

Eq. of straight line

① slope = m , Passes Through $(0,0)$

$$\rightarrow y = mx$$

② The y-intercept of line

$$\rightarrow y = mx + c$$

③ slope = m , Passing Through (x_1, y_1)

$$\rightarrow y - y_1 = m(x - x_1)$$

④ Passing Through (x_1, y_1) , (x_2, y_2)

$$m = \frac{y_2 - y_1}{x_2 - x_1} \rightarrow (y - y_1) = m(x - x_1) \rightarrow (y - y_1) = \frac{(y_2 - y_1)}{(x_2 - x_1)}(x - x_1)$$

$$\frac{(y - y_1)}{(y_2 - y_1)} = \frac{(x - x_1)}{(x_2 - x_1)}$$

⑤ The most general eq. of st. line

$$ax + by + c = 0 \quad y = \frac{-ax}{b} - \frac{c}{b}$$

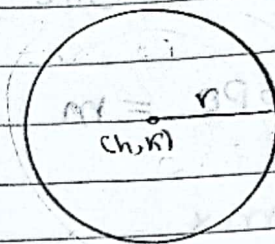
Horizontal st. line $\rightarrow a = 0$

Vertical st. line $\rightarrow b = 0$

$$\text{Slope} = \frac{-a}{b}$$

★ Circle

→ Center (h, k) , $r > 0$



$$\rightarrow r = \sqrt{(x-h)^2 + (y-k)^2}$$

→ The standard eq. $\rightarrow (x-h)^2 + (y-k)^2 = r^2$

Center $(h, k) \rightarrow$ reverse the sign from eq.

★ to write the eq. of a circle in standard form

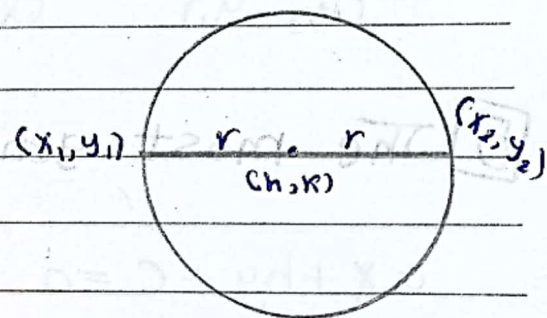
① Group the same variable together and position constant on other side

② Complete the square on both variable as needed.

③ Divide the both sides by the coefficients of squares.

Mid point \rightarrow (center) $(h, k) :$

$$(h, k) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$



Distance \rightarrow diameter or radius :

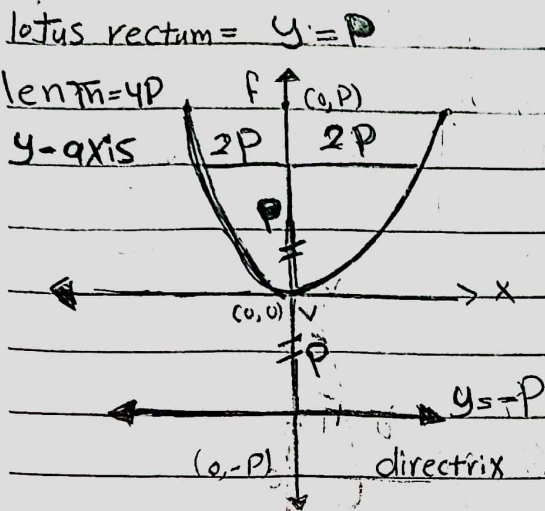
$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}, \quad r = \frac{1}{2} \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

unit circle :

Center $= (0, 0)$ with $r = 1$

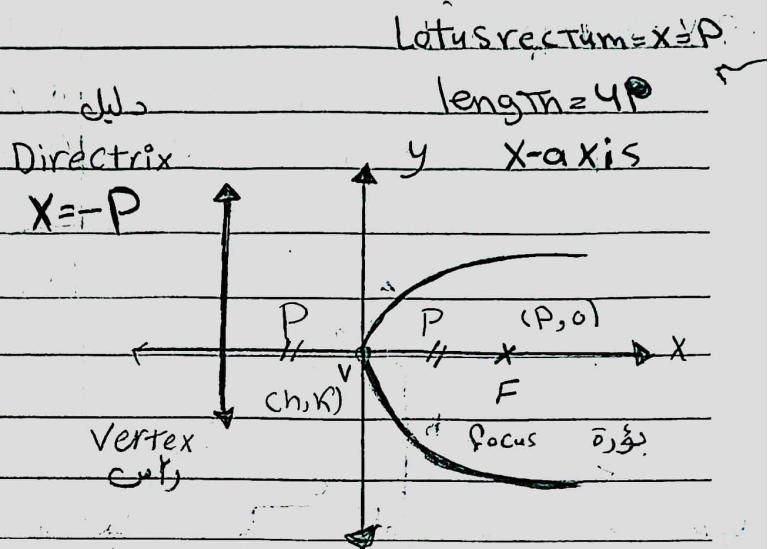
→ standard eq. $= x^2 + y^2 = 1$

Parabola: Let F Point in the plane and D be a line not containing F . it's the set of all points equidistant from F and D . The Point F is called focus of the parabola and, D is called the directrix of parabola.



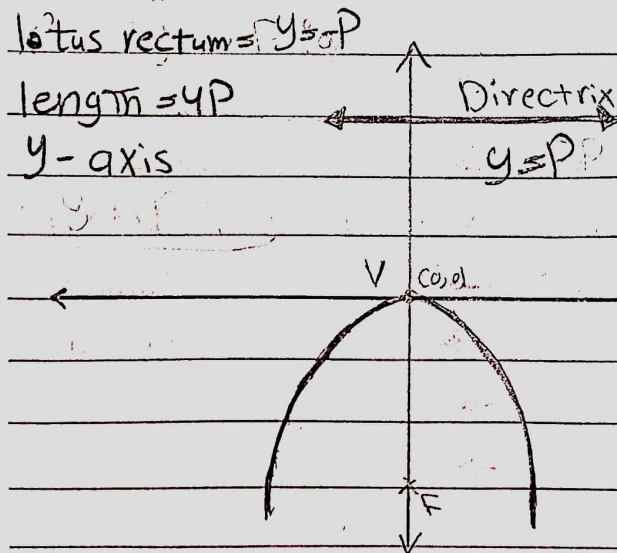
$$\sqrt{(x-0)^2 + (y-p)^2} = \sqrt{x^2 + (y+p)^2}$$

$$x^2 = 4py, \quad x^2 = ny$$



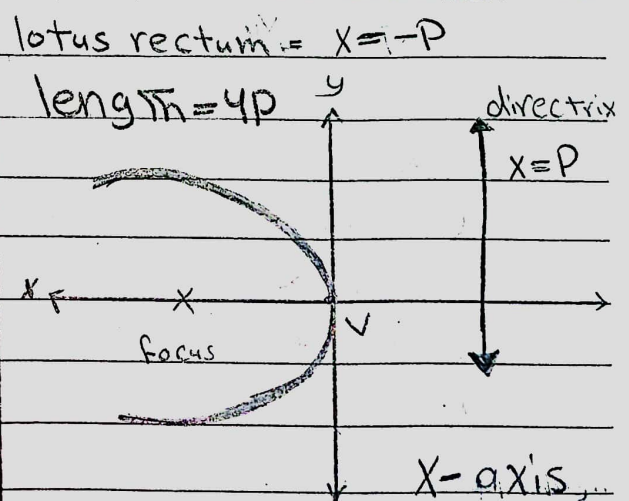
$$(y-k)^2 = 4p(x-h), \quad p > 0$$

$$y^2 = 4px, \quad y^2 = nx$$



$$(x-h)^2 = 4p(y-k), \quad p < 0$$

$$x^2 = 4py, \quad x^2 = -ny, \quad p < 0$$



$$y^2 = 4px, \quad y^2 = -nx, \quad p < 0$$

$x^2 \rightarrow$ upward, downward
 $y^2 \rightarrow$ left, right

$+p \rightarrow$ Right, upward
 $-p \rightarrow$ left, downward

$$x^2 \rightarrow F(h, k+p)$$

$$y^2 \rightarrow F(h+p, k)$$

★ Ellipse

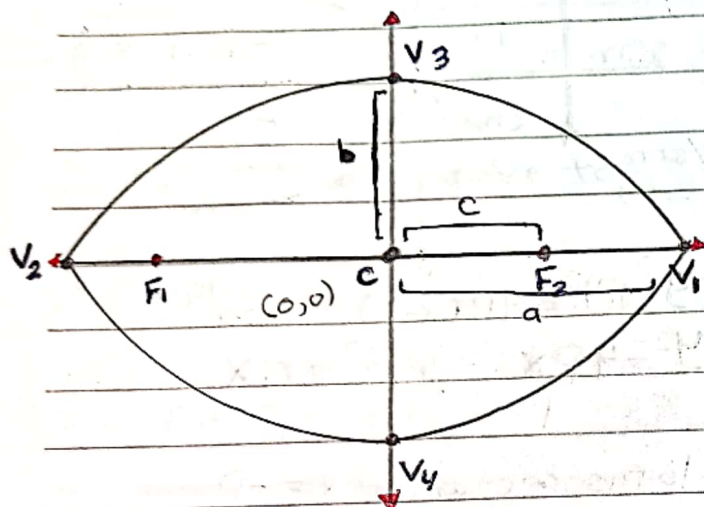
→ (Horizontal ellipse) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (Vertical ellipse) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$a > b$

$$c^2 = a^2 - b^2, e = \frac{c}{a}$$

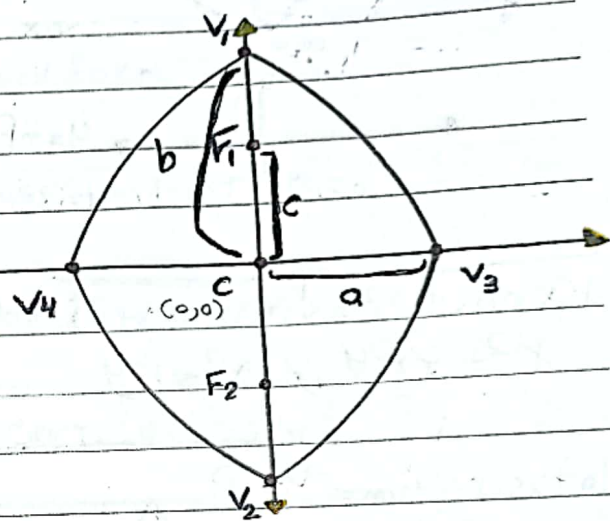
★ Changeable x , constant y

- ① Vertex - Foci - center
- ② Major axis - minor axis
- ③ EndPoints of minor axis



★ Changeable y , constant x

- ① Vertex - Foci - center
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Center → From eq. $(0,0)$

V_1, V_2 (Major) → $(a,0), (-a,0)$

V_3, V_4 (Minor) → $(0,b), (0,-b)$

F_1, F_2 (Foci) → $(c,0), (-c,0)$

Major → $y = \text{---}$

Minor → $x = \text{---}$

Center → From eq. $(0,0)$

V_1, V_2 → $(0,b), (0,-b)$ (Major)

V_3, V_4 → $(a,0), (-a,0)$ (minor)

F_1, F_2 → $(0,c), (0,-c)$ (Foci)

Major → $x = \text{---}$

minor → $y = \text{---}$

★ Center (x_1, y_1) , Vertex (x, z) , Focus (x, t)

$$b = \sqrt{(x_1 - x)^2 + (y_1 - z)^2} = \text{---}$$

$$c = \sqrt{(x_1 - x)^2 + (y_1 - t)^2} \rightarrow \frac{(x - x_1)^2}{a^2} + \frac{(y - y_1)^2}{b^2} = 1 \quad \text{Standard eq.}$$

$$a = \sqrt{b^2 - c^2} = \text{---}$$

★ Sketch The graph of The given Parabola, Find The vertex, Focus, directrix and The endpoint of The latus rectum

① $(x-3)^2 = -16y \rightarrow x^2, -P$ (downward)

$V = (3, 0)$

$4P = -16 \rightarrow P = -4$

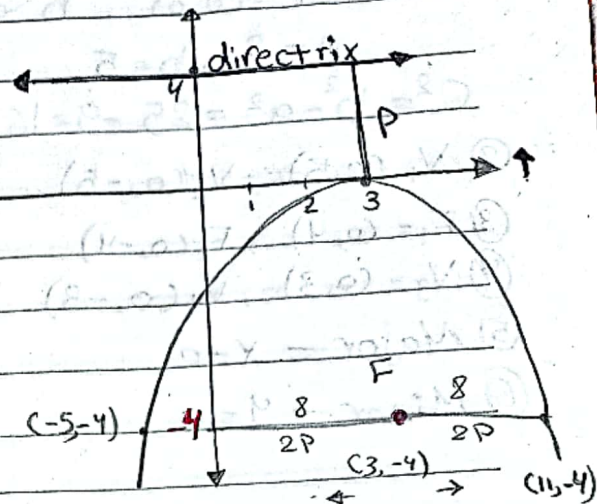
$F(h, k+P) = (3, -4)$

directrix $= y = 4$

end Points $= (-5, -4), (11, -4)$

lotus rectum (latus diameter) $\rightarrow y = -4 \rightarrow$

length $= |4P| = 16$



② $(y-2)^2 = -12(x+3) \rightarrow y^2, -P$ (left)

$V = (-3, 2)$

$4P = -12 \rightarrow P = -3$

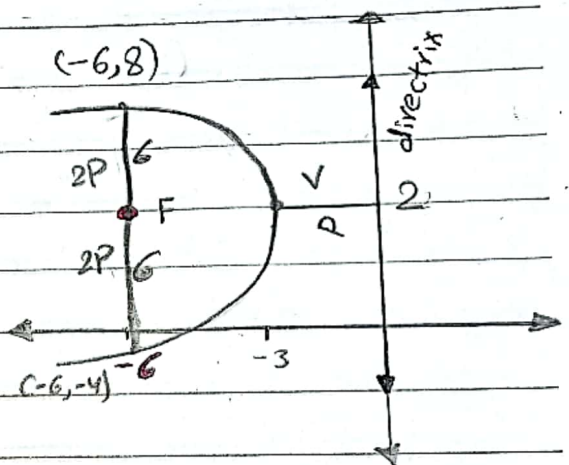
$F(h+P, k) = (-6, 2)$

directrix $= x = 0$

end Points $= (-6, 8), (-6, -4)$

lotus rectum $= x = -6 \rightarrow$

length $= |4P| = 12$



★ Ellipse \rightarrow Find The Center, The Line which contain Major, minor, The Vertex, Foci, and The end Points of The minor axis.

① $\frac{x^2}{169} + \frac{y^2}{25} = 1$ (Horizontal)

② Center $(0, 0)$, $a > b$, $a = 13, b = 5$

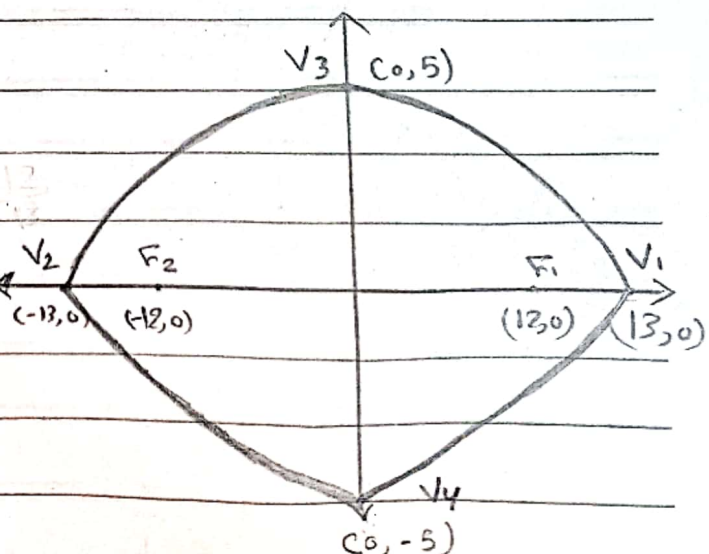
$c^2 = a^2 - b^2 = 169 - 25 = 144 \Rightarrow c = 12$ ③ $e = \frac{c}{a} = \frac{12}{13}$

④ Foci $= (12, 0), (-12, 0)$

⑤ $V_1, V_2 = (13, 0), (-13, 0)$

⑥ $V_3, V_4 = (0, 5), (0, -5)$

⑦ Major $= y = 0$, Minor $= x = 0$



② $\frac{x^2}{9} + \frac{y^2}{25} = 1$ (Vertical)

① Center = $(0, 0)$, $b > a$

$a = 3$, $b = 5$

$c^2 = b^2 - a^2 = 25 - 9 = 16 \rightarrow c = \pm 4$

② $V_1(0, 5)$, $V_2(0, -5)$

③ $F_1(0, 4)$, $F_2(0, -4)$

④ $V_3(0, 3)$, $V_4(0, -3)$

⑦ $e = \frac{c}{b} = \pm \frac{4}{5}$

⑤ Major = $x = 0$

⑥ Minor = $y = 0$

