Subject: Section 6 موضوع الدرس | Date التاريخ

*Use secant Method of finding roots of eq.

$F(x) = 3x^3 - x + 1$ $\qquad X_{-1} = -1,\ X_0 = 1$

a) Conduct two iteration to estimate the root of the above eq.
b) Find the absolute relative approximate error at end of each iteration

$$X_{n+1} = X_n - \frac{f(X_n)(X_n - X_{n-1})}{f(X_n) - f(X_{n-1})}$$

first iteration

$X_0 = 1 \rightarrow f(1) = 1$
$X_{-1} = -1 \rightarrow f(-1) = -3$

$$X_1 = X_0 - \frac{(X_0 - X_{-1})}{f(X_0) - f(X_{-1})} F(X_0)$$

$$X_1 = 1 - \frac{(1+1)}{1+3} \cdot 1 = \boxed{\frac{1}{2}} \qquad |\epsilon| = \frac{\frac{1}{2} - 1}{1/2} \times 100 = 100\%$$

Second iteration

$$X_2 = X_1 - \frac{(X_1 - X_0)}{f(X_1) - f(X_0)} f(X_1) = \frac{1}{2} - \frac{(\frac{1}{2} - 1)}{-\frac{9}{8} - 1}\left(-\frac{9}{8}\right) = \boxed{\frac{13}{17}}$$

$$|\epsilon| = \frac{13/17 - \frac{1}{2}}{13/17} \times 100 = 34.6\%$$

$\star\ f(x) = x^4 - x - 10$ (3 iterations) $\qquad X_{-1} = 1,\ X_0 = 2$

$f(X_{-1}) = -10 \qquad f(2) = 4$

① $$X_1 = 2 - \frac{4(2-1)}{4+10} = \frac{12}{7} \qquad \epsilon = \frac{\frac{12}{7} - 2}{\frac{12}{7}} \times 100 = 16.6\%$$

② $$X_2 = \frac{12}{7} - \frac{(\frac{12}{7}) - (2)}{(\frac{-7390}{2401}) - 4} \times \left(\frac{-7390}{2401}\right) = 1.84 \qquad \epsilon = \frac{1.84 - \frac{12}{7}}{1.84} \times 100 = 6.8\%$$

③ $$X_3 = 1.84 - \frac{(1.84) - (\frac{12}{7})}{(0.38) - (\frac{-7390}{2401})} \times (-0.38) = 1.86 \qquad \epsilon = \frac{1.86 - 1.84}{1.86} \times 100 = 1.08\%$$

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* $f(x) = x^3 - 2x - 5$ $x_{-1} = 2,\ x_0 = 3$

① $x_1 = 3 - \frac{(3-2)}{16+1} \times 16 = \frac{35}{17} \rightarrow \epsilon_1 = \frac{\frac{35}{17} - 3}{35/17} \times 100 = 45.7\%$

② $x_2 = \frac{35}{17} - \frac{(\frac{35}{17} - 3)}{(\frac{-1920}{4913}) - 16} \times \left(\frac{-1920}{4913}\right) = 2.081 \quad \epsilon_2 = \frac{2.081 - \frac{35}{17}}{2.081} \times 100 = 1.08\%$

③ $x_3\ 2.081 - \frac{(2.081 - \frac{35}{17})}{(-0.150) - (\frac{-1920}{4913})} \times (-0.150) = 2.09 \quad \epsilon_3 = \frac{2.09 - 2.081}{2.09} \times 100$

$= 0.66\%$

* Find The temperature at $z = -7.5$
using direct method For linear and Quadratic interpolation

Depth Z	$t\,°c$
-5	18.3
-6	18.2
-7	17.6
-8	11.7
-9	9.9

① linear

$t = a_0 + a_1 z \quad (-7, 17.6), (-8, 11.7)$

$17.6 = a_0 - 7a_1$ ①

$11.7 = a - 8a_1$

$a_1 = 5.9 \quad a_0 = 58.9$

$(t = 58.9 + 5.9z) \xrightarrow{-7.5} t = 14.65$

② Quadratic

$(-7, 17.6) \quad (-6, 18.2)$

$(-8, 11.7)$

$t = a_0 + a_1 z + a_2 z^2$

$17.6 = a_0 + -7a_1 + 49a_2 \quad a_0 = -89.5$

$11.7 = a_0 - 8a_1 + 64a_2 \quad a_1 = -33.85$

$18.2 = a_0 - 6a_1 + 36a_2 \quad a_2 = -2.65$

Cubic Sheet

$t = -89.5 - 33.85z - 2.65z^2 \rightarrow z \in [-8, -6]$

$\epsilon = \frac{\text{Quadratic} - \text{Linear}}{\text{Quadratic}} \times 100 = 4.32\%$

★ Find The velocity at $t=19$ using The direct Method for Linear, Quadratic, cubic interpolation

t_s (t)	v(t) m/s
0	0
10	227.04
15	362.78
20	517.35
22.5	602.97
30	901.67

① linear → $V = a_0 + a_1 t$

$(15, 362.78)\ (20, 517.35)$

$362.78 = a_0 + 15 a_1$ $\quad a_1 = 30.914$

$517.35 = a_0 + 20 a_1$ $\quad a_0 = -100.93$

$$\boxed{\begin{array}{l} V = (-100.93) + (30.914)t \\ V(19) = 486.436 \end{array}}$$

② Quadratic

$V = a_0 + a_1 t + a_2 t^2$ $\quad (15, 362.78)\ (20, 517.35)\ (22.5, 602.97)$

$362.78 = a_0 + 15 a_1 + 225 a_2$

$517.35 = a_0 + 20 a_1 + 400 a_2$ $\quad a_0 = 32.43$

$602.97 = a_0 + 22.5 a_1 + 506.25 a_2$ $\quad a_1 = 15.36$

$a_2 = 0.446$

$V(t) = 32.43 + (15.36)t + (0.446)t^2$

$V(19) = 484.459$

cubic root

$$\epsilon = \frac{484.459 - 486.436}{484.459} \times 100 = \boxed{0.41\ \%}$$

★ Newton divided difference method

① Linear $\quad f_1(x) = b_0 + b_1(x - x_0)$ $\quad b_0 = f(x_0) = F[x_0]$

② Quadratic $\quad F_2(x) = b_0 + b_1(x - x_0) + b_2(x - x_0)(x - x_1)$ $\quad b_1 = \dfrac{F(x_1) - F(x_0)}{x_1 - x_0} = F[x_0, x_1]$

$$b_2 = \frac{F[x_2, x_1, x_0]}{x_2 - x_0} = \frac{F[x_2 - x_1] - F[x_1 - x_0]}{x_2 - x_0}$$

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* Find y at x=0 by Newton divided difference method

x	-1	1	3	5
y	3	2	5	-7

① linear

$y = b_0 + b_1 (x - x_0)$

$y = 3 + \frac{2-3}{2} (x+1)$

$y = 3 - \frac{1}{2}x - \frac{1}{2} = -\frac{1}{2}x + \frac{5}{2}$

$y(0) = 2.5$

② Quadratic

$y = b_0 + b_1 (x - x_0) + b_2 (x - x_0)(x - x_1)$

		b_1	b_2	b_3
-1	3			
		-0.5		
1	2		0.5	
		1.5		-0.395
3	5		-1.875	
		-6		
5	-7			

$y = 3 - 0.5 (x+1) + 0.5 (x+1)(x-1)$

$y(0) = 2$