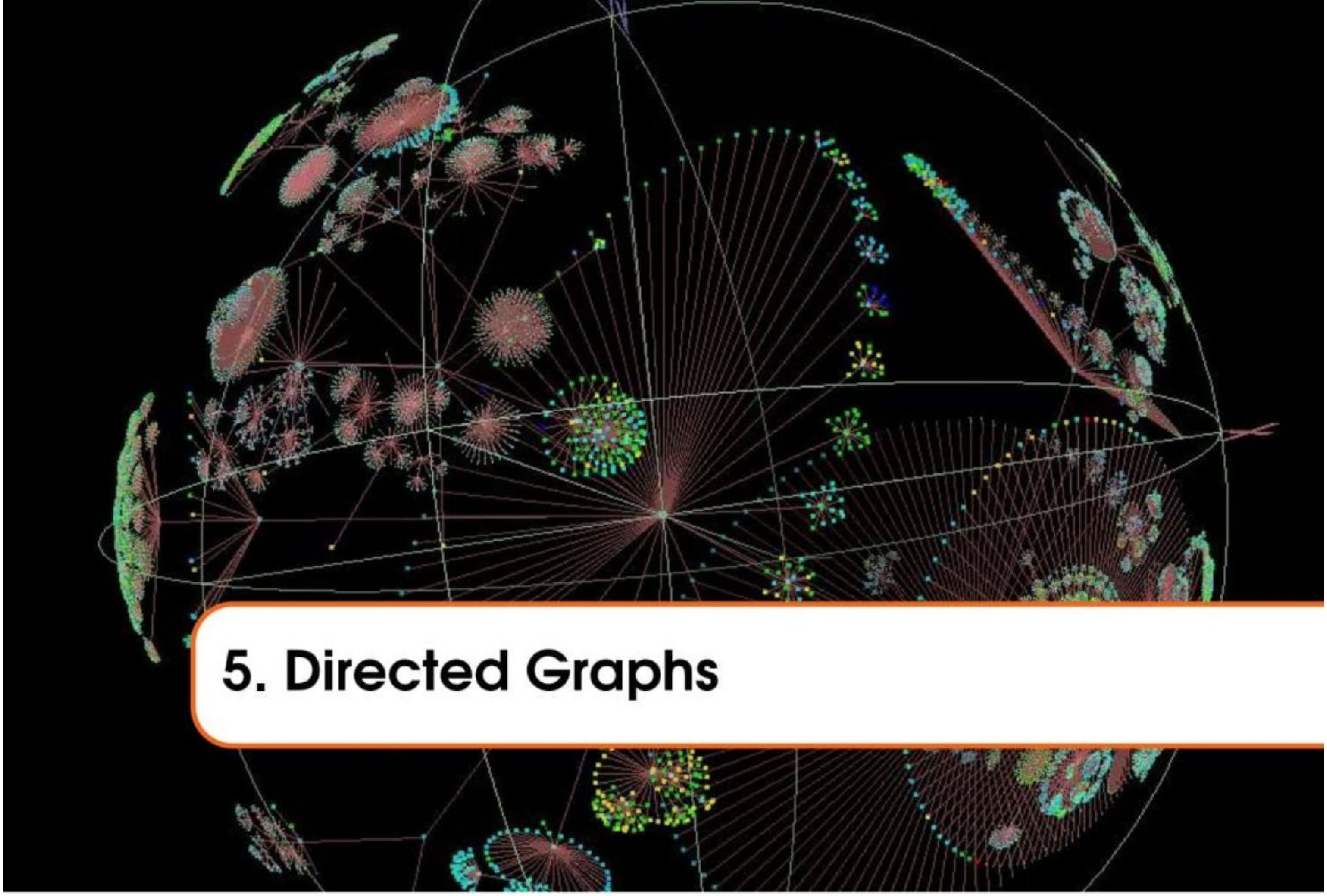


5. Directed Graphs

5.1 Directed Graphs

Definition 5.1.1 — Directed Graphs. A *directed graph* or *digraph* G consists of a set V of vertices and a set E of edges such that $e \in E$ is associated with an ordered pair of vertices. In other words, if each edge of the graph G has a direction, then the graph is called a directed graph.

The directed edges of a directed graph are also called *arcs*. The initial vertex of an arc a is called the *tail* of a and the terminal vertex v is called the *head* of the arc a. An arc $e = (u, v)$ in a digraph D is a *loop* if $u = v$. Two arcs e, f are *parallel edges* if they have the same tails and the same heads. If D has no loops or parallel edges, then we say that D is *simple*.

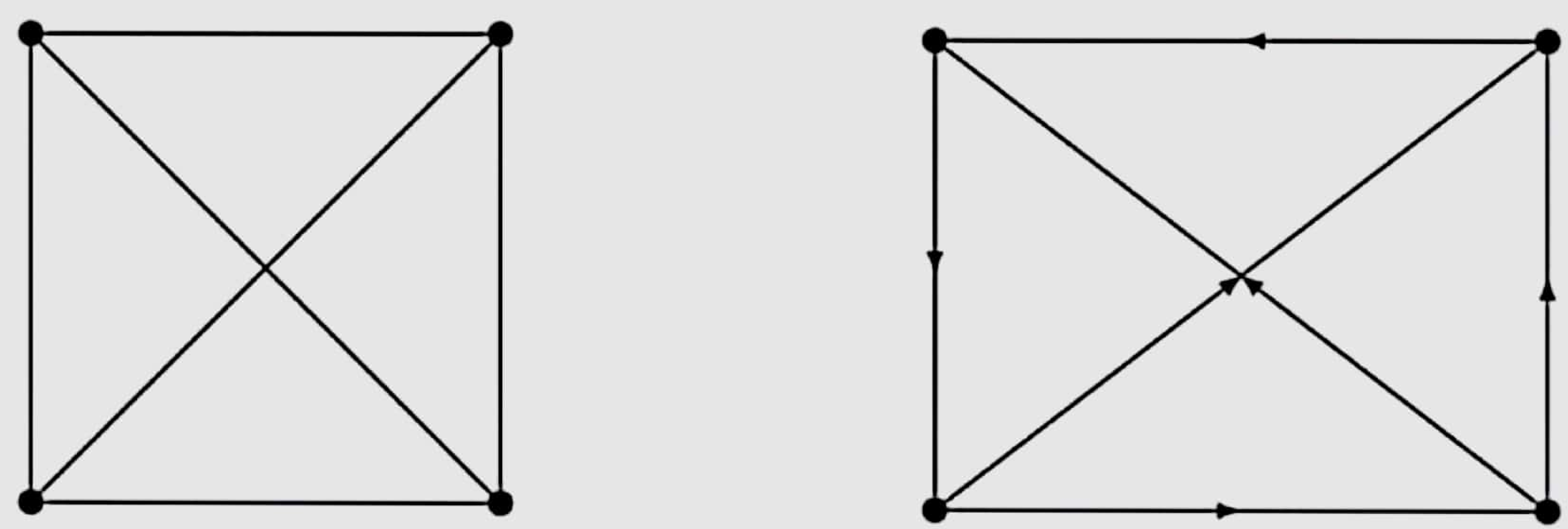
Definition 5.1.2 — Degrees in Digraphs. The *indegree* of vertex v in a directed graph D is the number of edges which are coming into the vertex v (that is, the number of incoming edges) and is denoted by $d^{-}(v)$. The *out-degree* vertex v in a directed graph D is the number of edges which are going out from the vertex v (that is, the number of outgoing edges) and is denoted by $d^{+}(v)$.

Definition 5.1.3 — Orientation of Graphs. If we assign directions to the edges of a given graph, then the new directed graph D is called an *orientation* of G.

Definition 5.1.4 — Underlying Graphs of Directed Graphs. If remove the directions of the edges of a directed graph D, then the reduced graph G is called the *underlying graph* of D.

Note that the orientation of a graph G is not unique. Every edge of G can take any one of the two possible directions. Therefore, a graph $G = (V, E)$ can have at most $2^{|E|}$ different orientations. But, a directed graph can have a unique underlying graph.

Figure 5.1b illustrates an undirected graph G and an orientation D of G.



(a) A graph G (b) Orientation of G

Figure 5.1: An undirected graph and one of its orientations.