Subject | Lecture 5 | موضوع الدرس | Date | التاريخ

Rule 1

IF $\frac{My - Nx}{N}$ is alone Function of x, say f(x) Then $I.F = e^{\int f(x)\,dx}$

Ex. Solve $(2x^2+y)\,dx + (x^2y - x)\,dy = 0$

$My = 1 \quad , Nx = 2xy - 1$

$\frac{My - Nx}{N} = \frac{2 - 2xy}{x^2y - x} = \frac{-2(xy-1)}{x(xy-1)} = \frac{-2}{x}$

$I.F = e^{\int f(x)\,dx} = e^{-2\ln x} = x^{-2}$

$\rightarrow (2 + yx^{-2})\,dx + (y - x^{-1})\,dy = 0$

$My = x^{-2} \quad , Nx = x^{-2} \rightarrow$

$\int 2 + yx^{-2}\,dx + \int y\,dy = c \rightarrow 2x - yx^{-1} + \frac{y^2}{2} + c$

Rule 2

IF $\frac{My - Nx}{-M}$ is a function of y alone, f(y), Then $I.F = e^{\int f(y)\,dy}$

Ex. $(y^4 + 2y)\,dx + (xy^3 + 2y^4 - 4x)\,dy = 0$

$My = 4y^3 + 2 \qquad Nx = y^3 - 4$

$\frac{My - Nx}{-M} = \frac{3y^3 + 6}{-y^4 - 2y} = \frac{3(y^3+2)}{-y(y^3+2)} = \frac{3}{-y}$

$I.F = y^{-3}$

$\int (y + 2y^{-2})\,dx + \int (x + 2y - 4xy^{-3})\,dy = 0$

$= yx + 2y^{-2}x + y^2 = c$

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Rule 3

IF M is of The Form $M = yf(x,y)$, and N is of The form $N = xf(x,y)$

Then $I.F = \frac{1}{Mx - Ny}$

Ex. $y(xy + 2x^2y^2)dx + x(xy - x^2y^2)dy = 0 \rightarrow ①$

$y(1+2xy)\,dx + x(1-xy)\,dy = 0 \rightarrow ①$

$M_y = 1 + 4xy$, $N_x = 1 - 2xy$ $\rightarrow$ $I.F = \frac{1}{xy + 2x^2y^2 - xy + x^2y^2} = \frac{1}{3x^2y^2}$

$\rightarrow \left(\frac{1}{3x^2y} + \frac{2}{3x}\right)dx + \left(\frac{1}{3xy^2} - \frac{1}{3y}\right)dy$

$M_y = \frac{-1}{3x^2y^2}$, $N_x = \frac{-1}{3x^2y^2}$

$\int \frac{dx}{3x^2y} + \int \frac{2}{3x}dx = \int \frac{dy}{3y} = c \rightarrow \frac{-1}{3xy} + \frac{2}{3}\ln x - \frac{1}{3}\ln y = c$