Subject: lecture 5 موضوع الدرس

Date التاريخ

Rule 4: If $M(x,y)\,dx + N(x,y)\,dy = 0$ be a homogenous Function

$\frac{dy}{dx} = \frac{x^3+y^3}{xy^2}$ The I.F. $\frac{1}{Mx+Ny}$

$(x^3+y^3)\,dx - (xy^2)\,dy = 0$

$M_y = 3y^2$, $N_x = -y^2 \rightarrow$ ① $\quad \frac{1}{Mx+Ny} = \frac{1}{x^4+\cancel{xy^3}-\cancel{xy^3}} = \frac{1}{x^4}$

$\left(\frac{1}{x} + \frac{y^3}{x^4}\right)dx - \frac{y^2}{x^3}\,dy = 0$

$M_y = \frac{3y^2}{x^4}$ $\quad N_x = \frac{3y^2}{x^4}$

$\int \frac{dx}{x} + y^3 \int \frac{dx}{x^4} = C \rightarrow \ln x - \frac{y^3}{3x^3} = C$

① $(2x\sinh\frac{y}{x} + 3y\cosh\frac{y}{x})\,dx - 3x\cosh(\frac{y}{x})\,dy = 0$

$M_y = 2\cancel{x}\cdot\frac{1}{\cancel{x}}\sinh\frac{y}{x} + 3y\cosh(\frac{y}{x})\cdot\frac{1}{x} + 3\cosh\frac{y}{x}$

$N_x = -3\cosh\frac{y}{x} + 3x\sin\frac{y}{x}\cdot\frac{y}{x^2}$

$I.F = \frac{1}{2x^2\sinh\frac{y}{x} + 3yx\cosh\frac{y}{x} - 3xy\cosh\frac{y}{x}} = \frac{1}{2x^2\sinh(\frac{y}{x})}$

$\left(\int\frac{dx}{x} + \int\frac{3y}{2x^2}\coth(\frac{y}{x})\,dx = C \rightarrow d(\frac{y}{x}) = \frac{y}{-x^2}\,dx\right.$

$\int \cot x = \int \frac{\cos x}{\sin x}\,dx$

$= \ln x - \frac{3}{2}\ln\sinh\frac{y}{x} = C$

Linear differential eq. of first order

$\frac{dy}{dx} + P(x)\, y = Q(x)$ $\qquad M = e^{\int P(x)\, dx}$

$y \cdot M = \int M \cdot Q\, dx + c$

★ Solve $\frac{dy}{dx} + \frac{2}{x} y = x^2$ $\qquad P(x) = \frac{2}{x}$, $Q(x) = x^2$

$M = e^{2\int \frac{dx}{x}} = e^{\overbrace{2\ln x}} = x^2$

$y \cdot x^2 = \int x^4\, dx + c \longrightarrow y \cdot x^2 = \frac{1}{5} x^5 + c$

★ $(x+1) \frac{dy}{dx} - y = e^x (x+1)^2$ $\qquad P(x) = -\frac{1}{x+1}$, $Q(x) = e^x (x+1)$

$\frac{dy}{dx} - \boxed{\frac{1}{(x+1)}}\, y = \boxed{e^x (x+1)} \longrightarrow M = e^{-\int \frac{dx}{x+1}} = (x+1)^{-1}$

$y (x+1)^{-1} = \int e^x\, dx + c \;=\; y (x+1)^{-1} = e^x + c$

★ $\frac{dy}{dx} - \frac{2}{x} y = \frac{1}{x^2}$ $\qquad P(x) = \frac{-2}{x}$, $Q(x) = \frac{1}{x^2}$

$M = e^{\int \frac{-2}{x}} = e^{-2\ln x} = x^{-2} = \frac{1}{x^2}$

$y \cdot \frac{1}{x^2} = \int \frac{1}{x^2} \cdot \frac{1}{x^2}\, dx + c \rightarrow y \cdot \frac{1}{x^2} = \frac{-1}{3x^3} + c$

★ $\frac{dy}{dx} + 2y \tan x = \sin x$ $\qquad P(x) = 2\tan x$, $Q(x) = \sin x$

$\tan x = \frac{\sin x}{\cos x}$

$M = e^{\int 2\tan x} = e^{-2\ln\cos x} = \cos^{-2} x = \sec^2 x$

$\boxed{y \cdot \sec^2 x = \sec x - 2}$

$y \cdot \sec^2 x = \int \frac{\sin x}{\cos^2 x}\, dx \Longrightarrow y \cdot \sec^2 x = \sec x + c \rightarrow 0 = 2 + c$

$\boxed{c = -2}$