18. Suppose not. Then there does not exists any $\mathbf{y} \neq \mathbf{0}$ such that $\mathbf{x} \cdot \mathbf{y} = \mathbf{0}$ for $k \geq 2$. Let $\mathbf{e}_i = (0, \cdots, 1, 0, \cdots, 0)$ be the k tuple with zeros at all coordinates except at i where it is 1. Since we assume the statement to be false, clearly $\mathbf{e}_i \cdot \mathbf{x} = x_i \neq 0$ since $\mathbf{e}_i \neq \mathbf{0}$. Put $\mathbf{y} = \mathbf{e}_1 + \alpha \mathbf{e}_2$ where $\alpha = -x_1/x_2$. We see that $|\mathbf{y}| = \sqrt{1 + \alpha^2} \geq 1$ which implies $\mathbf{y} \neq \mathbf{0}$ by Theorem 1.37. It follows that

$$\mathbf{y} \cdot \mathbf{x} = (\mathbf{e}_1 + \alpha \mathbf{e}_2) \cdot \mathbf{x} = x_1 + \alpha x_2 = x_1 - \frac{x_1}{x_2} x_2 = 0,$$

which is a contradiction since $\mathbf{y} \neq \mathbf{0}$ yet $\mathbf{x} \cdot \mathbf{y} = 0$.

If $k = 1$, then the statement is false for $x \neq 0$. To see this recall that $\mathbb{R}$ is a field for which Proposition 1.16 holds. Since both $x \neq 0$ and $y \neq 0$, we have that $xy \neq 0$.