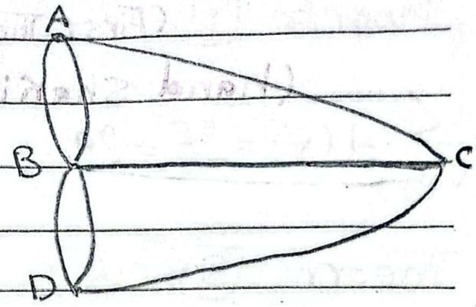
Page: Graph
Date: Theory

Ch. 1



★ vertex (V) = {A, B, D}

★ edge (E) = {DC, BC, --}

★ C, A (adjacent) while A, D (not adjacent)

deg (C) = 3
deg (A) = 3
deg (B) = 5
deg (D) = 3

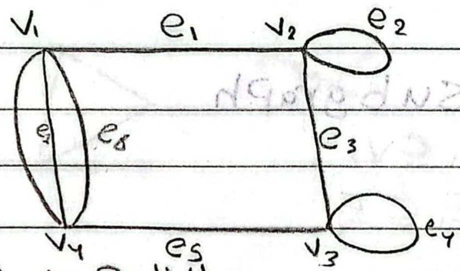
} odd no. → There isn't existance path

degree
عدد الخطوط الى داخله
vertex لل

---

adjacent → V, V
incident → E, V

G = (V, E) → (vertex, edge)



e_2, e_4 → loop - self loop ← counted twise
e_6, e_7, e_8 → Multi. edges - Parallel

order (P) → no. of vertex (رؤوس) → $p = |V|$
size (Q) → no. of edges (حروف) → $q = |E|$

★ one vertex → trivial graph ★ without edge → null - empty
★ Finite vertecies → Finite graph ★ infinite vertecies → infinite graph
★ Simple graph → without multi. edges or loop
★ vertex without edge → isolated vertex ← Zero degree
★ deg ( ) = 1 → Pendant ← end vertex
★ not simple graph → Multi. graph
★ deg( ) ≠ 1 ≠ 0 → internal vertex ← intermediate

$\delta(G)$ = min. degree , $\Delta(G)$ = max. degree

$$\delta(G) \leq d(v) \leq \Delta(G)$$

Date :

Theorm $\boxed{1}$ (First Theorem) (hand shaking lemma)

$$\sum_{V(G)} d(V) = 2E = 2q$$

Theorm $\boxed{2}$ $\delta(G) \leq \frac{2|E|}{|V|}$ or $\frac{2q}{p} \leq \Delta(G)$

مجموع ال deg لازم يطلع even

Theorm $\boxed{3}$

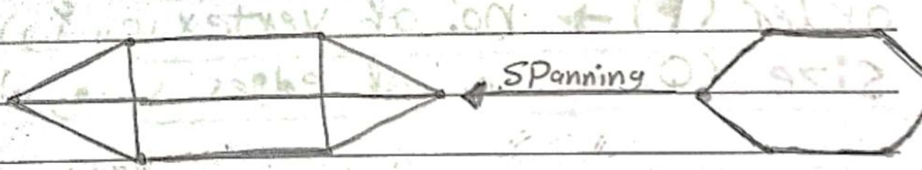
The no. of odd degree vertices is always even

★ $\sum d(v) \neq$ even → not graphical
★ Degree Seq. → (descending order)
★ regular graph → equal degrees

★ Neighbourhood of a vertex (open neighbourhood)
vertices connected to wanted vertex

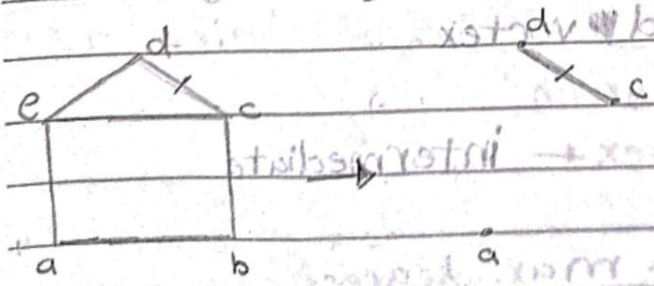
★ subgraph
$V_1 \subseteq V_2$
$E_1 \subseteq E_2$

★ Spanning graph
$V_1 = V_2$
$E_1 \subseteq E_2$

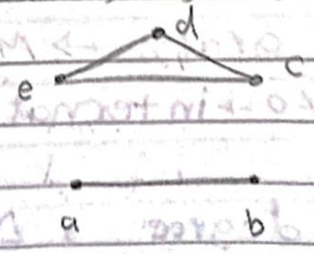


★ Induced subgraphs :

→ Induced subgraph [a, d, c]



→ Edge Induced subgraph
{ab, cd, ce, de}



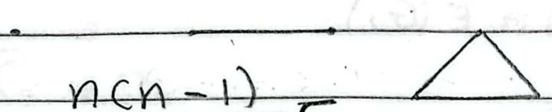
on n vertices is denoted by K_n → Complete grap

Date .

★ Complete graphs (K_n) n: no. of vertex

every vertex Connect to all vertices

K_1 K_2 K_3 $\longrightarrow \sum d = 2E$



$(n-1)+(n-1)+(n-1) = 2E$

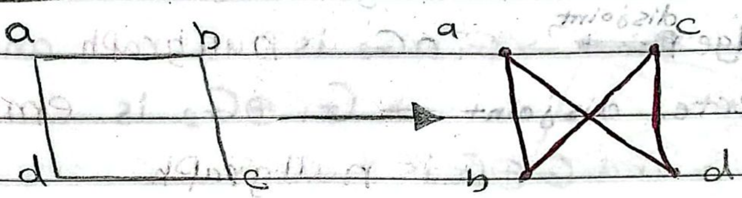
$n(n-1) = 2E$

$K_n = \frac{n(n-1)}{2} = E$ $\therefore E = \frac{n(n-1)}{2}$

★ every complete graph is $(n-1)$ regular graph

$K_4 \rightarrow$ 3-regular

★ Bipartite :- ثنائي التقسيم



★ Complete Bipartite

all Point up connect with all Point down

$K_{a,b} \rightarrow ab$ Edges $\longrightarrow |E(K_{a,b})| = ab$

$K_{2,2} \rightarrow E = 2 \times 2 = 4$

★ Regular graphs :-

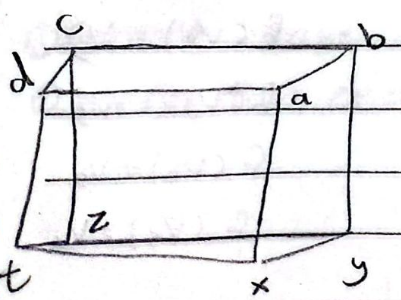
vertices have The same degree $\longrightarrow$ $(n-1)$ regular graph

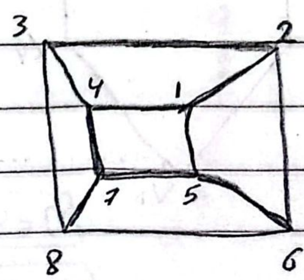
★ Isomorphism of two graphs

★ bijective $\rightarrow$ one to one and onto (احادية وفوقية)

$|V(G)| = |V(H)|$

$|E(G)| = |E(H)|$





$f(a)=1$ $f(t)=7$

$f(b)=2$ $f(z)=8$

$f(c)=3$

$f(d)=4$ $f(ax)=15$

$f(x)=5$ $f(cz)=38$

$f(y)=6$

Ch. 2

★ Union of graphs

$G = G_1 \cup G_2 \rightarrow V(G_1) \cup V(G_2) \rightarrow E(G_1) \cup E(G_2)$ بدون تكرار

★ Intersection of graph

$G = G_1 \cap G_2 \rightarrow V(G_1) \cap V(G_2) \rightarrow E(G_1) \cap E(G_2)$

★ Ringsum of graph (XoR operation)

$G = G_1 \oplus G_2 \rightarrow E(G_1) \oplus E(G_2) \rightarrow [G_1 \cup G_2] - [G_1 \cap G_2]$

→ Union, Intersection and Ringsum Commutative

→ G_1, G_2 are edge ~~joint~~ disjoint → $G_1 \cap G_2$ is null graph and $G_1 \oplus G_2 = G_1 \cup G_2$ ★

→ G_1, G_2 are vertex disjoint → $G_1 \oplus G_2$ is empty

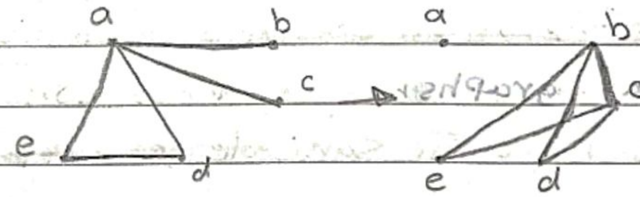
→ G, $G \cap G = G \cup G$ and $G \oplus G$ is nullgraph

★ Complement of graph : (Inverse) $(\bar{G})$ (Complete graph ★)

$V(G) = V = (\bar{G})$

→ $G_1 \cup \bar{G} = K_n$, $E(G) \cup E(\bar{G}) = E(K_n)$, $|E(G)| + |E(\bar{G})| = E(K_n) = \binom{n}{2} = {}^{n}C_2$
$= \frac{n(n-1)}{2}$

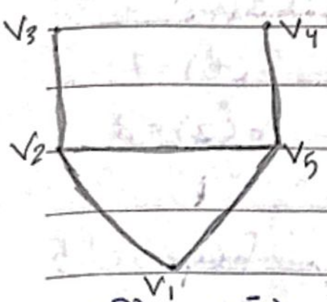
$\overset{Com.}{A} = U - A$, $\bar{G} = K - G$



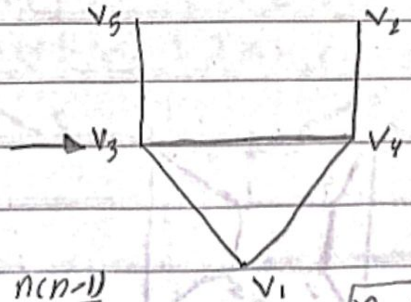
★ Self Complementary : IF The graph is Isomorphic to it's complement

$E(G) = E(\bar{G}) = \frac{1}{2}|E(K_n)| = \frac{1}{2}({}^{n}C_2) = \frac{n(n-1)}{4}$ $a \equiv b \pmod{n}$

$a - b / n$



$K_5 = 10$
$V = 5$
$E = 5$



$f(V_1) = V_1$
$f(V_2) = V_3$
$f(V_3) = V_5$
$f(V_4) = V_2$
$f(V_5) = V_4$

$V(G) = V(\bar{G})$, $E(G) = E(\bar{G}) = \frac{n(n-1)}{4}$ $\boxed{n = 4K}$

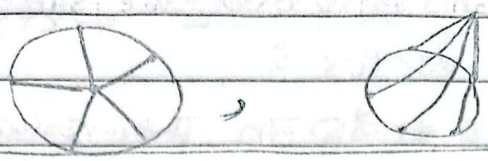
$E(G) + E(\bar{G}) = \frac{n(n-1)}{2}$ $\boxed{n = 4K+1}$

Date :

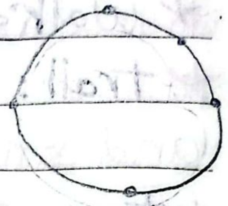
★ wheel graph

$W_n = K_1 + C_n$

$W_5 = K_1 + C_5$ =

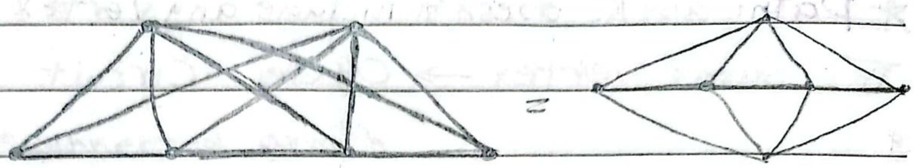


C_5 =



★

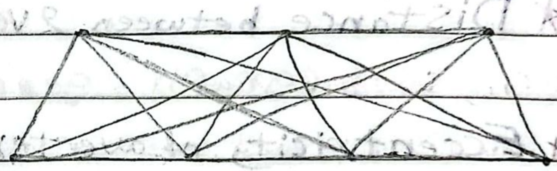
$\overline{K_2} + P_4$



★ Join of graphs : (G + H)

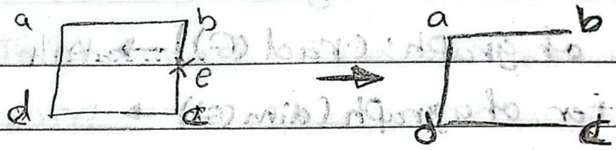
$V(G+H) = V(G) \cup V(H)$, $E(G+H) = E(G) \cup E(H)$

P_3 , P_4



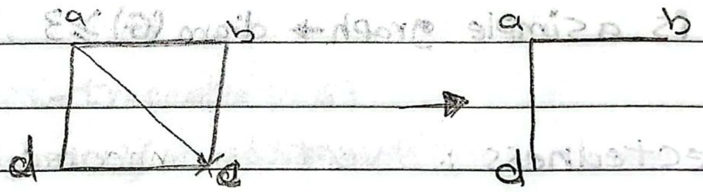
★ Edge deletion (G − e)

Edge only without vertex



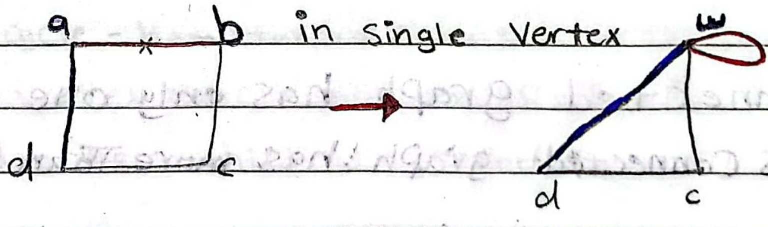
★ Vertex deletion (G − v)

remove vertex and edge together



★ Fusion of vertex (G − e)

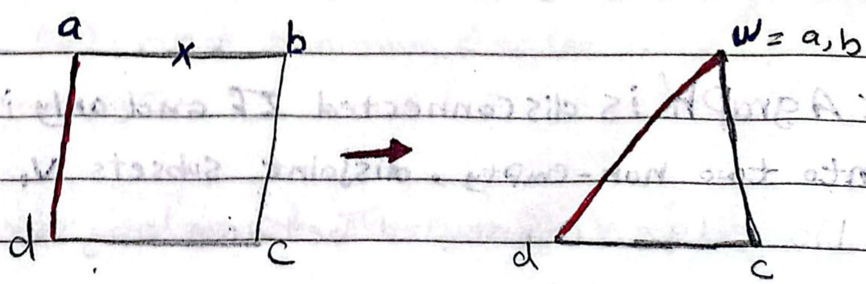
Merge two vertex without remove edge



★ Edge contraction (G∘e)

① remove edge

② merge two ends



not sub graph

★ Walks : Walk in any route from (V) to (V) along edges
★ trail : walk That doesn't pass over The same edge twice and might The same vertex
★ tour : trail begin and ends on The same vertex
★ Path : walk doesn't include any vertex twice and if it ends by The same vertex → Cycle / Circuit

★ length of cycle = n , length of path = n-1
odd length → odd cycle , even length → even cycle

★ Distance between 2 vertices : is The length of Shortest path (graph geodesic)
★ Eccentricity of a vertex $\mathcal{E}(v)$ → is The greatest geodesic distance between V and any other vertex.
★ Radius of graph : (rad (G)) → minimum eccentricity
★ Diameter of a graph (dim (G)) → maximum eccentricity
★ center of graph → vertex That equal eccentricity of radius.
★ IF G is a simple graph → diam (G) ≥ 3 , Then diam ($\bar{G}$) < 3

★ connectedness : 2 vertices connected if There is path between Them
★ Component : maximal connected subgraph of (G)

→ A connected graph has only one component
→ A disconnected graph has more Than one component.

★ Connected graph is bipartite if and only if has no odd cycles

★ A graph is disconnected IF and only if vertex can be partitioned into two non-empty , disjoint subsets V_1 and V_2
→ no edge connected between any vertices in V_1 with vertices V_2

★ In a graph (G) if order = 7 and edge = 21 Then The graph is connected

$\frac{(n-1)(n-2)}{2}$ edges → $\frac{(7-1)(7-2)}{2} = 15$ 20 > 15

★ edge deleted Subgraphs : $F \subseteq E$ → removing all edges in F
(Spanning sub graph)

★ Cut edge (G − e) → disconnected graph
★ Cut vertex (G − V) → disconnected graph

★ traversable (eulerian trail − euler walk) (semi-euler)
It use each edge exactly once (walk)
★ A connected graph is traversable If and only If it has 2 odd degree (V) only
★ Eulerian (eulerian cycle − eulerian circuit − euler tour)
It use each edge exactly once (cycle)
A connected graph is eulerian If and only if every (V) has even degree
" " " " " " " " it can be decomposed into edge disjoint cycles.

★ traceable (Hamiltonian path − traceable path)
(visit each vertex exactly ones), (Hamiltonian path → traceable graph)*

★ Hamiltonian (Hamiltonian cycle − Hamiltonian circuit − vertex tour)
It is a cycle That visit each vertex exactly once except for The vertex That is both Starts and ends which is visited twice

★ Dirac's Theorem :
every graph with $n \geq 3$ (V) and minimum degree $\delta(G) \geq n/2$ has a Hamilton cycle

★ directed graph: If each edge of The graph has direction

Ch. 6

★ Tree: graph is connected, no cycles → (circuitless)
★ Forest: components of Trees are disconnected

★ A graph is tree if and only if There is exactly one path b. every pair of vertices

★ trees are geodetic graphs: (Shortest Path)
★ tree with n verticies has n-1 edges
★ every edge in a tree is a cut edge
★ A graph is a tree IF and only if it is minimally connected
★ graph with n vertices, n-1 edges and no cycle is connected
★ Any tree with at least 2 vertices has at least two pendant (V)
★ A vertex in a tree is a cut vertex when $d(v) \geq 2$
★ every tree has either one or two centers

★ Spanning tree: (Skeleton - scaffold)
→ A Spanning tree of connected graph G is a tree containing all The Vertices of G and The Spanning tree is maximal tree subgraph
→ every connected graph G has spanning tree

★ on Counting trees:
→ labelled trees → (V_1, V_2, -- A, B, C)
→ un labelled trees

no. of (V)	1	2	3	4	5	6	7	8
no. of trees	1	1	1	2	3	6	11	23