Subject Lecture 3 موضوع الدرس Date التاريخ

(1) $\frac{dy}{dx} = \frac{y\cos x}{1+2y^2} \rightarrow \int \frac{1+2y^2}{y}\,dy = \int \cos x\,dx$

$\ln y + y^2 = \sin x + c$

(2) $y = x - \frac{A}{x}$ $\quad (y'-1)x^2 = A$

$xy' = -y + 2x$

$y' = 1 + \frac{A}{x^2}$ $\quad y = x - (y'-1)x^2$

(3) $(2ye^{\frac{y}{x}} - x)\frac{dy}{dx} + 2x + y = 0 \rightarrow \frac{dy}{dx} = \frac{-2x-y}{2ye^{\frac{y}{x}} - x} \overset{(x-1)}{=} \frac{2x+y}{x - 2ye^{\frac{y}{x}}}$

$= \frac{2+\frac{y}{x}}{1 - 2\frac{y}{x}e^{y/x}}$

Put $y = zx$ $\quad z = \frac{y}{x}$

$\frac{dy}{dx} = z + x\frac{dz}{dx} \rightarrow z + x\frac{dz}{dx} = \frac{2+z}{1-2ze^z}$

$x\frac{dz}{dx} = \frac{2+z}{1-2ze^z} - z = \frac{2+2z^2e^z}{1-2ze^z} \rightarrow \frac{1-2ze^z}{2(1+z^2e^z)}\,dz = \frac{dx}{x}$

$\frac{e^{-z} - 2z}{e^{-z} + z^2}\,dz = z\frac{dx}{x} \Rightarrow -\ln|e^{-z} + z^2| = 2x + c$

(4) $-\ln|e^{-\frac{y}{x}} + (\frac{y}{x})^2|^{-1} = 2\ln x + \ln c$

$(e^{-\frac{y}{x}} + (\frac{y}{x}))^{-1} = cx^2$

Subject

*Exact differential equations

$\frac{dM}{dy} = \frac{dN}{dx}$

$M(x,y)\,dx + N(x,y)\,dy = 0$

$\int M(x,y)\,dx + \int N$ (terms not containing x) $dy = C$

① $(2x\cos y + 3x^2 y)\,dx + (x^3 - x^2\sin y - y)\,dy = 0$

M

$\frac{dm}{dy} = -2x\sin y + 3x^2$, $\frac{dn}{dx} = 3x^2 - 2x\sin y$

$\frac{dm}{dy} \neq \frac{dn}{dx}$ is exact

$\int (2x\cos y + 3x^2 y)\,dx + \int -y\,dy = C$

$x^2\cos y + x^3 y - \frac{y^2}{2} = C$

② $(2y^2 + 4xy - x^2)\,dx + (2x^2 + 4xy - y^2)\,dy = 0$

$M_y = 4y + 4x$ =, $N_x = 4x + 4y$ Is exact

$2xy^2 + 2x^2 y - \frac{x^3}{3} - \frac{y^3}{3} = C$

③ $(xy\cos xy + \sin xy)\,dx + (x^2\cos xy + e^y)\,dy = 0$

Parts

$M_y = +x^2\sin(xy) + x\cos xy + x\cos xy$ = $N_x = 2x\cos xy + x^2\sin xy$

$u = xy$

$dv = \cos xy$

$x\sin xy + \frac{\cos xy}{y} - \frac{\cos xy}{y} + e^y = C$

$\sin xy + e^y = C$

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