

**15.** We use the same notation as in the proof of the Schwarz inequality Theorem 1.35. We already know from the proof that

$$\sum |Ba_j - Cb_j|^2 = B(AB - |C|^2) \geq 0.$$

Equality only holds if the expression above equals to 0. If  $B = 0$  the solution is trivial and therefore we assume  $B > 0$ .

The sum  $\sum |Ba_j - Cb_j|^2$  consists of non-negative terms and can only equal 0 if  $|Ba_j - Cb_j|^2 = 0$  for any  $1 \leq j \leq n$ . The absolute value of any complex number can only equal to 0 if that complex number is 0 by Theorem 1.31 (d). Hence

$$Ba_j - Cb_j = 0 \quad \Rightarrow \quad a_j = \frac{C}{B}b_j.$$

We can see here that if  $a_j$  satisfy the relation above, then equality holds in Schwarz inequality.