differential eq. → ordinary → $y=f(x) \to y', y'', y'''$,
→ Partial → $Z=Z(x,y) \to Z_x, Z_y, Z_{xx}, Z_{xy}$,

order: أعلى مشتقة

function $\underset{Int.}{\overset{Derv.}{\rightleftarrows}}$ ordinary (ode) degree: أعلى Power ل

1. Seperation (D.E) (There isnot $y \pm x$)
2. Homogenous (D.E) (All Variables Same degree) y^2, x^2, xy,
3. non homo. (D.E)
4. Exact (D.E)
5. Not Exact (D.E)

★ Seperation: نقدر نفصل كل متغير في طرف

$\frac{dy}{dx} = f(x)\, g(y) \to \frac{1}{g(y)}\, dy = f(x)\, dx \to \frac{dy}{g(y)} = f(x)\, dx$

$\to \int \frac{1}{g(y)}\, dy = \int f(x)\, dx$

$\to \int \frac{dy}{g(y)} = \int f(x)\, dx$

$\sin^2 x = \frac{1}{2}(1-\cos 2x)$
$\cos^2 x = \frac{1}{2}(1+\cos 2x)$
$\int \csc x\, dx = \ln|\csc - \cot|$
$\int \sec\, dx = \ln|\sec - \tan|$

★ Homogenous: $\boxed{=}$ equal مش هنقدر نفصل متغيرات بس يكونوا نفس الدرجة

$y = f(x)$ $\quad \frac{dy}{dx} = f\left(\frac{y}{x}\right)$ $\quad y = ux$ $\quad \frac{dy}{dx} = u + x\frac{du}{dx}$

$x = f(y)$ $\quad \frac{dx}{dy} = f\left(\frac{x}{y}\right)$ $\quad u = \frac{y}{x}$

★ Exact: مش هنقدر نفصل متغيرات ودرجة كل حد مختلفة

$M(x,y)\,dx + N(x,y)\,dy = 0$ $\quad \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ (exact)

$u_1 = \int M\,dx + c_1$

$C = u_1 + u_2$

$u_2 = \int N\,dy + c_2$

without repeating

★ Not exact :: مش هنقدر نفصل متغيرات ودرجه كل حد مختلفه

1] $Mx + Ny = 0$

② $f(x) = \frac{M_y - N_x}{N}$ OR $g(y) = \frac{N_x - M_y}{M}$ → That has easier Integration

3] $M = e^{\int f(x)\,dx}$ OR $N = e^{\int g(y)\,dy}$

4] Mult. The result to The eq.

5] $M_y = N_x$ → Exact

★ Rule3: IF $M(x,y)\,dx + N(x,y)\,dy = 0$ be ahomogenous Fun (Same degree)

The I.F. $= \frac{1}{Mx + Ny}$

1] $M_y = $ —, $N_x = $ — → (homogenous, not exact)

2] $= \frac{1}{Mx + Ny} = x \xrightarrow{x}$ Fun

3] $M_y = N_x$ exact

4] Integrate

★ Rule 4 :- If M is of The Form $M = y\, f(x,y)$, and N is of The Form $N = x\, f(x,y)$ Then

$$I.F = \frac{1}{Mx - Ny}$$

① $M_y =$ ______ , $N_x =$ ______

② $\frac{1}{Mx - Ny}$ = ⓧ $\xrightarrow{\text{Mult.}}$ fun.

③ $M_y = N_x \longrightarrow$ exact

④ Integrate

* Linear differential eq. of First order :-

1) $\frac{dy}{dx} + P(x)\,y = Q(x)$

2) $M = e^{\int P(x)\,dx}$

3) $x \cdot m = \int M \cdot Q\,dx + c$

or

3) $My = \int M\,q(x)\,dx$ (section)

K.M.S

★ Bernoulli's eq. :

$\rightarrow \frac{dy}{dx} + P(x)y = Q(x)\ y^n$

① $\times y^{-n} \rightarrow y^{-n}\frac{dy}{dx} + P(x)\,y^{1-n} = Q(x)$

② Put $Z = y^{1-n} \rightarrow \frac{dz}{dx} = (1-n)\,y^{-n}\frac{dy}{dx}$

$\rightarrow \boxed{\frac{1}{n-1}\frac{dz}{dx} + P(x)Z = Q(x)}$

OR section

$y' + P(x)\,y = q(x)\,y^n$

$P(x) = \_\_\_$, $q(x) = \_\_\_$

$n = \_\_\_$, $Z = y^{1-n}$

$M = e^{\int p(x)\,dx} = \_\_\_$

$MZ = \int M\,q(x)\,dx + c$

★ D.E of First order, higher degree solve in P [Solve For X]

$P_1 = F_1(x, y)$ ____

★ $P^2 + P - 6 = 0 \rightarrow (P+3)(P-2) = 0 \rightarrow P = -3$, $P = 2$

$P = \frac{dy}{dx}$

$\frac{dy}{dx} = -3$ | $\frac{dy}{dx} = 2$

$\int dy = \int -3\,dx$ | $\int dy = \int 2\,dx$

$y = -3x + c$

$y = 2x + c$

$\rightarrow \boxed{y + 3x + c = 0\ ,\ y - 2x + c = 0}$