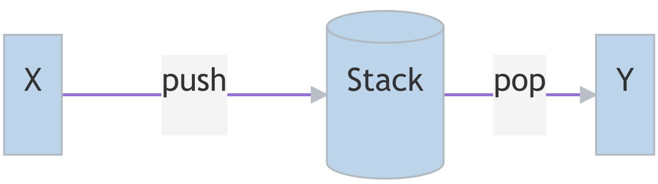
Abstract Data Types (ADTs)

Stack

Methods

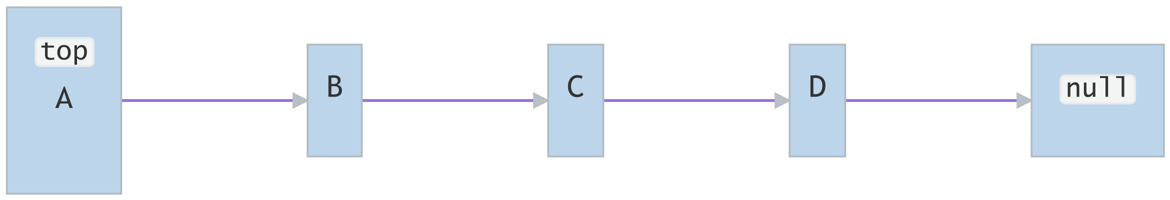
- `push(x, S)`: Puts `x` onto the stack `S`
- `pop(S)`: Remove (and returns) the top element of the stack `S`
- `top(S)`: Returns the top element of the stack `S`

Visualization



Structure

Linked List:



Runtime

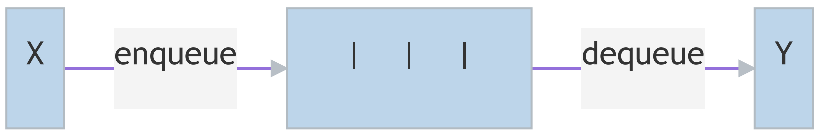
- `push(x, s)` $\in \mathcal{O}(1)$
- `pop(s)` $\in \mathcal{O}(1)$
- `top(s)` $\in \mathcal{O}(1)$

Queue

Methods

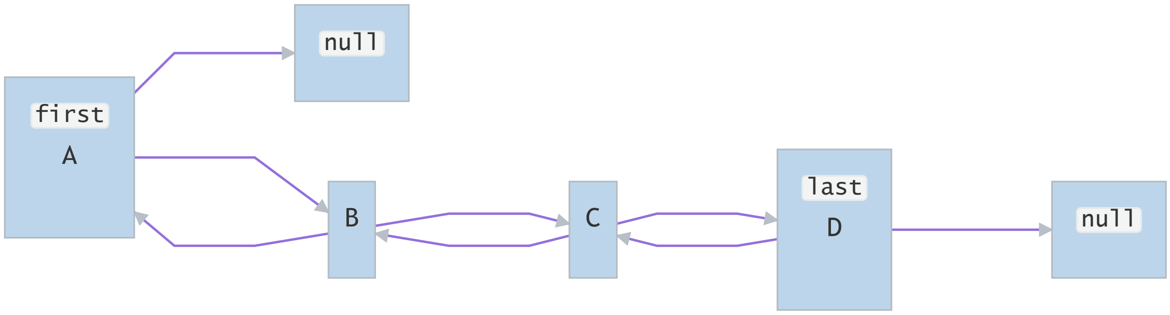
- `enqueue(x, s)`: Add `x` to the queue `s`
- `dequeue(s)`: Remove the first element of the queue `s`

Visualization



Structure

Doubly Linked List:



Runtime

- `enqueue(x, S)`: $\in \mathcal{O}(1)$
- `dequeue(S)`: $\in \mathcal{O}(1)$

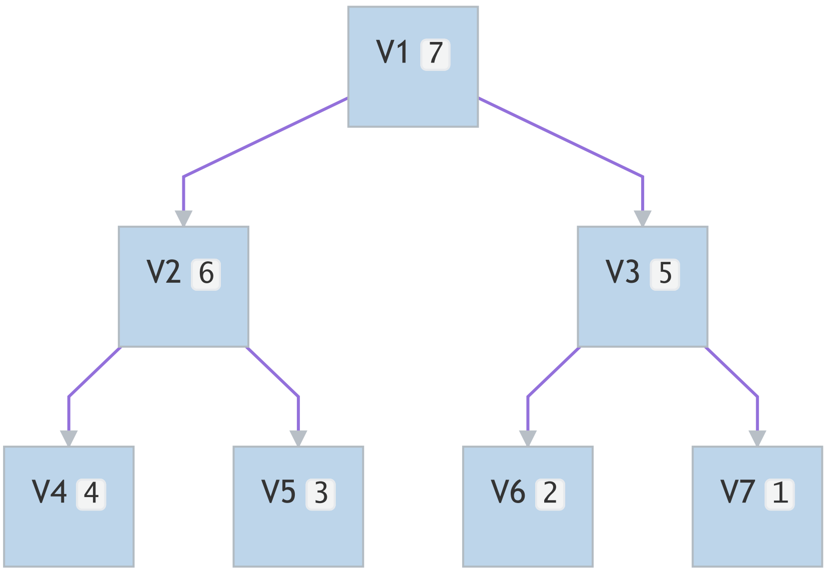
Priority Queue

Methods

- `insert(x, p, P)`: Insert `x` with priority `p` into the queue `P`
- `extractMax(P)`: Extracts the elements with maximal priority from the queue `P`

Structure

Max-Heap:



Runtime

- `insert(x, p, P)`: $\in \mathcal{O}(log(n))$
- `extractMax(P)`: $\in \mathcal{O}(log(n))$

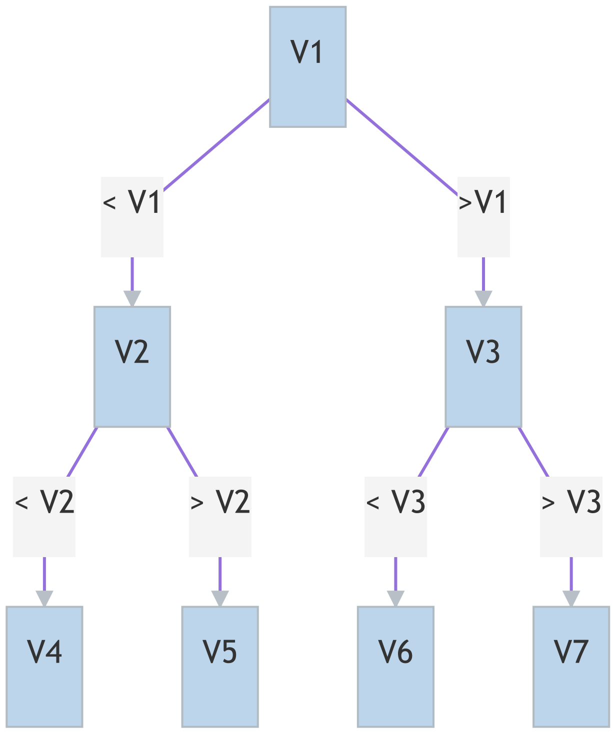
Dictionary

Methods

- `search(x, W)`: Finds `w` in dictionary `W`
- `insert(x, W)`: Insert `x` in dictionary `W`
- `remove(x, W)`: Remove `x` from the dictionary `W`

Structure

Search Tree:



Union-Find

Data structure used to compare ZHKs of a given graph.

Methods

- `make(V)`: Create a data structure for $F = \emptyset$
- `same(u, v)`: Test whether u, v are in the same ZHK of F
- `union(u, v)`: Merge ZHKs where u and v are

Structure

List `rep[]` which stores the identifiers of all the vertices. `rep[u]` = `rep[v]` if and only if THK(v) = ZHK(u).

Implementation

```
make(V):
    for (v in V):
        rep[v] = v

same(u, v):
    return rep[u] == rep[v]

// members[rep[u]] is a list containing all the nodes in ZHK(u)
union(u, v):
    for (x in memebers[rep[u]]):
        rep[x] = rep[v]
        members[rep[v]].add(x)
```

Runtime

- `make(V)` $\in \mathcal{O}(|V|)$
- `same(u, v)` $\in \mathcal{O}(1)$
- `union(u, v)` $\in \mathcal{O}(|ZHK(u)|)$

Weighted Quick-Union

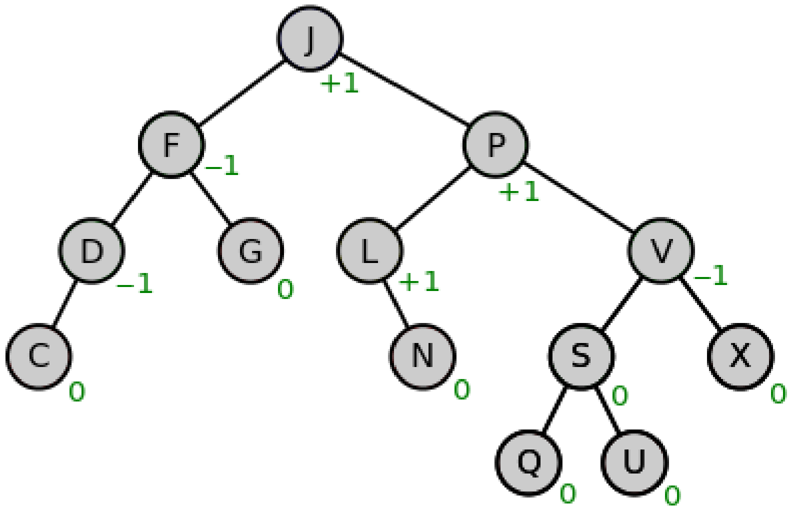
```
class UnionFind {
    int[] id;
    int[] size;

    public UnionFind(int N) {
        create(N);
    }

    void create(int N) {
        id = new int[N];
        for (int i = 0; i < N; ++i) {
            id[i] = i;
            size[i] = 1;
        }
    }

    private int root(int i) {
        while (i != id[i]) {
            id[i] = id[id[i]];
            i = id[i];
        }
        return i;
    }

    public int find(int x, int y) {
        return root(x) == root(y);
    }

    public void union(int x, int y) {
        if (size[x] < size[y]) { id[x] = y; size[y] += size[x]; }
        else { id[y] = x; size[x] += size[y]; }
    }
}
```

AVL Trees

Description

Most of the BST operations (e.g., `search`, `max`, `min`, `insert`, `delete`,...) take $\mathcal{O}(h)$ time where h is the height of the BST. The cost of these operations may become $\mathcal{O}(n)$ for a skewed Binary tree. If we make sure that height of the tree remains $\mathcal{O}(log(n))$ after every insertion and deletion, then we can guarantee an upper bound of $\mathcal{O}(log(n))$ for all these operations. The height of an AVL tree is always $\mathcal{O}(log(n))$ where n is the number of nodes in the tree

We define the balance of a vertex v, $bal(v) = h(T_r(v)) - h(T_l(v))$. For a Search Tree to fulfill the AVL-condition, we need $\forall v \; bal(v) \in \{-1, 0, 1\}$



An AVL Tree with every balance value written below the corresponding node

We distinguish three states of a node p before inserting a node:

- $bal(p) = -1$: not possible
- $bal(p) = 0$
- $bal(p) = 1$

Insertion

Left and right rotation

```
T1, T2 and T3 are subtrees of the tree rooted with y (on the left side) or x (on the right side)

                                    y                               x
                                   / \     Right Rotation          /  \
                                  x   T3   - - - - - - - >        T1   y
                                 / \       < - - - - - - -            / \
                                T1  T2     Left Rotation            T2  T3

Keys in both of the above trees follow the following order:
                               keys(T1) < key(x) < keys(T2) < key(y) < keys(T3)
So BST property is not violated anywhere.
```

Insertion and rotations

Steps to follow for insertion
Let the newly inserted node be w

- Perform standard BST insert for w.
- Starting from w, travel up and find the first unbalanced node. Let z be the first unbalanced node, y be the child of z that comes on the path from w to z and x be the grandchild of z that comes on the path from w to z.
- Re-balance the tree by performing appropriate rotations on the subtree rooted with z. There can be 4 possible cases that needs to be handled as x, y and z can be arranged in 4 ways. Following are the possible 4 arrangements:

- y is left child of z and x is left child of y (**Left Left Case**)
- y is left child of z and x is right child of y (**Left Right Case**)
- y is right child of z and x is right child of y (**Right Right Case**)
- y is right child of z and x is left child of y (**Right Left Case**)

a) Left Left Case

```
T1, T2, T3 and T4 are subtrees.
         z                                      y
        / \                                   /   \
       y   T4      Right Rotate (z)          x      z
      / \          - - - - - - - - - ->    /  \    /  \
     x   T3                               T1  T2  T3  T4
    / \
  T1   T2
```

b) Left Right Case

```
     z                               z                           x
    / \                            /   \                        /  \
   y   T4  Left Rotate (y)        x     T4  Right Rotate(z)    y      z
  / \      - - - - - - - - - ->  /  \       - - - - - - - - ->  / \    / \
T1   x                          y    T3                   T1  T2 T3  T4
    / \                        / \
  T2   T3                    T1   T2
```

c) Right Right Case

```
  z                                y
 /  \                            /   \
T1   y     Left Rotate(z)       z      x
    /  \   - - - - - - - - ->  / \    / \
   T2   x                     T1  T2 T3  T4
       / \
     T3  T4
```

d) Right Left Case

```
   z                            z                            x
  / \                          / \                          /  \
T1   y   Right Rotate (y)    T1   x      Left Rotate(z)   z      y
    / \  - - - - - - - - - ->    /  \   - - - - - - - - ->  / \    / \
   x   T4                      T2    y                  T1  T2  T3  T4
  / \                               /  \
T2   T3                           T3   T4
```