

12. We shall prove the statement using induction on n . The case $n = 1$ is clearly true. Assume the statement holds for $n = k$. Recall that $w = \sum_{i=1}^k z_i$ is a complex number since \mathbb{C} is closed under addition. Then we have that

$$\begin{aligned}
 |z_1 + z_2 + \cdots + z_{k+1}| &= \left| \sum_{i=1}^k z_i + z_{k+1} \right| \\
 &= |w + z_{k+1}| \\
 &\leq |w| + |z_{k+1}| \\
 &= |z_1 + z_2 + \cdots + z_k| + |z_{k+1}| \\
 &\leq |z_1| + |z_2| + \cdots + |z_k| + |z_{k+1}|,
 \end{aligned}$$

where we have used Theorem 1.33 (e) for the first inequality and the induction hypothesis for the latter.

□