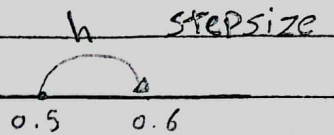


★ Find The value of  $f(0.6)$  by Taylor series if  $f(0.5) = 1$

$f'(0.5) = 2.5$ ,  $f''(0.5) = 2$  and all other higher order derivatives

of  $f(x)$  at  $x = 0.5$  are zero



$$f(x+h) = f(x) + hf'(x) + \frac{h^2}{2!} f''(x)$$

$$h = 0.6 - 0.5 = 0.1$$

$$f(0.6) = f(0.5) + 0.1 f'(0.5) + \frac{(0.1)^2}{2!} f''(0.5) = 1.26$$

\* Find  $y(0.1)$  if  $y' = -y + x + 1$ ,  $y(0) = 1$  by using Taylor series to fourth order

$$y(x+h) = y(x) + h(y'(x)) + \frac{h^2}{2!} (y''(x)) + \frac{h^3}{3!} y'''(x)$$

$$h = 0.1 - 0 = 0.1, \quad x_0 = 0, \quad y_0 = 1 \quad \rightarrow \quad y' = (-x + x + 1)$$

$$y(0.1) = y(0) + 0.1 \times y'(0) + \frac{(0.1)^2}{2!} y''(0) + \frac{(0.1)^3}{3!} y'''(0)$$

$$y(0.1) = 1 + 0.1 \times (-1 + 0 + 1) + \frac{(0.1)^2}{2!} (1) + \frac{(0.1)^3}{3!} (-1)$$

$$y'' = -y' + 1 = 1, \quad y''' = -y'' = -1$$

$$y(0.1) = 1.0048332$$

\*Maclauren

$$f(h) = f(c) + h(f'(c)) + \frac{h^2}{2!} f''(c) +$$

$$f(x) = e^x$$

$$f(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} +$$

error

$$R_n = \frac{(x-h)^{n+1} \cdot f^{(n+1)}(c)}{(n+1)!}$$

$$c \in [x, x+h]$$

\*How many terms would it require to get an approximation of  $e^1 (h=1)$  within a magnitude of absolute error of less than  $10^{-6}$

$$R_n = \frac{(x-h)^{n+1}}{(n+1)!} f^{(n+1)}(c)$$

$$c \in [x, x+h]$$

$$R_n = \frac{(-1)^{n+1}}{(n+1)!} e^c \rightarrow \frac{1}{(n+1)!} \leq |R_n| \leq \frac{e}{(n+1)!}$$

$$\frac{e}{(n+1)!} \leq 10^{-6} \xrightarrow{\div 10^{-6}} 10^6 e \leq (n+1)! \rightarrow 10^6 e \leq n(n+1)$$

$$n \geq 9$$



→ Forward elimination

→ Back sub.

→ number of steps :: of forward elimination  $(n-1)$ 

Solve The system by Gauss elimination

$$\left[ \begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 2 & -3 & 2 & 4 \\ 3 & -1 & -1 & -2 \end{array} \right] \xrightarrow{\substack{2R_1 - R_2 \\ 3R_1 - R_3}} \left[ \begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 0 & 7 & 4 & 8 \\ 0 & 7 & 10 & 20 \end{array} \right] \xrightarrow{R_2 - R_3}$$

$$\left[ \begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 0 & 7 & 4 & 8 \\ 0 & 0 & -6 & -12 \end{array} \right] \quad \begin{cases} -6Z = -12 \\ Z = 2 \end{cases} \quad \begin{cases} 7Y + 4(2) = 8 \\ Y = 0 \end{cases} \quad \begin{cases} X + 2(0) + 2(2) = 6 \\ X = 0 \end{cases}$$

Assignment 1:

$$\left[ \begin{array}{ccc|c} 2 & -3 & 1 & 5 \\ 1 & 0 & -4 & 3 \\ 0 & 5 & 3 & 8 \end{array} \right] \xrightarrow{\substack{\frac{1}{2}R_1 - R_2 \\ \frac{10}{3}R_2 - R_3}} \left[ \begin{array}{ccc|c} 2 & -3 & 1 & 5 \\ 0 & -\frac{3}{2} & \frac{9}{2} & \frac{1}{2} \\ 0 & 0 & 18 & \frac{19}{3} \end{array} \right]$$

$$18Z = \frac{19}{3} \quad \begin{cases} -\frac{3}{2}(Y) + \frac{9}{2}(\frac{19}{54}) = \frac{1}{2} \\ Y = \frac{25}{18} \end{cases} \quad \begin{cases} 2(X) - 3(\frac{25}{18}) + (\frac{19}{54}) = 5 \\ X = \frac{119}{27} \end{cases}$$

$$\boxed{Z = \frac{19}{54}}$$

$$\boxed{Y = \frac{25}{18}}$$

$$\boxed{X = \frac{119}{27}}$$