Monitoring committee progress report #6

Numerical representation of mountains in atmospheric models

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Monitoring committee: Paul Williams, Maarten Ambaum

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1 Introduction

Numerical weather and climate models are using increasingly fine meshes to resolve small-scale processes in order to improve forecasts. As meshes become finer, traditional latitude-longitude meshes become computationally inefficient (Staniforth and Thuburn, 2012). Fine meshes also resolve small-scale, steeply-sloping terrain that are poorly represented by traditional terrain-following meshes (Schär et al., 2002). These difficulties have motivated research into alternative horizontal meshes (Staniforth and Thuburn, 2012), better vertical meshes including improved terrain-following meshes (Schär et al., 2002; Klemp, 2011) and cut cell meshes (Jähn et al., 2015; Yamazaki et al., 2016), and improved numerical methods (Ullrich and Jablonowski, 2012; Steppeler and Klemp, 2017).

This PhD project makes three contributions to improve numerical atmospheric simulations, particularly in the vicinity of steeply-sloping terrain. First, we have created the finite volume transport scheme, 'cubicFit', which is stable and accurate over steep slopes represented by arbitrary meshes. We are currently developing an improved transport scheme which uses the method of Devendran et al. (2017), and which is expected to be more accurate than cubicFit in the domain interior. Second, we have created a new type of mesh, the 'slanted cell' mesh, in order to represent steeply-sloping terrain while avoiding severe meshes distortions associated with terrain-following meshes and avoiding severe time-step constraints associated with explicit numerical methods on cut cell meshes. Third, we have created a new two-dimensional test case to excite the Lorenz computational mode, based on the work of Arakawa and Konor (1996). The new test case is better suited for dynamical core evaluation than existing tests that are either very expensive to simulate (Untch and Hortal, 2004) or have no horizontal discretisation (Arakawa and Konor, 1996). We use the new test case to compare results from a fully-compressible Euler model with Lorenz staggering (Weller and Shahrokhi, 2014) with those from a newly-developed model variant with a Charney–Phillips staggering that has been generalised for arbitrary meshes.

Since our fifth monitoring committee meeting, my supervisors and I have been pursuing two lines of research. First, we have performed further experiments using the new, fully-compressible Euler model with a generalised Charney–Phillips staggering. Most results have been unstable or inaccurate, likely because potential temperature is insufficiently conserved by the advective-form transport scheme. Given that this first line of research has been unpromising, we have begun developing a high-order flux-form transport scheme in collaboration with Hans Johansen at Lawrence Berkley National Lab. We have already obtained high-order convergence using a one-dimensional test case, and we have just embarked upon a two-dimensional implementation.

2 A new test case for exciting the Lorenz computational mode

Monitoring committee report #5 presented results of a new test case designed to excite the Lorenz

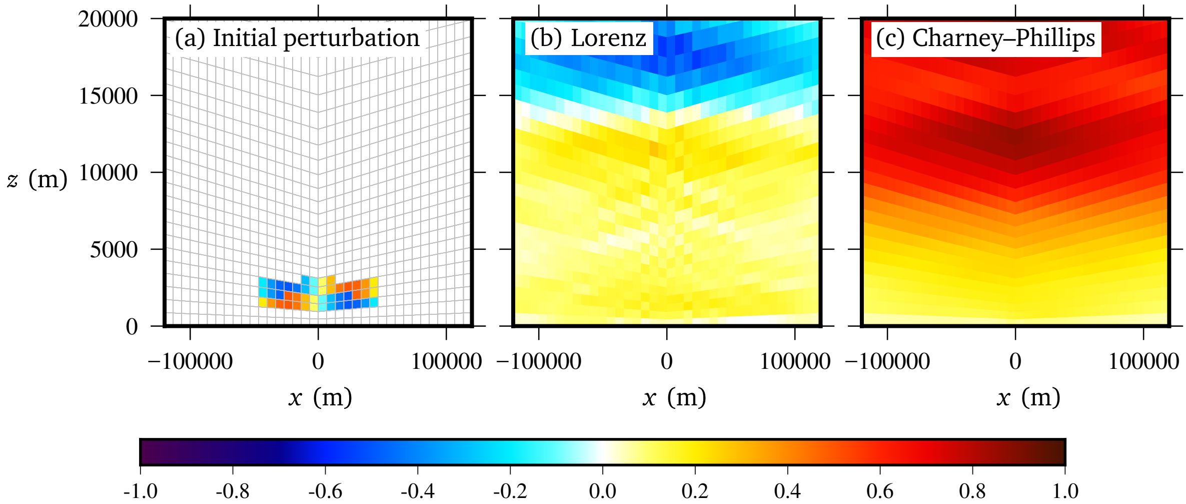

Figure 1: Potential temperature perturbations (K) from a stably stratified background profile. (a) An initial wave-shaped perturbation is applied near the ground in the centre of the domain such that the integral of the perturbation over the whole domain is zero. The perturbation after a two-day simulation is shown for (b) the Lorenz model, and (c) the Charney–Phillips model. Only the lowest 20 km in the central region is shown. The entire domain is 800 km wide and 30 km high.

computational mode. A fully-compressible Euler model included a newly-formulated generalisation of the Charney–Phillips staggering for arbitrary meshes. Unlike the model with Lorenz staggering, the Lorenz computational mode was not present in the new Charney–Phillips model. These results were obtained on a uniform mesh using a low-accuracy advective-form scheme for transporting potential temperature.

We have performed the same tests using distorted meshes in order to exercise the generalised Charney–Phillips staggering more thoroughly. Distorted meshes were constructed by tilting the horizontal mesh surfaces in order to retain a rectangular domain and avoid introducing any orographic effects. The cell edges are shown in figure 1a with the initial potential temperature perturbation shown in colour.

A large-scale response to the initial perturbation is observed in the final potential temperature perturbation after the 2-day simulation using the Lorenz model (figure 1b) and the Charney–Phillips model (figure 1c). This response is very different from the fine-scale waves observed using a uniform mesh (see figure 2 in monitoring committee report #5), and no evidence of the Lorenz computational mode is found using the distorted mesh. As seen in figure 1c, using the Charney–Phillips model, the internal energy gradually increases throughout the domain, and this error is larger for more highly-distorted meshes. This gradual increase in energy is clearly seen in figure 2a, and only occurs using the Charney–Phillips model with distorted meshes. In the other fully-compressible Euler tests, energy changes are of the order 10^{-7}: three orders of magnitude smaller than those found using the Charney–Phillips model with distorted meshes.

The Charney–Phillips model transports potential temperature in advective-form, not flux-form, so we do not expect potential temperature to be conserved. Hence, the non-conservative transport scheme is a likely cause for the increase in internal energy. To examine the transport scheme we perform another test case, transporting a tracer in a prescribed horizontal wind, using the same uniform and distorted meshes as before. The advective-form potential temperature transport scheme was extracted from the fully-compressible Euler model in order to solve the transport equation on Charney–Phillips staggered meshes. The results were compared to the flux-form multidimensional linear upwind transport scheme documented by Shaw et al. (2017).

As expected, the flux-form scheme with Lorenz staggering conserves total tracer to machine precision (figure 2b). Non-conservation errors are also small using the advective-form scheme using the Charney–

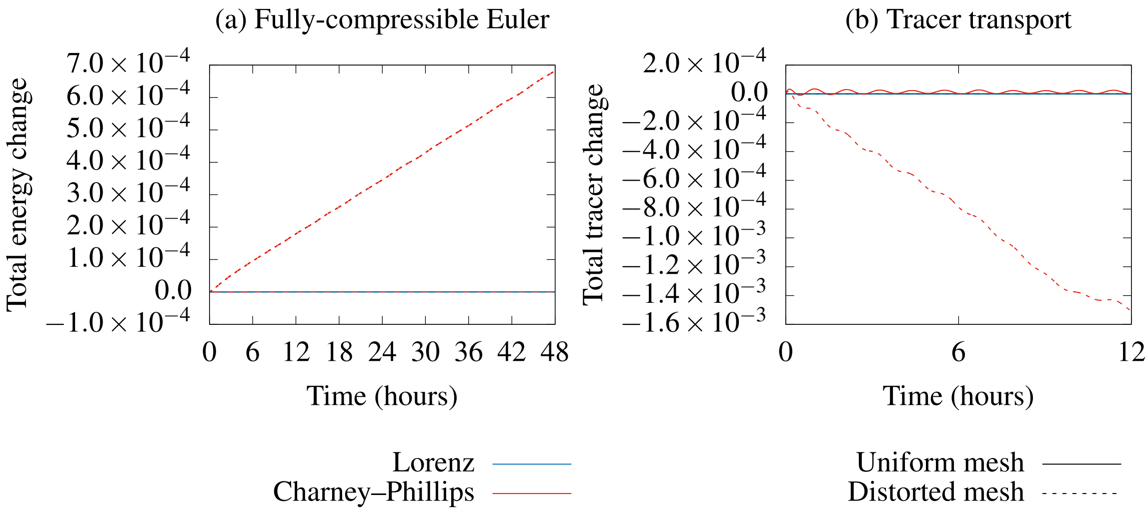

Figure 2: Conservation diagnostics for (a) the fully-compressible Euler equation models and (b) the tracer transport models. Energy changes are normalised by the total initial energy, and total tracer changes are normalised by the total initial tracer.

Phillips staggering and a uniform mesh. In contrast, total tracer is gradually lost using the Charney–Phillips staggering and a distorted mesh. In both fully-compressible Euler and tracer transport tests, non-conservation is evident using a Charney–Phillips staggering and distorted meshes. Interestingly, internal energy increases in the fully-compressible Euler test while total tracer decreases in the tracer transport test. We do not know why these behaviours differ.

We have developed some different advective-form transport schemes in an attempt to improve conservation. Inspired by cubicFit, we tried using larger stencils and higher-order polynomials to approximate the gradient at temperature points on Charney–Phillips staggered meshes. However, we have been unable to obtain a transport scheme that is stable with better conservation than the existing low-accuracy transport scheme. While it has been difficult to make progress on this research, we have had more success developing a high-order flux-form transport scheme in collaboration with Hans Johansen. The next section documents these developments.

3 High-order transport for arbitrary meshes

The cubicFit transport scheme solves the transport equation by approximating fluxes through faces using a multidimensional polynomial reconstruction (Shaw et al., 2017). The reconstruction fits a polynomial over known values stored at cell centre points, and this point-wise approach limits the cubicFit scheme to second-order convergence. Devendran et al. (2017) have developed a high-order finite volume scheme for solving Poisson's equation, and we apply their approach to obtain a transport scheme with high-order convergence. Since it has much in common with the cubicFit scheme, we name this high-order scheme 'highOrderFit'.

Instead of a point-wise reconstruction that uses cell centre positions, the highOrderFit scheme constrains the polynomial fit so that the average of the polynomial integrated over a cell volume equals the cell average value. Applying this constraint to each cell in the reconstruction stencil produces an overdetermined linear system that is solved using a weighted least squares approach in a similar manner to the cubicFit scheme (Shaw et al., 2017). Since we use the same reconstruction stencil, the highOrderFit scheme has the same computational cost per time-stage as the cubicFit scheme.

In one dimension, the reconstruction stencil comprises three upwind cells and one downwind cell. Every cell has a left and right face, each with its own four-point stencil (figure 3). Hence, assuming a suitably high-order time-stepping scheme, highOrderFit should achieve fourth-order convergence because

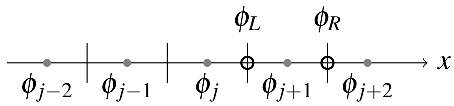

Figure 3: One-dimensional fluxes through a cell ϕ_{j+1} using four-point upwind-biased stencils to approximate fluxes ϕ_L and ϕ_R.

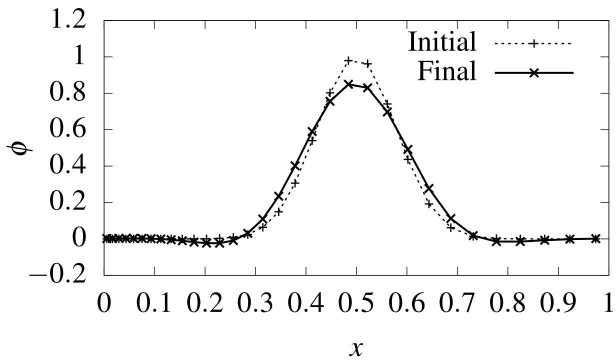

Figure 4: One-dimensional transport of a Gaussian hill across a periodic domain using the highOrderFit scheme. The initial tracer is transported in a uniform velocity field exactly once around the domain so that the analytic solution is equal to the initial solution marked by a dotted line. The numerical solution is marked by a solid line and cell centre averages are marked by crosses.

the flux divergence is approximated with five points.

We verify that highOrderFit achieves fourth-order convergence using a simple one-dimensional transport test. The periodic domain is defined over the interval $[0,1]$ with a uniform velocity field $u = 1$ and an initial tracer field ϕ being defined as

$$\phi(x) = \exp\left(-b\,(x - x_0)^2\right) \tag{1}$$

where the Gaussian hill is centred at $x_0 = 0.5$ with $b = 80$.

Since we are interested in achieving high-order convergence on arbitrary meshes, the one-dimensional domain is divided into N unequal intervals such that Δx_i, $i = 1 \,..\, N$ becomes gradually coarser with increasing i. The nonuniform mesh is smooth except for a discontinuity between Δx_1 and Δx_N at the periodic boundary.

The test is integrated forward by one time unit, transporting the tracer exactly once around the periodic domain such that the analytic solution is equal to the initial solution. A solution on a mesh with $N = 32$ is illustrated in figure 4 with the initial solution marked by a dotted line and the final numerical solution marked by a solid line, with cell centre averages marked by crosses.

Convergence tests are performed comparing cubicFit and highOrderFit transport schemes using meshes with cell counts between $N = 2^4$ and $N = 2^8$. The classical fourth-order Runge–Kutta time-stepping scheme (Durran, 2013, p. 53) is used for both transport schemes. The cubicFit scheme is limited to second-order convergence while the highOrderFit scheme is fourth-order convergent (figure 5).

4 Future research

Our attempts to develop a sufficiently conservative scheme for transporting potential temperature on meshes with a generalised Charney–Phillips staggering have been unsuccessful. As such, we have decided to invest no further effort researching a generalised Charney–Phillips staggering. Instead, having recently

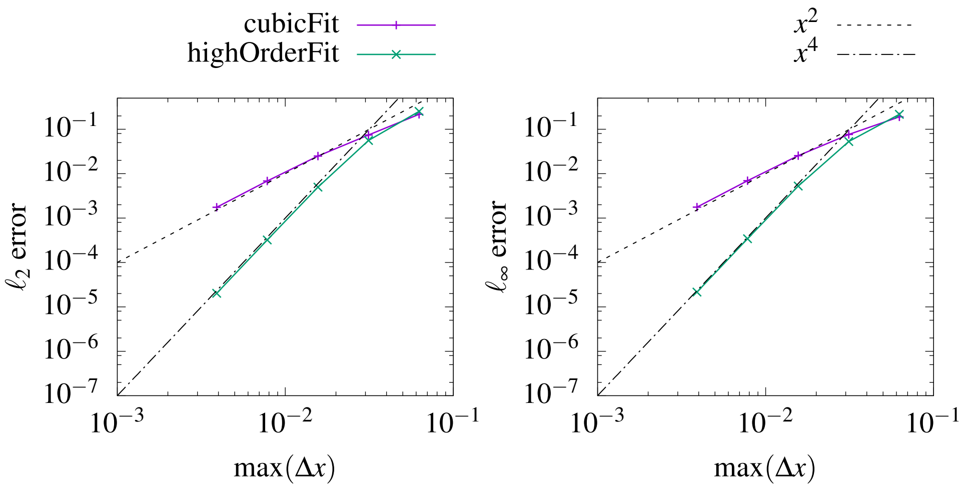

Figure 5: Convergence of the cubicFit and highOrderFit schemes in the one-dimensional transport test. The error norms are defined as $\ell_2 = \left(\sum\left((\phi-\phi_T)^2\Delta x\right)/\sum\left(\phi_T^2\Delta x\right)\right)^{1/2}$ and $\ell_\infty = \max|\phi-\phi_T|/\max|\phi_T|$ where ϕ is the numerical solution and ϕ_T is the analytic solution.

achieved high-order convergence in a one-dimensional transport test case, I hope to invest the next few months developing, testing and documenting a two-dimensional implementation of highOrderFit. In the interests of time, we will avoid any special boundary treatment as found in cubicFit by using periodic boundaries or tracers defined well away from boundaries.

A revised thesis plan is presented in Appendix A. Chapter 2 and chapter 4 are complete, requiring only minor revisions following supervisor comments. If future work on two-dimensional highOrderFit is successful, this will likely form a new thesis chapter 3. If unsuccessful, I will instead include additional sections in chapter 2 documenting the one-dimensional highOrderFit formulation and test results. I no longer intend to publish a third journal article, but I will still document our existing Charney–Phillips research in thesis chapter 5. In our fifth monitoring committee we agreed to extend my submission date by three months from 12 January 2018 to mid-April 2018. The revised plan presented here does not alter this extended submission date.

December 2017 Complete chapter 5 documenting existing Charney–Phillips research

January 2017 Complete the two-dimensional highOrderFit implementation

February 2017 Complete chapter 3 documenting the highOrderFit formulation and test results

March 2017 Complete introduction and discussion chapters

5 Personal development

I continue to give regular talks at group meetings and I have arranged to give a lunchtime seminar in January 2018. I have also been invited by James Kent to speak at a mathematics departmental seminar at the University of South Wales where Dr Kent lectures.

I also continue searching for postdoctoral vacancies. I was unsuccessful in my application for a Mozilla science fellowship, though I was very happy to be placed amongst the top 5% of more than 1000 applicants. I was offered a position as a scientific software engineer at the Met Office, but chose to decline since the role focused on software development with few opportunities for scientific research. I was also interviewed for a postdoctoral position developing numerical methods for flood forecasts at the University of Sheffield, but the position was given to the other, more experienced candidate.

I have worked with David Dritschel at the University of St Andrews and Alan Blyth, Doug Parker and Steven Böing at the University of Leeds to submit an EPSRC proposal that extends the Moist Parcel-in-a-Cell model (Böing et al., 2017) for studying cloud turbulence. We expect the EPSRC to reach a decision around March 2018. I continue to monitor job boards for new vacancies.

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Appendix A: Thesis plan

A work-in-progress draft is available at http://www.datumedge.co.uk/publications/phd-thesis.pdf.

1. Introduction

Not started	This project is motivated by the need for alternative horizontal and vertical representations of Earth's atmosphere in the proximity to mountainous terrain

2. Numerically stable transport over steep slopes

The cubicFit transport scheme is stable and accurate over steep slopes with arbitrary, distorted meshes.

Complete	Review of existing transport schemes and motivation for the cubicFit transport scheme
Complete	Document the cubicFit transport scheme
	Test results comparing a standard linear upwind scheme and the cubicFit transport scheme:
Complete	• horizontal transport test above steep slopes
Complete	• terrain-following transport test above steep slopes
Complete	• deformational transport tests on a spherical Earth

3. High-order transport for arbitrary meshes

The cubicFit transport scheme is limited to second-order convergence because it uses a point-wise reconstruction of cell centre values. If instead the reconstruction uses cell centre volume averages then higher-order convergence is achievable.

Not started	Document the high-order transport scheme
	Test results comparing cubicFit and highOrderFit transport schemes:
Not started	• two-dimensional solid body rotation of a Gaussian tracer on distorted meshes (Chen et al., 2017)
Not started	• horizontal transport of a cosine bell tracer above steep slopes as in chapter 2

4. A new mesh for representing the atmosphere above terrain

The slanted cell mesh reduces errors associated with pressure gradient calculations and avoids severe time-step constraints.

Complete	Introduction to pressure gradient calculations and existing methods to reduce errors associated with pressure gradient calculations
Complete	Describe the new slanted cell method
Complete	Transport test over a mountainous lower boundary using terrain-following, cut cell and slanted cell meshes
Complete	A two-dimensional test of a quiescent atmosphere above steep slopes, comparing terrain-following, cut cell and slanted cell meshes

5. A new test case for exciting the Lorenz computational mode

Existing test cases that excite the Lorenz computational mode are unsuited for assessing modern dynamical cores. Based on the work of (Arakawa and Konor, 1996), we develop a new two-dimensional test case for evaluating dynamical cores with Lorenz or Charney–Phillips staggerings.

In development	Describe the generalised Charney–Phillips formulation
In development	Document the necessary changes to the nonhydrostatic model including the advective-form transport scheme
In development	Compare the Charney–Phillips model variant with the Lorenz model using a standard mountain waves test case (Schär et al., 2002)
Not started	Document the new two-dimensional standing waves test case
In development	Compare standing waves test results using Lorenz and Charney–Phillips model variants on a uniform mesh and rectangular meshes with tilted faces

6. Discussion

Not started

Appendix B: Training record

Mathematics modules

Spring 2017	M5A47	Finite elements: numerical analysis and implementation	unassessed, partially completed
Spring 2016	MA3NAT	Numerical Analysis II	unassessed
Spring 2015	MAMNSP	Numerical Solution of Partial Differential Equations	78%

RRDP modules

I have completed the requisite 11 RRDP modules.

23 June 2017	Graduate school conference
3 May 2017	Effective CVs
28 Feb 2017	Getting your first post-doc position
9 Nov 2016	Open Access and research data management
24 Mar 2016	Voice coaching: looking after your voice
26–27 Jan 2016	Preparing to teach (introduction, marking & feedback, leading small groups)
2 Dec 2015	An essential guide to critical academic writing
17 Nov 2015	Understanding the UK higher education context
19 May 2015	How to avoid plagiarism
10 Mar 2015	How to write a literature review
19 Feb 2015	How to write a paper

External courses

June 2016	Dynamical core intercomparison project summer school, NCAR
13 May 2016	Peer review: the nuts and bolts, Sense about Science
June 2015	Advanced numerical methods for Earth-system modelling, ECMWF

Conferences and workshops

September 2017	Speaker	International conference on scientific computation and differential equations, University of Bath
August 2017	Co-organiser	Frontiers in natural environment research, Imperial College London
July 2017	Speaker	Met Office GungHo network meeting, University of Exeter
June 2017	Participant	Docker containers for reproducible research, University of Cambridge
April 2017	Speaker	PDEs on the Sphere, École normale supérieure, Paris
March 2017	Attendee	Open in practice: inspirations, strategies and methods for open research, University of Reading
March 2017	Participant	Effective quadratures workshop, University of Cambridge
February 2017	Invited speaker	Numerical methods for geophysical fluid dynamics, Imperial College London
January 2017	Attendee	Research software management, sharing and sustainability, British Library
December 2016	Invited speaker	South-East local centre meeting, Royal Meteorological Society
October 2016	Speaker	Numerical and computational methods for simulation of all-scale geophysical flows, ECMWF
November 2015	Attendee	GungHo workshop on next generation weather and climate prediction, Met Office
June 2015	Attendee	Hoskins@70
June 2015	Poster	SCENARIO DTP conference
March 2015	Speaker	Galerkin methods with applications in weather and climate forecasting, ICMS

Teaching

Oct 2016	Teaching assistant	MTMW11 fluid dynamics
Oct 2015	Teaching assistant	MTMG02 atmospheric physics
Sep 2015	Teaching assistant	NCAS summer school
Sep 2014	Course teacher	MPE python and linux short course

Visits and collaborations

July 2016	Organised visit from Simon Clark, stratospheric PhD researcher and YouTube vlogger
Summer 2016	Worked with Hilary's MSc student, Christiana Skea, studying variable time-stepping for ODEs
June 2016	Visited NCAR, hosted by Ram Nair
2015 – 2017	Coauthoring an article about dimensionally-split and multidimensional transport schemes, written with Hilary, her former student Yumeng Chen, and Stephen Pring at the Met Office

Outreach

17 Mar 2017	"The advection process: simulating wind on computers", Social Metwork blog article
14 Jul 2015	Schools physicist of the year awards
14 Jun 2015	East Reading festival
15 Feb 2015	Brighton science festival

Presentations

2 Feb 2018	University of South Wales	
22 Jan 2018	HHH group	
16 Jan 2018	Lunchtime seminar	
26 Oct 2017	Comp. Atmos. Dyn. group	High-order finite volume advection
17 Oct 2017	Mesoscale group	Numerical advection of chemical species
27 Sep 2017	DARC group	Subsampling for uncertainty quantification
6 Jul 2017	Comp. Atmos. Dyn. group	Charney–Phillips review and automated testing for OpenFOAM applications
19 Jun 2017	HHH group	Subsampling for uncertainty quantification
17 Nov 2016	Comp. Atmos. Dyn. group	A review of atmospheric transport schemes
9 Nov 2016	PhD group	Replicable computational atmospheric science
31 Oct 2016	HHH group	Advection over steep slopes
22 Sep 2016	PhD poster session	Improving numerical accuracy over steep slopes
23 Mar 2016	Quo Vadis	Numerical representation of orography in dynamical cores (honourable mention)
17 Feb 2016	PhD group	Multidimensional advection schemes for arbitrary meshes
9 Feb 2016	Mesoscale group	Curl-free pressure gradients for accurate modelling of cold air pools
19 Oct 2015	HHH group	Improving modelled mountain flows with alternative representations of terrain
27 Apr 2015	HHH group	A like-for-like comparison between terrain following and cut cell grids
21 Apr 2015	PhD group	Discrete vector calculus on Arakawa C grids
12 Feb 2015	Met Office	Poster presentation for Met Office Academic Partnership
18 Jan 2015	PhD group	Python and linux tips
17 Dec 2014	MPECDT jamboree	Poster presentation for Mathematics for Planet Earth Centre for Doctoral Training jamboree
12 Sep 2014	Lunchtime seminar	Gain control of your documents and code: hands-on with revision control and build automation

Appendix C: Publication milestones

10 Jun 2015	First MWR manuscript submitted
19 Aug 2015	Major revisions required to MWR manuscript
29 Oct 2015	Second MWR manuscript submitted
9 Dec 2015	Major revisions required to MWR manuscript
5 Feb 2016	Third MWR manuscript submitted
2 Feb 2017	First JCP manuscript submitted
13 Mar 2017	Minor revisions required to JCP manuscript
21 Apr 2017	Second JCP manuscript submitted