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lecture 2

Circle :

A circle with center (h,k) and $r>0$ is the set of all point (x,y) in the plane whose distance to (h,k) is r

★ Standard eq. → $(x-h)^2+(y-k)^2=r^2$

Ex.

① $(h,k)=(-2,3)$, $r=5$ → $(x+2)^2+(y-3)^2=25$

② $(x+2)^2+(y-3)^2=4$ → $(-2,3)$, $r=2$

★ to write the eq. of a circle in standard form

① Group the same variable together on one side of the eq. and position the constant on the other side

② Complete the square on both variable as needed

③ Divide the both sides by the coefficients of the squares

Ex. Complete the square to find the center and radius

$3x^2-6x+3y^2+4y-4=0$

$\div 3$ → $3(x^2-2x)+3(y^2+\frac{4}{3}y)=4$

$\Rightarrow (3x^2-6x)+(3y^2+4y)=4$

$(x^2-2x)+(y^2+\frac{4}{3}y)=\frac{4}{3}$

$(x-1)^2-1+(y+\frac{2}{3})^2-\frac{4}{9}=\frac{4}{3}$

$(x-1)^2+(y+\frac{2}{3})^2=\frac{25}{9}$ → $(h,k)=(1,-\frac{2}{3})$ $r=\frac{5}{3}$

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$(x-2)^2+(y+5)^2=4$

Ex. $x^2-4x+y^2+10y=-25$

$(x^2-4x)+(y^2+10y)=-25$

$(x-2)^2-4+(y+5)^2-25=-25$

$(h,k)=(2,-5)$

$r=2$

Ex.

$x^2+x+y^2-\frac{6}{5}y=1$

$(x^2+x)+(y^2-\frac{6}{5}y)=1$

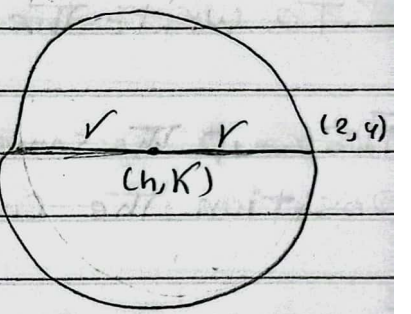
$(x+\frac{1}{2})^2-\frac{1}{4}+(y-\frac{3}{5})^2-\frac{9}{25}=1$

$(x+\frac{1}{2})^2+(y-\frac{3}{5})^2=\frac{161}{100}$

$(h,k)=(-\frac{1}{2},\frac{3}{5})$

$r=\sqrt{161/100}$

Ex. write the standard eq. which has $(-1,3)$ and $(2,4)$ as the end point of diameter



by midPoint

$(h,k)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)=\left(\frac{-1+2}{2},\frac{4+3}{2}\right)=\left(\frac{1}{2},\frac{7}{2}\right)$

by distance $=\frac{1}{2}\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}=\frac{1}{2}\sqrt{(2-(-1))^2+(4-3)^2}=\frac{\sqrt{10}}{2}$

$\rightarrow (x-\frac{1}{2})^2+(y-\frac{7}{2})=\frac{10}{4}$

The unit Circle : is the Circle Centered at $(0,0)$ with $r=1$

$\rightarrow$ Standard eq. $\rightarrow x^2+y^2=1$

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Ex. find the point of the unit circle y-coordenate $\frac{\sqrt{3}}{2}$

$x^2+y^2=1 \rightarrow x^2+(\frac{\sqrt{3}}{2})^2=1 \rightarrow x^2=\frac{1}{4} \rightarrow x=\pm\frac{1}{2}$

Ex. find the standard eq. of the circle which satisfy the given condition, end point of the diameter $(\frac{1}{2},4)$, $(\frac{3}{2},-1)$

$(h,k)=\frac{(x_1+x_2)}{2}+\frac{(y_1+y_2)}{2}=\frac{(\frac{1}{2}+\frac{3}{2})}{2}+\frac{(4-1)}{2}=(1,\frac{3}{2})$.

distance $=\frac{1}{2}\sqrt{(\frac{3}{2}-\frac{1}{2})^2+(-1-4)^2}=\sqrt{26}\times\frac{1}{2}=\frac{\sqrt{26}}{2}$

$(x-1)^2+(y-\frac{3}{2})^2=13$

Ex. Center (3,5) passes Through (-1,-2)

$r=\sqrt{(-1-3)^2+(-2-5)^2}=\sqrt{65}$

$(x-3)^2+(y-5)^2=65$

Ex. center (3,6) passes Through (-1,4)

$(x-3)^2+(y-6)^2=20$

Ex. $-2x^2-36x-2y^2-112=0$

$(-2x^2-36x)+(-2y^2)=112$

(÷2) $x^2+18x-81+y^2=-56 \rightarrow (x+9)^2+y^2=25$

(-9,0) ← center $r=5$

Ex. $4x^2+4y^2-24y+36=0 \rightarrow 4x^2+4y^2-24y=-36$

$(4x^2)+(4y^2-24y)=-36$ (÷4) $\rightarrow x^2+y^2-6y=-9$

$x^2+(y^2-6y)-9=-9 \rightarrow x^2+(y-3)^2=0$

(0,3) no circle