

10. First note that $u \leq \sqrt{u^2} \leq \sqrt{u^2 + v^2} = |w|$ using Theorem 1.18 (d). Hence $|w| + u \geq 0$ and $|w| - u \geq 0$. Therefore, we may use the corollary to Theorem 1.21 in the simplification below

$$\begin{aligned} 2ab &= 2\left(\frac{|w| + u}{2}\right)^{1/2} \left(\frac{|w| - u}{2}\right)^{1/2} \\ &= \left((|w| + u)(|w| - u)\right)^{1/2} = \left(|w|^2 - u^2\right)^{1/2} \\ &= \left(u^2 + v^2 - u^2\right)^{1/2} = |v|. \end{aligned}$$

We also have the following simplification,

$$a^2 - b^2 = \frac{|w| + u}{2} - \frac{|w| - u}{2} = u.$$

Using the results above we have that

$$\begin{aligned} z^2 &= a^2 + i2ab - b^2 = u + i|v| = w, \text{ if } v \geq 0 \\ (\bar{z})^2 &= a^2 - i2ab - b^2 = u - i|v| = w, \text{ if } v \leq 0. \end{aligned}$$

By proposition 1.16 (d) we have that for any $x \in \mathbb{C}$ it holds that $(-x)(-x) = (-x)^2 = x^2$. Hence we can conclude that for every complex number w , with exception for 0, there exists two square roots. They are either $\pm z$ or $\pm \bar{z}$, depending on wheter $\text{Im}(w)$ is positive or negative respectively.

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