

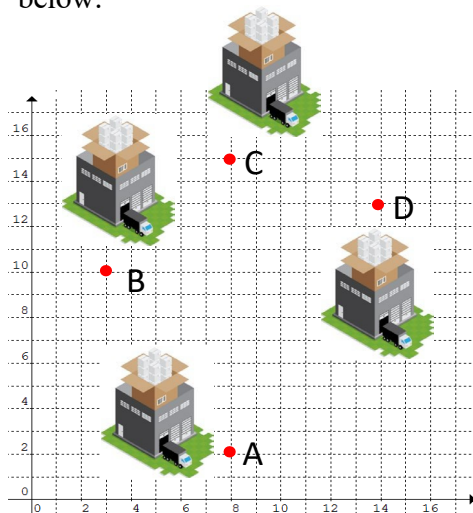


WEEK 4- NON-LINEAR PROGRAMMING QUESTIONS

Exercise 1. A company distributes specialty paper to stores (sales centres) in 4 major metropolitan areas and plans to consolidate its warehouses (distribution centres) into one national distribution centre.

To identify a suitable site, the logistic manager first maps the 4 stores (sales centres) on a two-dimensional grid, so that coordinates (X, Y) can be associated with each site.

The daily number of deliveries that must be made to each sales centre from the new distribution centre, as well as the (x, y) coordinates of each of the sales centre are tabulated below:



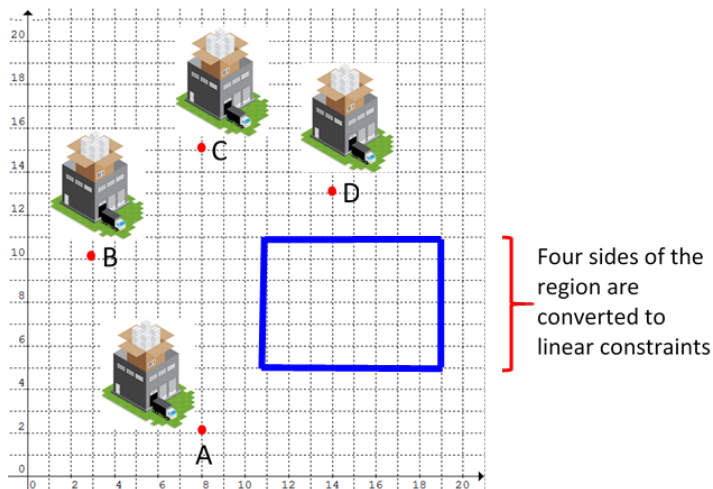
SALES CENTRE	DAILY NO. OF TRUCK DELIVERIES	X COORDINATE	Y COORDINATE
A	9	8	2
B	7	3	10
C	2	8	15
D	5	14	13

Suppose that truck delivery costs are \$1 per km for trucks that travel between the distribution centre and the sales centres. The company wishes to determine where to locate the new distribution centre, so as to service the sales centres with the lowest cost.

Note: The straight-line distance (also known as the Euclidean distance) between two points

(x_1, y_1) and (x_2, y_2) is equal to $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

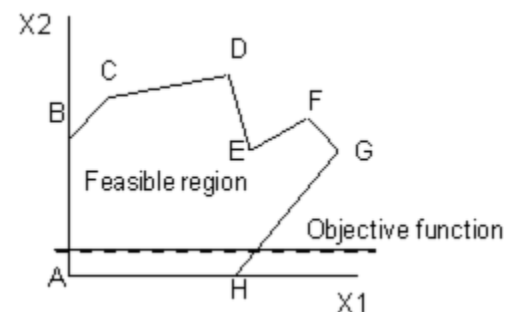
Exercise 2. Suppose that zoning laws restrict the site of the distribution centre to lie in the four-sided region as shown below. Formulate this problem.



Quiz

- Which point or points are global optima in this diagram? The dashed line represents the objective function and the objective is to maximize the value of the objective function.

- B
- D
- E
- F



- Which point or points are local optima in above diagram? The dashed line represents the objective function and the objective is to maximize the value of the objective function.

- D
- E
- F
- D,F

WEEK 4 SOLUTIONS

Exercise 1

Let $P = (x, y)$ be the location of the new distribution centre in the coordinate plane. Then, the distance from P to sales centre A is given by the distance formula: $\sqrt{(x - 8)^2 + (y - 2)^2}$.

The straight-line distance (also known as the Euclidean distance) between two points (x_1, y_1) and (x_2, y_2) is given by $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

For any distribution centre site (x, y) , it is possible to calculate the distance from the new distribution centre to each of the stores and to sum the distances.

This total can be thought as a proxy for the total annual cost incurred, since they will make regular trips to the individual stores.

Minimising the sum of distances therefore represents an objective that is consistent with minimising the annual distribution cost. The company wishes to determine the location that achieves the minimum sum of distances.

Since sales centre A must receive 9 truck deliveries per day, the travel cost associated with the sales centre A is given by: $2 \times 9 \sqrt{(x - 8)^2 + (y - 2)^2}$.

The number “2” appears above because each truck must travel to the sales centre from the distribution centre and back again. i.e. 9 round trips or 18 one-way trips.

If we derive the travel cost associated with the other three sales centres in a similar manner, the total travel cost:

$$2 \times 9 \sqrt{(x - 8)^2 + (y - 2)^2} + 2 \times 7 \sqrt{(x - 3)^2 + (y - 10)^2} \\ + 2 \times 2 \sqrt{(x - 8)^2 + (y - 15)^2} + 2 \times 5 \sqrt{(x - 14)^2 + (y - 13)^2}$$

NLP Model:

The coordinate values x and y of the location P of the distribution centre that will yield the minimum value of the daily travel costs.

Decision variables:

x : location of the new distribution Centre with respect to the X-axis

y : location of the new distribution Centre with respect to the Y-axis.

Objective function: minimise

$$2 \times 9 \sqrt{(x - 8)^2 + (y - 2)^2} + 2 \times 7 \sqrt{(x - 3)^2 + (y - 10)^2} \\ + 2 \times 2 \sqrt{(x - 8)^2 + (y - 15)^2} + 2 \times 5 \sqrt{(x - 14)^2 + (y - 13)^2}$$

Constraints: $x, y \geq 0$.

Exercise 2**NLP Model:**

The coordinate values x and y of the location P of the distribution centre that will yield the minimum value of the daily travel costs subject to the constraints that the point P must lie in the four-sided region.

Decision variables:

x : location of the new distribution centre with respect to the X-axis

y : location of the new distribution centre with respect to the Y-axis.

Objective function: minimise

$$2 \times 9\sqrt{(x-8)^2 + (y-2)^2} + 2 \times 7\sqrt{(x-3)^2 + (y-10)^2} \\ + 2 \times 2\sqrt{(x-8)^2 + (y-15)^2} + 2 \times 5\sqrt{(x-14)^2 + (y-13)^2}$$

Constraints: _____

$$y \geq 5$$

$$y \leq 11$$

$$x \geq 11$$

$$x \leq 19$$

$$x, y \geq 0.$$

Quiz

1) b. 2) d.