

# **A Scalable Nonhydrostatic Model Featuring Variably Stepped Topography Part 1: Formulation and Evaluation of Performance on Simple Obstacle Flow Problems**

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## **ABSTRACT**

This paper describes the formulation and performance of a nonhydrostatic model featuring a unique variable stepped topography representation capable of capturing steep topographical slopes, while at the same time accurately representing subtle topography variations. The proposed enstrophy conserving formulation and the quasi-compressible closure cast on a multiply nested rotated spherical grid are presented. The numerical differencing scheme and methodologies to treat the variably stepped lower boundary and satisfying integral constraints are described. The adherence to integral constraints throughout the prescription of the stepped boundary formulation maintains the dynamical integrity of the solution.

Model simulations of classical fluid dynamics problems involving flow relative to an obstacle that has known analytical or laboratory-simulated solutions are presented to evaluate the stepped topography system. In certain situations, such as when the topography becomes excessively severe, conventional terrain-following coordinate systems used in many atmospheric models contain large errors making a stable integration possible only if the topography is smoothed. In other cases, conventional stepped coordinate models, because of limitations resulting from the discrete terrain stepping algorithms, also encounter difficulties reproducing the known solution in regions of slowly varying topography. Overall, the evaluation results presented show the variable stepped UW-NMS overcomes the major limitations of both conventional approaches making this proposed formulation ideally suited for a scalable model that must accurately represent both subtle and strong terrain features.

# 1. Introduction

With the increasing use of models spanning several orders of magnitude of horizontal scale, fully scalable formulations are required. As the horizontal scales become finer, and flows become nonhydrostatic and fully three-dimensional, the scalable model must be able to faithfully represent flows in the presence of arbitrarily rough topography while still maintaining the ability to capture more subtle impacts of weak topography.

Topographical features can spawn evanescent, buoyancy or inertial responses within passing airflow. The topographical features may dam and cause pooling of stable flows, or force the stable flows to divert around or over the obstacle. The nature of the responses is strongly dependent on the topographical shapes together with the vertical structure of the atmospheric flows.

The debate over how atmospheric models should best represent topography together with the attributes of various vertical coordinate systems has been ongoing for over 4 decades. Early attempts to represent topography by Orville (1965) were based on simple idealized topographical slopes incorporated into a grid point model by the blocking of full or triangular shaped portions of grid cells that could be simply incorporated into the elliptical numerical solvers employed. Phillips (1956) introduced the concept of a generalized terrain-following pressure coordinate that came into fruition by the middle 1960's as primitive equation prediction models began to appear. A limitation of the slope is always necessary for this system as a singularity is reached as slopes become vertical. Even more severe slope restrictions are made in most applications not using a full contravariant transformation (Pielke and Martin, 1981)

The implementation of terrain-capable prediction systems for Cartesian based nonhydrostatic model formulations were not common in atmospheric science until the 1970's. Gal-Chen and Somerville (1974a,b) introduced two terrain-following approaches to the nonhydrostatic problem both involving a terrain following  $\sigma_z$  vertical coordinate. One method involved the complete transformation of all velocities to the new coordinate system such that gravity actually had projections on all three-wind components. The mathematical formulation was complex involving Cristoffel symbols for contravariant components of the transformations. This coordinate system allowed for relatively steep slopes, but was ultimately limited by the singularity as slopes exceed 45 degrees. Moreover, the rather complex mathematical transformation introduced and the realities of finite differencing on grids of differing horizontal and vertical resolution make the system error prone and computationally expensive. Gal-Chen and Somerville also proposed a second alternative, whereby the prediction of Cartesian velocities is retained, but on a  $\sigma_z$  coordinate system. This allowed for the elimination of the Cristoffel symbols and the formulation of a simple flux-conservative transformation functions, making the system behave much like the  $\sigma_p$  systems, that had become the standard among hydrostatic modelers. The system was first adopted by Clark (1977), then Tripoli and Cotton (1982), and others. Unfortunately, this system was limited to small slopes, also like the  $\sigma_p$  system. Even for the case of subtle slopes, the representation of critically horizontal derivatives have been found to produce unacceptable errors that are corrected by circumventing the mathematical chain rule form of the derivative (Kurihara, 1968, Carroll et al., 1987, and so on). Today, the  $\sigma_p$  and  $\sigma_z$  systems involving the prediction of Cartesian velocities have become what might be considered the “conventional”

technique for topography representation, although other methods are continually being proposed.

In response to the slope limitations of the hydrostatic  $\sigma_p$ , the “eta” coordinate proposed by Messinger et al., 1988 has become one of the more successful alternatives to the terrain following approach. Since the number of vertical levels employed by modern prediction models has dramatically increased, Messinger et al (19?) found that it had become practical to explicitly represent topography with sufficient accuracy by simply “blocking out” grid cells much as Orville (1965) did with specific simple topographical slopes. The challenge was in formulating proper boundary conditions for the primitive equations in the vicinity of the stepped topography. This problem was solved by the use of the Arakawa “E” grid. The “eta” model, used for the past decade as the United States regional forecast model, was able to now represent the most severe of topography using this method. Critics point out, however, that subtle topography slopes are poorly represented in the form of infrequent “steps” that distort along slope flows to contain step-ups rather than a continuous rise or descent along the slope. The “eta” system was also attempted for a nonhydrostatic formulation (Gallas and Rancic, 1996) although not implemented operationally.

The arguments for a pressure (or entropy) based vertical coordinates generally do not scale down to the fine vertical horizontal resolutions now being achieved in this era of powerful inexpensive computational platforms. The finest scale simulated flows may become isotropic and certainly nonhydrostatic at any height above the surface, and perhaps governed by turbulence mechanics independent of atmospheric stratification that clearly invalidate the basis for entropy based coordinates. In addition, the use of a

terrain-following coordinate becomes problematic as severely sloped terrain features become increasingly important particularly at fine horizontal resolutions. A true solution to these problems must involve a coordinate system and topography representation that becomes truly isotropic at high resolutions but still represents the largest global scale flows. Moreover, to achieve computational feasibility in the presence of potentially decades of resolved scales, it is advantageous for any finite difference scheme to be local, at least in the horizontal direction, so as to be easily set up on distributed memory computer platforms.

This paper proposes a new scalable model that meets these requirements. The University of Wisconsin-Nonhydrostatic Modeling System (UW-NMS) has evolved over several decades in an attempt to reach these goals. In recent years, the development of a stepped topography system has led to a major improvement in the ability to represent topography over previous formulations based on a terrain following system. Together with a dynamics conservation and thermodynamics conservation based approach to the formulation for a spherical grid, we believe that we have achieved the scalability and so versatility sought. In Part 1 this paper, the model formulation is presented in sufficient detail to describe the subtle infrastructure necessary to make this variable stepped topography approach viable. We make no attempt to validate the formulation for the plethora of weather phenomena on scales ranging from 1s of meters to 1,000s of meters for which the basic formulation can be applied. Such tests will appear in Part 2 of this paper. In this paper, we will focus our evaluations on the ability of the variably stepped coordinate to tackle some classical flow problems that demonstrate both the proposed model's accuracy and its versatility.

Section 2 of this paper describes the form of the governing equations used. This is particularly important because of the enstrophy conserving form of the dynamics, more common in spectral models, allows not only for enstrophy conserving finite differencing, but for the accurate and stable representation of flows around stepped topography. The third section describes the finite differencing and in particular the method by which boundary conditions are defined along the topographical steps. In the fourth section, the model performance on classical flow problems involving not only mountains but cylinders is presented. Besides summarizing the results of this paper, the conclusions also referred the reader to an Internet Web page, where twice-daily forecasts by this model are provided on the synoptic scale and mesoscale.

## 2. Governing Equations

The model is formulated on a spherical grid having coordinates latitude, longitude and geopotential height  $(\lambda, \phi, z)$  that are equivalenced to a locally defined Cartesian coordinate  $(x_1, x_2, x_3)$  by the relationships:  $x_1 = \lambda / m$ ,  $x_2 = \phi / n$ , and  $x_3 = g'(r - r_{msl}) / g$ , where  $g'$  is the acceleration of gravity at Earth radius  $r$ ,  $g = 9.8 \text{ ms}^{-2}$ , and  $r_{msl}$  is the Earth radius of mean sea level. The metric factors  $m$  and  $n$  are defined as:  $n = 1/a$  and  $m = 1/(a \cos \phi)$  where  $a$  is the mean radius of the earth. We have neglected the variation of the earth's radius with atmospheric height. The three velocity components are then defined to be  $u_i = dx_i / dt$ , where  $t$  is time.

These spherical coordinates can be rotated relative to the standard definition of latitude and longitude of the Earth to allow the origin of the spherical grid to be placed at any arbitrary location so that coordinate distortion can be limited. The “*rotated spherical*

*grid*” is defined relative to the Earth latitude and longitude by its point of origin (in Earth latitude and longitude) and by the angle formed between a meridian of the rotated spherical grid and a standard Earth meridian at the origin of the rotated spherical grid.

## 2.1 Equations of Motion

The equations of motion are given by:

$$\frac{\partial u_i}{\partial t} + B_u G_i = I_i + S_i + F_i^1 + F_i^2 - \delta_{i3} g \quad (1)$$

where  $I_i$  are the inertial accelerations for  $u_i$ ,  $\delta_{ij}$  is the Kronecker delta,  $g$  is the acceleration of gravity,  $S_i$  represents any source of momentum such as from a cumulus parameterization ( $C_{u_i}$ ),  $F_i^1$  represents parameterized turbulence mixing tendencies, and  $F_i^2$  is the tendency on velocity from a numerical filter used to control numerical noise and aliasing.

The inertial operator which includes advection and Coriolis effect, is formulated in the enstrophy conserving tensor form:

$$I_i = \varepsilon_{i,j,k} m_j \eta_k - \frac{\partial k}{\partial x_i}, \quad (2)$$

and where the three components of momentum are given by:

$$m_i = \rho_T u_i \cos \phi. \quad (3)$$

$\rho_T$  is the total density of the air parcel defined by:

$$\rho_T = \rho(1 + q_{liq} + q_{ice}) = (\rho_d + \rho_v)(1 + q_{liq} + q_{ice}), \quad (4)$$

where  $\rho$  is the density of the vapor,  $\rho_v$ , added to the density of dry air,  $\rho_d$ . The specific mass (ratio of mass of variable to mass of moist air) of vapor, liquid and ice are given by

$q_v$ ,  $q_{liq}$ , and  $q_{ice}$  respectively. The three components of vorticity are defined to be:

$$\eta_i = \frac{\varsigma_i + f_i}{\rho_T \cos \phi} , \quad (5)$$

where the  $f_i$  are the three components of Coriolis along the rotated coordinate axes, and the three components of relative vorticity are defined as:

$$\varsigma_i = \varepsilon_{ijk} \frac{\partial u_j}{\partial x_i} . \quad (6)$$

The specific kinetic energy is given by:

$$k = \frac{1}{2} (u_i^2) + \frac{2}{3} e , \quad (7)$$

where  $e$  is the turbulent kinetic energy, which may be either diagnosed or explicitly predicted. The pressure gradient accelerations ( $BG_i$ ) are composed of the buoyancy coefficient,

$$B_u = \theta_{vv} , \quad (8)$$

and the pressure gradient,

$$G_i = \frac{\partial \pi}{\partial x_i} . \quad (9)$$

The exner function,  $\pi$ , is related to pressure (p) by:

$$\pi = c_p \left( \frac{p}{p_{oo}} \right)^{\frac{R}{c_p}} , \quad (10)$$

where  $R$  is the gas constant for dry air and  $c_p$  is the specific heat of dry air at constant pressure., and  $p_{oo} = 1000\text{hPa}$ . The water loading virtual potential temperature is given by:



$$\theta_{vv} = \theta \frac{(1 + 0.61q_v)}{(1 + q_{liq} + q_{ice})} = \frac{\theta_v}{(1 + q_{liq} + q_{ice})}, \quad (11)$$

where  $\theta$  is the potential temperature, related to temperature ( $T$ ) by Poisson's equation,

$$T = \theta \left( \frac{p}{p_{oo}} \right)^{\frac{R}{c_p}}, \quad (12)$$

and  $\theta_v$  is the virtual temperature. The density of moist air is related to  $\theta_v$  and  $p$  by the ideal gas law:

$$p = \left( \frac{\rho R \theta_v}{\frac{R}{c_p} p_{oo}} \right)^{\gamma}, \quad (13)$$

where  $\gamma = c_p / c_v$ , and  $c_v$  is the specific heat of dry air at constant volume.

## 2.2 Quasi-Compressible Closure

The nonhydrostatic dynamics system is closed with a quasi-compressible closure. The continuity equation may be combined with the ideal gas law to yield:

$$\frac{\partial \pi}{\partial t} = D_\pi + S_{\pi\theta} + S_{\pi q_v}, \quad (14)$$

where the divergence term is defined as:

$$D_\pi = -\frac{R}{c_v} \frac{\pi}{\cos \phi \rho \theta_v} \frac{\partial m_j \theta_v}{\partial x_j}. \quad (15)$$

The last two terms on the right hand side of eq. (14) are source terms for  $\pi$  resulting from sources of  $\theta$  and  $q_v$ . This quasi-compressible closure, similar to that described by

Klemp and Wilhelmson (1978) and Tripoli (1992), is designed to capture the anelastic evolution of the pressure field by modeling a sufficient portion of the acoustic tendencies to enable anelastic adjustment ahead of any other inertial or gravitational adjustment processes. As a consequence, the source terms can be dropped from eq. (14) and the coefficients to the adjustment term can be approximated to allow for increased computational efficiency:

$$D_{\pi} \cong -\alpha_s^2 \frac{R}{c_v} \frac{\pi_o}{\cos \phi \rho_o \theta_o} \frac{\partial m_j \theta_v}{\partial x_j}, \quad (16)$$

where  $\alpha_s$  is the fraction of the base state sound speed similar to the method of Drogemier and Wilhelmson (1985). It is important that the modeled sound speed be significantly faster than the largest meteorological phase speed in the system to prevent the formation of an acoustic shock wave and possible unphysical interaction between acoustic waves and other modeled waves. We therefore require:

$$M_s = \frac{c_m}{\sqrt{\gamma R T}} < 0.8, \quad (17)$$

where  $M_s$  is the Mach number, and  $c_m$  is the maximum phase speed of any meteorological wave, such as an advective wave, a gravity wave and so on. Because of the quasi-compressible approximation, the predicted exner function will have properties similar to a diagnostic anelastic pressure with Neuman boundary conditions. In other words, the predicted Exner function will be unique only to within a constant. To find this constant, an additional condition is necessary, being the external specification of the mean perturbation exner function. When using a one-way nest, the outer parent nest can provide this constant. Within a non-nested model the mean exner function can be held

constant or its evolution predicted from the mean divergence of mass across the model domain.

### **2.3 Conservative Scalar Tendency Equation**

Time dependent equations are also solved for a number of thermodynamic and other scalar quantities to complete the system. As described by Tripoli (1992), the tact is taken to form equations for the most highly conserved thermodynamic variables to minimize the number of finite difference approximations of source tendencies that lead to truncation errors. For conservation, the scalars are normally formulated per mass of moist air. They include ice-liquid water potential temperature (proportional to moist entropy per unit mass), specific mass (mass of constituent per unit mass of moist air), specific concentration (concentration of a constituent per unit mass of moist air), or turbulent kinetic energy per unit mass. The tendency equation for all of these conserved scalar quantities can be written in the general form:

$$\frac{\partial A}{\partial t} = I_A + P_A + F_A^1 + F_A^2 + S_A, \quad (18)$$

where A is the conserved scalar. The inertial tendencies of advection ( $I_A$ ) and precipitation settling ( $P_A$ ) are given by:

$$I_A = -\frac{1}{\rho \cos \phi} \frac{\partial m_j A}{\partial x_j} + \frac{A}{\rho \cos \phi} \frac{\partial m_j}{\partial x_j}, \quad (19)$$

and

$$P_A = -\frac{1}{\rho \cos \phi} \frac{\partial \rho v_A A}{\partial x_j} . \quad (20)$$

where  $v_A$  is the magnitude of mean terminal velocity (mass weighted average movement of the scalar relative to the dry air). The second term on the right hand side of equation (19), represents the effect of three-dimensional momentum divergence ( $\partial m_j / \partial x_j$ ) on the conservation of the scalar. The averaged effect of this term vanishes over a period of a few minutes (Dutton and Fichtl, 1969), but is on the order of the flux divergence terms at any given instant. As a result it must be retained to maintain stability, although its net impact on conservation over time is small.

The terms  $F_A^1$  and  $F_A^2$  represent physical turbulence mixing and a numerical filter respectively. The numerical filter is used control numerically generated noise at the most poorly resolved scales and to progressively damp poorly resolved features. The physical turbulence mixing is based on a physical turbulence closure that models the effects of turbulence mixing in response to production of turbulence by resolved processes. The general source term,  $S_A$ , represents all other sources to the otherwise conserved scalar  $A$ . This may include diabatic terms, microphysics conversions, chemical conversions and other source terms.

### 2.3 Thermodynamics Closure

The thermodynamics closure involves an equation for moist entropy ( $\theta_{il}$ ) is predicted,

where:

$$\theta = \theta_{il} \left( 1 + \frac{L_v q_{liq} + L_{il} q_{ice}}{c_p \max(T, 253)} \right) . \quad (21)$$

Additional conservation equations for specific mass of total moisture ( $q_T$ ) and possibly specific masses of selected categories of ice or liquid precipitating hydrometeors are specified where the total water specific humidity is given by:

$$q_T = q_v + q_{liq} + q_{ice} \quad (22)$$

. Tripoli and Cotton, 1981 (TC) described the diagnostic system linking these predicted quantities to potential temperature, temperature, density and cloud water.

## 2.4 *Physical Parameterizations and Filters*

Physical turbulence associated with diagnosed down gradient subgrid scale motion is represented by a level 1 (or optionally level 2) closure based on the formulation of Redelsperger and Sommeria (1982). In the case of the level 1 closure, turbulent kinetic energy ( $e$ ) is diagnosed from the predictive equation for  $e$  assuming a steady state balance along a Lagrangian trajectory. For level 2 closure,  $e$  is predicted using the scalar prediction equation (18).

Fluxes of entropy, moisture and momentum resulting from exchanges with the underlying surface enter the simulation domain as a vertical surface boundary flux of physical turbulence. The fluxes are calculated as bulk fluxes of the form:

$$\overline{\rho A'' w''} = -\rho C_{DA} (A_{abv} - A_{sfc}) \quad (23)$$

where  $A$  is the predictive variable for which a surface flux is determined,  $A_{abv}$  is the variable defined at a prescribed height,  $z_{abv}$ , above the surface,  $A_{sfc}$  is the variable

defined at the surface,  $C_{D_A}$  is the bulk drag coefficient defined for variable  $A$ . The value of  $z_{abv}$  is set to a constant height given by:

$$z_{abv} = \min\left(\frac{\Delta z_1}{2}, 35\right)$$

where  $\Delta z_1$  is the depth of the lowest grid box over zero topography. The variable  $A$  is interpolated between model levels and the surface value to this height. For scalars, the interpolation is linear, while for the two horizontal velocity components the interpolation is log-linear relative to the surface roughness height. The surface value of the variable can be determined from a soil/vegetation model or coupled ocean/lake model. Several choices of surface layer parameterizations are used to determine the bulk drag coefficient,  $C_{D_A}$ , including a simple prescribed coefficient (Emanuel, 1986), The Businger (19??) scheme and the Louis (1979) scheme. The later two schemes are cast to define the fluxes directly, however the NMS uses the calculated surface fluxes to determine the effective bulk coefficient. This enables an implicit calculation of surface flux, described in the next section, designed to overcome potential stability problems which could be encountered as the depth of the lowest grid box becomes small.

A numerical filter is employed to control nonlinear instability and other spurious numerical noise. A selective filter, designed to control the numerically induced accumulation of small-scale variance for any variable  $A$ , is given by:

$$F_{A2} = F_H \left[ \frac{\partial}{\partial x} \left( \frac{\partial}{\partial x} \right) \right]^{n_h/2} A' + F_H \left\{ \frac{\partial}{\partial y} \left[ \cos \phi \left( \frac{\partial}{\partial y} \right) \right] \right\}^{n_h/2} A' + F_V \left\{ \frac{\partial}{\partial z} \left[ \rho_o \left( \frac{\partial}{\partial z} \right) \right] \right\}^{n_v/2} A', \quad (24)$$

where  $n_h$  and  $n_v$  are even integers representing the order of the horizontal and vertical filter respectively. Note that the filter is formulated as a repeated application of a second order filter. In this way, only a single boundary condition is needed at each second order application rather than a reduction of order as the boundary is approached. This is especially useful with the stepped topography model where numerous lateral boundaries occur in low levels over rough terrain.

Similar to Klemp and Wilhelmson's (1978) application of a similar numerical filter, the smoothed parent grid solution can be subtracted from the scalar field being smoothed to reduce unphysical transport caused by the application of this filter. Typically, a 4<sup>th</sup> order filter is assigned for the horizontal and a more selective 6<sup>th</sup> order filter is assigned to the vertical. The filter coefficients  $F_H$  and  $F_V$  are set to be a prescribed fraction of their maximum damping value for the shortest wave. Typical values are 0.1 for the horizontal and 0.005 for the vertical directions.

The UW-NMS employs several physical parameterizations to simulate cloud physics, the surface layer, radiative processes, subgrid scale moist convection. It is beyond the scope of this presentation to present these schemes in any detail here. However, some discussion of the implementation of the surface layer scheme with variable stepped topography will be discussed in the Section 4.

### **3. Finite Differencing**

The grid spacing is taken to be of constant latitude and longitude increments in the horizontal. The vertical grid may be stretched with variable spacing. In order to

optimally conserve mass and energy, a box method of differencing similar to that developed (Kurihara and Holloway, 196?) is followed. Predictive variables including velocity and all scalars such as  $\theta_{ii}$  and specific masses are taken to be intensive variables, since when multiplied by density they represent a conserved quantity, i.e. moist entropy, mass, and momentum respectively. The values of these quantities are taken to represent the average value over the grid box, and when multiplied by density and grid volume, represent the total amount of that quantity within the box. Keeping with that philosophy, the scalar value is assumed constant throughout a grid box. At the interfaces between grid boxes, a discrete jump of the variable quantity defined in the box to the value within the neighboring box occurs. Fluxes across box boundaries contribute to strict budgets of these conserved quantities within each grid cell, so that all mass leaving a grid box is accounted as entering another box.

The Exner function is unique from other scalars because it represents a continuously varying intensive property. Its gradients tend to balance with intensive properties, such as in the hydrostatic or geostrophic balance. As a result, the gradient of exner function is constant over a grid box rather than its value. Since gradients of exner function are needed at box interfaces, where they form balances typically with inertial forces, care must be taken in the averaging procedures of scalars and velocities participating in these balances. Therefore,  $\theta_{vv}$ , appearing in equation (8), as a coefficient of the pressure gradient, is averaged to the velocity location in its inverse form, i.e.

$$\overline{\theta_{vv}}^{x_i} = \frac{1}{\left( \frac{1}{\theta_{vv}} \right)^{x_i}} \quad (25)$$



This preserves the linear relationship between exner function and  $\theta_{vv}$  under balanced conditions. Hence in a grid cell under hydrostatic balance, exner function will decrease at a constant rate with height over the region where  $\theta_{vv}$  is constant.

The model employs the hybrid time-split, leapfrog-forward space-time-differencing scheme similar to that discussed by Tripoli (1992) and Klemp and Wilhelmson (1978). The dynamics, as formulated in the Exner Function system, enables a clean decoupling of the acoustic containing fluctuations from the more slowly evolving inertial and gravitational fluctuations. This allows the acoustic terms to be integrated on a separate small time step. The inertial tendencies for velocity are integrated with a leap frog scheme, finite differenced in a fashion to optimally conserve enstrophy, kinetic energy and mean vorticity. The scalars are integrated with a Crowley forward scheme that requires the archiving of only one time level of information while maintaining second order accuracy in time. Moreover, the Crowley scheme does a good job maintaining positive definiteness while remaining a linear scheme, is cast in flux conservative form, and can be easily set up to be 2<sup>nd</sup>, 4<sup>th</sup>, 6<sup>th</sup> or higher order accuracy. These schemes, as they are interleaved with the numerical schemes for the remaining terms in the predictive equations are illustrated in full detail in Appendix 1.

## **4. Variable Stepped Topography**

Although an explicit small slope approximation is not made, terrain-following coordinates suffer from serious errors in the vicinity of strong slopes. This is illustrated in Figure 2a. In the vicinity of very steep slopes, the transformed coordinate directions

along the slope and vertical become increasingly nonorthogonal. Since the horizontal derivative, necessary in conjunction with horizontal Cartesian velocity, is derived from one vertical and one nearly vertical derivative, making the result often on the order of the numerical truncation error. Moreover, the numerical truncation error in the two directions may be substantially different as a result of differing horizontal and vertical spacing, leading to the failure of the chain rule of transformation numerically where horizontal and vertical truncation errors will not cancel and so not vanish. These effects are clear in Figure 2. In the vicinity of strong slopes, it can be seen that a second order along-slope derivative may span the equivalent of several vertical grid boxes, making the comparison of a vertical derivative and the along-slope derivative for the purpose of attaining a horizontal derivative error prone. This effect is made even more serious when it is recognized that many quantities, such as pressure, moisture and temperature vary much more strongly in the vertical than in the horizontal, making the horizontal derivative a small difference between two much larger quantities.

Step topography, such as used in the “eta” model, overcomes many of the shortcomings of the transformed coordinate systems discussed above, but introduces new concerns. Figure 2b, depicts the appearance of a height based vertical coordinate with step topography equivalent to the topography representation used by the eta model in conjunction with the  $\sigma_p$  vertical coordinate. The steps are defined so that if the mean topography height exceeds the height of the center point of a grid cell, the grid cell is blocked out. Topography then appears as discrete steps, with step height equal to the vertical grid spacing. Note that the coordinate surfaces above the topography are unaffected and remain numerically orthogonal. Moreover, there are no practical limits to

slope in this system. In fact, any resolved structure can be represented. The major disadvantage of the step coordinate, is that terrain slope is represented by discrete steps. Therefore, surface topographical variations must be resolved by the vertical grid spacing in order to be captured numerically. Subtle variations, which are at least partially represented with a terrain following system, are lost.

The proposed *variable step coordinate* of the UW-NMS is depicted in Figure 2c. Unlike the step coordinate system, the step is defined so that the grid box intersecting the surface in each vertical column has a depth adjusted so that the bottom surface of the box is coincident with the mean topography height of the grid cell column. Figure 3 depicts the location of the velocity and scalar points as they appear along the topography steps.

Note that the first above-ground “u” point is located at the maximum height of the center points of the two adjacent surface grid cell columns. The lowest “u” velocity grid cell is taken to have the depth equivalent to the most shallow of the two adjacent scalar grid cells. The lateral boundaries of the lowest above ground grid cells may then have a partial-wall boundary beneath the extent of the adjacent “u” point. The flux divergence across the lowest scalar grid box is formed by the box method Kurihara and Holloway (1967), by adding together all open fluxes multiplied by the area of the open boundary and divided by the area normal to the flux at the center of the scalar box.. The box method conservation of mass, entropy and mass weighted scalars on the “C” grid. Potential numerical instabilities of the flux divergence form of the advection equation arising because of the existence of very “thin” grid boxes in some surface locations are avoided also with these rules as vertical motion will scale to the box depth. Additional rules are:

1. **Topography Interface Velocities.** All velocity points located along a vertical or horizontal grid interface adjacent to a blocked out topography grid cell are required to be zero.
2. **Topography Interface Fluxes.** All advective and turbulence fluxes across any grid surface adjacent to blocked out topography box are zero, except for specified diabatic surface fluxes of heat, moisture, momentum and radiation.
3. **Vertical Exner Function Gradient.** This is applied implicitly using the Crank-Nicholson operator for vertical derivatives and therefore remains stable, even as grid boxes become “thin”.
4. **Vertical Mass Divergence for Exner Function Tendency.** Treated implicitly with Crank Nicholson scheme on small time step. Also, total mass divergence vanishes proportionally to grid cell depth.
5. **Vertical Filter and Physical Turbulence Between Surface Box and Above.**  
To prevent stability problems, high order vertical filtering is halted one grid cell above lowest box. The prescribed coefficient of vertical filtering is scaled to second order filtering and added to the second order physical filtering coefficient at the lowest grid cell. That coefficient maintains full grid cell mixing lengths and so acts to smooth the vertical transition from a thin to a full grid cell depth between the surface grid cell and the grid cell just above. Differencing is Crank Nicholson for the physical turbulence application to maintain stability.
6. **Surface Fluxes.** As mentioned earlier, surface fluxes are recast into the bulk form allowing them to be treated implicitly as the surface boundary condition

to the vertical Crank Nicholson calculation for physical turbulence. This keeps the system stable, and together with the specification of full filter values at the upper interface of the surface box just described, enables surface fluxes to pass through the “thin” boxes stably.

7. **Absolute Vorticity** A free slip condition could require either relative vorticity vanish or relative vorticity gradients vanish. Both conditions have been used in the testing and both conditions give good results. In practice, UW-NMS is defaulted to the zero gradient relative vorticity condition, which is closest to the spirit of true free slip. On the other hand, it is more common in terrain following systems to assume a vanishing velocity gradient at the surface, which is equivalent to the zero vorticity condition. In either case, in a terrain following system, the vorticity is fully specified and not left to drift, which is shown later to lead to anomalous vorticity generation. The zero gradient condition is applied approximately by averaging all vorticity points above the ground and adjacent to point along the topography interface, working down from above.
8. **Kinetic Energy** Kinetic energy outside the grid cells above the surface is not needed. However, kinetic energy in grid cells adjacent to a topography boundary would include normal velocities at the vertical or horizontal surface boundary interface if calculated routinely. Therefore, the kinetic energy for grid cells that have at least part of one lateral cell boundary partially adjacent to topography is calculated by assuming a zero gradient of the normal velocity over that fraction of the box below the topography level. This prevents large

false kinetic energy gradients in regions of transition from a thin step to a full box.

9. **Horizontal Exner Function and Kinetic Energy Gradients.** Horizontal Exner function gradient and kinetic energy gradient at velocity points along interfaces with a surface based box are calculated by linearly interpolating exner function (or kinetic energy) to the height of the velocity point, assuming a zero vertical gradient below the lowest above ground point. This is consistent with the requirement that Exner function (or kinetic energy) gradient be constant across a grid box, rather than exner function itself.

Applying these boundary conditions as stated above eliminates the need for the specification of any variables at grid points filled with topography under the constraints of second order differencing. In the case where higher order differencing is used, as it is when a 6<sup>th</sup> order Crowley is employed, the differencing order must be reduced as the boundary is approached. In the case of the high order filter, the boundary condition is applied recursively, as a series of second order filters. The velocities are set to zero at the boundary which eliminates the need to specify any further condition for the velocity filter, while the scalar flux across the boundary is required to vanish, completing the need for further boundary specification for any even order of scalar filter.

The surface layer forcing, i.e. surface friction and surface heat and moisture flux, derived from surface layer theory, is formulated as a function of the vertical profiles of humidity, temperature and wind. To apply this theory to a sloped surface represented by discrete steps and with the first atmospheric model levels at varying heights above the steps requires some special conditions.:

1. Values of atmospheric wind, temperature and humidity representative of the air just above the friction layer are needed to drive the calculation of surface fluxes. These values are interpolated or extrapolated to a height  $z_{abv}$ . For surface layer computations, the horizontally-staggered horizontal velocity components are to  $z_{abv}$  at their staggered location and then averaged back to the
2. In order to account for the slope of the terrain, the wind speed used to calculate friction is projected along the terrain slope calculated between the centers of grid boxes using a centered difference. The thermal, moisture and momentum fluxes calculated are then divided by the cosine of the slope angle in order to account for the larger area of the surface emitting the flux. Because the net change of the quantity across the grid volume is only dependent on the total flux emitted into the box, this effectively accounts for the fluxes coming in from the vertical faces of the box. Of course this technique meets with some increasing error as the terrain slope approaches  $90^\circ$ .
3. Solar radiative fluxes are also adjusted so the intensity intercepting the surface are adjusted by the slope angle. The effects of topographical shadowing are ignored.
4. Because of the variable depth of the first grid box, and the existence of some very “shallow” boxes, it is even more important to apply these surface fluxes to the model in a numerically stable fashion. For instance, as the depth of the box approaches zero, the effect of a discrete surface flux on a box over a discrete timestep goes to infinity. This is clearly unacceptable and is prevented by applying the surface flux implicitly. As mentioned in the previous section, we express the surface derived flux as a bulk flux operating down gradient between a skin temperature, humidity or zero wind and

the model value at the lowest point. Holding the skin condition constant, the flux is then represented implicitly along with the other vertical turbulent fluxes. The result is an always stable, always down-gradient surface flux that closely matches that determined by the surface layer theory, but which adjusts to maintain stability and down gradient transport. This prevents the wind from slowing down so much that it reverses, or the lowest layer heating so much over a time step that it becomes warmer than the surface doing the heating. These three surface flux conditions effectively overcome any disadvantages resulting from the variable depth of the lowest model grid cell and the lack of an explicitly sloped lower boundary.

For display purposes, and for purposes of interpolation to other inner nested grids, a representative value of predicted variables under the surface is sometimes needed. For this, thermodynamic variables are extrapolated to below-ground grid points assuming a constant prescribed lapse rate for  $\theta_{il}$ , constant value for  $q_v$ , zero for condensate and other specific mass variables. Pressure is then integrated downward hydrostatically from the surface to sea level.

## 5. One Way Nesting Scheme

The model domain is one-way nested within an externally specified parent grid. One-way nesting implies that the model is influenced by the parent grid, but has no feedback on the parent grid. This grid may be horizontally and temporarily varying, or may be a prescribed horizontally homogeneous state. The nesting occurs through specification of the initial, lateral and upper boundary conditions and the mean model



pressure. The parent grid is typically assumed to be hydrostatic, although this would not be necessary if non-hydrostatic information is available. The hydrostatic balance is finite differenced to be consistent with the model's finite differencing of vertical pressure gradient forcing. The parent grid may be taken directly from analysis of data for the initial condition, may be interpolated from another model run such as the NECP or ECMWF models, or may be built from a single sounding providing a horizontally homogeneous initial state.

The parent grid is interfaced over time using a combination of Rayleigh damping and radiative boundary conditions. Several options for radiative lateral boundary conditions are made available to allow the outward passage of short term perturbations from the domain which are not represented or resolved by the parent grid. These options include the Orlanski (1974), the Klemp and Wilhelmson (1978), The Klemp and Lilly (1980) and the Hack and Schubert (1981) normal mode conditions. These radiative conditions are most important for mesoscale and tropical simulations where rapidly propagating gravity waves are generated which may not be adequately damped by a limited Rayleigh damping zone.

Although Tripoli and Cotton (1982) described a vertical acoustic radiation condition, the quasi-compressible prediction of exner function in lieu of the prediction of air density allows for the use of an upper wall where  $u_3 = 0$  in conjunction with an upper level Rayleigh damping zone. Moreover, the Klemp and Durran (1984) normal mode upper gravity wave radiation condition is available but seldom used because of the difficulty in applying it to multiply nested grids and potential instabilities associated with

the free and open upper boundary.. Instead, a simple absorbing layer is used, where the tendency of  $\theta_{il}$ ,  $u_1$ ,  $u_2$ ,  $q_T$  are given by:

$$\left(\frac{\partial A}{\partial t}\right)_{x_3}^{rayleigh} = \frac{(A - A_B)}{\tau_z(x_3^{top} - x_3)} , \quad (26)$$

where  $A$  is the predicted variable,  $A_B$  is the outer grid value and  $\tau_z(\delta z)$  is a damping time scale assumed to be maximum at a height of  $z_{top}$ , the model height, and linearly decreasing to zero at the height prescribed to be the bottom of the friction layer.

Similarly, a lateral Rayleigh damping layer is prescribed such that:

$$\left(\frac{\partial A}{\partial t}\right)_{x_i}^{rayleigh} = \frac{(A - A_B)}{\tau_{x_i}(|x_i^B - x_i|)} , \quad (27)$$

where  $i = 1, 2$ , and  $x_i^B$  is the boundary value of distance and  $A_B$  is spatially and temporally varying parent grid value of variable  $A$ . The horizontal time scale for damping is  $\tau_{x_i}(x_H)$ , which is a prescribed maximum at the boundary, i.e.  $x_i^B$ , decreasing to zero at some prescribed distance. That distance is a function of grid spacing and wave movement. For typical synoptic simulations with 40 km grid spacing, the damping zone is taken to be about 8 grid points, or 320 km. Fewer damping zone points may be considered with coarser spacing or very frequently updated outer grid information.

The domain mean pressure is set from the parent grid. This is accomplished by comparing the predicted mean sea level exner function, horizontally averaged over the model domain,  $\overline{\pi_{msl}}^{x_1, x_2}$ , to the average of the parent grid and then adding the difference to the exner function at all points within the model on all grids. Since the dynamic impact of exner function is only a result of its gradient, this adjustment will only impact

the model thermodynamically, i.e. through subtle changes in actual temperature, which will be minor. This adjustment is made once each long model timestep.

The relative roles played by the lateral radiation boundary condition and the Raleigh damping zone depend on the spatial scale and the temporal availability of a prescribed parent grid. For synoptic and mesoscale domains, the damping zone tends to be of dominant importance, while for cloud scale domains, often only a single sounding is available and the strict adherence to the horizontally homogeneous parent grid is relaxed in favor of more open boundary conditions.

In certain circumstances, periodic lateral boundary conditions can also be used, so long as the boundary grids match. This is often used for idealized simulations, but can be used for simulations that wrap around the globe 360 degrees. In simulations that reach the pole of the rotated spherical grid, the pole is handled using conditions described by Arakawa and Lamb (1981). This involves placing the pole at a scalar point, avoiding the singularity resulting from prescribing a velocity locally. This becomes a close three-dimensional analogue of an axisymmetric model at the center of the circular domain.

## **5. Performance of the Variably Stepped Topography on Classical Problems**

In this section several sets of experiments are performed with the UW-NMS to evaluate the ability of the variably stepped topography scheme to reproduce the solutions classical fluid flow problems involving the interaction of the fluid with an obstacle. The problems

were chosen also to demonstrate the strong versatility of this model for capturing not only conventional atmospheric flow problems that have been successfully simulated by atmospheric models in the past, but also flow problems which previously could only be tackled by physical laboratory models or specialized numerical models.

***Irrotational Flow around a cylinder.*** Perhaps one of the most studied fluid flow phenomena has been flow around a cylinder. For an infinitely long and frictionless cylinder, irrotational flow must remain irrotational. Laboratory experiments, discussed by Tritton (1988) show such flow to separate symmetrically around the cylinder creating two flow speed minima on the windward and leeward side of the cylinder.

Numerical simulation of this phenomenon is impossible for models employing a coordinate transform to represent the cylinder because of the vertical cylinder walls. For step topography models, the problem is an especially difficult test because of the challenge involved in maintaining irrotationality as the flow passes the obstruction. Spurious vorticity can be introduced numerically by the specification of lateral boundary conditions on velocity, particularly on an Arakawa “C” grid. Messinger et al. (1988) overcame these problems with the ETA model by casting the equations on an Arakawa “E” grid. In the case of NMS, the problem is solved with the enstrophy conserving form of equation (1) that requires boundary conditions on vorticity rather than momentum. By requiring the *relative* vorticity to vanish at the surface of the cylinder, the no new vorticity is introduced and precise conservation is maintained.

To demonstrate vorticity conservation, the NMS model was restricted to horizontal two-dimensional flow by requiring vertical variation to vanish. An infinitely

tall cylinder, measuring 10 km in diameter was placed in the center of a domain measuring 65 km in the along wind (zonal) direction and 40 km in the meridional direction. Horizontal grid spacing was 500 m. The grid was centered over the Earth's equator, where the rotated grid was also centered. Periodic lateral boundary conditions were used along the north and south boundaries while open radiative boundaries were used on the east and west boundaries. The initial condition was an isothermal atmosphere with a constant westerly wind of 10 meters per second.

Figure 4 shows the simulated flow after 1 hour of integration. Note the symmetric spreading of the streamlines around the cylinder. The initially irrotational flow remains irrotational as evidenced by vorticity computations showing peak vorticities staying within the range of those expected from simple roundoff error, given the irrotational but curved 10 m/s flow. Other experiments (not shown) where the cylinder was confined to a finite depth in a fully three dimensional framework showed some vorticity developing from the formation of a vertical circulation around the cylinder, as one would expect under those circumstances. These results clearly demonstrate the model's excellent vorticity conservation properties for flow moving around objects with very steep slopes. Its success is attributed to the careful formulation of the boundary conditions for vorticity on the topography surface, as is encountered on the side of a cylinder.

***Rotational flow over a cylinder: Taylor-Proud Problem.*** The above experiment for flow around a cylinder in an irrotational fluid was extended to the case of a strongly rotating fluid. The experiment conducted in the previous section was modified to simulate the cylinder in a three dimensional domain of 500 m vertical resolution

extending to 15 km elevation. In this experiment, the prescribed cylinder extended from the surface to a height of 4 km. The domain was set at 45 degrees Earth latitude, with the origin of the rotated grid remaining coincident with the Earth's standard origin along the Greenwich meridian at the equator. This way, the specified westerly wind runs parallel to the model's x axis. In order to simulate the strongly rotating fluids, which produce a Taylor column in the laboratory (Litton, 1988), the angular velocity of the simulated Earth was set to be a factor of 1000 greater than the real Earth. In addition, the vertical atmospheric structure was taken to be neutral. The simulation was run for a period of 1 hour.

The results showed the development of a region of vanishing flow directly above the cylinder extending to the domain top. A massless tracer was introduced within the column of air above cylinder and found to remain essentially steady, although a small plume did leak away to the south east in reaction to some turbulent transport created by the strong deformation. Figure 5 shows a visualization of the simulated tracer depicting the formation of a “Taylor” column. The present ability to capture this rather elegant flow regime is a testimonial to both the vorticity conservation properties of the NMS and its ability to competently capture the most severe topographical features.

***Linear Mountain Waves (1 m ridge).*** A common criticism to step coordinate models is their apparent inability to capture subtle topography features that are captured by terrain following models. A powerful aspect of the variable stepped topography of the UW-NMS is its ability to capture the subtlest of topography features. In this experiment, the linear mountain wave model test discussed by Durran and Klemp (1984) (DK) is

repeated using the variable stepped topography. The grid spacing is 2 km in the horizontal and 250 m in the vertical. A 20 m/s flow in an isothermal atmosphere is simulated passing over a 1 meter high bell shaped mountain. The spinup time was 8 hours. Because the two-dimensional design, the domain is periodic in the meridional direction and consists of only 6 points. Results discussed are for the meridonal average across those points. The upper boundary condition employed was the Klemp and Durran (1983) upper normal mode condition which allows gravity wave energy to escape upwards without allowing the downward propagation of spurious energy.

The analytical solution for the linear gravity wave field that develops is well known (see DK) and can be directly compared to the model prediction. Figure 6 depicts the analytical and model simulated vertical velocity fields that can be seen to be strikingly similar. As DK pointed out, a more sensitive parameter to study is the average vertical transport of horizontal momentum simulated compared to analytical calculations. This effectively provides a more quantitative comparison of the performance. A model has to be physically and numerically flawless to consistently achieve transports within 10% of the analytical values.

Figure 7 depicts a comparison of the NMS performance using three different lateral boundary condition formulations. Lateral boundary conditions are important to these experiments because of the need for gravity wave energy to escape the domain laterally to reproduce the analytical result in the limited domain used. Note that the results are very good, comparable to or better than those reported by DK. These results are a testimonial to the success of the variably steeped topography, which this shows, eliminates, perhaps the most celebrated of the perceived disadvantages of the step

coordinate system. Note there is a slight sensitivity to the choice of lateral boundary condition although all three tested conditions perform well.

An upper absorbing layer top boundary condition, is more commonly employed, in nesting and real data applications of the NMS, than the normal mode top condition tested above. The results of linear mountain experiment using the upper absorbing layer are compared to those using the upper normal mode condition in figure 8. There is a slight degradation of results, although the solution is still quite accurate.

A particular concern of atmospheric modelers is the impact of stretching the vertical grid resolution on the integrity of vertically propagating gravity waves. Stretching is often used to economically improve the resolution of the low level atmospheric structure. The linear mountain experiment is an excellent platform to evaluate to what extent the simulation deteriorates from stretching. It is emphasized here also because of the unique vertical averaging scheme described in the second section of this paper which is meant to improve the models ability to represent flow with a vertically stretched grid. The results for stretching ratios ranging from 1 to 1.5 (where the stretching ratio is the ratio of the vertical grid depth of a grid box to the depth of the grid box just below) are given in figure 9. In this test the grid spacing is taken to be 100 m at the lowest level and stretched to 500 m after which it is held constant, the top of the model remaining at 8 km as in the first test of the linear mountain experiment.

These results show a surprisingly consistent result until the stretching ratio approaches 1.5, where serious solution degradation begins to take place. In practice vertical stretching ratios seldom exceed 1.15. These results suggest that the impacts of stretching the vertical grid on linear vertical gravity wave propagation and structure are



minimal provided the structural features continue to be resolved by coarsest of the stretched vertical resolutions.

***Nonlinear Mountain Waves (500 m mountain ridge).*** The UW-NMS with variable stepped topography was also tested to determine if a non-linear mountain could be simulated accurately. The test employed was the simulation of a 500 m high bell shaped mountain ridge which produces an analytical structure described by Long's equation as proposed by Durran (1987). Following the initialization procedure described by Durran (1987), the simulated wave pattern is given by figures 10 and 11.. The UW-NMS with variably stepped topography again produces a good solution for the 500-m high mountain, and equal in accuracy to the solution of a coordinate transform model described by Durran (1984).

***Flow over isolated mountain peak.*** In this set of two experiments, the UW-NMS simulates flow over an isolated 4 km high by 2 km half-width bell shaped mountain. The first test is designed to demonstrate an important advantage of the variable stepped topography over a transformed topography. A domain of 40 by 40 grid points of 500 m spacing in the horizontal and a vertical grid of 32 points of 500 m spacing was selected. A stable sounding was used to initialize the vertical temperature structure. Humidity was taken to be zero. A constant  $30 \text{ ms}^{-1}$  westerly flow was prescribed at heights above the 350 hPa pressure level only. The upper level westerly flow was prescribed to decrease to 0 by 400 hPa pressure. This wind profile, interpolated to the model spherical grid, was assumed to be horizontally homogeneous, as was the initial thermal and pressure field. The prescribed bell-shaped mountain was placed at the domain center.

Since the prescribed wind field only existed only well above the mountain top in a statically stable environment, the model should not develop any fluctuations in the flow

as a result of the mountain. Indeed the variable stepped solution developed no flow fluctuations. For comparison, the same simulation was performed with an older version of the UW-NMS model with a terrain following vertical coordinate. This resulted in The deformation of the horizontal coordinate surface to follow the terrain. The horizontal advection operator was represented by a transformed derivative involving a difference between a derivative along the sloped surface and a vertical derivative (see Tripoli and Cotton, 1982). Hence because of differing truncation errors in the horizontal and vertical, a disturbance in the flow can be excited by the coordinate system itself. Figure 12 depicts the solution achieved for the transformed model. Note that a  $.5 \text{ ms}^{-1}$  disturbance is produced high above the mountain, which resembles the physical characteristics of a vertically propagating mountain wave. It might have been believed had the experiment not been designed so that no environmental flow can actually interact with the topography. Hence this experiment demonstrates a clear disadvantage of the terrain following system compared to the variably stepped topography system.

For the second experiment, the flow confined to the upper level was extended downward all the way to the surface. The result, should be flow around and partially over the isolated mountain. Figure 13 depicts such a solution for both the coordinate transformed version of the model and for the variably stepped model. Note the solution for the coordinate transformed mountain features a notable swirl in the flow downstream of the mountain that does not exist in the variably stepped topography simulation. Since an irrotational flow set at the equator of the rotated grid was specified for this experiment and surface friction was neglected, the flow should remain irrotational as it moves around the mountain. Therefore, the vortices should not exist, as in the variably stepped model.

This suggests that the proposed variably stepped model handles the flow more accurately. The improvement can be directly attributed to the explicit application of vorticity boundary conditions along the topography in the presence of appropriate dynamical constraints implied by the enstrophy and vorticity conserving finite difference algorithms.

It is also apparent that the structures of vertically propagating gravity waves emanating from the two experiments differ, as the terrain following coordinate solution seems to produce a longer wave length solution. We attribute these differences to the merging of the physical gravity wave with the numerically generated wave, isolated in the previous experiment, in the terrain-following system.

## **6. Conclusions**

In this paper we have described a new scalable nonhydrostatic model featuring variably stepped topography. A set of boundary conditions to the vertical and horizontal differencing of the model has been described that allow for the application of a surface grid box of variable depth to remain numerically stable under all model applications. Moreover the prescribed boundary formulation allows for strict conservation of momentum and vorticity along the stepped model boundary. Methods to facilitate the representation of surface radiative heating and frictional dissipation in the stepped terrain system were presented.

We have demonstrated the ability of the variably stepped topography formulation to represent the even subtle second order dynamical aspects of flow over simple subtle and strong topography at least as well as systems involving a terrain-following vertical coordinate. At the same time we demonstrated that the proposed variably stepped

topography system accurately simulated flow interacting with very steep topography, even vertical walls, which cannot be represented at all with a coordinate transformed system. Finally, for this increased versatility, the UW-NMS pays no price computationally. In fact, for typical simulations involving regions of topography, such as a continental scale simulation over North America, the variably stepped topography system is computationally less expensive by about 15% over the terrain-following system. This is mainly because of the elimination of additional computations for the transformation functions.

In conclusion we have demonstrated considerable advantage of the stepped system over a terrain following coordinate in a scalable model. The resulting ability to represent steeply sloped as well as subtle terrain features allows for considerable versatility while not incurring the considerable cost of a fully transformed topographical system. Although not currently programmed in NMS, the variably stepped system would theoretically be capable of representing complex topographical features with cliffs and overhangs, buildings or any structure so long as its shape can be created under the prescribed grid resolution.

Part 2 of this paper will demonstrate quantitatively evaluate the performance of NMS simulating weather phenomena on a variety of scales ranging from large eddy simulations to predictions of synoptic scale flows. Twice daily synoptic and mesoscale predictions of current weather made by NMS in collaboration with the National Weather Service are available on the NMS web page (<http://mocha.meteor.wisc.edu>).

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## Appendix

### 1.0 Time Differencing Scheme

The time-split compressible hybrid scheme employs several forward, centered and implicit operators to numerically difference the equations. Table A. lists the different numerical schemes as they apply to the governing equations given in the text.

SEQ.	TENDENCY TERM	VAR.	UP- DATE?	NAME	EQ.	O	SPACE DIF.	TIME DIF.	TIME- STEP
1	Buoyancy	$u_i$	-	$B_u$	1	-	Hydrostatic Averaging	Leap Frog	$\Delta t_l$
2	Radiative Transfer	$\theta_{il}$	-	$R_\theta$	18	2	Centered	Forward	$\Delta t_l (\Delta t_r)$
3	Soil Model and Surface Flux	$T_{gd}, r_{gd}$	-	$F_A^1, F_i^1$	1, 18	2	Centered	Crank-Nicholson	$\Delta t_l (\Delta t_g)$
4	Horizontal Turbulent Mixing	$u_i$	-	$F_i^1$	1	2	Centered	Forward	$2\Delta t_l$
5	Vertical Turbulent Mixing	$u_i$	-	$F_i^1$	1	2	Centered	Crank-Nicholson	$2\Delta t_l$
6	Horizontal Turbulent Mixing	$A$	-	$F_A^1$	18	2	Centered	Forward	$\Delta t_l$
7	Vertical Turbulent Mixing	$A$	-	$F_A^1$	18	2	Centered	Crank-Nicholson	$\Delta t_l$
8	Cumulus Parameterization	$\theta_{il}, q_T$ $u_1, u_2$	- - -	$C_\theta, C_{q_v}$ $C_{u_i}$	18	2	Upstream	Forward	$\Delta t_l (\Delta t_c)$
9	Numerical Filter	$u_i$	-	$F_i^2$	1	n	Dimension-Split Centered	Forward	$2\Delta t_l$
10	Numerical Filter	$A$	-	$F_A^2$	18	n	Dimension-Split Centered	Forward	$\Delta t_l$
11	Advection and Coriolis	$u_i$	-	$I_i$	1	2	Enstrophy (Sadourny)	Leap Frog	$\Delta t_l$
12a	Horizontal Pressure Gradient	$u_1, u_2$	-	$G_1, G_2$	1	2	Centered	Forward- Backward	$\Delta t_s$
12b	Vertical Pressure Gradient, Momentum Divergence	$u_3, \pi$	-	$G_3, D_\pi$	1	2	Centered	Crank-Nicholson	$\Delta t_s$
13	Scalar Advection	$A$	-	$I_A$	18	2	Dimension-Split Crowley	Forward	$\Delta t_l$
14	Thermodynamic Diagnosis	$\theta, T, p$ $q_v, q_c$	- - -	$T_A$	18	-	-	-	-
15	Microphysics Tendencies	$q_r, q_{ice}$	-	$M_A$	18	-	None	Forward	$\Delta t_l$
16	Precipitation Settling	$q_r, q_{ice}$ $\theta_{il}$	- -	$P_A, P_{\theta_{il}}$	20	1	Upstream	Forward	$\Delta t_l (\Delta t_p)$



17	Thermodynamic Diagnosis	$\theta, T, p$ $q_v, q_c$	- -	$T_A$	<sup>12,13</sup> , <sup>21</sup> , 22	-	-	-	-
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**Table A.1 Shows the time differencing scheme applied to the governing equations.**