Subject | موضوع الدرس
Section

① Absolute error = exact value − approximate value (E_t)

② Relative error = $\frac{\text{Absolute}}{\text{exact}}$ (ε_t)

Ex.

The derivative $f'(x)$ of function $f(x)$ can be approximated by the equation

$$f'(x) = \frac{f(x+h) - f(x)}{h}$$

If $f(x) = 3x^2$ and $h = 0.1$

a) find the approximate value of $f'(2) \rightarrow \frac{f(2+0.1) - f(2)}{0.1} = \frac{3(2.1)^2 - 12}{0.1}$
$= 12.3$

b) exact value of $f'(2) \rightarrow f'(x) = 6x \rightarrow f'(2) = 12$

c) Absolute error for part (a) → exact − app. → $12 - 12.3 = -0.3$

③ Approximate error = Present app. − previous app.
(E_a)

ex.

$f(x) = 3x^2$ at $x = 2$

find the app. error for the value of $f'(2)$ for part b

a) $f'(2)$ using $h = 0.1$

b) $f'(2)$ using $h = 0.2$

Subject موضوع الدرس | Date التاريخ

a) $f'(2) = 12.3$

b) $f'(2) = \dfrac{f(2+0.2) - f(2)}{0.2} = \dfrac{3(0.2)^2 - 12}{0.2} = \boxed{12.6}$

APP. error $= 12.6 - 12.3 = \boxed{0.3}$

$\boxed{4}$ Relative error $= \dfrac{\text{approximate}}{\text{present}}$
(E_a)

example:

Calculate of e^1 with an relative app. error of less Than 1%

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \text{———}$$

Sol.

	n	e^1	E_a	$E_a\%$	
Previous ←	1	1	–	–	
Present ←	2	$1+x=2$	$2-1=1$	50%	$\rightarrow \frac{1}{2} \times 100 =$
	3	$1+x+\frac{x^2}{2!}$	$2.5-2=0.5$	20%	$\rightarrow \frac{0.5}{2.5} \times 100$
	4	$1+x+\frac{x^2}{2!}+\frac{x^3}{3!}$	0.16667	6.25%	$\rightarrow \frac{0.16667}{2.6}$
	5	$1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\frac{x^4}{4!}$	$\frac{1}{24}$	1.53%	
	6	// $+ \frac{x^5}{5!}$	$\frac{1}{120}$	0.3%	

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* The max. error for function of multi variable

$f(x_1, x_2, x_3, x_4, \text{———})$

$$\Delta F = \frac{\partial f}{\partial x_1} \Delta x_1 + \frac{\partial f}{\partial x_2} \Delta x_2 + \text{———} \rightarrow \Delta F \sum_{i=1}^{n} \left|\frac{\partial f}{\partial x_i}\right| \left|\Delta x_i\right|$$

Example:

Find The max. error for The function $f(x,y,z) = \frac{xy}{z}$

$x = 1 \pm \underset{\Delta x}{0.01}$ $y = 2 \pm \underset{\Delta y}{0.03}$, $z = 3 \pm \underset{\Delta z}{0.04}$

$\frac{\partial f}{\partial x} = \frac{y}{z}$

$\Delta F_{max} = \left|\frac{\partial f}{\partial x}\Delta x\right| + \left|\frac{\partial f}{\partial y}\Delta y\right| + \left|\frac{\partial f}{\partial z}\Delta z\right|$

$\frac{\partial f}{\partial y} = \frac{x}{z}$

$\frac{\partial f}{\partial z} = -xyz^{-2} = \frac{-xy}{z^2}$

$\Rightarrow \left|\frac{\partial f}{\partial x}\right|_{(1,2,3)} = 2/3$

$\Rightarrow \left|\frac{\partial f}{\partial y}\right|_{(1,2,3)} = 1/3$

$\Rightarrow \left|\frac{\partial f}{\partial z}\right|_{(1,2,3)} = \frac{2}{9}$

$\Delta F_{max} = \frac{2}{3}(0.01) + \frac{1}{3}(0.03) + \left(\frac{2}{9}\right)(0.04) = \frac{23}{900}$

relative maximum error $\rightarrow \left|\frac{\Delta F}{F}\right|_{max} = \frac{23}{900} \times \frac{3}{2} = \frac{23}{600}$