Motivation for the cubicFit transport scheme

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Stability

- The slanted cell method improves the numerical representation of hydrostatic balance over steep orography compared to terrain-following meshes but requires a new advection scheme that is stable (and accurate) — TODO: *but the spurious velocities are small on terrain-following meshes anyway, so why should we use slanted cells?*
- Application of von Neumann stability analysis is unique to the best of my knowledge — TODO: *but it's not entirely effective, I'm still finding meshes and wind fields that satisfy the stability criteria but are actually unstable*
- The stability analysis technique could be applied to any equation in any domain — TODO: *perhaps, but this hasn't been done*

Computationally cheap

- Computationally cheap, just a vector dot product (unlike most swept-area schemes that require a matrix-vector multiply (Lashley, 2002; Skamarock and Menchaca, 2010; Thuburn et al., 2014)) — TODO: *but have we traded accuracy for computational efficiency and was it really worth it?*
- Small stencil permits lots of parallelisation — TODO: *surely true of any scheme with a Courant number* ≤ 1

Multidimensionality

- Not susceptible to splitting errors (unlike models such as CAM-FV, UKMO Dynamo) — TODO: *but (Chen et al.) shows that splitting errors are negligible for all but the steepest of terrain, so does it really matter, especially given the gains in computational efficiency by using a dimensionally-split transport scheme?*
- No special treatment for the corners of cubed-sphere panels (unlike TODO: *citation, check weller2017*) or for hexagonal geometry (unlike TODO: *citation, gassmann?*) — TODO: *that's nice, but if the special treatments are effective, then does it matter?*
- Multidimensional in arbitrary dimensions. We use the scheme on an x–z plane but no special treatment would be needed for 3D multidimensional — TODO: *many other schemes would generalise to 3D multidimensional, too, but is there any motivation for doing so?*

Arbitrary meshes

- Suitable for arbitrary meshes including terrain-following meshes, cut cells, cubed-spheres and hexagonal icosahedra — TODO: *but real models choose their grid upfront and choose a suitable transport scheme to go with it, do they really need this flexibility?*

- Suitable for steep slopes — TODO: *is this really a problem? e.g. ECMWF say they have no problem with steep slopes*

Miscellaneous

- Second-order accurate on distorted meshes (really?) unlike (Skamarock and Gassmann, 2011) — TODO: *only demonstrated properly in 1D, in 2D sometimes the scheme diverges, it doesn't even converge!*
- Conservative (unlike UKMO EndGame which suffers from eternal fountain of moisture) — TODO: *so are all other finite volume transport schemes*
- Eulerian schemes are less restrictive than semi-Lagrangian schemes for small-scale rotational flow because non-simply connected domains are permitted (Lauritzen et al., 2011) — TODO: *other Eulerian schemes already exist*
- Eulerian schemes allow choice in timestepping, e.g. optimized Runge–Kutta schemes could allow Courant numbers > 1 (as mentioned by John Thuburn) — TODO: *true for any Eulerian scheme*
- Possibility of super-convergence — TODO: *only been achieved in 1D, not in 2D/3D*
- Standard limiters could be applied — TODO: *hasn't been done, true of many schemes*

References

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