Theorema 2.0: A First Tour

This is a Theorema notebook. **If this notebook is displayed within the Help-system, it is not possible to perform any actions in the notebook!** In order to follow the examples in this tutorial "live" in your Theorema installation, we recommend to open the Theorema notebook "FirstTour.nb" in the directory "TheoremaNotebooks" in the documentation directory of your Theorema installation using the "Open"-button in the Theorema Commander.

You can still browse through the notebook within the Help-system and it will be fully functional (after enabling dynamic objects).

For Wolfram Workbench users: We recommend to move the entire directory “TheoremaNotebooks” including all subdirectories) from the documentation directory of your Theorema installation into a location outside the Theorema Workbench project. Otherwise, the workbench will notice a change within the project as soon as the auxiliary files in the associated directory are updated and start reloading the project over and over.

We consider “proving”, “computing”, and “solving” as the three basic mathematical activities.

Proving

We want to prove

$$\left(\underset{x}{\forall}\,(P[x] \vee Q[x])\right) \wedge \left(\underset{y}{\forall}\,(P[y] \Rightarrow Q[y])\right) \Leftrightarrow \left(\underset{x}{\forall}\,Q[x]\right).$$

To prove a formula like the above, we need to enter it in the context of a Theorema environment.

Proposition (First Test, 2014)

In[ ]:= $\left(\left(\underset{x}{\forall}\,(P[x] \vee Q[x])\right) \wedge \left(\underset{y}{\forall}\,(P[y] \Rightarrow Q[y])\right)\right) \Leftrightarrow \left(\underset{x}{\forall}\,Q[x]\right)$ ((P or Q ∧ if P then Q) ⇔ Q)

☑ Proof of ((P or Q ∧ if P then Q) ⇔ Q) #1: *Show proof*

Computing

Definition (Lexical Ordering)

In[∘]:= $\forall_{\substack{a,b \\ |a| == |b|}}$

In[∘]:= $a <_{lex} b :\Longleftrightarrow \left(\exists_{i=1,\ldots,|a|} \left(a_i < b_i \wedge \left(\forall_{j=1,\ldots,i-1} (a_j == b_j) \right) \right) \right)$ (<lex)

In[∘]:= $\langle 1, 1, 1 \rangle <_{lex} \langle 1, 2, 0 \rangle$

Domain Definitions

Definition (Monomials)

In[∘]:= $\forall_{K}$

In[∘]:= $\mathrm{Mon}[K] := \underset{M}{\Delta}$

In[∘]:= $\forall_{m1,m2}$

In[∘]:= $m1 \underset{M}{*} m2 := \left\langle m1_1 \underset{K}{*} m2_1, \left\langle (m1_2)_i \underset{\mathbb{N}}{+} (m2_2)_i \;\Big|_{i=1,\ldots,|m1_2|} \right\rangle \right\rangle$ (1)

In[∘]:= $\left(m1 \underset{M}{<} m2 \right) :\Longleftrightarrow (m1_2 <_{lex} m2_2)$ (2)

In[∘]:= $\langle 3, \langle 1, 2 \rangle \rangle \underset{\mathrm{Mon}[\mathbb{Q}]}{*} \langle 3/4, \langle 3, 7 \rangle \rangle$

In[∘]:= $\langle 3, \langle 1, 2 \rangle \rangle \underset{\mathrm{Mon}[\mathbb{Q}]}{<} \langle 3/4, \langle 3, 7 \rangle \rangle$

Set Theory

Definition (subset)

In[∘]:= $\forall_{x,y}$

In[∘]:= $x \subseteq y := \left(\forall_{z} (z \in x \Rightarrow z \in y) \right)$ (3)

Proposition (transitivity of ⊆)

In[∘]:= $\underset{a,b,c}{\forall}\left(\left(a \subseteq b \land b \subseteq c\right) \Rightarrow a \subseteq c\right)$ (4)

⚠ Proof of (4) #1: *Show proof*

☑ Proof of (4) #2: *Show proof*