13. For any complex numbers z, w it holds that $\overline{z}w$ is the conjugate of $z\overline{w}$. Hence $z\overline{w} + \overline{z}w = 2\text{Re}(z\overline{w})$ by Theorem 1.31. We have that

$$\begin{aligned}
\big||x| - |y|\big|^2 &= \big(|x| - |y|\big)\overline{(|x| - |y|)} = |x|^2 - 2|x||y| + |y|^2 \\
&= |x|^2 - 2|x||\overline{y}| + |y|^2 = |x|^2 - 2|x\overline{y}| + |y|^2 \\
&\leq |x|^2 - 2\text{Re}(x\overline{y}) + |y|^2 = x\overline{x} - 2\text{Re}(x\overline{y}) + y\overline{y} \\
&= x\overline{x} - x\overline{y} - \overline{x}y + y\overline{y} = (x - y)\overline{(x - y)} \\
&= |x - y|^2,
\end{aligned}$$

where we used Theorem 1.33 for the following: $|\overline{z}| = |z|$, $|zw| = |z||w|$ and $\text{Re}z \leq |\text{Re}z| \leq |z|$.

$\square$