Subject موضوع الدرس Section 4

Date التاريخ

Linear D.E

$y' + P(x)\, y = q(x)$ $\qquad$ $y' - \frac{1}{x} y = 1 \quad P(x) = -\frac{1}{x}$

$M = e^{\int P(x)\, dx}$ $\qquad$ $My = \int M\, q(x)\, dx$

(1) $\frac{dy}{dx} + \frac{2}{x} y = x^2$

$P(x) = \frac{2}{x}$, $q(x) = x^2$

$M = e^{\int \frac{2}{x} dx} = x^2$

$My = \int x^2 \cdot x^2 dx$

$x^2 y = \int x^4 dx + C$

$x^2 y = \frac{x^5}{5} + C$

(2) $\frac{dy}{dx} - y \tan x = \sec x$

$P(x) = -\tan x$, $q(x) = \sec x$

$M = e^{-\int \tan x\, dx} = \cos x$

$My = \int \cos x \cdot \sec x\, dx + C$

$y \cos x = x + C$

(3) $(1+x^2)\, y' + 4xy = x \quad \div (1+x^2)$

$y' + \frac{4x}{1+x^2} y = \frac{x}{1+x^2}$ $\qquad$ $P(x) = \frac{4x}{1+x^2}$, $q(x) = \frac{x}{1+x^2}$

$M = e^{\int \frac{4x}{1+x^2} dx} = (1+x^2)^2$ $\qquad$ $My = \int (1+x^2)^{\cancel{2}} \cdot \frac{x}{\cancel{1+x^2}}\, dx + C$

$(1+x^2)^2 y = \int (x + x^3)\, dx + C \rightarrow (1+x^2)^2 y = \frac{x^2}{2} + \frac{x^4}{4} + C$

☆ Bernoulli D.E

$P(x)$, $q(x)$

$y' + P(x)y = q(x)y^n$

$Z = y^{1-n}$

$M = e^{\int P(x)dx} \rightarrow MZ = \int M q(x)\,dx + c$

(1) $\frac{dy}{dx} + \frac{2}{x}y = xy^2$

$P(x) = \frac{2}{x}$, $q(x) = x$, $n = 2$

$M = e^{\int \frac{2}{x}dx} = x^2$

$x^2 y^{1-2} = \int x^2 \cdot x\,dx + c$

$x^2 y^{-1} = \frac{x^4}{4} + c$

(2) $2xy\frac{dy}{dx} - y^2 + x = 0$

$\div 2xy$

$\frac{dy}{dx} - \frac{1}{2x}y = -\frac{1}{2}y^{-1}$

$P(x) = -\frac{1}{2x}$, $q(x) = -\frac{1}{2}$, $n = -1$

$Z = y^{1-n} = y^{1+1} = y^2$

$M = e^{\int -\frac{1}{2x}dx} = x^{-\frac{1}{2}}$

$x^{-\frac{1}{2}}y^2 = \frac{1}{2}\int x^{-\frac{1}{2}}\,dx + c$

(3) $\frac{dy}{dx} - \frac{3}{x}y = x^4 y^{1/3}$

$P(x) = -\frac{3}{x}$, $q(x) = x^4$, $n = \frac{1}{3}$

$Z = y^{1-1/3} = -2/3$

$M = e^{\int -\frac{3}{x}dx} = 1/x^3$

$\frac{1}{x^3} \times y^{\frac{2}{3}} = \int \frac{1}{x^3} \cdot x^4\,dx + c$

$\frac{1}{x^3}y^{-2/3} = \frac{x^2}{2} + c$

محمد و أميرة

Subject	موضوع الدرس	Date	التاريخ

D.E of first order, higher degree solve in P: $P=F(x,y)$,

1) $P^2+P-6=0 \to (P+3)(P-2) \to P=-3,\ P=2$

$P=\frac{dy}{dx}$

$\frac{dy}{dx}=-3$ | $\frac{dy}{dx}=2$

$dy=-3dx$ | $dy=2dx$

$y=-3x+c \rightarrow y+3x+c=0$

$y=2x+c \rightarrow y-2x+c=0$

2) $x^2P^2+xyP-6y^2=0$

$(xP-2y)(xP+3y)$

$xP=2y \to P=\frac{2y}{x}$

$xP=-3y \to P=\frac{-3y}{x}$

$dy=\frac{2y}{x}dx \Rightarrow \int \frac{1}{y}dy=\int \frac{2}{x}dx \to \cancel{\ln} y=\cancel{\ln} x^2+\ln c$

$dy=\frac{-3y}{x}dx \Rightarrow \int \frac{1}{y}dy=\int -\frac{3}{x}dx \to \cancel{\ln} y=\cancel{\ln} x^{-3}+\ln c$

$y=x^2+c$

$y=x^{-3}+c$ $\rightarrow (y-x^2c)(y-x^3c)=0$

Solve in x :

$P=\frac{dy}{dx}$

① $x=P^3 \to \frac{dx}{dy}=3P^2\frac{dP}{dy} \Rightarrow \frac{1}{P}=3P^2\frac{dP}{dy}$

(×P) $\int dy=\int 3P^3\,dp \rightarrow y=\frac{3}{4}P^4+c$

② $x=4(P^3+P) \to \frac{dx}{dy}=(12P^2+4)\frac{dP}{dy}$ $\quad P=dy/dx$

(×P) $\frac{1}{P}=(12P^2+4)\frac{dP}{dy} \to \int dy=\int 12P^3+4P\,dP \to y=3P^4+2P^2+c$

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