Subject lecture 6 موضوع الدرس

Date التاريخ

* Bernoulli's eq.

$$\frac{dy}{dx} + P(x)\, y = Q(x)\; y^n \xrightarrow{\times y^{-n}} y^{-n}\frac{dy}{dx} + P(x)\, y^{1-n} = Q(x) \quad (2)$$

(1)

Put $z = y^{1-n} \rightarrow \frac{dz}{dx} = (1-n)\, y^{-n}\frac{dy}{dx}$ (3)

$\rightarrow \frac{1}{n-1}\frac{dz}{dx} + P(x) z = Q(x)$

Ex: $\frac{dy}{dx} + \frac{2}{x} y = x\, y^{+2}$

$y^{-2}\frac{dy}{dx} + \frac{2}{x} y^{-1} = x \rightarrow (1)$ Put $z = y^{-1}$

$\frac{dz}{dx} = -y^{-2}\frac{dy}{dx} \rightarrow (2)$

$-\frac{dz}{dx} + \frac{2}{x} z = x$

$\frac{dz}{dx} - \frac{2}{x} z = x$

$M = x^{-2}$

$z.m = \int M.\, Y(x)\, dx + c$

$z.x^{-2} = -\int x^{-2}.\, x\, dx + c \longrightarrow z x^{-2} = -\ln x + c$

$y^{-1} = x^{-2} = \ln x + c \longrightarrow y = \frac{1}{x^2(\ln x + c)}$

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Ex① $2xy\,dy - (x-y^2)\,dx = 0$

$M_y = 2y \quad , \quad N_x = 2y$

$-\frac{x^2}{y} - y^2 x = C$

$$\boxed{-\frac{x^2}{y} - y^2 x = C}$$

By Bernoulli's

$\frac{dy}{dx} - \frac{1}{2x} y = -\frac{1}{2y}$

$y\frac{dy}{dx} - \frac{1}{2x} y^2 = -\frac{1}{2} \rightarrow z = y^2$

$\frac{dz}{dx} = 2y\frac{dy}{dx} \rightarrow \frac{1}{2}\frac{dz}{dx} - \frac{1}{2x} z = -\frac{1}{2}$

$\boxed{\frac{dz}{dx} - \frac{1}{x} z = -1}$

$M = x^{-1}$

$z.x^{-1} = -\int x^{-1} dx + C$

$z.x^{-1} = -\ln x + C$

$y^2 = (-\ln x + C)x$

③ $y' + y = y^2(\cos x - \sin x)$

$y^{-2}\frac{dy}{dx} + y^{-1} = \cos x - \sin x \rightarrow z = y^{-1}$

$\frac{dz}{dx} = -y^{-2}\frac{dy}{dx}$

$-\frac{dz}{dx} + y^{-1} = \cos x - \sin x \rightarrow \frac{dz}{dx} - z = \cos x . \sin x$

$M = e^{\int -1 dx} = e^{-x}$

by parts

$z.e^{-x} = \int e^{-x}(\sin x - \cos x)dx + C$

④ $\left(\frac{dy}{dx}\right)^2 - \left(\frac{dy}{dx}\right) - 6y = 0$

$p^2 - p - 6 = 0$

$(p-3)(p+2) = 0$

$\frac{dy}{dx} = 3$	$\frac{dy}{dx} = 2$
$dy = 3dx$	$dy = -2dx$
$y - 3x + C = 0$	$(y + 2x - C) = 0$

$(y - 3x - c)(y + 2x + c) = c$

$x^2p^2 + xyp - 6y^2 = 0$

$(xp + 3y)(xp - 2y) = 0$

$x\frac{dy}{dx} = -3y$	$x\frac{dy}{dx} = 2y \rightarrow \frac{dy}{y} = 2\frac{dx}{x}$
$\frac{dy}{y} = \frac{3dx}{x}$	$\ln y = 2\ln x + \ln c$
$\ln y = -3\ln x + \ln c$	$(y - cx^{-2}) = 0$
$(y - cx^3) = 0$	

⑤ $P^2-(x+e^x)P+xe^x=0$

$(P-x)(P-e^x)=0$

$\frac{dy}{dx}=x$ | $\frac{dy}{dx}=e^x$

$dy=x\,dx$ | $dy=e^x\,dx$

$y=\frac{x^2}{2}+c$ | $y=e^x+c$

⑥ $P^2-(2\cosh x)P+1=0$

$P^2-\cancel{2}\frac{(e^x+e^{-x})}{\cancel{2}}P+e^x\cdot e^{-x}$

$(P-e^{-x})(P-e^x)=0$

$\frac{dy}{dx}=e^{-x}$ | $\frac{dy}{dx}=e^x$

$y=-e^{-x}+c$ | $y=e^x+c$

⑦ $x=P^3$

$\frac{1}{P}\frac{dx}{dy}=3P^2\frac{dP}{dy}$

$dy=3P^3\,dP$

$y=\frac{3}{4}P^4+c$

$y=\frac{3}{4}P^{4/3}+c$