

# Exercises

## 7.4 ELLIPSES

525

### 7.4.1 EXERCISES

In Exercises 1 - 8, graph the ellipse. Find the center, the lines which contain the major and minor axes, the vertices, the endpoints of the minor axis, the foci and the eccentricity.

1.  $\frac{x^2}{169} + \frac{y^2}{25} = 1$

2.  $\frac{x^2}{9} + \frac{y^2}{25} = 1$

3.  $\frac{(x-2)^2}{4} + \frac{(y+3)^2}{9} = 1$

4.  $\frac{(x+5)^2}{16} + \frac{(y-4)^2}{1} = 1$

5.  $\frac{(x-1)^2}{10} + \frac{(y-3)^2}{11} = 1$

6.  $\frac{(x-1)^2}{9} + \frac{(y+3)^2}{4} = 1$

7.  $\frac{(x+2)^2}{16} + \frac{(y-5)^2}{20} = 1$

8.  $\frac{(x-4)^2}{8} + \frac{(y-2)^2}{18} = 1$

In Exercises 9 - 14, put the equation in standard form. Find the center, the lines which contain the major and minor axes, the vertices, the endpoints of the minor axis, the foci and the eccentricity.

9.  $9x^2 + 25y^2 - 54x - 50y - 119 = 0$

10.  $12x^2 + 3y^2 - 30y + 39 = 0$

11.  $5x^2 + 18y^2 - 30x + 72y + 27 = 0$

12.  $x^2 - 2x + 2y^2 - 12y + 3 = 0$

13.  $9x^2 + 4y^2 - 4y - 8 = 0$

14.  $6x^2 + 5y^2 - 24x + 20y + 14 = 0$

In Exercises 15 - 20, find the standard form of the equation of the ellipse which has the given properties.

15. Center (3, 7), Vertex (3, 2), Focus (3, 3)

16. Foci (0, ±5), Vertices (0, ±8).

17. Foci (±3, 0), length of the Minor Axis 10

18. Vertices (3, 2), (13, 2); Endpoints of the Minor Axis (8, 4), (8, 0)

19. Center (5, 2), Vertex (0, 2), eccentricity  $\frac{1}{2}$

20. All points on the ellipse are in Quadrant IV except (0, -9) and (8, 0). (One might also say that the ellipse is "tangent to the axes" at those two points.)

# Answers

## 7.4.2 ANSWERS

1.  $\frac{x^2}{169} + \frac{y^2}{25} = 1$

Center  $(0, 0)$

Major axis along  $y = 0$

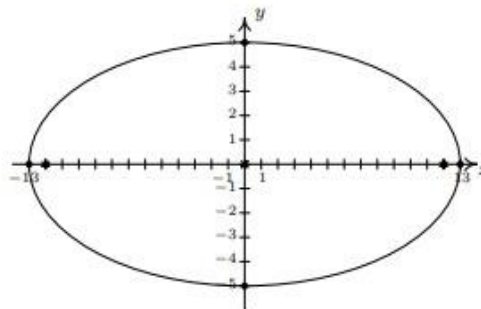
Minor axis along  $x = 0$

Vertices  $(13, 0), (-13, 0)$

Endpoints of Minor Axis  $(0, -5), (0, 5)$

Foci  $(12, 0), (-12, 0)$

$e = \frac{12}{13}$



2.  $\frac{x^2}{9} + \frac{y^2}{25} = 1$

Center  $(0, 0)$

Major axis along  $x = 0$

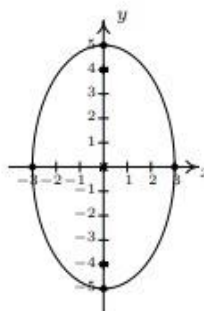
Minor axis along  $y = 0$

Vertices  $(0, 5), (0, -5)$

Endpoints of Minor Axis  $(-3, 0), (3, 0)$

Foci  $(0, -4), (0, 4)$

$e = \frac{4}{5}$



3.  $\frac{(x-2)^2}{4} + \frac{(y+3)^2}{9} = 1$

Center  $(2, -3)$

Major axis along  $x = 2$

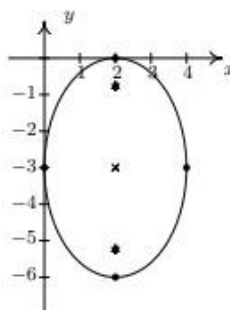
Minor axis along  $y = -3$

Vertices  $(2, 0), (2, -6)$

Endpoints of Minor Axis  $(0, -3), (4, -3)$

Foci  $(2, -3 + \sqrt{5}), (2, -3 - \sqrt{5})$

$e = \frac{\sqrt{5}}{3}$



4.  $\frac{(x+5)^2}{16} + \frac{(y-4)^2}{1} = 1$

Center  $(-5, 4)$

Major axis along  $y = 4$

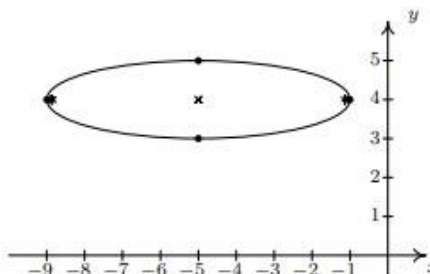
Minor axis along  $x = -5$

Vertices  $(-9, 4), (-1, 4)$

Endpoints of Minor Axis  $(-5, 3), (-5, 5)$

Foci  $(-5 + \sqrt{15}, 4), (-5 - \sqrt{15}, 4)$

$e = \frac{\sqrt{15}}{4}$



$$5. \frac{(x-1)^2}{10} + \frac{(y-3)^2}{11} = 1$$

Center  $(1, 3)$

Major axis along  $x = 1$

Minor axis along  $y = 3$

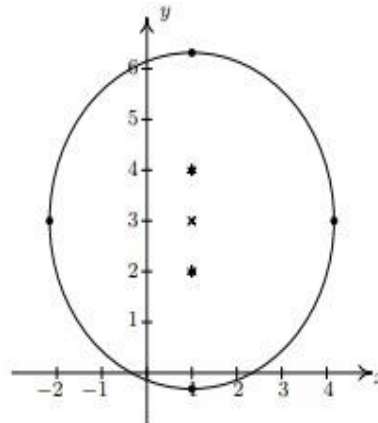
Vertices  $(1, 3 + \sqrt{11})$ ,  $(1, 3 - \sqrt{11})$

Endpoints of the Minor Axis

$(1 - \sqrt{10}, 3)$ ,  $(1 + \sqrt{10}, 3)$

Foci  $(1, 2)$ ,  $(1, 4)$

$$e = \frac{\sqrt{11}}{11}$$



$$6. \frac{(x-1)^2}{9} + \frac{(y+3)^2}{4} = 1$$

Center  $(1, -3)$

Major axis along  $y = -3$

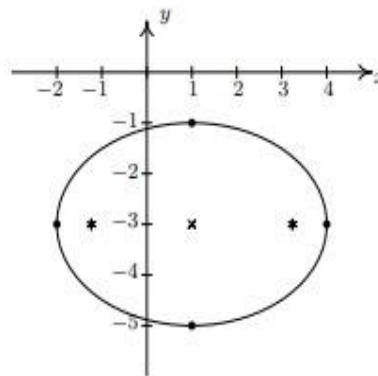
Minor axis along  $x = 1$

Vertices  $(4, -3)$ ,  $(-2, -3)$

Endpoints of the Minor Axis  $(1, -1)$ ,  $(1, -5)$

Foci  $(1 + \sqrt{5}, -3)$ ,  $(1 - \sqrt{5}, -3)$

$$e = \frac{\sqrt{5}}{3}$$



$$7. \frac{(x+2)^2}{16} + \frac{(y-5)^2}{20} = 1$$

Center  $(-2, 5)$

Major axis along  $x = -2$

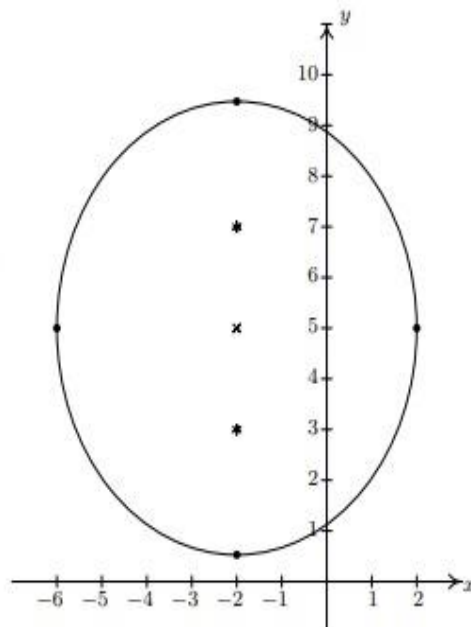
Minor axis along  $y = 5$

Vertices  $(-2, 5 + 2\sqrt{5})$ ,  $(-2, 5 - 2\sqrt{5})$

Endpoints of the Minor Axis  $(-6, 5)$ ,  $(2, 5)$

Foci  $(-2, 7)$ ,  $(-2, 3)$

$$e = \frac{\sqrt{5}}{5}$$



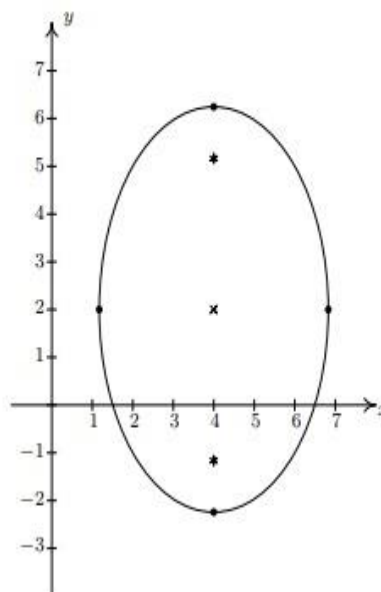
$$8. \frac{(x-4)^2}{8} + \frac{(y-2)^2}{18} = 1$$

Center  $(4, 2)$ Major axis along  $x = 4$ Minor axis along  $y = 2$ Vertices  $(4, 2 + 3\sqrt{2})$ ,  $(4, 2 - 3\sqrt{2})$ 

Endpoints of the Minor Axis

 $(4 - 2\sqrt{2}, 2)$ ,  $(4 + 2\sqrt{2}, 2)$ Foci  $(4, 2 + \sqrt{10})$ ,  $(4, 2 - \sqrt{10})$ 

$$e = \frac{\sqrt{5}}{3}$$



$$9. \frac{(x-3)^2}{25} + \frac{(y-1)^2}{9} = 1$$

Center  $(3, 1)$ Major Axis along  $y = 1$ Minor Axis along  $x = 3$ Vertices  $(8, 1)$ ,  $(-2, 1)$ Endpoints of Minor Axis  $(3, 4)$ ,  $(3, -2)$ Foci  $(7, 1)$ ,  $(-1, 1)$ 

$$e = \frac{4}{5}$$

$$10. \frac{x^2}{3} + \frac{(y-5)^2}{12} = 1$$

Center  $(0, 5)$ Major axis along  $x = 0$ Minor axis along  $y = 5$ Vertices  $(0, 5 - 2\sqrt{3})$ ,  $(0, 5 + 2\sqrt{3})$ Endpoints of Minor Axis  $(-\sqrt{3}, 5)$ ,  $(\sqrt{3}, 5)$ Foci  $(0, 2)$ ,  $(0, 8)$ 

$$e = \frac{\sqrt{3}}{2}$$

$$11. \frac{(x-3)^2}{18} + \frac{(y+2)^2}{5} = 1$$

Center  $(3, -2)$ Major axis along  $y = -2$ Minor axis along  $x = 3$ Vertices  $(3 - 3\sqrt{2}, -2)$ ,  $(3 + 3\sqrt{2}, -2)$ Endpoints of Minor Axis  $(3, -2 + \sqrt{5})$ , $(3, -2 - \sqrt{5})$ Foci  $(3 - \sqrt{13}, -2)$ ,  $(3 + \sqrt{13}, -2)$ 

$$e = \frac{\sqrt{26}}{6}$$

$$12. \frac{(x-1)^2}{16} + \frac{(y-3)^2}{8} = 1$$

Center  $(1, 3)$ Major Axis along  $y = 3$ Minor Axis along  $x = 1$ Vertices  $(5, 3)$ ,  $(-3, 3)$ Endpoints of Minor Axis  $(1, 3 + 2\sqrt{2})$ , $(1, 3 - 2\sqrt{2})$ Foci  $(1 + 2\sqrt{2}, 3)$ ,  $(1 - 2\sqrt{2}, 3)$ 

$$e = \frac{\sqrt{2}}{2}$$

13.  $\frac{x^2}{1} + \frac{4(y - \frac{1}{2})^2}{9} = 1$   
 Center  $(0, \frac{1}{2})$   
 Major Axis along  $x = 0$  (the  $y$ -axis)  
 Minor Axis along  $y = \frac{1}{2}$   
 Vertices  $(0, 2)$ ,  $(0, -1)$   
 Endpoints of Minor Axis  $(-1, \frac{1}{2})$ ,  $(1, \frac{1}{2})$   
 Foci  $(0, \frac{1+\sqrt{5}}{2})$ ,  $(0, \frac{1-\sqrt{5}}{2})$   
 $e = \frac{\sqrt{5}}{3}$

15.  $\frac{(x-3)^2}{9} + \frac{(y-7)^2}{25} = 1$

17.  $\frac{x^2}{34} + \frac{y^2}{25} = 1$

19.  $\frac{(x-5)^2}{25} + \frac{4(y-2)^2}{75} = 1$

14.  $\frac{(x-2)^2}{5} + \frac{(y+2)^2}{6} = 1$

Center  $(2, -2)$   
 Major Axis along  $x = 2$   
 Minor Axis along  $y = -2$   
 Vertices  $(2, -2 + \sqrt{6})$ ,  $(2, -2 - \sqrt{6})$   
 Endpoints of Minor Axis  $(2 - \sqrt{5}, -2)$ ,  
 $(2 + \sqrt{5}, -2)$   
 Foci  $(2, -1)$ ,  $(2, -3)$   
 $e = \frac{\sqrt{6}}{6}$

16.  $\frac{x^2}{39} + \frac{y^2}{64} = 1$

18.  $\frac{(x-8)^2}{25} + \frac{(y-2)^2}{4} = 1$

20.  $\frac{(x-8)^2}{64} + \frac{(y+9)^2}{81} = 1$