

MT131

Discrete Mathematics

Midterm Examination (MTA)
Fall Semester 2023/2024

Date: 07 /Nov./2023

Number of Exam Pages: ...
(including this cover sheet)

Time Allowed: 90 Minutes

Instructions:

- Total Marks: **30**
- This exam consists of **four questions**.
- **ALL questions** must be answered in the External Answer booklet.
- Be sure you write your **name and ID** on the External Answer booklet.
- **Calculators** are allowed.

Q-1: [5×3 marks] Choose the correct answer:

1.1: Which of the following option is true?

- a) If the Sun is a planet, elephants will fly
- b) $3 + 2 = 8$ if $5 - 2 = 7$
- c) $1 > 3$ and 3 is a positive integer
- d) $-2 > 3$ or 3 is a negative integer
- e) None of the above

1.2: Let $A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$ be Boolean matrices. Find $A \vee B$

a) $\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

c) $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

d) $\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

e) None of the above

1.3: If set $C = \{1, 2, 3, 4\}$ and $C - D = \phi$ then set D can be

- a) $\{1, 2, 4, 5\}$
- b) $\{1, 2, 3\}$
- c) $\{1, 2, 3, 4, 5\}$
- d) ϕ
- e) None of the above

1.4: The inverse of function $f(x) = x^3 + 2$ is

- a) $f^{-1}(y) = (y - 2)^{\frac{1}{2}}$
- b) $f^{-1}(y) = (y - 2)^{\frac{1}{3}}$
- c) $f^{-1}(y) = (y)^{\frac{1}{3}}$
- d) $f^{-1}(y) = (y - 2)$
- e) None of the above

1.5: For the sequence 1, 7, 25, 79, 241, 727, ... simple formula for $\{a_n\}$ is

- a) $3^{n+1} - 2$
- b) $3^n - 2$
- c) $(-3)^n + 4$
- d) $n^2 - 2$
- e) None of the above

Each question in below worth 15 marks. Answer all the following:

Q-2:

a) [9 marks] Show that $\neg p \rightarrow (q \rightarrow r)$ and $q \rightarrow (p \vee r)$ are logically equivalent.

p	q	r	$\neg p$	$q \rightarrow r$	$\neg p \rightarrow (q \rightarrow r)$	$p \vee r$	$q \rightarrow (p \vee r)$
F	F	F	T	T	T	F	T
F	F	T	T	T	T	T	T
F	T	F	T	F	F	F	F
F	T	T	T	T	T	T	T
T	F	F	F	T	T	T	T
T	F	T	F	T	T	T	T
T	T	F	F	F	T	T	T
T	T	T	F	T	T	T	T

b) [3 marks] Determine the truth values of the statement where the domain of discourse is the set of positive integers, justify your answers:

$$\forall x((x \neq 4) \rightarrow (x - 5 > 1))$$

False [1 mark], take example $x = 1$ [2 marks]

c) [3 marks] Translate the following statement into logical expressions using predicates, quantifiers, and logical connectives.

"Everything is in the correct place and in excellent condition."

$$\forall x (C(x) \wedge E(x))$$

Q-3:

a) [3×2 marks] Suppose that the universal set $U = \{1,2,3,4,5,6,7,8,9,10\}$, and the sets A, B and C be represented as a bit string as following:

A : 0011100000

B : 1010010001

C : 0111001110

Find

i. The elements of the sets A, B, C

$$A = \{3, 4, 5\}, B = \{1, 3, 6, 10\}, C = \{2, 3, 4, 7, 8, 9\}$$

ii. $A \cap (B \oplus C)$

$$(B \oplus C) = \{1, 2, 4, 7, 8, 6, 9, 10\}$$

$$A \cap (B \oplus C) = \{4\}$$

b) [3×2 marks] Consider the functions $f(x) = 2x - 3$ and $g(x) = x^2$, where x a real number. Find the following:

i. $g \circ f = (2x - 3)^2$

ii. Determine whether the function $f(x)$ is one-to-one (injective) and onto (surjective). Provide an explanation for each.

$$f(x) \text{ is one-to-one: Let } f(a) = f(b) \Rightarrow 2a - 3 = 2b - 3 \Rightarrow a = b$$

$$f(x) \text{ is Onto: Let } y \in \mathbb{R}, \text{ and take } x = \frac{y+3}{2}, \text{ then } f(x) = f\left(\frac{y+3}{2}\right) = 2\left(\frac{y+3}{2}\right) - 3 = y$$

- c) [3 marks] Given the terms $a_1 = 18$ and $a_2 = 54$, what is a_3 in a geometric sequence? Find the sum of the first three terms.

To find the common ratio of the geometric sequence, we can divide the second term by the first term:

$$r = a_2 / a_1$$

$$r = 54 / 18$$

$$r = 3$$

Now that we know the common ratio ($r = 3$), we can use it to find the third term (a_3) by multiplying the second term by the common ratio:

$$a_3 = a_2 * r$$

$$a_3 = 54 * 3$$

$$a_3 = 162$$

Therefore, the third term of the geometric sequence is $a_3 = 162$.

To find the sum of the first three terms of the geometric sequence

$$S^3 = 18 + 54 + 162 = 234$$

Q-4:

- a) [5 marks] Consider the integers $a = 675$ and $b = 891$. Find:

- i. The prime factorization of a .

$$675 = 3^3 \times 5^2$$

- ii. The prime factorization of b .

$$891 = 3^4 \times 11$$

- iii. The greatest common divisor (GCD) of a and b .

$$GCD(675, 891) = 3^3 = 27$$

- iv. The least common multiple (LCM) of a and b .

$$LCM(675, 891) = 3^4 \times 5^2 \times 11 = 22275$$

- v. Based on this information, determine if 675 and 891 are relatively prime?

They are not relatively prime because $GCD(675, 891) \neq 1$

- b) [10 marks] Using the encrypting function $f(x) = (x + 5) \bmod 26$,

$0 \leq x \leq 25$, encrypt the message "HELLO". Provide details of the solution.

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25

"MJQQT"

Message	H	E	L	L	O
x	7	4	11	11	14
$(x + 5) \bmod 26$	12	9	16	16	19
Encrypted message	M	J	Q	Q	T

End of questions