

Exercise 1.35: We show that the golden ratio φ is a fixed point to $x \mapsto 1 + 1/x$. If φ is a fixed point, then successive function applications should not alter the value. Let $f(x) = 1 + 1/x$, then

$$f(\varphi) = 1 + \frac{1}{\varphi} = \frac{\varphi + 1}{\varphi} = \frac{\varphi^2}{\varphi} = \varphi,$$

where we have used the fact $\varphi^2 = \varphi + 1$. Clearly, applying f again will result in the same value.

□