**11.** If $z = 0$, then any $w \in \mathbb{C}$ such that $|w| = 1$ together with $r = 0$ satisfies $z = rw$. For example, if $w$ equals $i$ or 1, then $rw$ satisfy the condition. In this case $w$ is not uniquely determined by $z$, but $r$ is.

Assume that $z \neq 0$. Put $r = |z|$ and $w = z \cdot 1/|z|$. Since the real number $1/|z|$ is the multiplicative inverse of the real number $|z|$, then clearly $|w| = 1$ by field axiom (M5) and Theorem 1.31 (b). Now we show that $z$ can be written as $rw$,

$$z = z \cdot \frac{|z|}{|z|} = |z| \left( z \cdot \frac{1}{|z|} \right) = rw.$$

We may do so because multiplication is commutative and $z \neq 0 \Rightarrow |z| > 0$ by Theorem 1.31 (d).

We shall now demonstrate that whenever $z \neq 0$, then $r$ and $w$ are uniquely determined by $z$. Suppose not. Then there exists $q > 0$, $v \neq w$ such that $|v| = 1$ and $z = qv = rw$. It follows that $|z| = |rw| = |r||w| = r = |qv| = |q||v| = q$. Hence $r = q$ and since $z = rw = qv$,

$$0 = rw - qu = r(w - q).$$

Because $r > 0$ we have that $w = v$ and we get a contradiction since we assumed otherwise.

$\square$