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Review of Mathematical Optimization in Federated Learning

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Abstract. Federated Learning (FL) has been becoming a popular interdisciplinary research area in both applied mathematics and information sciences. Mathematically, FL aims to collaboratively optimize aggregate objective functions over distributed datasets while satisfying a variety of privacy and system constraints. Different from conventional distributed optimization methods, FL needs to address several specific issues (e.g., non-i.i.d. data distributions and differential private noises), which pose a set of new challenges in the problem formulation, algorithm design, and convergence analysis. In this paper, we will systematically review existing FL optimization research including their assumptions, formulations, methods, and theoretical results. Potential future directions are also discussed.

AMS subject classifications: 90C26, 90C31, 68W15, 68T05

Key words: Federated Learning, distributed optimization, convergence analysis, error bounds.

1 Introduction

With the increasingly stringent privacy regulations [1, 2], data isolation has been becoming the key bottleneck of data sciences and artificial intelligence. To address this issue, Federated learning (FL) emerges as a popular privacy-preserving distributed Machine Learning (ML) paradigm, which enables multiple data owners to jointly train ML models without sharing the raw data [3–5]. It has gained extensive interests from both academia and industry, and demonstrated great success across multiple domains, including medicine [6], finance [7], and industry [8], etc.

Mathematically, FL training tasks are essentially distributed optimization problems, which aim to minimize aggregate global objectives (e.g., the mean empirical loss), across a set of distributed data owners by exchanging model parameters trained on their local datasets [9–11]. Despite being similar in essence, FL optimization has many distinct characteristics from traditional distributed optimization. Their main difference lies in the

communication environment. Specifically, traditional distributed optimization, mainly in the form of distributed ML [12–14] (some used in distributed resource allocation [15–17]), is often used for high-throughput ML speedup in data centers. Here, multiple homogeneous computing nodes with uniformly distributed data partitions are connected by reliable networks with Gigabytes of bandwidth. However, FL is often applied to achieve collaborative and privacy-preserving ML in wide-area networks, where geographically distributed clients with naturally generated data collaborate to train ML models over bandwidth-constrained communication channels.

Due to the drastic differences, FL needs to address a variety of specific and complicated issues in optimization. For example, naturally generated data at the FL clients are commonly heterogeneous, i.e., non-balanced and non-i.i.d. distributed, which leads to biased model aggregation. Considering limited communication bandwidth, FL often performs multiple local updates before the global aggregation, further amplifying the model bias caused by data heterogeneity. Furthermore, model dissemination and aggregation give rise to concerns of private information leakage and falsification. The decentralized communication architecture also leads to partially local approximations of global objective functions. These issues pose new challenges in optimization in FL, including problem formulation, algorithm design, and convergence analysis. In particular, typical challenges are summarized as follows:

- **Biased local objectives from non-i.i.d. datasets.** The potentially unknown and non-i.i.d. data distributions among distributed nodes* could result in biased local objectives [5, 18]. Thus, the global objective function in FL optimization cannot be decomposed into trainable local objectives without bias. This means that the gradients of local models may largely deviate from the steepest descent direction of the global objective, leading to significant degradation of convergence speed and model accuracy [19].
- **Perturbed gradients with DP noises.** The Differential Privacy (DP) mechanisms [20] are commonly adopted in the FL optimization process to protect exchanged parameters or gradients, by introducing statistically unbiased noises (e.g., Gaussian [21] or Laplacian [22] noises). Despite the statistical unbiasedness, random noises may overwhelm the useful gradient information, causing unstable and slow convergence towards sub-optimal solutions [23–25].
- **Partially approximated objectives under decentralized topologies.** In FL systems with decentralized typologies, the model exchange has to rely on limited peer-to-peer neighborhood communication. The global objective for each data owner is thus partially approximated by consolidating the local objectives of their adjacent neighbors [26]. Consequently, the aggregated gradient in each step is also a partial approximation to that of the global objective. The approximation error can lead to significantly slower convergence [3, 27], especially in non-i.i.d. scenarios.

*The terms "node", "client" and "data owner" are used interchangeably within this survey.

- **Obsolete solutions in online settings.** Online FL optimization aims to learn a series of global functions with minimized cumulative losses from distributed sequential data [28–31]. However, the coming data may have a time-evolving distribution, resulting in inevitable generalization errors and instability of models on new data. Meanwhile, the inherent time-varying and cumulative constraints, e.g. communication, computing, and memory capabilities, may also complicate the problem modeling and solving [32].

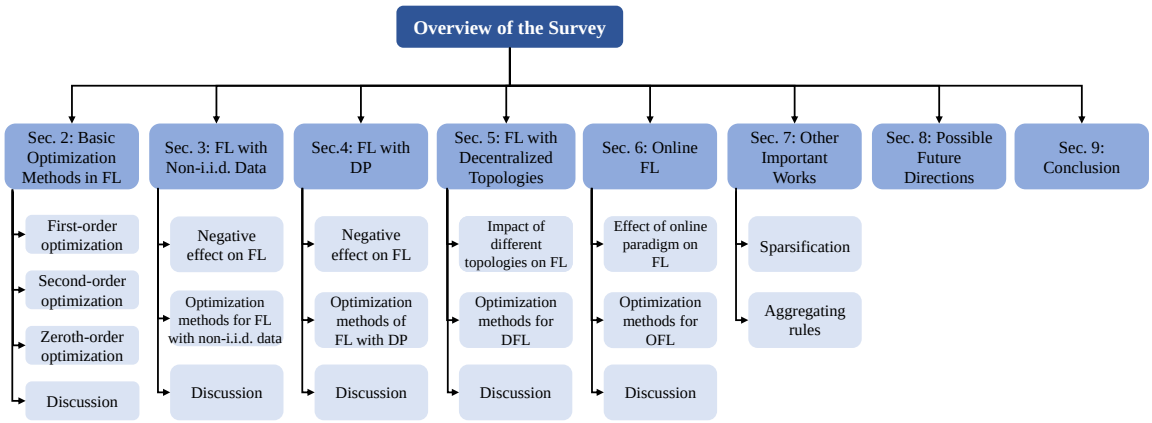


Figure 1: The overview of the survey.

This paper presents a systematic survey of the mathematical optimization in FL, summarizing the assumptions, problem formulations, optimization methods, and theoretical results. It is worth noting that although there exist several FL surveys [5, 33–37], all of them are presented from the perspective of information sciences, including the FL architectures [33], algorithms [34], and applications [38], rather than mathematical optimizations like our survey. Fig. 1 illustrates the overview of this survey. We first review typical optimization methods in FL in Section 2. The optimization challenges and solutions when dealing with non-i.i.d. data, DP noises, decentralized network topologies, and online FL optimizations are presented in Sections 3-6, respectively. We then summarize other important works in Section 7 and discuss possible future directions in Section 8. Finally, we conclude the survey in Section 9.

2 Basic optimization methods in Federated Learning

The common FL optimization is mostly studied in the context of horizontal FL (HFL) where participants possess the same feature space but different samples [5, 39]. We will

Algorithm 1 Algorithmic procedures of FL optimization

Input: Client number N, objective function $f(x)=\frac{1}{N}\sum_{i=1}^{N} f_i(x)$, initialized x_0, total communication rounds T, local learning rate η_l, global learning rate η_g, number of local updates E in one round, distributed dataset $\{\xi_1,\ldots,\xi_N\}$.

Output: x^{T+1}.

1: **for** $t\in\{1,\ldots,T\}$ **do**
2: Server samples a subset S^t of K clients and server sends x^t to these clients;
3: **for** each client $i\in S^t$ **do**
4: x_i^{t+1}, ∇_i^t=**LocalOPT**(x^t,η_l,E,ξ_i);
5: **end for**
6: Server aggregates $\nabla^t=\frac{1}{|S^t|}\sum_{i\in S^t}\nabla_i^t$;
7: Update global model $x^{t+1}=x^t+\eta_g\nabla^t$.
8: **end for**

use HFL as the default FL setting † to introduce the general workflow of FL optimization.

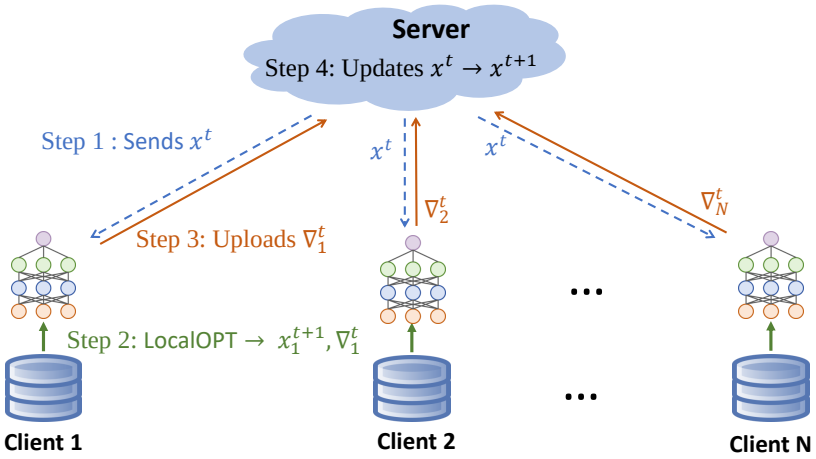


Figure 2: General framework and workflow of FL.

As shown in Algorithm 1 and Fig. 2, a typical FL system consists of multiple distributed clients and one central server. At each iteration round t, the server randomly samples a subset S^t from N clients and broadcasts the current global model $x^t\in\mathbb{R}^d$ to these clients. Each client indexed by i in S^t then executes LocalOPT upon receiving x^t, which involves applying an optimization method to minimize their respective local objectives, subsequently obtaining a local model x_i^{t+1} and uploading the local gradient ∇_i^t to the server. Once all gradients from S^t are received, the server aggregates all local

†Another important category of FL is vertical FL (VFL), where participants share the same sample space but different features. More details can be referred to in [40–42].

gradients to obtain the global gradient ∇^t and accordingly updates the global model as $x^{t+1}=x^t+\eta_g\nabla^t$, where η_g is the global learning rate.

Given its pivotal role in local data information extraction, the choice of LocalOPT would significantly affect the performance of FL training process. Since LocalOPT essentially executes machine learning (ML) tasks in each round on the client side, its candidates encompass all ML optimization methods, including the first-order, second-order, and zeroth-order ones. Each type of these optimization methods exhibits unique convergence characteristics when being adapted to FL settings, which are summarized in Table 1. The following are common assumptions used in the convergence analysis of FL optimization algorithms [11, 43, 44].

- **Lipschitz Objective Function (LOF):** $f(x)$ is β-Lipschitz continuous if there exists $\beta\geq 0$ such that for all $x_1,x_2\in\mathbb{R}^d$,

$$|f(x_1)-f(x_2)|\leq\beta\|x_1-x_2\|. \tag{2.1}$$

- **Smooth Objective Function (SOF):** $f(x)$ is L-smooth if $f(x)$ has L-Lipschitz continuous gradient, i.e., for all $x_1,x_2\in\mathbb{R}^d$,

$$\|\nabla f(x_1)-\nabla f(x_2)\|\leq L\|x_1-x_2\|. \tag{2.2}$$

  When the second-order gradients of $f(x)$ is Lipschitz continuous, we call the condition **Lipschitz Hessian (LH)**.

- **Strongly Convex Objective Function (SCOF):** $f(x)$ is μ-strongly convex if there exists $\mu\geq 0$ such that for all $x_1,x_2\in\mathbb{R}^d$,

$$f(x_1)\geq f(x_2)+(x_1-x_2)^T\nabla f(x_2)+\frac{\mu}{2}\|x_1-x_2\|_2^2. \tag{2.3}$$

- **Convex Objective Function (COF):** $f(x)$ is convex if for all $x_1,x_2\in\mathbb{R}^d$, it holds that

$$f(x_1)\geq f(x_2)+(x_1-x_2)^T\nabla f(x_2). \tag{2.4}$$

- **Coercive Function (CF):** $f(x)$ is coercive if $\lim_{\|x\|\to\infty}f(x)\to\infty$.

- **Bounded Gradient (BG):** The gradient of $f(x)$ is G-bounded if there exists $G\geq 0$ such that for all $x\in\mathbb{R}^d$, $\|\nabla f(x)\|\leq G$.

- **Bounded Variance (BV):** The variance of each stochastic gradient $\nabla f_i(x;\xi)$ is bounded if there exists $\sigma\in\mathbb{R}$, such that

$$\mathbb{E}_\xi\|\nabla f_i(x;\xi)-\nabla f_i(x)\|^2\leq\sigma^2, \tag{2.5}$$

  where $f_i(\cdot)$ denotes the local objective function of the i-th client, x is the current model parameter and ξ is the data sampled in the current round of local training.

- **Bounded Gradient Dissimilarity (BGD):** Local gradients $\{\nabla f_i(x), \forall i\}$ satisfy (G,B)-bounded gradient dissimilarity if there exist constants $G \geq 0$ and $B \geq 1$ such that

$$\frac{1}{N}\sum_{i=1}^{N}\|\nabla f_i(x)\|^2 \leq G^2 + B^2\|\nabla f(x)\|^2, \quad \forall x \in \mathbb{R}^d. \tag{2.6}$$

In above assumptions, LOF, SOF, and LH describe the smoothness of the objective function. SCOF and COF characterize the convexity of objective functions. CF ensures that the objective function has a global minimum. BG, BV, and BGD capture the properties of gradients.

Table 1: Comparison of basic optimization methods in FL.

Categories	Methods	Assumptions	Convergence rates
First-order Methods	SGD (e.g.,[45, 46])	SOF, BV, BG	$\mathcal{O}(\frac{\sigma}{\sqrt{KT}})$
	Momentum-based SGD (e.g., [47, 48])	SOF, BV	$\mathcal{O}(\frac{\sigma}{\sqrt{KT}})$
	Adam (e.g., [44, 49])	SOF, BV, BG	$\mathcal{O}(\frac{\sigma}{\sqrt{KT}})$
	ADMM (e.g., [50–52])	SOF, CF	$\mathcal{O}(\frac{1}{T})$
Second-order Methods	Newton (e.g., [53–56])	SOF, LH, SCOF	$\mathcal{O}(\gamma^{-2^T})(\gamma > 1)$
Zeroth-order Methods	ZO-SGD (e.g., [57–59])	LOF	$\mathcal{O}(\sigma\sqrt{\frac{d}{KT}})$

2.1 First-order optimization methods

First-order optimization methods [60] rely only on first-order gradients for the model updating in LocalOPT. They are widely adopted in FL due to their lower computational requirements for gradient estimation, especially in settings with pronounced computational heterogeneity. Here, we introduce several representative first-order optimization methods in FL.

The most common choice for the first-order method is Stochastic Gradient Descent (SGD) [19, 27, 39, 61–63]. SGD works by iteratively updating the model as

$$x^{t+1} = x^t - \eta \nabla f(x^t; \xi^t), \tag{2.7}$$

starting from an arbitrary point x^0, where $\nabla f(x^t;\xi^t)$ is the stochastic gradient at x^t estimated on ξ^t. Here, ξ^t can be a single data, or a minibatch of data uniformly sampled from

the whole training data at random. FedAvg [39] is an extensively employed adaptation of SGD to FL settings. In FedAvg, the LocalOPT in Algorithm 1 adopts SGD and the server takes the weighted average of local gradients to update the global model. It has been demonstrated that FedAvg can converge at a sub-linear rate of $\mathcal{O}(\frac{\sigma}{\sqrt{KT}})$ when using a decaying learning rate [45, 46].

Despite the statistically optimal convergence rate, SGD suffers from the problem of converging to saddle points and difficulty of adjusting its learning rate. These drawbacks are also inherited by FedAvg. Momentum-based SGD methods [47, 64–67] can accelerate model convergence and escape saddle points by incorporating historical gradient information into the current gradient. [48] has shown that introducing Nesterov's method into FL leads to the same theoretical convergence rate as FedAvg. However, when the learning rate satisfies mild conditions, momentum-based SGD achieves faster actual convergence speed. Building upon this, a recent study [44] has further integrated typical methods for adaptive learning rate into momentum-based SGD to achieve higher convergence speed. It has been demonstrated that integrating Adam [68] into FL to optimize the global learning rate η_g can achieve the convergence rate of $\mathcal{O}(\frac{\sigma}{\sqrt{KT}})$ in non-i.i.d. scenarios, matching the rate of FedAvg. Variance reduction methods [19, 69–71] can also be integrated into federated frameworks to reduce the gradient variance term σ in FedAvg's convergence rate. Additionally, the Alternating Direction Method of Multipliers (ADMM) algorithm [50–52] can be used to handle constrained federated optimization problems. Research [50] has shown that ADMM achieves a convergence rate of $\mathcal{O}(\frac{1}{T})$ when dealing with topology-constrained federated optimization problems in decentralized FL.

2.2 Second-order optimization

Second-order optimization methods [72, 73] invest additional computational resources to compute the Hessian matrix [74], thereby offering a detailed representation of the local optimization landscape. Despite the increased computation and communication overhead, they can deliver a more precise optimization path and often surpass first-order methods with faster convergence speed. The mainstream second-order algorithm in FL is the Newton's method [53–56], which typically updates the model parameters by combining the Hessian matrix (e.g.,$\nabla^2 f(x^t)$) and the first-order gradient information:

$$x^{t+1} = x^t - \nabla^2 f(x^t)^{-1} \nabla f(x^t). \tag{2.8}$$

Considering the aggregation process in FL, i.e., $\nabla f(x^t) = \frac{1}{N}\sum_{i=1}^{N} \nabla f_i(x^t)$, the server updates the global model by

$$x^{t+1} = x^t - \Big(\frac{1}{N}\sum_{i=1}^{N} \nabla^2 f_i(x^t)\Big)^{-1} \Big(\frac{1}{N}\sum_{i=1}^{N} \nabla f_i(x^t)\Big). \tag{2.9}$$

However, as mentioned, the adaptation of Newton's method to FL suffers from heavy communication and computation of the Hessian matrix. To relieve the burden, one intu-

itive approach is to use advanced compression methods such as the top-k selection technique [54]. Furthermore, to avoid computing the inverse of the Hessian matrix, the server can leverage ADMM to approximate Hessian inverse-gradient product $\nabla^2 f(x^t)^{-1}\nabla f(x^t)$ (also called global Newton direction) [53]. Some other iterative methods, like Richardson iteration [55], can also be applied to get local Newton direction, i.e., $\left(\nabla^2 f_i(x^t)\right)^{-1}\nabla f_i(x^t)$, which is regarded as a solution of a linear system. It has been proved that FL with these adapted Newton's methods can converge at the quadratic speed as centralized Newton's methods [53, 55].

2.3 Zeroth-order optimization

The gradient (including the first-order and second-order) information may be inaccessible or computationally prohibitive in some scenarios, e.g., black box models [75] and reinforcement learning [76, 77]. Zeroth-order (ZO) optimization (also called derivative-free optimization) methods [78, 79] works for such scenarios by utilizing the zeroth-order information, i.e., the variation of objective function value along some directions over a mini-batch of data samples. They can naturally be adopted in FL and named as federated ZO [57–59]. Particularly, in federated ZO, the gradients can be approximated by the variation of objective function value as

$$\widetilde{\nabla} f_i\left(x_i^t;\{\xi_{i,m}^t\}_{m=1}^{b_1},\{v_{i,l}^t\}_{l=1}^{b_2},\mu\right)=\frac{1}{b_1 b_2}\sum_{m=1}^{b_1}\sum_{l=1}^{b_2}\frac{dv_{i,l}^t}{\mu}(f_i(x_i^t+\mu v_{i,l}^t;\xi_{i,m}^t)-f_i(x_i^t;\xi_{i,m}^t)), \quad (2.10)$$

where $\{\xi_{i,m}^t\}_{m=1}^{b_1}$ is a set of i.i.d. random samples, $\{v_{i,l}^t\}_{l=1}^{b_2}$ is a set of i.i.d. random direction vectors (sampling from the d-dimensional uniform distribution), and μ is a positive step size. Then the local model update in LocalOPT of federated ZO algorithms can be written as

$$x_i^{t+1}=x_i^t-\eta_l\widetilde{\nabla} f_i\left(x_i^t;\{\xi_{i,m}^t\}_{m=1}^{b_1},\{v_{i,l}^t\}_{l=1}^{b_2},\mu\right). \quad (2.11)$$

Although federated ZO algorithms adopt a similar framework to FedAvg, the convergence analysis of federated ZO algorithms is slightly different from that of FedAvg. That is because that the gradient estimator of ZO algorithms does not preserve the unbiasedness of stochastic gradients, i.e., $\widetilde{\nabla} f_i\left(x_i^t;\{\xi_{i,m}^t\}_{m=1}^{b_1},\{v_{i,l}^t\}_{l=1}^{b_2},\mu\right)$ is a biased approximation to the real gradient $\nabla f_i(x_i^t)$ [57, 79]. Therefore, in the analysis of federated ZO algorithms, the gradient estimator is usually decomposed into two components, i.e., the difference between the real gradient and its expected estimator, the divergence between the expected and its ZO-approximated estimator [80]. It has been shown that federated ZO algorithms can achieve sub-linear convergence under the assumptions of non-smooth convex loss functions in both i.i.d. and non-i.i.d. settings [58]. Notably, under the assumption of L-smooth local objective functions, federated ZO algorithms can also achieve sub-linear convergence in the non-convex setting [59].

2.4 Discussion

In federated optimization, one criterion for selecting appropriate optimization methods is the trade-off between iteration complexity and communication cost [81]. A larger volume of communication data, such as the Hessian matrix, can reduce the total number of iterations, leading to faster model convergence. On the other hand, first-order methods with higher iteration complexity, often incur lower communication costs. Zeroth-order methods are primarily employed in specific FL scenarios where gradients are unavailable, showing no significant advantage in communication or iteration complexity.

3 Federated Learning with Non-i.i.d. Data

Table 2: Non-i.i.d. classification based on different feature and label distributions [5].

Non-i.i.d. categories	**Probability distributions**
Feature distribution skew	$\mathcal{P}_i(y\|x)=\mathcal{P}_j(y\|x)$, $\mathcal{P}_i(x)\neq\mathcal{P}_j(x)$.
Label distribution skew	$\mathcal{P}_i(x\|y)=\mathcal{P}_j(x\|y)$, $\mathcal{P}_i(y)\neq\mathcal{P}_j(y)$.
Varying features under one label	$\mathcal{P}_i(y)=\mathcal{P}_j(y)$, $\mathcal{P}_i(x\|y)\neq\mathcal{P}_j(x\|y)$.
Identical features with different labels	$\mathcal{P}_i(x)=\mathcal{P}_j(x)$, $\mathcal{P}_i(y\|x)\neq\mathcal{P}_j(y\|x)$.
Data imbalance	Data amount significantly varies across clients.

In FL, data is generated naturally and stored locally, giving rise to data heterogeneity, i.e., non-independent and identically distributed (non-i.i.d.) data across diverse clients. Formally, assume any data sample (x,y) of the i-th client is drawn from a distribution $\mathcal{P}_i(x,y)$, where x and y denote the feature vector and label respectively. $\mathcal{P}_i(x,y)$ can be rewritten as

$$\mathcal{P}_i(x,y)=\mathcal{P}_i(y|x)\mathcal{P}_i(x)=\mathcal{P}_i(x|y)\mathcal{P}_i(y),\quad i\in[N]. \tag{3.1}$$

Then the non-i.i.d. settings in FL can be classified into five categories according to different feature and label distributions [5, 18], which are summarized in Table 2.

3.1 Negative effect of non-i.i.d. data on FL

In real-world FL tasks, the data heterogeneity comes from the complicated mixture of the above five non-i.i.d. issues. Regardless of the category, data heterogeneity inherently leads to the decomposition of the global gradient into multiple biased local gradients over non-i.i.d datasets. The biases in local gradients then compromise the convergence rate and error bound of the global model through the aggregation process [19, 62]. In the following, we will present some theoretical results as well as an intuitive explanation for the performance differences in model convergence between i.i.d. and non-i.i.d. scenarios.

3.1.1 Theoretical results on FL with non-i.i.d. data

In the following, we highlight the degraded model convergence by comparing the theoretical convergence rates and error bounds in both i.i.d and non-i.i.d. settings.

- **Faster convergence to the first-order stationary point in i.i.d. settings.** Under the assumption of the smooth objective function, the convergence results are as follows:
  - **For strongly convex objectives**, traditional distributed algorithms such as local SGD [82] demonstrate a convergence rate of $\mathcal{O}\left(\frac{1}{T}\right)$ towards the optimal point [82, 83].
  - **For non-convex objectives**, the model may get stuck in an extreme point or a saddle point, and require more iterations to escape these points [84]. Therefore, traditional distributed algorithms converge towards the first-order stationary point at a sub-optimal convergence rate of $\mathcal{O}\left(\frac{1}{\sqrt{T}}\right)$ [85].
- **Slower convergence to a neighborhood of the first-order stationary point in non-i.i.d. settings.** In the theoretical analysis, the non-i.i.d. issue is always characterized by the assumption of (G,B)-bounded gradient dissimilarity [19]. By further imposing the smoothness assumption on the objective function, we can obtain the following analytical convergence results:
  - **For strongly convex objectives**, the error bound of FedAvg is $\mathcal{O}\left(\frac{1}{\mu T}+\frac{G^2}{\mu T}+\frac{B^2}{\mu}\right)$, where μ is a coefficient to describe the convexity [19]. This implies that although FedAvg also achieves a convergence rate of $\mathcal{O}(\frac{1}{T})$, non-i.i.d. data distributions still result in a reduced convergence rate and an increased convergence error due to the existence of terms $\frac{1+G^2}{\mu T}$ and $\frac{B^2}{\mu}$ respectively.
  - **For non-convex objectives**, FedAvg exhibits a sub-linear convergence rate of $\mathcal{O}\left(\frac{1+G^2}{\sqrt{T}}+\frac{B^2}{T}\right)$ [19].

3.1.2 Intuitive explanations

Here we provide an intuitive understanding of the negative impact of non-i.i.d. data on the model convergence by comparing the model updating trajectories in the FL optimization process in both i.i.d. and non-i.i.d. settings.

- **Faster convergence to the first-order stationary point in i.i.d. settings.** In ideal i.i.d. settings, as illustrated in Figure 3 (a), the proximity between client optima and global optimum ensures unbiased model updates and consistent convergence under the identical initial models. Consequently, the global model achieves a fast convergence to the first-order stationary point.

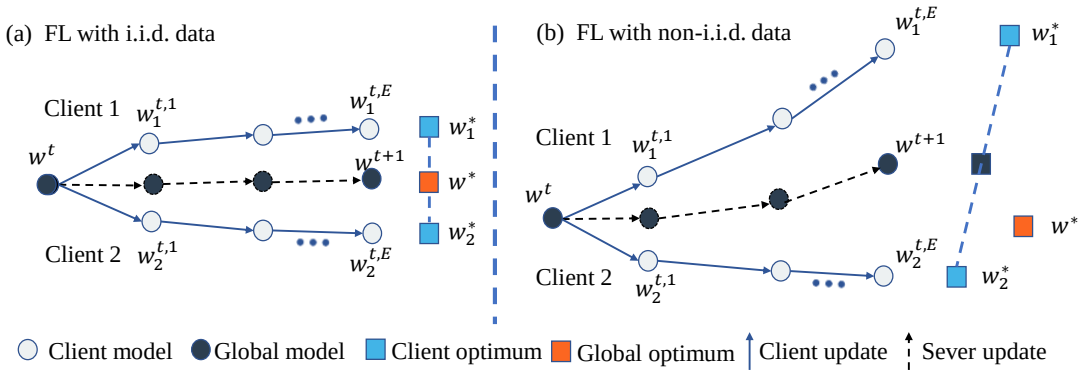


Figure 3: Illustration of model update trajectories under i.i.d. (a) and non-i.i.d. (b) settings in FL for two clients with E local iterations [19].

- **Slower convergence to a neighborhood of the first-order stationary point in non-i.i.d. settings.** In practical non-i.i.d. settings, as illustrated in Figure 3 (b), due to disparities among multiple client optima, FL algorithms such as FedAvg exhibit a decreased convergence rate and increased error bound [19]. On the client side, the local model overfits the local data with biased distribution, causing inevitable biases in gradients [62]. This can be characterized as

$$\mathbb{E}[\nabla f_i(x)] \neq \nabla f(x), \quad i \in [N]. \tag{3.2}$$

  On the server side, the biases cannot be eliminated in the model aggregation and updating process due to the inaccessibility to local data distributions [62]. After multiple training rounds, the biases accumulate, which causes the global model to update along a largely biased direction [86]. Eventually, the non-i.i.d. data significantly degrades the model utility [87].

3.2 Optimization methods for FL with non-i.i.d. data

The above analysis and explanations show the routine along which the biases in data propagate to the model. Existing methodologies [63, 88–90] aim to mitigate the non-i.i.d. issue by correcting the biases in different components along the routine. They can be summarized as follows.

- **Regularization** [63, 89–92] methods manipulate the objective function to constrain the parameter space, ensuring that the optimal parameters across different clients closely resemble one another. Specifically, regularization methods try to constrain the biases by introducing a regularization term related to the distance between the local and global model parameters. The empirical loss function for the i-th client is then represented as

$$\min_x F_i(x, x^{t-1}) := f_i(x) + \alpha l_{\text{reg}}(x, x^{t-1}), \tag{3.3}$$

where α is the regularization parameter and $l_{\text{reg}}(x, x^{t-1})$ is the regularization term, e.g., the L_2-regularization term $\|x-x^{t-1}\|^2$ [63]. The regularization term enforces the local model to remain in a limited neighboring region around the global model and reduces the model deviation in the FL optimization process. It has been proved that the loss reduction in each round is lower bounded by $\rho^t\|\nabla f(x^t)\|^2$ under assumptions of the smooth and non-convex loss function [63], where

$$\rho = \mathcal{O}\left(\frac{1-\gamma}{\alpha} - \frac{\gamma(1+\alpha)}{(\alpha-\theta)\alpha} - \frac{\gamma^2}{(\alpha-\theta)^2}\right), \tag{3.4}$$

γ is a parameter positively correlated with the level of data heterogeneity and θ is a constant. This theoretical result indicates that the convergence accelerates with the increase of α due to the constrained model biases. However, when α exceeds a certain threshold, local models tend to align closely with the received global model and stop learning from local data, thereby slowing down the convergence. By selecting an appropriate α, regularization-based algorithms can achieve a convergence rate of $\mathcal{O}(\frac{1}{T})$ [89].

- **Model interpolation** [91, 93, 94] combines the biased local model and global model to reduce the distances among different local models, improving the trade-off between model personalization and generalization. Specifically, each client trains a composite model $\lambda_i x_i + (1-\lambda_i)x$ in each round, where λ_i is the interpolated weight for the i-th client. Interpolation facilitates the alignment of each client's model towards the global average model, thereby fostering consistency across different local models throughout the convergence process. Theoretical analysis [91] reveals that the interpolation methods can affect the generalization error bound of each local model x_i through a scaling term $\mathcal{O}\left(\sqrt{\frac{\lambda_i^2}{n_i} + \frac{(1-\lambda_i)^2}{n}}\right)$, where n_i and n represent the number of local data and global data, respectively. The generalization error bound scales markedly without interpolation (i.e., λ_i equals 0 or 1). Instead, by setting an appropriate λ, model interpolation may significantly reduce the generalization error bound.

- **Adaptive optimizer** [44] enhances the exploration capability of the optimizer (e.g., SGD in FedAvg) by adaptively adjusting the learning rates of both local and global models, thereby reducing the biases. This is motivated by the theoretical analysis that FedAvg with a fixed learning rate cannot minimize the convergence error incurred by the non-i.i.d. data [44]. The basic idea is to incorporate advanced adaptive optimizers, e.g., Adagrad [95], Yogi [96], and Adam, into the FedAvg framework. Taking Adam as an example, the local and global learning rate is set as $\mathcal{O}(\frac{1}{L\sqrt{T}})$ and $\eta_g = \mathcal{O}(\frac{\sqrt{K}}{\sqrt{v_t}+\frac{G}{L}})$ respectively, and the global updating rule is

$$x^{t+1} = x^t + \eta_g m^t, \tag{3.5}$$

where m^t denotes the momentum and v^t is a time-varying term that combines the historical and current gradient information. Then the convergence rate of the global model in non-i.i.d. settings can reach $\mathcal{O}(\frac{1}{\sqrt{KT}})$, under the assumptions of non-convex and L-smooth objective function and G-bounded gradients. This rate matches the standard convergence rate in i.i.d. settings [97].

- **Variance reduction** [19, 69–71] methods reduce the variance of local gradients among clients, enhancing the consistency among each other. They are motivated by some theoretical results that a larger gradient variance induces decreased convergence rate and increased convergence error [19]. Variance reduction is always accomplished by fusing the unbiased global gradient with each local biased gradient. In particular, in each round of local training, each client combines historical unbiased global gradients and newly calculated local gradients to yield the local update [70, 71]. The local updating rule for the i-th client is:

$$x_i^{t+1} = x_i^t - \eta_l \left(\nabla f_i\left(x^t\right) - \nabla f_i\left(x^{t-1}\right) + \frac{1}{N}\sum_{i=1}^{N} \nabla f_i\left(x^{t-1}\right) \right). \tag{3.6}$$

  Theoretical results reveal that applying variance reduction methods in non-i.i.d. settings can achieve a similar convergence performance to i.i.d. settings. That is, the convergence rate of $\mathcal{O}\left(\frac{1}{T}\right)$ for strongly convex objective functions and a sublinear rate of $\mathcal{O}\left(\frac{1}{\sqrt{T}}\right)$ for non-convex objective functions [19].

- **Momentum** [64, 98, 99] utilizes both the historical information and current model to correct the update direction, thereby alleviating the bias induced by non-i.i.d. data. Specifically, this optimization method aggregates the last round gradients and the current update by assigning an appropriate weight. Existing research [64, 98] has demonstrated that momentum can alleviate client drift and accelerate the convergence of FL. This is attributed to the fact that the accumulated historical information can enhance the representativeness of data from multiple clients and mitigate the inherent biases stemming from non-i.i.d. data. Specifically, the momentum method can achieve a convergence rate of $\mathcal{O}(\frac{1}{\sqrt{NET}} + \frac{1}{T})$ [98, 99] under the assumptions of L-smooth functions.

- **Discrepancy-aware aggregation** methods [100] allocate aggregation weights that exhibit a negative correlation with local discrepancy levels. These methods are driven by the theoretical analysis of the expected gradient error. It suggests that, to minimize the upper bound of gradient error in FedAvg, aggregation weights should demonstrate a negative correlation with local discrepancy levels while being positively correlated with dataset size. In the implementation, each client first computes its local discrepancy using the local category distribution. Then, the server assigns distinctive aggregation weights based on these discrepancy values.

Table 3: Summary of optimization methods for FL with non-i.i.d. data.

Optimized components	**Methods**	**Advantages**	**Disadvantages**
Objective function	Regularization	Easy deployment	Difficulty in selecting appropriate α
Model	Model interpolation	Balance between generalization and personalization	High computation complexity and memory consumption
	Discrepancy-aware aggregation	Comprehensive theoretical foundation	Difficulty in selecting weight-related hyper-parameters
Optimizer	Adaptive optimizers	Enhanced model exploration capability	High computation complexity and memory consumption
Gradients	Momentum	Enhanced model convergence	High memory consumption
	Variance reduction	Tight analysis in non-i.i.d. settings	High communication cost and memory consumption

3.3 Discussion

As summarized in Table 3, the regularization methods are convenient to deploy since they can be achieved by simply manipulating the objective function. However, it is challenging to select an appropriate regularization parameter α in prior. Model interpolation methods can strike a balance between generalization and personalization, but require additional computational resources to learn the interpolation parameter λ and extra memory to store the global model for each client. Discrepancy-aware aggregation stems from a rigorous theoretical analysis of optimizing error bounds and has a tight theoretical analysis. However, the model performance relies on the properly selected metric for local discrepancy levels and hyper-parameters for discrepancy-aware aggregation weights. Adaptive optimizers enhance model exploration but introduce complexity, risking bias-variance dilemmas and demanding more computational and memory resources. Momentum methods improve the convergence via utilizing the historical information, which however requires additional memory consumption for both clients and the server. Variance reduction methods offer rigorous theoretical guarantees in non-i.i.d. scenarios. However, they necessitate additional communication costs and memory consumption for transmitting and storing historical gradients.

4 Federated Learning with Differential Privacy

Differential Privacy (DP) provides a mathematical framework to formulate and control privacy loss in FL [23, 25]. Despite no raw data exchange, the communicated models in FL may still disclose sensitive information about training data. By incorporating the DP constraint [101–103], the FL optimization can be regularized to converge to a model with limited information disclosure.

Formally, let $\mathcal{M}$ be a differentially private FL (DP-FL) algorithm that takes the distributed dataset $\mathcal{D}=\{\mathcal{D}_1,\ldots,\mathcal{D}_N\}$ as input and outputs the well-trained model x. Then $\mathcal{M}$ satisfies (ε,δ)-differential privacy if it holds that

$$Pr[\mathcal{M}(\mathcal{D})=x]\leq e^{\varepsilon}Pr[\mathcal{M}(\mathcal{D}')=x]+\delta, \tag{4.1}$$

where $\mathcal{D}'$ is a virtual dataset which differs in $\mathcal{D}$ with only one record. According to the different granularity of $\mathcal{D}'$, the privacy guarantee can be classified into *sample-level* (i.e., $\mathcal{D}'$ differs from $\mathcal{D}$ in a single sample [104]) and *client-level* (i.e., $\mathcal{D}'$ differs from $\mathcal{D}$ in a client's dataset [104]).

Besides the granularity of privacy protection, it is also necessary to consider different adversary/threat models. According to whether the server is trusted or not, there are two main DP models, namely centralized DP (CDP) and local DP (LDP) [105–107]. In CDP, the server is assumed trustworthy, while the threat comes from external malicious analysts or clients. These adversaries may have access to global models from the server to infer the sample or client-specific information. To address this, the server introduces noise into the global model to safeguard privacy in FL with CDP. In contrast, in LDP, the server is assumed honest-but-curious, meaning it follows the protocol but may attempt to infer the information of training samples via the received intermediate results from clients. In this case, FL clients have to locally perturb their gradients or models to defend against this threat in FL with LDP [107–110]. Note that, with the same privacy parameter ϵ, LDP provides stronger privacy protection than CDP, but causes larger utility loss. To improve the privacy-utility tradeoff, distributed DP (DDP) [111] integrated with secure shuffling [25] or secure aggregation [112, 113] has also been proposed, in which the local model updates are perturbed with a smaller noise and then shuffled or securely aggregated to achieve a sufficient CDP noise.

We use the sample-level DP as the default setting to illustrate the general workflow of FL with DP. As shown in Fig. 4, each client first conducts local training and then clips the local gradients $\nabla f_i(x^t)$ with a positive constant c:

$$\overline{\nabla f_i(x^t)}=\nabla f_i(x^t)\cdot\min\left\{1,\frac{c}{\|\nabla f_i(x^t)\|_2}\right\}. \tag{4.2}$$

Gradients clipping [21] is a common method to limit the gradients' *sensitivity*, i.e., the change of the gradient norm due to adding or deleting an individual sample. Afterward, a certain level of DP can be achieved by injecting carefully calibrated noises (e.g.,

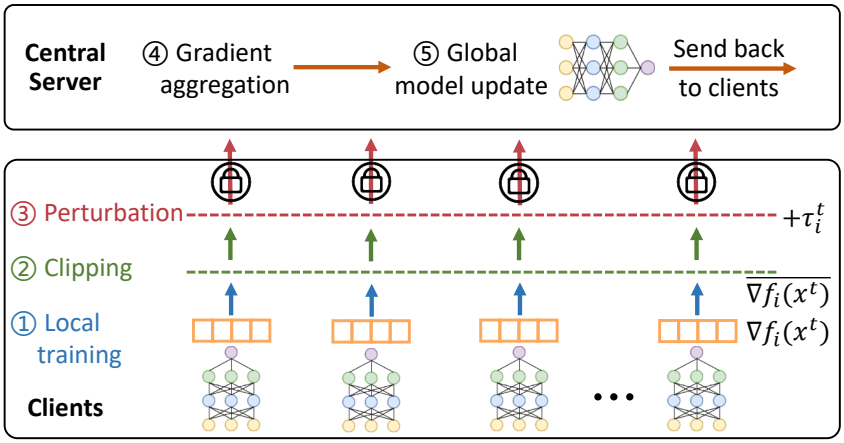


Figure 4: Typical workflow of FL with DP.

Gaussian [21] or Laplacian [22] noises) into the clipped gradients. Taking the most commonly used Gaussian mechanism as an example, the scale of Gaussian noise is specified as $\sigma^2 = \frac{2c^2\log(1.25/\delta)}{\epsilon^2}$ to achieve $(\epsilon,\delta)-$DP [21]. Finally, the perturbed gradient $\overline{\nabla f_i(x^t)}+\tau_i^t$ is sent to the central server for updating the global model [104].

4.1 Negative effect of DP on FL optimization

As discussed, realizing DP in FL involves the operations of gradient clipping and noise addition to the vanilla FL algorithms. While guaranteeing DP, these operations inevitably lead to a reduced convergence rate and increased error bound, reflecting the privacy-utility trade-off [24, 25]. Specifically, their major impacts on the model convergence can be summarized as follows:

- **Decreased convergence rate.** Both gradient clipping and noise addition introduce addition error, thus impacting the convergence of FL algorithms [23–25]. As analyzed in [25], the convergence rate of DP-enhanced FL can be represented as

$$\mathcal{O}\left(\frac{\log(T)\max\{d^{\frac{1}{2}-\frac{1}{p}},1\}}{\sqrt{T}}\sqrt{\frac{cd}{q}\left(\frac{e^\epsilon+1}{e^\epsilon-1}\right)}\right), \tag{4.3}$$

  where p represents the norm exponent adopted to clip the gradient, q is the data sampling ratio in a mini-batch, and d is the dimension of model parameters. Specifically, it is This theoretical result indicates that a larger sampling ratio q and privacy budget ϵ can speed up the convergence, while the larger gradient dimension d and clipping bound c can slow down the convergence.

- **Increased error bound.** Besides the convergence rate, DP also harms the final model utility. By achieving DP in FedAvg, an error bound of the trained model [23]

can be derived as follows with a small clipping bound c.

$$\frac{1}{T}\sum_{t=1}^{T}\mathbb{E}\left[\alpha^t\left\|\nabla f\left(x^t\right)\right\|^2\right]\leq\mathcal{O}\left(\frac{1}{\sqrt{KET}}\right)+\mathcal{O}(c^2E)+\mathcal{O}(\frac{d\sigma^2}{EK}), \tag{4.4}$$

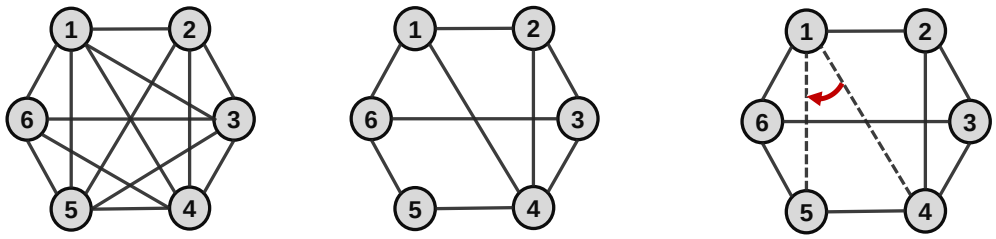
where α is a constant, K is the number of participant clients, and σ^2 and d represent the noise scale and the dimension respectively. The term $\mathcal{O}(c^2E)$ in the right side of Eq. (4.4) captures the impact of clipping, wherein a larger clipping bound c indicates a higher error bound. Similarly, the last term $\mathcal{O}(\frac{d\sigma^2}{EP})$ demonstrates that the error bound increases linearly with the dimension d and noise scale σ^2.

4.2 Optimization methods of FL with DP

Section 4.1 reveals that an additional DP guarantee compromises the model utility in terms of both error bound and convergence rate. Recent studies [114, 115] propose several strategies to improve the model utility in FL without sacrificing the DP guarantee, including the clipping bound calibration to reduce sensitivity [116], sampling for amplifying the privacy preservation [25], and gradient sparsification to enhance the siginal-noise-ratio [115].

- **Sensitivity reduction by fine-grained clipping.** As analyzed in Section 4.1, a small clipping bound can effectively improve the model utility through limiting the added noises. However, an undersized clipping bound c can lead to severe biases in the aggregated results [116, 117], also diminishing the model utility. Therefore, the clipping bound should be calibrated carefully. To this end, recent study [116] empirically observes the reduction of gradient norms with increasing T and accordingly proposes an adaptive norm-aware clipping algorithm. The fine-grained clipping algorithm effectively reduces the aggregation bias and DP noises, achieving a convergence rate of $\mathcal{O}(\frac{1}{T})$ under the assumption of smooth objective function [116].

- **Privacy amplification by sampling.** As a common method, sampling [25, 118], e.g., the client sampling and data sampling in FedAvg, can amplify the privacy protection capability of a randomized FL algorithm by introducing additional randomness. Given ϵ_0 as the privacy budget to protect gradients in each round, the total T rounds of training can be proved to satisfy $(\mathcal{O}(\epsilon_0\sqrt{qT\log(qT)}),\delta)$-DP [25], where q is the sampling ratio. This theoretical result indicates that by setting a smaller sampling ratio, the integration of the sampling method can significantly reduce the DP noises required to perturb the gradients [114], thereby improving the model utility. However, this is conflicted with the analysis that a larger sampling ratio speeds up the convergence, as discussed in Section 4.1. Therefore, we conclude that there exists a trade-off between the model convergence and the sampling ratios [25]. Thus, through appropriate calibration of the sampling ratios, substantial enhancements in model utility can be achieved.



(a) Fully connected topology. (b) Partially connected topology. (c) Time-varying connected topology.

Figure 5: Illustration of network topology for fully connected topology (a), partially connected topology (b), time-varying connected topology (c).

- **Dimension reduction by sparsifying the gradients.** Eq. (4.3) demonstrates that a high dimension d of gradients can degrade the model convergence [119]. A direct solution involves randomly perturbing k out of d dimensions in the model gradients with Gaussian noise while setting the remaining dimensions to zero [120], based on which, the dimension-related terms in the error bound (Eq (4.4)) can then be scaled by $\frac{k}{d}$, thus decreasing the error bound. However, such a random sparsification algorithm introduces significant biases into gradients, thus also degrading the model utility. To this end, recent work [115] first discovers the effect of zeroing out different dimensions of gradients on the model utility, and accordingly proposes to identify and zero out $d-k$ dimensions with the least impact.

4.3 Discussion

The core challenge of FL optimization with DP is to strike a satisfactory balance between privacy protection and model performance (e.g., model errors and convergence rates). Under a certain level of privacy protection, reducing the intensity of DP noises is a core idea to achieve better model performance. Generally, the noise intensity is proportional to both the model sensitivity and the size of communicated data in the whole training process [121, 122]. Given this fact, optimization methods described in Section 4.2 utilize gradient clipping to reduce model sensitivity, and employ gradient sparsification or sampling to reduce the size of communicated data, thereby decreasing the added noise. Additionally, sampling, which introduces extra randomness, further enhances the level of privacy protection. Besides, these methods are complementary and can be combined to achieve a better trade-off between privacy protection and model performance.

5 Federated Learning with Decentralized Topologies

Conventional FL, which is also named server-client FL [123], enables a central server to coordinate the learning task by communicating with multiple distributed clients. However, server-client FL encounters several problems, such as single-point failure [124], communication bottleneck [125], and potential malicious server [126], etc. As an effective alternative, Decentralized FL (DFL) [127–129] can mitigate these problems.

In DFL, each client can only communicate with its neighboring nodes. Formally, the network topology of DFL can be modeled as an adjacent matrix $\mathbf{A} \in \mathbb{R}^{N\times N}$, where $\mathbf{A}_{i,j} = 1/0$ indicates the presence/absence of the communication link between nodes i and j. As shown in Figure 5, the network topology can fall into three categories according to different links among nodes, including fully connected topology [130], partially connected topology [129] and time-varying connected topology [17, 131].

Under fully connected topology, each pair of nodes maintains direct link [130], and the adjacent matrix $\mathbf{A} = \mathbf{E}$, where $\mathbf{E}$ is the all-ones matrix. Under partially connected topology, each node only maintains direct links to a subset of nodes [129] and the adjacent matrix is a time-invariant symmetric $0/1$ matrix. Classical partially connected topologies include ring-structured [132] and clique topologies [133, 134], etc. Under time-varying connected topology, links among nodes are randomly or selectively determined based on specific factors [131] such as resource availability. Here, the adjacent matrix is a time-varying symmetric $0/1$ matrix.

In DFL with the above topologies, due to the lack of a central server and limited communication range, it becomes challenging to obtain global gradient information for each client in each round of aggregation. Several DFL methods have been proposed to mitigate this issue for different underlying topologies. Some typical ones are summarized as follows.

- **Decentralized FedAvg** [126] is an adaptation of FedAvg in server-client FL. It enables each node, e.g., the i-th node, to aggregate neighboring gradients to update its own model x_i. Through multiple local aggregations, x_i can gradually fuse the information from the nodes that is not in $\mathcal{N}_i$. Such information propagation in the decentralized network can facilitate the model consistency and convergence.

- **Cyclic learning**, originally proposed in decentralized and consensus optimizations [135, 136], has been also adapted into DFL [137, 138]. It allows different clients to train a single model sequentially and cyclically in the FL system with ring-structured topology. Specifically, in each round, each client trains the model received from the previous client and then passes the trained model to the next client. Such a process proceeds for multiple rounds. However, cyclic learning may suffer from poor model utility due to *catastrophic forgetting* [139–141].

- **Swarm learning** [131, 142] dynamically elects the leader among nodes to aggregate local models based on the consensus mechanism of blockchain. It combines

the strengths of server-client FL and DFL, eliminating the need for a trusted third party while guaranteeing consistency among local models. It can achieve the same model utility as the server-client FL with additional computation and communication costs in the elected leader in each round.

5.1 Impact of different topologies on FL

Decentralized FL is essentially an extension of the well-established field of decentralized optimization [143, 144]. In traditional decentralized optimization, the influence of topologies has been revealed explicitly [145]. And it is apparent that the connectivity of the graph topology impacts the model performance in DFL [127, 146] as well. However, it encounters new challenges in analyzing the impact of decentralized topologies on DFL due to the distinct characteristics of FL scenarios, such as constrained communication, heterogeneous data, privacy concerns, etc. Specifically, the lower node connectivity coupled with non-i.i.d. dataset, even DP noise, will significantly hinder the information propagation, and degrade the model convergence and generalization performance [26, 27, 146].

- **Decreased convergence rate.** It has been proved that static topology can achieve a convergence rate of $\mathcal{O}(\frac{1}{\sqrt{T}}+\frac{1}{(1-\lambda)^2T^{3/2}})$ [127], where $\lambda\in(0,1)$ is the second largest absolute value of the eigenvalues of the mixing matrix (i.e., maximum-degree matrix and metropolis-hastings matrix [147]) associated with network topology. A smaller λ corresponds the higher graph connectivity of the topology. The convergence rate reveals that a smaller λ can result in a faster convergence [26, 27, 127, 148], meaning that a higher graph connectivity can facilitate the model convergence. A randomly connected topology probably corresponds to a larger λ, thereby decreasing the model convergence rate. Furthermore, as a common issue in FL, the data heterogeneity is also proven to be an important factor to degrade the model convergence in DFL [127, 145, 149].
- **Decreased generalization ability.** It has been proved that the *generalization error* of DFL, i.e., the difference between the true risk of model on the real data distribution and the empirical risk on the training data distribution, is a monotone-increasing function with respect to λ. This implies that the generalization ability of DFL can be degraded by the low connectivity of the underlying topology [26, 27].

5.2 Optimization methods for DFL

Extensive optimization methods for improving the model convergence and generalization ability in DFL have been proposed [127, 146]. We review these methods from the aspects of topology-aware optimization [26], and accelerated optimization [17, 150].

- **Topology-aware optimization** methods [26] attempt to design graph topology with higher connectivity to facilitate model convergence and enhance model generalization ability. However, a highly connected graph inevitably incurs a large overhead

on communications [151, 152]. To speed up model convergence with limited communication resources, existing work proposes a sparse network topology design method based on d-regular expander graphs to optimize the trade-off between connectivity and communication, where d is a pre-specified threshold of graph degree. This method gradually densifies the network topology based on a set of virtual coordinates and recursive queries until the degree of each node reaches d.

Considering the topology of DFL, multiple gossip steps (MGS), which means more frequent communication, can result in improved consensus among the participating clients. In view of this, a balance between the communication cost and generalization ability can be ensured [153].

- **Accelerated optimization** methods [17, 146, 150] adapt advanced optimization techniques to facilitate model convergence in DFL settings. By approximating the global unbiased gradient using neighboring local gradients, the variance reduction method corrects the local gradient for each client [146, 154, 155]. This reduces model inconsistency caused by data heterogeneity and achieves a convergence rate of $\mathcal{O}(\frac{1}{\sqrt{KT}})$. Combining the Nesterov gradient descent method with gradient compression, a contractive compression operator is derived for the time-varying decentralized topology [150]. Theoretical analysis demonstrates the accelerated model convergence. Besides, by modeling the DFL task as a constrained optimization problem:

$$\begin{aligned} &\min_{x_i,\, i\in[N]} \ \frac{1}{N}\sum_{i=1}^{N} f_i(x_i), \\ &s.t. \ \ x_i = x_j, \ \ \forall (i,j)\in\mathcal{E}, \end{aligned} \tag{5.1}$$

  where $\mathcal{E}$ represents the set of node pairs that maintain links, ADMM can be applied to decouple and solve the DFL optimization problem, effectively enhancing model consistency among nodes [17, 156]. Theoretical results demonstrate that the iteration sequence generated by the ADMM algorithm converges to ϵ-suboptimality with a manageable iteration complexity $\mathcal{O}(\frac{1}{\epsilon})$ under time-varying (un)directed network topologies.

  By utilizing sharpness aware minimization (SAM), i.e., introducing a small perturbation to the models, both generalization and robustness are enhanced [153]. Meanwhile, the improved convergence rate $\mathcal{O}(\frac{1}{\sqrt{KT}}+\frac{1}{T}+\frac{1}{K^{1/2}T^{3/2}(1-\lambda)^2})$ can be achieved in the non-convex settings.

5.3 Discussion

Table 4 summarizes the advantages and disadvantages of the two optimization methods. Topology-aware optimization methods improve the model convergence and generalization ability by optimizing the underlying topology directly. However, it is only applicable

Table 4: Summary of optimization methods in DFL.

Methods	**Advantages**	**Disadvantages**	**Network topologies**
Topology-aware optimization	Enhanced model convergence and generalization	Constraints of connectivity among clients	Partially connected topology
Accelerated optimization	Enhanced model convergence	High computation cost	All topologies

to settings where the topology is allowed to change. In practice, the generated topology potentially imposes higher communication costs on neighboring nodes located at substantial geographic distances. Accelerated optimization methods can enhance the model convergence by manipulating the gradients in each round of training. It is applicable to all types of topologies but suffers from relatively higher computation costs to correct the gradients in each round.

6 Online Federated Learning

The above discussions of FL optimization assume that each client possesses a fixed and static dataset, which is also referred to as batch-based FL. However, in numerous real-world scenarios, the training data of clients are often generated in a streaming mode [157]. Consequently, online FL (OFL) [28–31] has been introduced by combining online learning (OL) [158, 159] and FL. OFL performs online optimization in FL over the time-evolving data, and makes a prompt label prediction or decision upon receiving incoming data.

OFL aims to learn a sequence of global models from distributed streaming data at local devices. At timestamp t, each client indexed by $i\in[N]$ receives a new data (u_i^t,v_i^t), where u_i^t is the feature and v_i^t is the label, and the latest global model x^t from the server. The global model is used to predict the label of newly incoming data. Thus, the i-th client has a local loss $\mathcal{L}(f(u_i^t;x^t),v_i^t)$, where $\mathcal{L}(\cdot,\cdot)$ is a loss function that measures the error between true and predicted labels. Leveraging the local loss, the i-th client optimizes its local model x_i^{t+1} and sends it to the server. After receiving all local models, the server updates the global model x^{t+1} by averaging the local models $\{x_i^{t+1}\}_{i=1}^N$. OFL aims to seek a sequence of global models $x^1,x^2,\ldots,x^t$ that minimizes the cumulative regret (i.e., the difference between cumulative loss of the algorithm and that of the static optimal function) over T timestamps:

$$\text{Regret}_T=\sum_{t=1}^{T}\sum_{i=1}^{N}\mathcal{L}(f(u_i^t;x_i^t),v_i^t)-\min_{x\in\mathbb{R}^d}\sum_{t=1}^{T}\sum_{i=1}^{N}\mathcal{L}(f(u_i^t;x),v_i^t). \tag{6.1}$$

6.1 Effect of online paradigm on federated optimization

The online paradigm introduces novel optimization problems and requires new optimization methods for handling streaming data. Here, we discuss the effects of the online paradigm on federated optimization from the following two aspects.

- **Exacerbated issue of obsolete solutions.** In online settings, the model is required to be updated exclusively with newly acquired data and continuously give solutions for dynamic environments [28, 31]. However, general FL methods may be unable to update the model continuously according to new data. This produces obsolete solutions, as the data distribution may have changed, leading to poor model performance (e.g., regret). Furthermore, the constraints of computing and communication in FL aggravate the difficulty of continuously updating the model, which causes a more serious issue of obsolete solutions.

- **Severe catastrophic forgetting.** In the scenario of continuously learning a sequence of tasks based on the online learning paradigm, a model may exhibit degraded performance on old tasks if it only learns new tasks when acquiring new data. This is known as the *catastrophic forgetting* issue [139–141]. More seriously, different FL clients may have different task sequences, which may lead to the mutual interference of task knowledge between different clients [140, 141, 160]. This further aggravates the catastrophic forgetting of clients.

6.2 Optimization methods for OFL

To alleviate the negative effects caused by the online paradigm, many optimization methods for OFL have been proposed [28–31]. We review these methods from the aspects of addressing obsolete solutions and mitigating catastrophic forgetting issues.

- **Kernel-based methods for addressing obsolete solutions**. Current OFL methods [28, 30, 31] mainly rely on online gradient descent method (OGD) [158], an adaptation of SGD for online optimization, to update local models. For better solving non-linear optimization problems (e.g., deep learning tasks) and addressing the issue of obsolete solutions in OFL, some works [28, 30, 161] integrate OGD, FedAvg and multi-kernel learning (MKL). MKL is an advanced method in OL and has exhibited superior performance [162, 163]. The kernel that maps the original input space to a higher-dimensional feature space can significantly improve the generalization capabilities of the model, thereby addressing the issue of obsolete solutions. It is analyzed that the existing OFL methods with multiple kernels can achieve an optimal sub-linear regret $\mathcal{O}(\sqrt{T})$ by setting the learning rate as $\mathcal{O}(\frac{1}{\sqrt{T}})$.

- **Mitigating catastrophic forgetting at the data and model levels.** Existing methods mitigate the catastrophic forgetting issue from both data and model perspectives.

Data-based methods aim to leverage historic training samples or generated samples with the similar distribution for training models with new classes. One way is to simply store and replay old samples at the client [164]. The other is to use generative models to simulate training data with the similar distribution to the historic data [165, 166]. Model-based methods try to balance the model stability on old tasks and generalization on new tasks [139]. For exmaple, weighted averaging of new and old models [164] and decreasing learning rates [167] are beneficial in avoiding catastrophic forgetting for local model updates. The specific mitigating methods can be further divided into the regularization-based and knowledge distillation-based ones. The former usually adds penalty terms to the loss function [164] while the latter applies knowledge distillation to old training samples [165, 166, 168]. Regarding the knowledge distillation-based methods, prototype-based learning approaches [141, 160, 169] are often utilized to collect prototype data of each class and monitor global model performance, which helps to choose the best old global model for knowledge distillation to mitigate the impact of inter-client interference. Also, a recent method of parameter decomposition [164] separates the network parameters into the global and task-specific parameters, enabling clients to selectively learn from each other.

6.3 Discussion

Currently, research on OFL is still in its early stages. The basic framework integrates FedAvg and OGD. Existing works in OFL address obsolete solutions using OL methods like kernel-based approaches [28, 30, 161]. They also tackle the problem of catastrophic forgetting by adopting centralized continual learning methods such as regularization-based approaches [164–166, 168]. However, OFL research has yet to explore the important aspects of heterogeneity and privacy issues in FL from both challenges and optimization methods perspectives.

7 Other important works

In addition to FL optimization studies discussed in the above sections, there are other important research topics, such as sparsification methods [170, 171] and gradient aggregation rules [172, 173]. Sparsification methods aim to optimize the communication complexity of FL systems while maintaining the model performance. Gradient aggregation rules aim to evaluate and aggregate gradients from different local clients to generate a high-quality (e.g., Byzantine robust) global model update.

7.1 Sparsification

Reducing communication complexity is another core challenge in FL optimization [170, 174], especially in the context of massive participants and complex models. Sparsifica-

tion [174–176], as a class of communication-efficient methods, can cut down the communication costs by reducing the number of exchanged parameters, which can also improve the generalization ability of the model [174]. However, these methods have a significant impact on the convergence rate and errors of the model. Thus, the essential problem is to optimize the model convergence while satisfying the communication constraint via calibrating the sparsification parameter. Next, we will review the advanced studies from three perspectives, including client sparsification, temporal sparsification and gradient sparsification.

7.1.1 Client sparsification

Exchanging model updates with abundant participating clients contributes to the communication bottleneck during an FL training round. Client sparsification (also called client sampling) [39, 177, 178], i.e., a random selection of a subset of clients, is a viable solution, but that randomness may result in a lot of missed potential. In most FL implementations, the clients vary in design and capability, a diversity that extends to the quality of communication mediums. Choosing the clients that meet the most favorable communication conditions in each round should help achieve a higher convergence rate [170]. It is analyzed in [171] that a larger selection skew results in faster convergence at the rate $\mathcal{O}(\frac{1}{T\rho})$, where ρ represents the selection skew towards clients with higher local losses, which reveals that biasing client selection towards clients with higher local loss achieves faster convergence.

7.1.2 Temporal sparsification

To leverage all the available data samples on the data owners, standard FL methods generally let the clients synchronize their models through the server in each training iteration. However, this implies many rounds of communication between the clients and the server which results in communication contention over the network. Instead, some works [31, 179] propose that participating clients conduct several local updates and synchronize through the server periodically. Specifically, once clients pull an updated model from the server, they update the model locally by running τ iterations of the SGD method and then send proper information to the server for updating the aggregate model. Under a strongly convex setting, for a total number of iterations $T = K\tau$, where K is a positive integer, the convergence rate is $\mathcal{O}(\frac{\tau}{T}) + \mathcal{O}(\frac{\tau^2}{T^2}) + \mathcal{O}(\frac{(\tau-1)^2}{T}) + \mathcal{O}(\frac{\tau-1}{T^2})$ [179]. In particular, any pick of $\tau = \mathcal{O}(\sqrt{T})$ ensures the convergence of the FL to the global optimal. For smooth non-convex loss functions, the convergence rate is $\mathcal{O}(\frac{1}{\sqrt{T}}) + \mathcal{O}(\frac{\tau-1}{T})$.

7.1.3 Gradient sparsification

Gradient sparsification methods [180, 181] convert a dense gradient into a sparse one by retaining only a subset of significant elements and setting the remaining coordinates to zero. Two commonly used techniques are rand-k sparsification [181] and top-k sparsification [182–184]. In particular, rand-k sparsification randomly selects k elements from

the gradients, whereas top-k sparsification retains the k elements with the highest absolute values. It has been proved [185] that rand-k sparsification is an unbiased compression operator which, however, results in larger compression errors and therefore makes it less effective in practice compared to top-k sparsification when high compression is required [186].

Quantization [31, 179, 187] is another classic sparsification method that reduces the model size by lowering the bit width from 32-bit floating-point to a smaller precision. For now, numerous quantization methods (e.g., stochastic quantization [188], rotation-based quantization [189], etc.) have been proposed to compress model gradients of each client into a discrete set. Theoretical analysis shows that the convergence rate of FL with quantization has the order of $\mathcal{O}(\frac{1}{\sqrt{T}})$, which is the same as that of a classical FL framework for a non-convex loss function [179].

Generally, top-k sparsification methods can reduce communication complexity more efficiently compared to quantization. It has been demonstrated that the top-k sparsification with error feedback can accelerate convergence and accuracy with over 99% gradient elements zeroed out [190].

7.2 Aggregation Rules

The most widely used aggregation rule is FedAvg, which takes weighted average over all local gradients according to local data sizes. However, FedAvg implicitly assumes the equal quality of all gradients, limiting its effectiveness in scenarios like asynchronous FL (e.g., some gradients are stale [191]) and Byzantine attack settings (e.g., some gradients are fake [192]). To overcome these limitations, a variety of aggregation rules [193] have been proposed to optimize the model utility (e.g., model convergence and Byzantine robustness). Advanced aggregation methods can be roughly divided into two categories: weighting-based aggregation and statistic-based aggregation.

7.2.1 Weighting-based aggregation

Weighting-based aggregation rules aim to optimize the model performance (e.g., convergence, fairness, etc.) by assigning differentiated weights to local gradients according to specific statistical indicators (e.g., gradient staleness, model loss. etc.). The aggregation rule is as follows:

$$\nabla f(x) = \frac{\sum_{i=1}^{N} a_i \nabla f(x_i)}{\sum_{i=1}^{N} a_i}, \tag{7.1}$$

where $a_i \geq 0$ represents the weight for local gradient $\nabla f(x_i)$.

In asynchronous FL, with the objective of improving model convergence rate and decreasing errors, the weights are mainly determined by staleness [172, 194] or descent direction [191]. Stale gradients typically exhibit biased descent directions and larger norms than normal gradients. Consequently, simply averaging the stale gradients would result in their dominance during the training process, leading to a sub-linear convergence rate

of $\mathcal{O}(\frac{1}{\sqrt{T}})$ under the assumptions of general convexity [195] or bounded gradient [196]. Motivated by the intuition that low-staleness gradients are more reliable and accurate, it is reasonable to have the server take the weighted average over all gradients according to their staleness to estimate an unbiased gradient. Nonetheless, some high-staleness gradients also exhibit consistent descent directions to unbiased gradients, which can be utilized to accelerate model convergence [191, 197]. Consequently, existing research enables the server to first evaluate the consistency of stale gradients with the estimated unbiased gradient, and accordingly assigns differentiated weights to those gradients to improve the convergence rate [191]. Through increasing the contributions of consistent gradients to the aggregation, such weighted aggregation method can significantly decrease the error in each round. It has been demonstrated that weighted asynchronous FL can achieve a convergence rate of $\mathcal{O}(\frac{1}{T})$ even under the assumption of non-convexity [191, 198].

In fair FL, to achieve performance consistency, most fair algorithms primarily enhance optimization objectives by amplifying the dominance of the large losses, which ultimately influences the aggregation weights in the training process [89, 199, 200]. The fundamental idea is to encourage the global model to demonstrate a favorable inclination towards disadvantaged clients (i.e. clients with smaller losses) by augmenting their weights. A straightforward method is to assign larger weights to gradients with larger loss values [199, 200]. Besides, several research introduces fairness constraints (e.g., the limited difference of predicted label probability distribution under different sensitive attributes [200]) to the FL optimization problem [199, 200]. To solve the constrained problem, the server first assigns different fairness budgets to clients and then adjusts the weights of different gradients according to whether the model performance inconsistency surpasses the assigned fairness budget.

7.2.2 Statistic-based aggregation

Statistic-based aggregation rules [173, 192, 193, 201–204] generate some robust statistics over the local gradients to perform aggregation, aiming to optimize the model utility with the presence of some fake gradients. The typical FedAvg, which takes the arithmetic mean of gradients to update the model, may diverge in the optimization process due to its vulnerability to outliers. In contrast, robust averaging methods [173, 201–203] demonstrate improved robustness, i.e., median/trimmed-mean [201] can achieve order-optimal error rates while maintaining a convergence rate of $O(\frac{\sigma}{\sqrt{KT}})$ under the assumption of non-convexity. However, these methods assume that honest gradients are close to each other and overlook data heterogeneity, making them vulnerable in non-i.i.d. FL settings.

To address the impact of non-i.i.d. data on robustness, recent studies [193, 204] leverages a bucketing step that groups heterogeneous gradients and computes the average within each bucket. This pre-processing step generates more homogeneous gradients, enabling robust aggregation and producing more resilient results. The bucketing-based methods have been shown to converge with a rate of $O(\frac{\sigma}{\sqrt{KT}})$, even under the assumptions of data heterogeneity and non-convexity. Furthermore, a large dimension of model

gradients can also amplify the effects of malicious gradients in non-i.i.d. data settings. To this end, GAS [192] splits high-dimensional gradients into multiple low-dimensional subsets to mitigate these impacts. Robust statistic-based rules can then be applied to each subset, followed by concatenation. This splitting method is proven to be effective in reducing the impact of large gradients dimension d.

Existing studies [205, 206] have demonstrated that the variance of stochastic gradients significantly impacts the Byzantine robustness of FL and a higher variance in stochastic gradients leads to weaker robustness. Hence, some research [205–207] combines existing variance reduction algorithms with robust statistics to mitigate the impact of gradient variances on Byzantine robustness. Specifically, Byra-SAGA [205] proposes the integration of the distributed SAGA algorithm with the geometric median statistics achieving enhanced robustness. Theoretical results suggest that Byra-SAGA can achieve a convergence rate of $O(\frac{1}{T})$. Byz-VR-MARINA [206] introduces the combined gradient compression algorithm VR-MARINA, variance reduction algorithm SARAH/PAGE, and robust aggregation rules, such as the geometric median statistics-based rule, to simultaneously achieve communication compression and robustness enhancement. Theoretical analysis indicates that under general non-convex assumptions, Byz-VR-MARINA can achieve a convergence rate of $O(\frac{1}{T})$.

Additionally, recent research [98, 208–212] has explored the extension of statistic-based aggregation rules to decentralized FL. For example, BRIDGE [209] extends the coordinate-wise trimmed-mean to decentralized FL. Theoretical findings indicate that, in a statistical sense, BRIDGE can converge at a convergence rate of $O(\frac{1}{T})$ to the optimal solution and a first-order stationary point in convex and non-convex settings respectively. IOS [213] provides general guidelines to design Byzantine robust statistics for aggregation in decentralized FL, and proposes an effective method that iteratively discards models that are farthest away from the weighted average of models from neighboring nodes.

8 Possible Future Directions

8.1 Discussions for FL Development

Along with extensive academic research, FL has also shown great potential and even practical applications in several industrial scenarios. For instance, Google has deployed FL on billions of Android systems to enable precise next-word prediction for mobile keyboards [4]. NVIDIA has applied FL in medical image analysis across multiple institutions [214]. As artificial intelligence has been becoming pervasive while privacy consciousness is ever increasing, FL is believed to thrive in the following future application scenarios.

- **FL in autonomous cars and robots.** Both self-driving cars and robots are expected to bloom soon, which will accumulate huge amounts of data in diverse environments. On the other hand, both applications are highly driven by various ML tech-

nologies, which require repeated training over massive data. At the same time, these applications may experience varying communication channels in the real world. Clearly, FL is very promising to be applied for accelerating their model learning while limiting data transfer and privacy disclosure [215, 216].

- **Vertical federated Learning.** Besides these applications based on traditional horizontal FL [35] (where the feature space is identical but the sample space is different), there are also explorations for vertical FL [40] (where the sample space is overlapped but the feature space is orthogonal). For example, Webank has applied vertical FL in financial risk controls by sharing knowledge between banks and insurance companies [35, 217]. Also, many internet companies like Bytedance has adopted vertical FL for intelligent recommendation in e-commence [218].

- **FL for large models (LMs).** Large Models (LMs) have recently demonstrated astonishing AI abilities, gaining massive attention. To exploit the full potential of LMs, it often has to fine-tune them to domain-specific tasks or adapt them with domain-specific knowledge [219]. However, the fine-tuning of LMs not only requires rather powerful computing resources but also relies on a large amount of high-quality domain-specific data [220], which however may be scattered among multiple sites and cannot be centralized. In this case, FL has been becoming a promising technology for achieving privacy-preserving LMs fine-tuning, thus truely grounding the large foundation models [221].

8.2 Future Directions for FL Optimization

Despite the numerous advanced studies and promising applications, there still remains considerable space for the foundational FL optimization in terms of the optimization methods, and the practical system and privacy constraints. In the following, we discuss the possible future directions in FL optimization from three perspectives: optimization, system, and privacy.

- **New theories and methods for black-box optimization.** As we try to optimize the increasingly complicated AI systems, many practical learning tasks are essentially black-box optimization [75, 78, 79], which can hardly give the analytic expression of the loss function or the gradient information. In such cases, besides zeroth-order optimization [78, 79] without the gradient information, we sometimes need more sophisticated optimization theories and methods. For example, hyper-parameter optimization [222–224] and neural architecture search [225, 226] are very promising for optimizing the complex deep learning models. These problems are often modeled as Bi-level optimization where an optimization problem contains another optimization problem as a constraint [90, 227–231] in essence. However, how to adapt these advanced optimizations to the FL setting remains largely unexplored, thus being a promising research direction.

- **Optimization under practical FL system constraints.** FL optimization differs from conventional distributed optimization in many practical system constraints, including the underlying topology [127, 130, 131], parallel mode [38, 232], resource limit [11, 32], and data distribution [5, 18] etc. Existing studies have extensively explored the issue of non-i.i.d. data distribution, but still lack the deep consideration of many system constraints like topology and resource, especially their combinations. As discussed before, the underlying topologies can impact both the convergence rate and stability of FL optimization. In fact, the possible imbalance of resources across computing nodes, can impact not only the choice of the FL parallel mode but also the convergence performance. Therefore, future work may consider how to achieve faster and more accurate FL optimization by simultaneously optimizing the topology and resource allocation among FL nodes.

- **FL optimization aware of privacy-preserving techniques.** FL optimization is proposed as a privacy-aware optimization method by design. Therefore, beyond DP, FL is often incorporated with many other privacy-preserving technologies like multi-party computation [233, 234] and homomorphic encryption [235, 236]. However, similar to the degraded convergence incurred by DP constraints, the additional constraints in these privacy-preserving methods bring new challenges for FL optimization. For example, encryption-based methods often require quantization of exchanged gradient information to reduce the computing complexity [237–239]. This would further result in a degraded performance of FL optimization. Also, the cryptographic primitives significantly burden the computation and communication of FL optimization, which again complicates the trilemma among privacy, utility and efficiency. Therefore, an important question is how to design FL algorithms to optimize the tradeoff among the accuracy, privacy and complexity.

9 Conclusion

As an important interdisciplinary research area in both applied mathematics and information sciences, FL still lacks a summarization of advanced studies in terms of mathematical optimization. To this end, we presented the first systematic survey on the assumptions, formulations, methods, and theoretical results in FL optimization, mainly focusing on the optimization challenges induced by non-i.i.d. data, rigorous privacy guarantee, decentralized topology, and online settings. Besides, we also reviewed some other important works on sparsification methods and aggregation rules, which can also improve FL optimization. Finally, we envisioned the applications of FL in the AI era and discussed several broader future directions from the perspectives of optimization, system, and privacy respectively.

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