

Section 1

$$① y''' + 2xy'' + y = e^{2x}$$

أعلى مشتقة order

أعلى أوس degree

لأعلى مشتقة

$$0 = 3, D = 1$$

$$② y^{(5)} + 2x = y$$

$$0 = 5, D = 1$$

① Separation D.E (not y=f(x)) ④ Exact D.E

② Homo. D.E (same degree) ⑤ Not exact D.E

③ non homo. D.E

① (Separation)

$$① 2x(y^2+1)dx - 2y(x^2+1)dy = 0 \rightarrow 2x(y^2+1)dx = 2y(x^2+1)dy$$

$$\int \frac{2x}{(x^2+1)} dx = \int \frac{2y}{(y^2+1)} dy \rightarrow \ln|x^2+1| = \ln|y^2+1| + \ln C$$

$$\ln(x^2+1) = \ln(y^2+1) + \ln C \rightarrow x^2+1 = y^2+1 + C$$

(Separation)

$$② yy' = xe^{y^2} \rightarrow y \frac{dy}{dx} = xe^{y^2} \xrightarrow{x dx} y dy = xe^{y^2} dx$$

$$\int y e^{-y^2} dy = \int x dx \rightarrow -\frac{1}{2} e^{-y^2} = \frac{x^2}{2} + C$$

③ (Separation)

$$y' = e^{-x-y} \rightarrow \frac{dy}{dx} = \frac{e^{-x}}{e^y} \xrightarrow{x dx} dy = \frac{e^{-x} dx}{e^y} \rightarrow \int e^y dy = \int e^{-x} dx$$

$$e^y = -e^{-x} + C$$

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(separation)

$$\textcircled{1} \csc y \, dx + \sec^2 x \, dy = 0 \quad \textcircled{2} y' = \cos^2 x \cos y$$

$$\textcircled{3} y' = cy - 1 \quad \textcircled{4} y' = xy^2 + y^2 + xy + y$$

② Homo D.E

$$\star (y^2 + x^2) \, dx - 2xy \, dy = 0$$

$$\text{let } y = xu \rightarrow \frac{dy}{dx} = u + x \frac{du}{dx}$$

$$(y^2 + x^2) \, dx = 2xy \, dy \rightarrow \frac{dy}{dx} = \frac{y^2 + x^2}{2xy} \rightarrow \frac{dy}{dx} = \frac{(x^2 u^2) + x^2}{2x^2 u} = \frac{x^2(u^2 + 1)}{2x^2 u}$$

$$\frac{dy}{dx} = \frac{u^2 + 1}{2u} \rightarrow u + x \frac{du}{dx} = \frac{u^2 + 1}{2u} \rightarrow \frac{u^2 + 1}{2u} - u = \frac{u^2 + 1 - 2u^2}{2u}$$

$$\int \frac{2u}{1-u^2} \, du = \int \frac{1}{x} \, dx \rightarrow \ln(1-u^2)^{-1} = \ln(x) + \ln c$$

$$(1-u^2)^{-1} = xc \rightarrow \left(1 - \left(\frac{y}{x}\right)^2\right)^{-1} = xc$$

$$\star xy' - y = xe^{\frac{y}{x}}$$

$$x \frac{dy}{dx} = xe^{\frac{y}{x}} + y \rightarrow \frac{dy}{dx} = \frac{xe^{\frac{y}{x}} + y}{x}$$

$$y = xu$$

$$\frac{dy}{dx} = u + x \frac{du}{dx}$$

$$\frac{du}{dx} = \frac{xe^u + xu}{x} = \frac{x(e^u + u)}{x} \rightarrow e^u + u = x \frac{du}{dx} + u$$

$$e^u = x \frac{du}{dx} \rightarrow \int e^{-u} \, du = \int \frac{1}{x} \, dx \rightarrow -e^{-u} = \ln x + c$$

$$-e^{-\frac{y}{x}} = \ln x + \ln c$$

$$\star y' = \frac{x^2 + y^2}{xy}$$

$$y = xu$$

$$\frac{dy}{dx} = \frac{x^2 + x^2 u^2}{x(xu)} = \frac{x^2(1+u^2)}{x^2(u)}$$

$$u + x \frac{du}{dx} = \frac{1+u^2}{u} \rightarrow x \frac{du}{dx} = \frac{1+u^2}{u} - u$$

$$x \frac{du}{dx} = \frac{1+u^2 - u^2}{u} \rightarrow x du = \frac{1+u^2 - u^2}{u} dx$$

$$\int u du = \int \frac{1}{x} dx \rightarrow \frac{u^2}{2} = \ln x + C$$

$$\frac{u^2}{2} = \ln x + C$$

$$(3x^2y + y^3)dx + (x^3 + 3xy^2)dy = 0 \quad \text{Sheet}$$

$$y' = \frac{x-y}{x+y}$$

$$(x^2 + y^2)dx - xy dy = 0$$