Subject lecture 2, موضوع الدرس Date تاريخ

*Saparable differential equations

$$\frac{dy}{dx} = f(x)\,g(y)$$

$$dy = f(x)\,g(y)\,dx \quad (\div g(y))$$

$$\frac{dy}{g(y)} = f(x)\,dx$$

EX.1

$$\frac{dy}{dx} = -\frac{x}{y} \quad at \quad y(4) = 3$$

$$y\,dy = -x\,dx \leftarrow (\int)$$

$$\frac{y^2}{2} = -\frac{x^2}{x} + c \leftarrow (\times 2)$$

$$y^2 + x^2 = c$$

$$9 + 16 = c = 25$$

$$y^2 + x^2 = 25$$

eq. of Circle

EX.2

$$2x(y^2+1)\,dx - 2y(x^2+1)\,dy = 0 \rightarrow 2x(y^2+1)\,dx = 2y(x^2+1)\,dy$$

$$\div (x^2+1)(y^2+1)$$

$$\frac{2x}{x^2+1}\,dx = \frac{2y}{y^2+1}\,dy$$

$$\ln|x^2+1| + \ln|C| = \ln|y^2+1| \rightarrow C(x^2+1) = y^2+1$$

EX.3

$$\frac{dy}{dx} = (x+y-1)^2$$

$$\frac{du}{dx} = u^2 + 1$$

$$du = (1+u^2)\,dx \rightarrow \int \frac{du}{(1+u^2)} = \int dx$$

Put $u = x+y-1$

$$\frac{du}{dx} = 1 + \frac{dy}{dx}$$

$$\frac{du}{dx} - 1 = u^2$$

$$\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a}$$

$$\tan^{-1} u = x + c \rightarrow \tan^{-1}(x+y-1) = x+c \rightarrow x+y-1 = \tan(x+c)$$

$$y = \tan(x+c) - x + 1$$

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Ex.4

$\frac{dy}{dx} = 6y^2x \rightarrow \int \frac{dy}{y^2} = \int 6x\,dx$

$-\frac{1}{y} = 3x^2 + C$

$\frac{1}{x^n} = \frac{-n}{x^{n+1}}$

$\int \frac{dx}{x^n} = \frac{1}{(1-n)x^{n+1}}$

$\int \frac{dx}{x^5} = \int x^{-5}\,dx$

$= \frac{x^{-4}}{-4} = \frac{-1}{4x^4}$

Ex.5

$y' = \cos^2 x \cos y$

$\frac{dy}{\cos y} = \cos^2 x\,dx$

$\sec y\,dy = \cos^2 x\,dx$

$\ln|\sec y + \tan y| = \frac{1}{2}(1+\cos 2x)\,dx$

$\ln|\sec y + \tan y| = \frac{1}{2}\left(x + \frac{\sin 2x}{2}\right) + C$

$\int \sec x\,dx = \ln|\sec x + \tan$

$\int \csc dx = \ln|\csc - \cot x$

$\cos^2 x = \frac{1}{2}(1+\cos 2x)$

$\sin^2 x = \frac{1}{2}(1-\cos 2x)$

Ex.6

$\csc y\,dx = -\sec^2 x\,dy \rightarrow \frac{dx}{\sec^2 x} = \frac{dy}{\csc}$

$\cos^2(x)dx = \sin(y)\,dy$

$\frac{1}{2}(1+\cos 2x)dx = \sin(y)\,dy \rightarrow \frac{1}{2}\left(x + \frac{\sin 2x}{2}\right) + C = -\cos(y)$

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★ Homogenous different equations

$y = f(x)$ $\quad \frac{dy}{dx} = f\left(\frac{y}{x}\right)$ (1)

$x = f(y)$ $\quad \frac{dx}{dy} = f\left(\frac{x}{y}\right)$

Put $u = \frac{y}{x} \rightarrow y = ux$

$\frac{dy}{dx} = u + x\frac{du}{dx} \rightarrow (2)$

$f(u) = u + x\frac{du}{dx} \rightarrow x\frac{du}{dx} = f(u) - u$

$\frac{du}{f(u)-u} = \frac{dx}{x}$

$(y^2+x^2)\,dx - 2xy\,dy = 0$

$\frac{dy}{dx} = \frac{y^2+x^2}{2xy} = \frac{(\frac{y}{x})^2+1}{2(\frac{y}{x})}$

$\frac{dy}{dx} = u + x\frac{du}{dx}$

$y = ux$

$u = \frac{y}{x}$

$u + x\frac{du}{dx} = \frac{u^2+1}{2u} \rightarrow x\frac{du}{dx} = \frac{u^2+1}{2u} - u$

$x\frac{dy}{dx} = \frac{u^2+1-2u^2}{2u} \Rightarrow \frac{x\,du}{dx} = \frac{1-u^2}{2u} \rightarrow \frac{2u}{1-u^2}\,du = \frac{dx}{x}$

$-\ln|1-u^2| + \ln C = \ln x$

$b = \frac{x}{(1-u^2)} \Longrightarrow b = \frac{x}{1-(\frac{y}{x})^2}$