Case Study III: A Data Center

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- Most of the figures in this set of slides come from the book “Performance by Design: computer capacity planning by example,” by Menascé, Almeida, and Dowdy, Prentice Hall, 2004. It is strictly forbidden to copy, post on a Web site, or distribute electronically, in part or entirely, any of the slides in this file.

The Data Center

- M functionally equivalent machines
- Staff of N people that repair machines that fail.
- Data center failure diagnostic system:
  – "heartbeat" for failure detection
  – Trouble tickets are automatically generated
  – Failed machine are placed in a queue for a repair person.
- Idle members of the repair staff continuously monitor the repair queue.

Questions

1. Given the machine failure rate, the number of machines, the size of the repair staff, and the average repair time, what is the probability that j machines are operational?
2. Given the machine failure rate, the number of machines, the size of the repair staff, and the average repair time, what is the probability that at least j machines are operational?

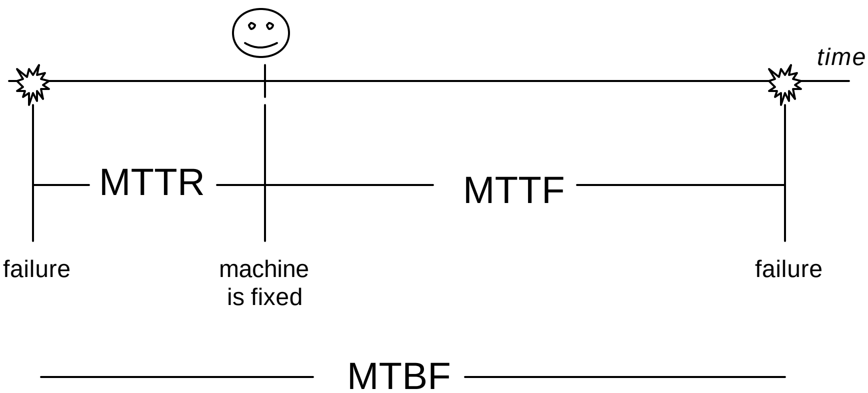
Questions (cont'd)

3. Given the machine failure rate, the number of machines, and the average repair time, how many people are needed to guarantee that at least j machines are operational?
4. What is the effect of the size of the repair team on the MTTR and on the percentage of machines that can be expected to be operational?

Questions (cont'd)

5. What is the effect of the average time it takes a repair person to fix a machine on the overall MTTR? How does a repair person's skill affect the percentage of operational machines?

MTTR, MTTF, and MTBF



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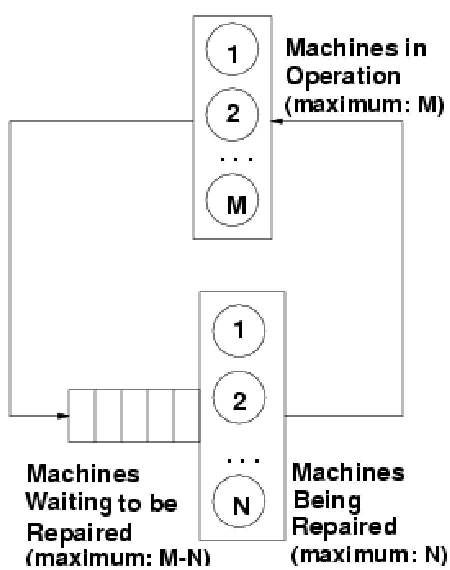
Failure-Recovery Model



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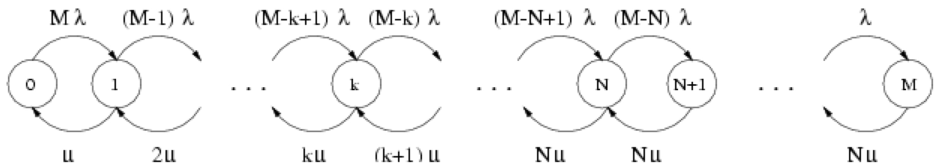
Markov Chain for the Data Center



State = number of failed machines.

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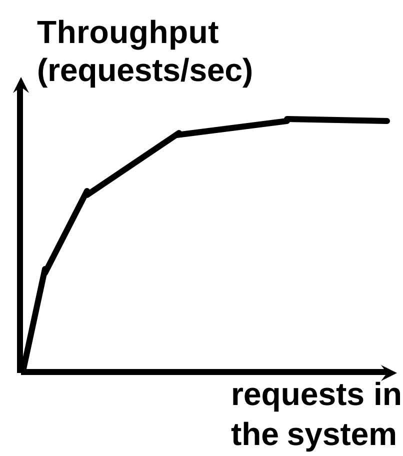
System-level Models

- System is seen as a black box.
- Only its input-output characteristics are considered.
- Inputs: arrivals of requests
- Output: throughput.



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System-level Example

- A Web server receives 10 requests/sec.
- The maximum number of requests in the server is 3.
- Requests that arrive and find three requests being processed are rejected.

System-level Example

- The measured throughput as a function of the number of requests is:

Number of requests	Throughput (req/sec)
0	0
1	12
2	15
3	16

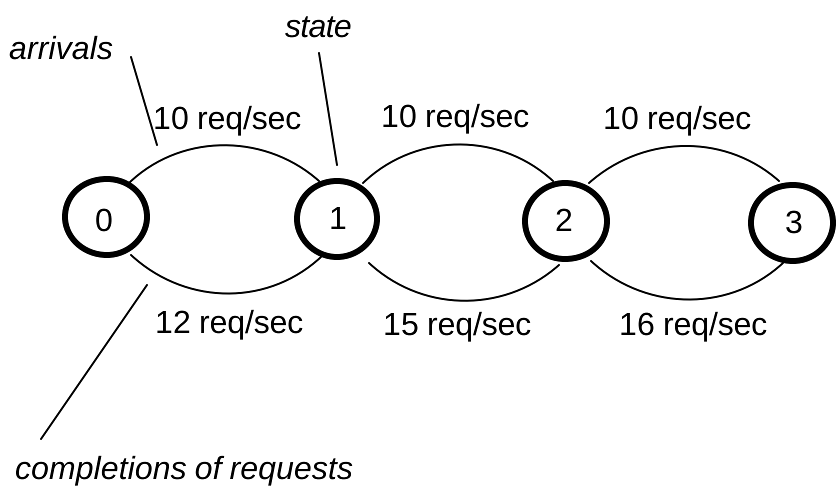
System-level Example: a few questions

- Q1: What is the probability that an incoming request is rejected?
- Q2: What is the average number of requests in execution?
- Q3: What is the average throughput of the Web server?
- Q4: What is the average time spent by an HTTP request in the Web server?

System-level Example

- Characterize the Web server by its state, i.e., the number *k* of requests in the Web server.
- Assumptions made:
  - homogeneous workload: all requests are equivalent
  - memoryless: how the system arrived at system *k* does not matter.
  - operational equilibrium: no. requests at beginning of interval = no. request at the end.

System-level Example



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System-level Example

- Assume we are able to find the values of:
  - P_k = probability that there are *k* requests in the Web server.
- Question: can we answer all the questions posed before as a function of the P_k's?

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System-level Example: a few questions

- Q1: What is the probability that an incoming request is rejected?
- A:

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System-level Example: a few questions

- Q1: What is the probability that an incoming request is rejected?
- A: It is the probability that an arriving HTTP request finds 3 requests already being processed. The answer is then P_3.

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System-level Example: a few questions

- Q2: What is the average number of requests in execution?
- A: using the definition of average:

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System-level Example: a few questions

- Q2: What is the average number of requests in execution?
- A: using the definition of average:

$$n_{req} = 0 \times P_0 + 1 \times P_1 + 2 \times P_2 + 3 \times P_3$$

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System-level Example: a few questions

- Q3: What is the average throughput of the Web server?
- A: again, using the definition of average:

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System-level Example: a few questions

- Q3: What is the average throughput of the Web server?
- A: again, using the definition of average:

$$X = 0 \times P_0 + 12 \times P_1 + 15 \times P_2 + 16 \times P_3$$

throughput value at each state

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System-level Example: a few questions

- Q4: What is the average time spent by an HTTP request in the Web server?
- A: It will be a function of the average number of requests, n_{req}, and the average throughput X. More on this later...

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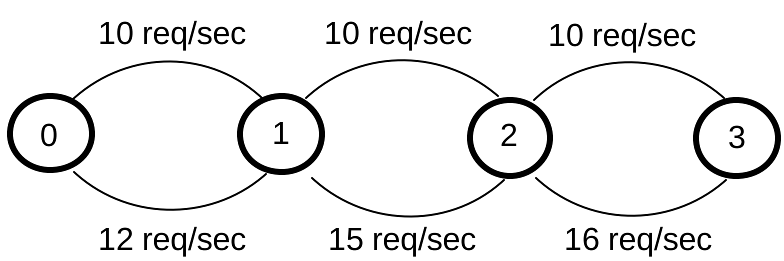
System-level Example: computing the P_k's



- use the flow in = flow out principle: the flow into a set of states is equal to the flow out of this set of states in equilibrium.

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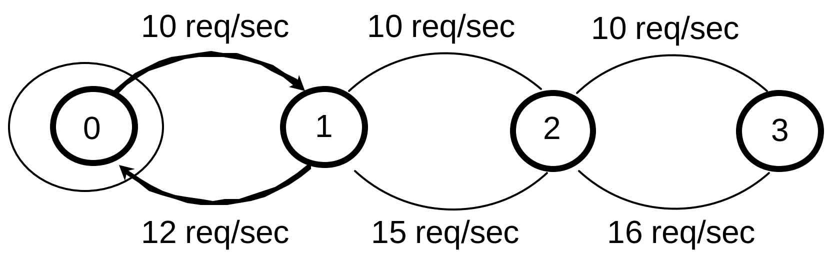
System-level Example: computing the P_k's



flow in = flow out

$$12 \times P_1 = 10 \times P_0$$

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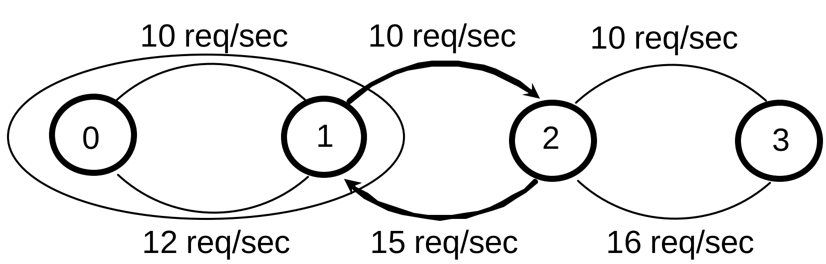
System-level Example: computing the P_k's



flow in = flow out

$$15 \times P_2 = 10 \times P_1$$

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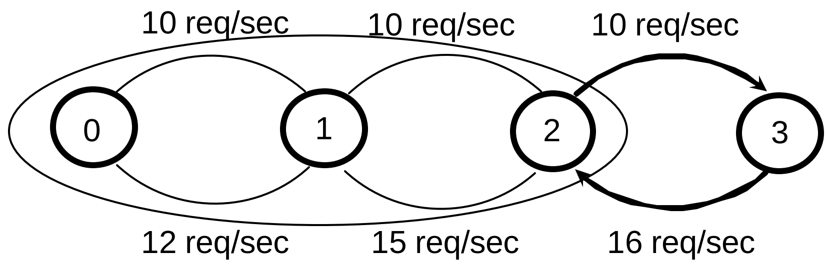
System-level Example: computing the P_k's



flow in = flow out

$$16 \times P_3 = 10 \times P_2$$

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System-level Example: computing the P_k's

- Putting it all together:

$$12 \times P_1 = 10 \times P_0 \Rightarrow P_1 = 10/12\ P_0$$

$$15 \times P_2 = 10 \times P_1 \Rightarrow P_2 = 10/15\ P_1 = \frac{10\times10}{15\times12} P_0$$

$$16 \times P_3 = 10 \times P_2 \Rightarrow P_3 = 10/16\ P_2 = \frac{10\times10\times10}{16\times15\times12} P_0$$

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System-level Example: computing the P_k's

- Putting it all together:

$P_1 = 10/12\ P_0$; $P_2 = \frac{10 \times 10}{15 \times 12} P_0$; and

$P_3 = \frac{10 \times 10 \times 10}{16 \times 15 \times 12} P_0$

- But, the Web server has to be in one of the four states at any time. So, $P_0 + P_1 + P_2 + P_3 = 1$.

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System-level Example: computing the P_k's

- Solving for P_0 and then for the other P_k's we get:

k	Pk
0	0.365
1	0.305
2	0.203
3	0.127

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System-level Example: answering the questions

- Q1: What is the probability that an incoming request is rejected?
- A: It is the probability that an arriving HTTP request finds 3 requests already being processed. The answer is then

  $P_3 = 0.127 = 12.7\%$.

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System-level Example: answering the questions

- Q2: What is the average number of requests in execution?
- A: using the definition of average:

$$
\begin{aligned}
n_{req} &= 0 \times 0.365 + 1 \times 0.305 + \\
&\quad 2 \times 0.203 + 3 \times 0.127 \\
&= 1.091 \text{ requests}
\end{aligned}
$$

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System-level Example: answering the questions

- Q3: What is the average throughput of the Web server?
- A: again, using the definition of average:

$$X = 0 \times 0.365 + 12 \times 0.305 + 15 \times 0.203 + 16 \times 0.127 = 8.731 \text{ requests/sec.}$$

System-level Example: answering the questions

- Q4: What is the average time spent by an HTTP request in the Web server?
- A: It is a function of the average number of requests, n_{req}, and the average throughput X. We need Little's Law to answer this question.

System-level Example: answering the questions

- Q4: What is the average time spent by an HTTP request in the Web server?
- A: From Little's Law,

$$R = n_{\mathbf{req}} / X = 1.091 / 8.731 = 0.125 \text{ sec.}$$

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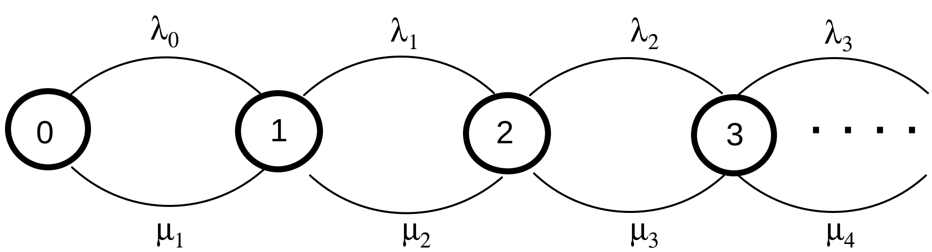
Generalized System-level Models



Generalized System-level Models can be solved using the ***flow in = flow out*** *principle!*

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Generalized System-level Models

$$p_k = \prod_{i=0}^{k-1} \frac{l_i}{m_{i+1}}$$

$$p_0 = \left[\sum_{k=0}^{\infty} \prod_{i=0}^{k-1} \frac{l_i}{m_{i+1}} \right]^{-1}$$

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Solving the Markov Model for the Failure-Recovery Case

$$l_k = (M - k)\, l$$

$$m_k = \begin{cases} km & k = 1, \ldots, N \\ Nm & k = (N+1), \ldots, M \end{cases}$$

$$p_k = \begin{cases} p_0 \left(\frac{l}{m} \right)^k \binom{M}{k} & k = 1, \ldots, N \\ p_0 \left(\frac{l}{m} \right)^k \binom{M}{k} \frac{N^{N-k} k!}{N!} & N < k \le M \end{cases}$$

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Solving the Markov Model for the Failure-Recovery Case

Average aggregate machine failure rate:

$$\overline{X}_f = \sum_{k=0}^{M-1} l_k \times p_k = \sum_{k=0}^{M-1} (M-k)\, l \times p_k$$

MTTR (from the Interactive Response Time Law):

$$MTTR = M / \overline{X}_f - MTTF = M / \overline{X}_f - 1/l$$

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Solving the Markov Model for the Failure-Recovery Case

Average number of failed machines (Little's Law):

$$N_f = \overline{X}_f \times MTTR = M - \overline{X}_f \times MTTF = M - \overline{X}_f / l$$

Average number of operational machines:

$$N_o = M - N_f = \overline{X}_f \times MTTF = \overline{X}_f / l$$

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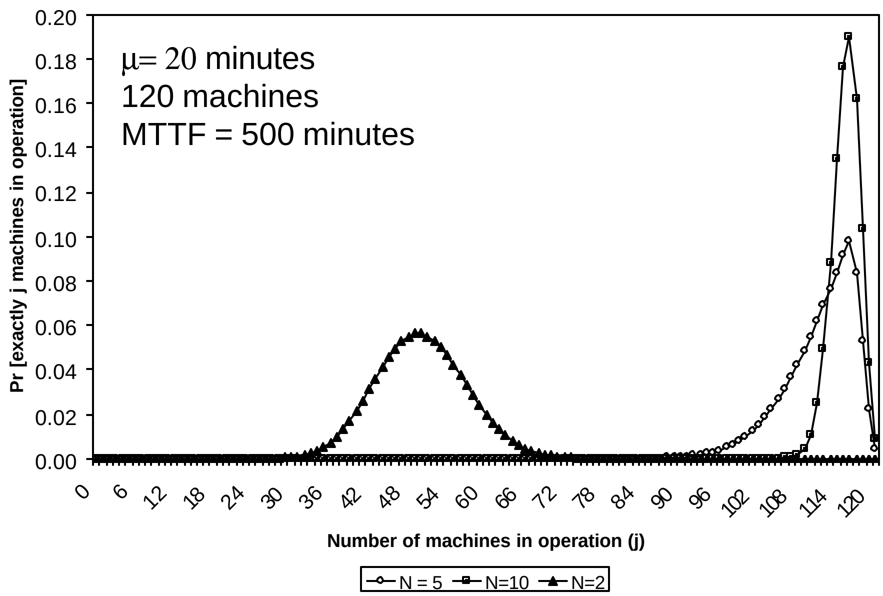
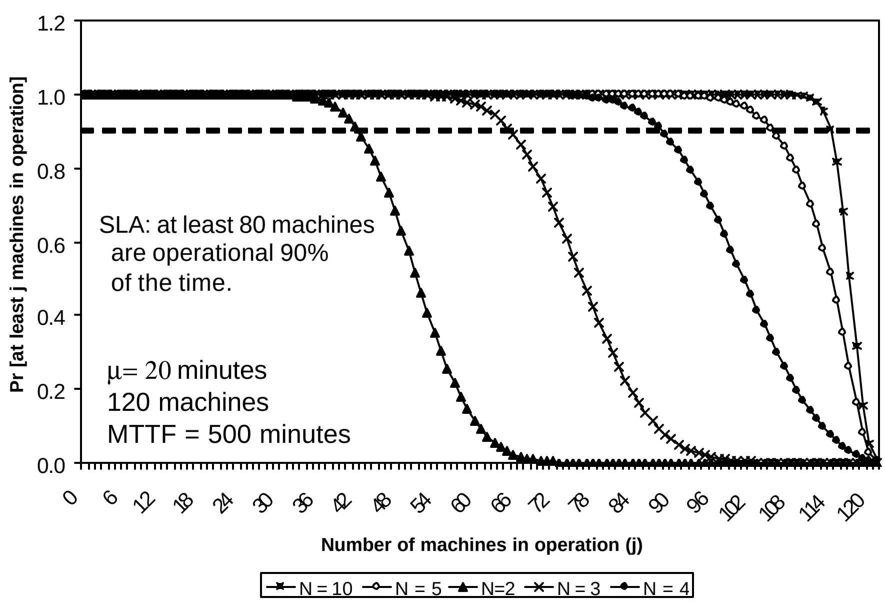


μ= 20 minutes
120 machines
MTTF = 500 minutes
Pr [exactly j machines in operation]
0.20
0.18
0.16
0.14
0.12
0.10
0.08
0.06
0.04
0.02
0.00
0 6 12 18 24 30 36 42 48 54 60 66 72 78 84 90 96 102 108 114 120
Number of machines in operation (j)
N = 5
N=10
N=2



SLA: at least 80 machines are operational 90% of the time.
μ= 20 minutes
120 machines
MTTF = 500 minutes
Pr [at least j machines in operation]
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0 6 12 18 24 30 36 42 48 54 60 66 72 78 84 90 96 102 108 114 120
Number of machines in operation (j)
N = 10
N = 5
N=2
N = 3
N = 4

Effect of Number of Repair People

N	N0	Nf	MTTR (min)	% Machines in Operation
1	25.00	95.0	1900.0	20.8%
2	50.00	70.0	700.0	41.7%
3	75.00	45.0	300.0	62.5%
4	99.20	20.8	104.8	82.7%
5	111.49	8.5	38.1	92.9%
6	114.27	5.7	25.1	95.2%
7	115.02	5.0	21.7	95.8%
8	115.26	4.7	20.6	96.0%
9	115.34	4.7	20.2	96.1%
10	115.37	4.6	20.1	96.1%
120	115.38	4.6	20.0	96.2%

Number of machines: 120; Failure Rate: 0.002 machines/min;
Repair rate: 0.05 machines/minute

Effect of Repair Rate

Average time to repair a machine (minutes)	Repair Rate (mu) (1/min)	N0	Nf	MTTR	% Machines in Operation
10	0.100	117.6	2.4	10.4	98.0%
12	0.083	117.0	3.0	12.9	97.5%
15	0.067	115.8	4.2	18.1	96.5%
18	0.056	113.8	6.2	27.2	94.8%
20	0.050	111.5	8.5	38.1	92.9%
25	0.040	99.1	20.9	105.5	82.6%

Number of machines: 120; Failure Rate: 0.002 machines/min;
Number of repair people: 5

Cost Analysis

- Cp: annual personnel cost per person in the repair staff.
- Cm: annual machine cost (per machine)
- Ca: total annual cost

  Ca = (N x Cp) + (M x Cm)
- Ra: annual revenue

  Ra = β(No – Mmin)
- Mmin: minimum no. of machines that need to be in operation for the data center to avoid paying a penalty.
- P: annual profit

  P = Ra – Ca = β(No – Mmin) - [(NxCp+MxCm)]

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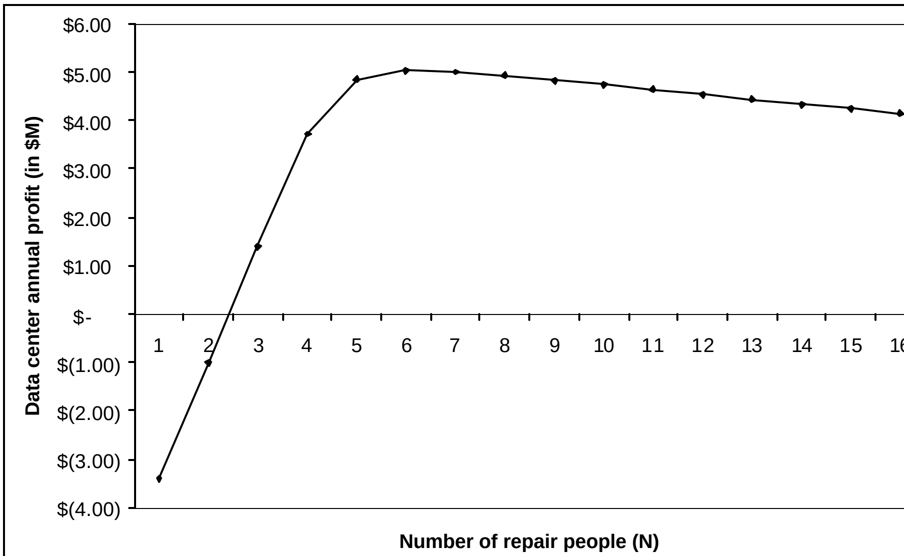

μ= 20 minutes; 120 machines

MTTF = 500 minutes

Mmin = 10; Cp = \$100,000; Cm=\$40,000

β=\$100,000

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