not exact

$M_x + N_y = 0$

(1) $f(x) = \frac{M_y - N_x}{N}$

(2) $g(y) = \frac{N_x - M_y}{M}$

(1) $M = e^{\int f(x)dx}$

(2) $N = e^{\int g(y)dy}$

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★ $(2x^2+y)dx + (x^2y - x)dy = 0$

$M_y = 1$ $N_x = 2xy - 1$

$f(x) = \frac{1 - 2xy + 1}{x^2y - x} = \frac{-2(xy - 1)}{x(xy-1)} = \frac{-2}{x}$

$M = e^{\int -\frac{2}{x}dx} = e^{-2\ln x} = e^{\ln x^{-2}} = \frac{1}{x^2}$ (*)

$\rightarrow (2 + \frac{y}{x^2})dx + (y - \frac{1}{x})dy = 0$

$M_y = \frac{1}{x^2}$ $N_x = \frac{1}{x^2}$ exact

★ $xy^3 dx + (x^2y^2 - 1)dy = 0$

$M_y = 3xy^2$ $N_x = 2xy^2$

$f(x) = \frac{3xy^2 - 2xy^2}{x^2y^2 - 1} = \frac{xy^2}{x^2y^2 - 1}$ X

$g(y) = \frac{2xy^2 - 3xy^2}{xy^3} = \frac{-xy^2}{xy^3} = -\frac{1}{y}$

$N = e^{\int g(y)dy} = e^{\int -\frac{1}{y}dy} = \frac{1}{y}$ X

$(xy^2)dx + (x^2y - \frac{1}{y})dy = 0$

$M_y = 2xy$ $N_x = 2xy$ Exact

**Subject** موضوع الدرس

$\star\ (\sin y + x^2 + 2x)\,dx + (\cos y)\,dy = 0$

$M_y = \cos y$ $\qquad N_x = 0$ $\qquad f(x) = \dfrac{\cos y - 0}{\cos y} = 1$

$\mu = e^{\int 1\,dx} = e^x$

$(e^x \sin y + x^2 e^x + 2xe^x)\,dx + e^x \cos y\,dy = 0$

$N_x = e^x \cos y$ $\qquad M_y = e^x \cos y \rightarrow$ Exact

$u = \int (e^x \sin y + x^2 e^x + 2xe^x)\,dx =$

| u | dv |
|---|---|
| $2x$ | $e^x$ |
| $2$ | $e^x$ |
| $0$ | $e^x$ |

| u | dv |
|---|---|
| $x^2$ | $e^x$ |
| $2x$ | $e^x$ |
| $2$ | $e^x$ |
| $0$ | $e^x$ |

$e^x \sin y + x^2 e^x - \cancel{2xe^x} + \cancel{2e^x} + \cancel{2xe^x} - \cancel{2e^x}$

$= e^x \sin y + x^2 e^x$

$u = \int e^x \cos y\,dy = e^x \sin y + c \Rightarrow e^x \sin y + x^2 e^x = C$

H.W

1. $(xy + y - 1)\,dx + x\,dy = 0$
2. $(xy + y^2)\,dx + (xy - x^2)\,dy = 0$
3. $(3y + 3e^x y^{2/3})\,dx + x\,dy =$
4. $2(2y^2 + 5xy - 2y + 4)\,dx + x(2x + 2y - 1)\,dy = 0$

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