint to the CIP-CSLR scheme on the Yin-Yang grid and successfully tested it with advection test cases involving advection by solid body rotation and by deformational flow, and a shallow water test case of steady state nonlinear geostrophic flow.

4.3 Quasi-uniform grids

Recently much effort is being devoted to developing new methods applied to quasi-uniform grids covering the sphere. Some are becoming operational production models, some are close to operational, but many are still only in the shallow water phase. The drive is again for better physical properties of the solutions as well as to avoid the pole problem.

a. Geodesic grids

Geodesic grids for atmospheric models were first resurrected by Masuda and Ohnishi (1986). Subsequently Thuburn (1997) developed a PV-based shallow-water model based on an icosahedral grid. Swarztrauber et al. (1997) developed a local method for solving differential equations on the sphere using Cartesian coordinates. They applied it to the shallow water equations using a geodesic grid. Stuhne and Peltier (1999) also developed a shallow-water model on an icosahedral grid. More recently spherical geodesic grids have been taken up again by several groups which has led to the development of several successful baroclinic cores being based on them. In fact, the effort at the Deutscher Wetterdienst has led to the first operational model based on an icosahedral grid (GME), which replaced their earlier global spectral transform model. This model is described by Majewski et al. (2002) who also include a brief summary of recent icosahedral grid developments. The model was built on earlier developments of Baumgardner at the Los Alamos National Laboratory. At each grid point it applies a local spherical coordinate system in which the equator goes through the point. Thus the polar singularities are far removed from the grid point neighborhood involved in the numerical approximations. Transformations are applied between the local coordinate systems when operators are applied to vector fields. Second-order-accurate gradient and Laplace operators are obtained by approximating a variable in the neighborhood of each grid point by a quadratic polynomial in the local coordinates. They apply a semi-Lagrangian method for water vapor and cloud water using bilinear interpolation for trajectory calculation and biquadratic for the prognostic variables. Ringler et al. (2000) also produced a dynamical core based on icosahedral grids. It was based on the vorticity and divergence equations in place of the momentum equation. The analytic horizontal operators are reduced to line integrals and numerically evaluated with second-order accuracy.

Tomita et al. (2001) developed a shallow water model on an icosahedral grid based on the finite volume method to obtain conservative transport. As with other recent applications the icosahedral grid was first based on recursively dividing the triangles, more or less halving the grid interval. This leads to a larger than desired variation in grid interval. To obtain better accuracy Tomita et al. (2001) applied spring dynamics to redefine their grid. They relocate the grid points to the centers of the control volumes, then for numerical stability they further modify the locations of the points. The points are connected by appropriate springs which move the points to their equilibrium positions. The latter modification reduced the grid-noise in the numerical integrations because the area and shape distortion exhibit a monotonic distribution on the sphere. They demonstrate the advantage of their modified grid over the standard grid for the numerical accuracy and stability. Tomita et al. (2002) further studied the spring dynamics and concluded that the most homogeneous grid is the best choice for numerical accuracy as well as computational efficiency. Most groups using icosahedral grids have now adopted their spring-dynamic modification. Tomita and Satoh (2004) extended their modeling framework to develop a nonhydrostatic global model extending the regional formulation of Satoh (2002, 2003) using the icosahedral grid. More recently Miura and Kimoto (2005) considered additional optimization methods for the icosahedral grid. They point out that each optimization has strengths and weaknesses. The grid optimization must be considered in conjunction with the numerical scheme used.

In comparing finite difference, and finite volume approaches, the flux form, finite volume approaches more naturally lead to conservative schemes. To date the finite difference and finite volume based schemes on the geodesic grid have tended to be lower order such as second-order it they were applied on a uniform grid. With these approaches it seems harder to derive higher order approximations on a geodesic grid than say with spectral element to be discussed next. As mentioned earlier, Williamson (1971) had trouble with higher order schemes exhibiting noise. Drake (personal communication) also had trouble with noise with higher order approximations following the Cartesian method introduced by Swarztrauber et al. (1997). Although the geodesic grid is quasi-uniform, the variation in grid lengths and grid angles is not very regular. This property might be responsible for the noise, especially when the equations admit divergence in the solutions which should remain small, such as the shallow water equations. The noise was less of a problem in the earlier tests with the filtered barotropic vorticity equation.

Giraldo (1997) developed a Lagrange-Galerkin finite element method for the spherical geodesic grid using Cartesian coordinates in three dimensions for linear triangles on the surface of the sphere. He first tested it for horizontal advection then applied it to the shallow water equations (Giraldo 2000). Giraldo (1998) developed a method combining Lagrange-Galerkin and spectral element methods for grid elements which are quadrilaterals. His method is derived for elements comprised of quadrilaterals. He applied it on the spherical geodesic grid by subdividing each triangle into quadrilaterals. This paper contains only horizontal advection on the sphere. Giraldo (2001) further developed the Lagrange-Galerkin spectral element method for the shallow water equations on the spherical geodesic grid, again divided into quadrilaterals, but reverted to the Cartesian coordinate he used earlier with the finite element approach (Giraldo 1997, 2000). Giraldo et al. (2002) developed a discontinuous Galerkin method for the shallow water equations, again on the spherical geodesic grid partitioned into quadrilateral elements. Giraldo et al. (2003) extended the approach and improved the efficiency by including a semi-implicit approximations for the time aspect.

Giraldo and Rosmond (2004) ultimately extended the method introduced in Giraldo (2001) to develop a spectral element dynamical core with the horizontal operators discretized in 3D Cartesian space. The horizontal operators are approximated by local high-order elements. By developing the equations in Cartesian coordinates any grid can be used. They applied the model on an icosahedral grid and on a hexahedral grid. The spectral elements require quadrilateral domains. These are constructed for the icosahedral grid by dividing each triangular element into three quadrilateral elements. The hexahedral is just the cube and each face is subdivided into a number of quadrilateral elements. The spectral element grids are constructed in gnomonic space.

In an approach related to that of Nair and Machenhauer (2002), Lipscomb and Ringler (2005) developed an incremental remapping scheme for transport on a spherical geodesic grid. Trajectories are projected backward from cell corners to define a departure region. Fields at the previous time are reconstructed over the grid and integrated over the departure regions to provide the forecast on the origination grid. They applied the method to the geodesic grid.

b. *Cubed sphere*

Cubed sphere grids for atmospheric models were first resurrected for atmospheric application by Ronchi et al. (1996). They developed a method for the shallow water equations, but rather than applying the cubic-gnomonic projection of Sadourny (1972), they used nonconformal mappings for the projected sides of the cube. Rather than using one-sided differences which led to problems in Sadourny's approach, they used a composite mesh similar to that illustrated in Fig. 7. Their grid consists of the

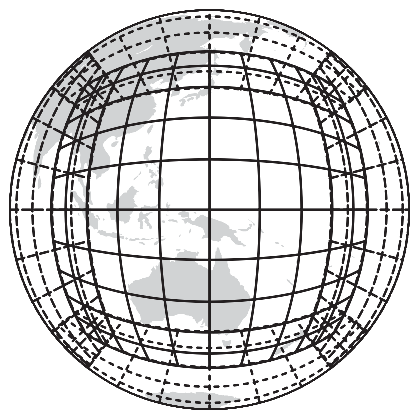

Fig. 7. A composite mesh form of the cubed sphere grid in which the projections of the cube faces are extended beyond the edges of the faces.

intersections of two sets of angularly equidistant great circles. Such a coordinate system is nonorthogonal. With their choice of grids, the coordinate lines that are "parallel" to the edges of the cube coincide on adjacent faces of the cubed sphere and only one-dimensional interpolation along those coordinate lines is required. (This grid is somewhat different than that illustrated in Fig. 7.) They solved the pure advection and shallow water equations by applying fourth-order centered differencing with a third-order Adams-Bashforth time integration scheme. They apply centered interpolations that are of the same accuracy of the finite difference approximations. An explicit hyperdiffusion term was included to free the solutions of small scale noise introduced at the internal boundaries by the interpolation procedure.

Rančić et al. (1996) applied Arakawa-type finite difference schemes on a B-grid to the shallow water equations on the cubed sphere. With the gnomonic projection they considered both equidistant and equiangular versions and concluded that the equidistant version was better. They also felt that the use of the B-grid, while producing better results than Sadourny (1972) obtained, did not remove all the problems on the edges. They speculated that the relative decrease in grid interval near the boundaries with the equidistant gnomonic version compensates for the lack of second-order accuracy at the edges. They also developed an alternative approach involving numerically generated conformal coordinates which are smooth and continuous across the edges. This eliminates the directional discontinuity of the gnomonic projections at the edges of the cube. They concluded that the orthogonalized conformal grid convergence of the solution with increasing resolution was faster than the equidistant gnomonic. Purser and Rančić (1997) also developed a conformal octagon based grid for global models. They demonstrated its feasibility with the same shallow water model as used in Rančić et al. (1996). Purser and Rančić (1998) proposed a variational method to generate smooth quasi-homogeneous grids in order to increase the minimum grid distance of grids such as the cubed sphere in order to increase the maximum time step allowed by explicit schemes. This generalizes their conformal cubic and octagonal grids.

McGregor (1997) applied semi-Lagrangian advection to the cubed sphere with the gnomonic projection with good results. He also added an additional transformation on the panels of the cube to stretch the grid and make it more uniform. This provided more accurate solutions. McGregor (1996) reported further improvements when he applied his scheme to the conformal-cubic grid devised by Rančić et al. (1996).

Several of these approaches have small scale noise in the solutions which required diffusion to control. This noise was thought to be introduced at the internal boundaries by the interpolation procedure or by the variation in the grid interval there. Noise might also be generated at the corners of the cube due to the strong change in grid directions between the adjacent sides.

Taylor et al. (1997) implemented the spectral element method for the shallow water equations on the sphere using gnomonic projections to map the sphere onto the cube. Thomas and Loft (2002) implemented a semi-implicit time stepping into the scheme into Taylor's scheme. Giraldo and Rosmond (2004) also applied their spectral element baroclinic model on a cubed sphere. Nair et al. (2005) developed a conservative transport scheme based on the discontinuous Galerkin method on the cubed sphere.

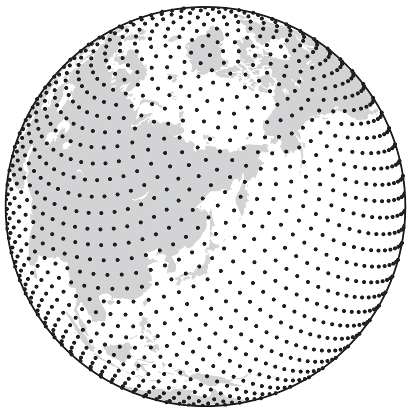

Fig. 8. A Fibonacci grid.

They applied a two-dimensional formulation on the cubed sphere using both equidistant and equiangular central projections to map between the inscribed cube and the sphere. They used a modal expansion in comparison to the nodal expansion of Giraldo et al. (2002). In test cases they found the equiangular to be more accurate than equidistant. Adcroft et al. (2004) developed a dynamical core for application to both atmospheric and oceanic flows. They formulated it in general orthogonal curvilinear coordinates and applied the finite-volume method on the cubed sphere using the conformal mapping of Rančić et al. (1996).

c. Fibonacci grids

Swinbank and Purser (2006) developed a spherical grid which is virtually uniform and highly isotropic, mimicking forms found in nature. They describe them as "mathematically ideal generalizations of the patterns occurring naturally in the spiral arrangements of seeds and fruit found in sunflower heads and pineapples" for example. An example of the grid points in such a grid is shown in Fig. 8. To identify the nearest neighbors of every grid point to be used in the differential operators they perform a Delaunay triangularization of the points in the Fibonacci spherical grid. They also find the dual Voronoi mesh yielding cells centered on the grid points. In most cases the Voronoi cells are irregular hexagons. However in some cases they are pentagons or heptagons. They implemented the shallow water equations on the grid. In solving the standard test cases they found that diffusion was required to maintain computational stability.

4.4 Energy conservation

An area that has not received enough attention in the past is conservation of total energy in baroclinic dynamical cores in practical applications such as climate change simulations. While of less importance for NWP, it is becoming critical for coupled climate system models. Thuburn (2006) discusses many issues associated with conservation in dynamical cores and trade-offs associated with different choices. Here we consider just one aspect which has become important in coupled climate system modeling. For coupled model climate change applications, the dynamical atmospheric component of climate system models must conserve energy to tenths of W m^{-2} or better (Boville 2000); a greater imbalance could produce drift of the deep ocean in a coupled system which is large enough to imply a non-equilibrium solution. Since the models have finite resolution, a means must be included to control the build up of kinetic energy at the smallest resolved scales in order to maintain a reasonable kinetic energy spectrum during long simulations. Many models including most based on spectral transform approximations add a horizontal diffusion term, often of ∇^4 form, to control the energy at the smallest resolved scales. Other cores with shape preserving approximations may be able to control the smallest scales via monotonicity constraints in the numerical scheme and do not require an explicit diffusion term. In semi-Lagrangian schemes the interpolants control the energy at the smallest resolved scales to some extent, but such models often include explicit diffusion as well. The energy loss associated with these damping mechanisms is around 2 W m^{-2} at resolutions normally applied to climate simulation. This is clearly not negligible. As a result some models include a frictional heating term that corresponds to the momentum diffusion to make the momentum damping process conservative, e.g., the CAM (Collins et al. 2004). Dissipative heating associated with vertical momentum diffusion is also an issue for energy conservation, although this is usually considered as part of the parameterization suite rather than as part of the dynamical component. Boville and Bretherton (2003) present a consistent formulation for the heating due to kinetic energy dissipation associated with the vertical diffusion of momentum.

Although the frictional heating associated with an explicit horizontal momentum diffusion seems a reasonable approach, it is somewhat arbitrary and does not capture the true energetics of the system. Such heating provides for a greater degree of energy conservation than described above, but terms such as diffusion on temperature lead to lack of conservation in the tenths of $W m^{-2}$ range. In formulations in which monotonicity constraints might control the smallest scales the energy loss due to the inherent damping is not explicitly known and cannot be compensated by a frictional heating term such as that associated with explicit diffusion. Thus to compensate for these unknown sinks an a posterior energy fixer has been included in some models to ensure conservation. An example is the CAM in which one is applied every time step (Collins et al. 2004). Care must be taken in defining such fixers to ensure they do not affect the flow in adverse ways. An example of where an energy fixer did just that is illustrated in Williamson et al. (2007). Tomita and Satoh (2004) avoid this problem by formulating their approximations to guarantee conservation of mass and total energy. It remains to be seen if the kinetic energy and temperature variance spectra in very long runs are consistent with atmospheric estimates.

5. Baroclinic core evaluation

In the past it has been very difficult and time consuming to insert a fundamentally different dynamical core into an existing model, in part because the data structures were tightly linked to the original core. This should not be an issue because the parameterizations deal with independent vertical columns and need no information from their neighbors. Nevertheless, assumptions about the grid structures were often embedded within the parameterizations. Thus model dynamical cores tended to evolve with incremental improvements within a modeling environment, rather than with wholesale replacement. This is expected to be less of a problem in the future since modeling groups are modifying their codes to cleanly isolate the dynamical core from the parameterizations, and to remove all assumptions about geographical location from the parameterizations.

Even with a clean separation, it will remain difficult to evaluate a dynamical core when coupled with parameterizations because of complex nonlinear and nonlocal feedbacks which can mask deficiencies in any of the components. It is very useful to be able to evaluate the qualities of a dynamical core before investing in the effort to develop a complete model around it with physical parameterizations. The core might have notable deficiencies which would indicate that it did not warrant inclusion in a full model. In addition there may be minor subtle flaws of formulation that are easily fixed but which are not exposed in weather forecasts or climate simulations with a complete model because other processes and feedbacks mask them. Williamson et al. (2007) present two such examples. While such flaws might not seem serious in the complete model especially if they are not detected in simulations, it is still desirable to eliminate them to prevent possible spurious interactions in the future.

Before developing a baroclinic core it is desirable to test the horizontal aspects of the approximations, assuming they easily separate out, upon which the baroclinic core will be built, especially with regards to the application on the surface of the sphere. Presumably, numerical schemes are originally developed and tested in Cartesian geometry and deemed to have properties suitable for atmospheric modeling before they are adapted to spherical geometry. Standard tests and evaluation metrics have been proposed for the shallow water equations (Williamson et al. 1992) and have generally been adopted by the community. Additional tests have been proposed more recently to evaluate additional properties of schemes. These include a barotropic instability case of Galewsky et al. (2004). Another potentially useful transport test in spherical geometry involving non-smooth deformational flow over the sphere following Doswell (1984) was used by Nair et al. (1999, 2002), Nair and Machenhauer (2002), Nair (2004) and Zerroukat et al. (2004). Stuhne and Peltier (1996, 1999) and Bates and Li (1997) employed a test case involving the erosion of a polar low following Juckes and McIntyre (1987). These could be added to the stable of standard shallow water tests.

5.1 Deterministic baroclinic tests

While most model development groups devise and apply tests during their model development and documentation, the tests themselves are often not specified in enough detail that another group can completely match the setup or analysis for comparison. To date there is no commonly adopted set of tests with specified metrics for baroclinic cores such as the set mentioned above and commonly used with the shallow water equations (Williamson et al. 1992). A standard set with concise specification is desirable to allow comparison of new schemes with previous results. Contributions toward such a set have recently been made by Polvani et al. (2004) and Jablonoski and Williamson (2006a, b). Both propose tests involving the growth of a specified perturbation in a baroclinicly unstable flow. The former strives for numerical convergence by including a dominant ∇^2 diffusion term with a coefficient which is independent of resolution, unlike the normal modeling approach in which the coefficient decreases with increasing resolution. They demonstrated numerical convergence with two rather similar models. The latter consider the diffusion as part of the dynamical core and/or numerics and propose that modeling groups specify any diffusion as they would in a full model. Thus they do not seek numerical convergence forced by horizontal diffusion, but rather attempt to evaluate schemes as they would be applied to the climate or NWP applications. They apply the test to four dynamical cores which represent a broad range of numerical approaches. At very high resolution these models provide independent reference solutions. Jablonoski and Williamson (2006a, b) consider the convergence-with-resolution characteristics of the four schemes and evaluate the uncertainty of the high resolution reference solutions.

Other tests that have been used by modeling groups that hold promise for general use include a three-dimensional Rossby wave applied by Giraldo and Rosmond (2004) following earlier work of Monaco and Williams (1975), and flow over an isolated, idealized mountain applied by Smolarkiewicz et al. (2001) and Tomita and Satoh (2004). Cases proposed for standard evaluation need a clear definition of all parameters, specification of evaluation metrics, and application to a few dynamical cores to demonstrate their utility at discriminating between the qualities of numerical schemes and to establish benchmark solutions with schemes currently used in models. Jablonowski (2006) is compiling a list of potential test cases and developing precise definitions and evaluation metrics for some of them. Details are available at http://www-personal.engin.umich.edu/~cjablono/dycore_test_suite.html.

5.2 Longer term statistical tests

a. Simplified forcing

Deterministic tests do not tell us all we need to know about a dynamical core especially ones intended for use in climate models. Longer term statistical characteristics of a numerical scheme are also important. Thus evaluation of a dynamical core when it is in a "climate-like" equilibrium after the initial conditions have long been forgotten is required. Such tests require some type of forcing to ultimately balance explicitly specified dissipation along with any inherent numerical diffusion which controls the accumulation of energy at smaller scales associated with the cascade of energy and enstrophy through the resolved scales. The goal is to have simple forcing instead of a very complicated parameterization suite.

Held and Suarez (1994) and Boer and Denis (1997) introduced such tests. Held and Suarez (1994) specified a simple Newtonian relaxation of the temperature field to a prescribed zonally symmetric "radiative equilibrium" state which is a specified function of latitude and height, with Rayleigh damping of low-level winds to represent boundary-layer friction. Instead of Newtonian relaxation, Boer and Denis (1997) forced the temperature equation with a dominant time-independent specified forcing that is a function of latitude and height plus a weak linear relaxation of the zonal average temperature to a specified state. They also specified a stress in the lowest model level. Both specifications did not include any subgrid-scale diffusivity. That is considered as part of the numerical scheme. Thus the tests can be used for conservative schemes that require explicit subgrid-scale mixing as well as for schemes that are dissipative by design.

While both approaches achieved "climate-like" structures and proved to be useful in examining dynamical cores, the Held-Suarez specification has won more favor and become a test routinely applied to baroclinic cores. Unfortunately, since an analytic solution of either test is not known, detailed evaluation of the results from the tests is somewhat problematical. Results from the Held-Suarez test are commonly described as "showing similar patterns" or as "in general being comparable to results given by many other models." A quantified analysis would be more valuable for strict core evaluation. It would be worthwhile to establish whether there is a convergence with increasing resolution with several models, taking into account sampling issues associated with determining statistical significance. The eddy statistics in particular should be considered. Wan et al. (2006) have made a start in this direction.

b. Complex forcing

Although the Held and Suarez (1994) and Boer and Denis (1997) tests seem useful, they do not capture all the interactions present in a complete model. To consider these Neale and Hoskins (2000) proposed a suite of aqua-planet simulations as a standard test for Global Atmospheric Models including their physical parameterizations. Their suite is intended to provide an experimental protocol which is less complex than a complete model but more complex than the idealized forcings of Held and Suarez (1994) and Boer and Denis (1997) which eliminate most of the feedback present in parameterization suites. The aqua-planet approach includes the full complexity of parameterizations, but simplifies the lower boundary exchange by defining a less complex surface with symmetries in its specification as well as in the external forcing. In an aqua-planet the earth is covered with water and has no mountains. The sea surface temperatures (SST) are specified, usually with rather simple geometries such as zonal symmetry.

Unfortunately evaluation of a dynamical core in the aqua-planet framework is somewhat subjective, there being no known correct answer. In addition, the behavior depends on both the core and the parameterization suite. However, it does allow comparative analysis if only the core is changed and the parameterization suite is left unchanged, or visa-versa. In addition it might expose potential problems with the interaction of a core and a parameterization package.

While evaluation is subjective, there is a growing accumulation of experience from many modeling groups for comparison. In addition an international intercomparison, the Aqua Planet Experiment (APE), in underway, and published results with soon be forthcoming (http://www.met.reading.ac.uk/~mike/APE/ape_home.html). The aqua planet allows further evaluation before coupling to complex land, sea-ice and ocean models. One can check for results that are not inconsistent with similar models, and are within the range of acceptable APE participants. In addition a few statistical properties that are primarily controlled by the dynamical approximations such as the kinetic energy spectra can be checked for reasonableness.

5.3 Perturbation growth

Another property of dynamical cores that should be checked, but which is usually ignored, is that small perturbations grow slowly with time, and consistently with turbulent atmospheric flow which implies a doubling time of a few days. The growth of rounding-level perturbations (relative order of 10^{-14}) should be at a rate expected for baroclinic flow. For such a test the initial conditions need to be atmospheric-like and consistent with the discrete approximations. Therefore this type of test should start from states created by the model itself when run as an aqua planet or when run in climate mode with a complete parameterization set. Thus the test cannot be performed until the core has been joined to a parameterization suite because that is needed to generate suitable initial conditions. For the growth test of the dynamical core the model itself is run adiabatically, i.e., with only growth arising from the dynamical core. Rosinski and Williamson (1997) show that the parameterizations can lead to very rapid promotion of rounding sized perturbations, masking the growth due only to the dynamical core. The adiabatic core alone exhibits slow growth. The short dashed line in Fig. 2 of Rosinski and Williamson (1997) illustrates such slow error growth from the dynamical core alone when starting from a representative state generated by a climate run. They provide more details of this type of test.

6. Coupling of dynamical cores to sub-grid scale parameterizations

One area related to dynamical cores that needs more attention is the coupling of the dynamical core to the sub-grid scale parameterization suite. In general this coupling is not well understood and can depend on the nature of the parameterizations themselves. Recent work such as that of Cullen and Salmond (2003) has introduced some of the issues, especially when longer time steps are involved. Nevertheless, many models couple the two components in technically convenient ways, rather than well understood ways. For example, the earlier NCAR CCM adopted a coupling strategy that had evolved over the life of the model with aspects of the coupling traceable back 20 years or more. As pointed out by Williamson (2002b) such a long evolution lead to a hesitancy to change the coupling since ramifications of the coupling were not well understood, but it worked, and everyone was comfortable with it.

Not all models have such a legacy, but often the method of coupling is chosen for convenience or economics without understanding the nature of the errors. Time splitting the dynamical core and parameterization suite is commonly adopted. Beljaars (1991) and Lenderink and Holtslag (2000) provide arguments for applying time split approximations when the time scales of the parameterizations are shorter than the time step of the model. There are many options for the actual form of splitting. A particular form is often chosen for convenience or economics without understanding the nature of the errors. For example the CAM3 uses a Process Split form for coupling with the spectral transform dynamical core and a Time Split form for coupling with the optional finite-volume core. In Process Split coupling, the two components are calculated from the same state and their tendencies are added to produce the updated state. In the Time Split coupling, the two components are calculated sequentially, each based on the state produced by the other. The Process Split form is convenient for spectral transform models. With Time Split approximations extra spectral transforms are required to convert the updated momentum variables provided by the parameterizations to vorticity and divergence for the Eulerian spectral core. The Time Split form is convenient for finite-volume core which adopts a Lagrangian vertical coordinate. Since the scheme is explicit and time-step restricted by its non-advective component, it sub-steps the dynamics through a longer parameterization time step. With Process Split approximations the forcing terms would have to be interpolated to an evolving Lagrangian vertical coordinate every sub-step of the dynamical core. Besides the expense involved, it is not completely obvious how to interpolate the parameterized forcing, which can have a vertical grid scale component arising from vertical grid scale clouds, to a different vertical grid.

Williamson (2002b) examined the errors introduced by the different coupling strategies in a practical sense by comparing the differences between long simulations with the dynamical core and parameterization suite of CCM3 combined via different coupling strategies. He compared Time Split and Process Split couplings. Overall the differences between simulations with the two coupling methods were relatively small with the time step used in the CCM3. However, there were small regions where the differences were statistically significant or where the balance of terms in the two cases was very different.

The simulations of Williamson (2002b) used a relatively short time step appropriate for explicit advection. With longer time steps allowed by semi-Lagrangian dynamical cores, because the parameterizations contain some fast processes, the coupling method may become more important. Dubal et al. (2004) analyze the time- and process-split methods in the context of simple model equations. They found that terms arising from the use of splitting techniques produced erroneous solutions when the time step was large, of the size typically used in semi-Lagrangian models. Caya et al. (1998) showed that splitting the parameterizations and dynamical core may introduce serious errors when used in conjunction with longer time step semi-Lagrangian methods. Murthy and Nanjundiah (2000) performed further analysis of the system introduced by Caya et al. (1998) and developed variants of their approximations which avoid certain of the splitting errors. Because of its expense, radiative heating is often calculated less frequently than every model time step. Pauluis and Emanuel (2004) have shown that this approach can introduce a destabilizing time lag which can give rise to instability.

Staniforth et al. (2002a, 2002b) studied four schemes for coupling physical parameterizations to the dynamical core using two simplified, canonical model problems which are modifications of the one introduced by Caya et al. (1998). The schemes are analyzed in terms of their numerical stability and the accuracy of both the transient and steady-state solution components. Dubal et al. (2005) considered a summarized splitting technique, and Dubal et al. (2006) considered a multiple-sweep predictor-corrector coupling scheme, both again in a simplified model environment. All these studies acknowledge that the simplified problems studied are considerable simplifications which limit the generality of the conclusions. Nevertheless they do provide some insight into issues of coupling. The bridge to the full models still needs to be made however.

Cullen and Salmond (2003) argue that implicit methods are well-suited to achieving coupling between different processes, but are not practical. They show that a predictor-corrector scheme can give some of the advantages of a fully-implicit scheme. Experiments with the ECMWF model showed that predictor-corrector scheme has a significant impact on the large-scale performance. Comparison at equal cost suggested that the predictor-corrector scheme is competitive. However, a different formulation for some parameterization components might be required for the method to perform at its best. Basically, the parameterizations need to vary smoothly with input data. e.g., convective parameterizations should not behave as adjustment schemes.

Another aspect that has received little attention is the spatial scales on which parameterizations should be calculated. Lander and Hoskins (1997) argue that parameterizations should be coupled to the dynamics by applying them to spatial scales that are larger than the smallest scales resolved by the dynamics. Their argument is that the parameterized processes should be treated on the scale that the model can handle properly. The smallest scales in the model, which in general are not accurately calculated by the dynamical core, should not be forced directly nor be used to calculate the forcing, but should be left to deal with the enstrophy cascade and the effects of the truncation. Their suggestion is to map to coarser grids for the parameterization calculations, then map the forcing back to the dynamical grid. They attempt to define which scales are well resolved for spectral models. Laprise (1992) also attempts to identify which scales are adequately resolved for spectral models while Pielke (1991) does so for grid point models. While in principle such an approach seems reasonable, in practice there may be problems. In a study that effectively calculated the parameterizations on a coarser scale than the dynamics, Williamson (1999) found that there could be instabilities attributable to spatial aliasing in the parameterizations and that some linear component of the parameterizations might need to be calculated on the dynamical grid to avoid the problem.

7. Summary

We have reviewed the evolution of global atmospheric model dynamical cores from the first developments in the early 1960s to present day. We concentrate on aspects involving the application to spherical geometry rather than the mathematical aspects of approximations needed to model atmospheric-like flows. Modeling on the sphere is not straight forward because the natural spherical coordinates of latitude and longitude have a singularity at the poles, and the physical distance between two lines of constant longitude goes to zero approaching the pole. This convergence imposes severe time step restrictions on many discrete numerical methods applied on latitude-longitude grids. Most of the development through the years specifically associated with global models has been devoted to avoiding such restrictions. The earliest attempts used a composite mesh approach combining two or more conformal projections with different Cartesian coordinates on each. Methods on grids based on spherical coordinates themselves were modified to either lengthen the longitudinal grid interval near the poles or shorten the time step there. Other approaches developed quasi-uniform grids based on subdividing triangles of an icosahedron into smaller triangles and projecting them onto the sphere or placing

Cartesian grids on each side of a cube inscribed in the sphere and projecting them onto the sphere. None of these approaches was entirely successful.

The spectral transform method provided a natural and very elegant solution to the sphere problem and dominated global modeling for some time. It was especially good for large scale, relative smooth dynamical motions and had advantages at coarse resolution that was common then. Development of semi-Lagrangian, semi-implicit methods coupled to either grid point or spectral transform approximations led to tremendous gains in model efficiency.

As models have become more complex and include more physical processes the spectral transform method began to fall out of favor, especially in the climate modeling community. Today the numerical methods development community is a hotbed of activity with emphasis placed on developing methods with more desirable physical properties, especially conservation and shape preservation, while retaining the accuracy and efficiency gained in the past. Much of the development is based on quasi-uniform grids.

While we have emphasized the need for better physical properties in the schemes, another aspect driving current development is the need to develop schemes which are capable of running efficiently on computers with thousands of processors and distributed memory. We have not emphasized this aspect here although it remains a driving force. While currently there is a very active community developing schemes for global models, only a few approaches have reached the maturity needed to justify inclusion in a full model. As yet, there is no dominant scheme such as the spectral transform was in the past. In addition, most of the newly developed grid point schemes contain many arbitrary parameters, making their application almost an art form.

In addition to reviewing scheme evolution, we also briefly consider tests for dynamical core evaluation. These range from well defined deterministic tests to longer term statistical tests with both idealized forcing and complete parameterization packages but simple geometries. Unfortunately the latter in particular require a somewhat subjective evaluation at the present. Finally we consider some aspects of coupling dynamical cores to parameterization suites.

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