Theorema 2.0: A First Tour

NB reached List of cells reached Cell Group
Data reached List of cells reached $\mathbf{Null}\mathbf{Cell}$ reached

We consider "proving", "computing", and "solving" as the three basic mathematical activities.

CellGroupData reached List of cells reached

1 Proving

We want to prove

$$(\displaystyle \mathop{\forall}_{x} (P[x] \vee Q[x])) \wedge (\displaystyle \mathop{\forall}_{y} (P[y] \Rightarrow Q[y])) \Leftrightarrow (\displaystyle \mathop{\forall}_{x} Q[x]).$$

To prove a formula like the above, we need to enter it in the context of a Theorema environment.

1.1 Proposition (First Test, 2014)

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Cell reached CellGroupData reached List of cells reached Cell reached List of cells reached

2 Computing

CellGroupData reached List of cells reached Cell reached

2.0.1 Global Declaration

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2.1 [?]

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2.1.1 Global Declaration

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2.1.2 Global Declaration

 $\mathrm{Mon}[\mathrm{K}]{:=} \underline{\Delta}_{M}$

2.1.3 Global Declaration

 $\underset{m1,m2}{\forall}$

2.2 [?]

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2.3 [?]

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■Cell reached CellGroupData reached List of cells reached Cell reached Cell reached CellGroupData reached List of cells reached

3 Set Theory

CellGroupData reached List of cells reached Cell reached

3.0.1 Global Declaration

 $\forall x,y$

[?]3.1

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3.2 Proposition (transitivity of \subseteq)

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