

1. Suppose $r + x$ is rational. Then there exists integers m, n such that

$$r + x = \frac{m}{n} \Rightarrow x = \frac{m}{n} - r.$$

Since \mathbb{Q} is closed under addition the RHS of the latter expression is rational. It follows then that x is rational, which is a contradicton. With the same line of reasoning it can be shown that rx is irrational.

□