Assignment 1 is available

- Assignment one support session Timetable – Please see the Announcements area on the unit site
- Assignment 1 queries will only be answered via the Discussion forum from Monday to Friday during business hours
- To be fair to all students, we are unable to answer Assignment one queries via email.

Week 7 - Easter vacation/intra-trimester break:
Friday 18th April –Sunday 27th April Sunday (inclusive)

What's the Plan for Week 7?

- No F2F seminars or online seminars on Friday 18th April (Public Holiday)
- **Week 7 Online seminar will be on Thursday, 17th April at 7 pm**
- **Friday On-Campus Seminar groups are very welcome to attend the week 7 online seminar or any other face to face seminar**

MIS775
Decision Modelling for Business Analytics

TOPIC 5: Network Modelling

Recap

- In **deterministic decision models** all parameters in the model are assumed to be fixed, i.e. there is no random variation associated with these parameters

- The objective is to develop optimal solutions to these models

- There are three main optimisation techniques:

  - **Linear programming**

  - **Integer Linear programming**

  - **Non-linear programming**

- This week we turn our attention to network models, which are special cases of LP and ILP models

Network Models

There are several reasons for distinguishing network models from other LP/ILP models:

- **Many companies have real problems, often extremely large, that can be represented as network models**

- **These models can be visualised using a network diagram, which users can intuitively understand, and which can also be helpful in developing the spreadsheet model**

Motivation

Many important optimisation models can be represented as a network

- Supply chain models are network models that are designed to satisfy customer demand for a product at <u>minimum cost</u>. Those who control the supply chain must make decisions such as where to produce a product, how much should be produced, and where it should be sent. Business models include:
  - **Transportation model**
  - **Transshipment model**
- Other business models can also be represented as networks, such as:
  - **Assignment model**
  - **Maximal Flow model**
 - **Shortest Path model**
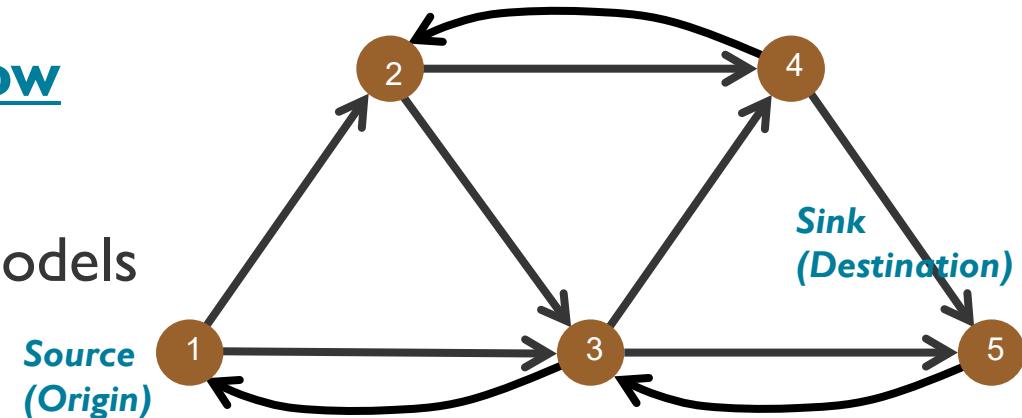
Learning Objectives

- Understand why applying optimisation techniques to supply chain problems is useful

- Understand how to structure special LP network flow models

- Become familiar with the basic features of the **transportation, transshipment, assignment, maximal flow, and shortest path** models

- Be able to set up and solve network models using Excel's Solver

Textbook reading: Chapter 5 (5.2-5.5)

What is a Network Model?

- A network model is comprised of nodes (locations, either **origins** or **destinations**) connected by arcs (arrows representing shipping routes between locations), and amounts (unit shipping costs) associated with the arcs and/or nodes

- Decision variables are called the **flows** and are the amounts shipped along the arcs

- An arrow pointing <u>into</u> a node is called an **inflow**

- An arrow pointing <u>out</u> of a node is called an **outflow**

- Some nodes have both inflows and outflows

- Network models can be formulated as LP or ILP models

Sink (Destination)

Source (Origin)

- **Source, Sink** refers to **flow systems** (e.g., fluid, data, power).
- **Origin, Destination** refers to **transport and movement** (e.g., vehicles, people, shipments).

Network Models

5.1 **Transportation Model**

5.2 **Assignment Model**

5.3 **Transshipment Model**

5.4 **Maximal Flow Model**

5.5 **Shortest Path Model**

5.1 Transportation Models*

- A company manufactures cars in three plants and ships them to four regions. The table below shows the unit costs of shipping a car from a plant to a region. Each plant has a limited capacity

- The company wants to minimise its total shipping cost while staying within its plant capacities and meeting regional demands

If Capacity >=Demand
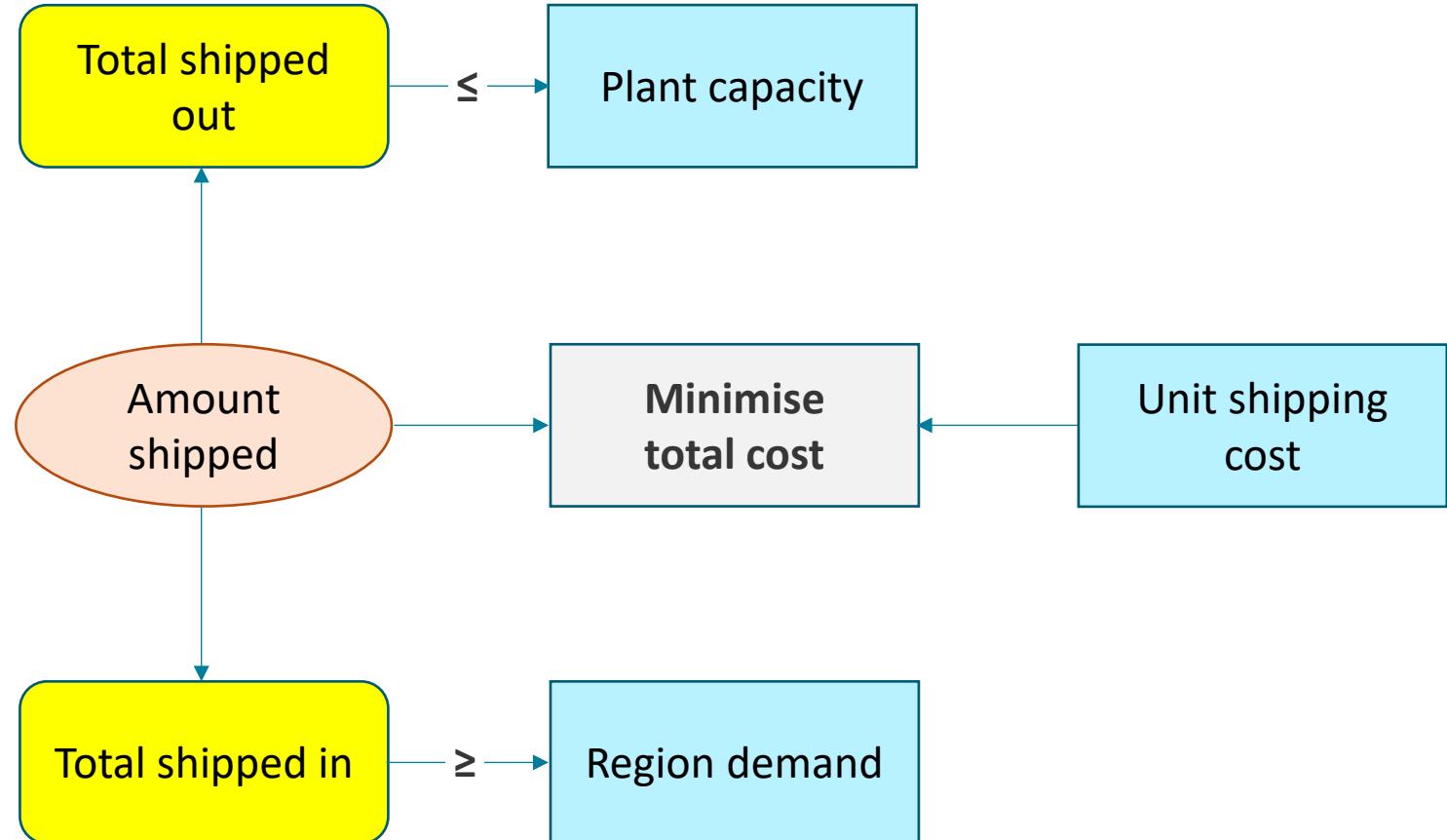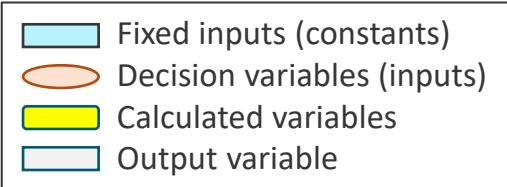
	Region 1	Region 2	Region 3	Region 4	Capacity (s_i)
Plant 1	$131	$218	$266	$120	450
Plant 2	$250	$116	$263	$278	600
Plant 3	$178	$132	$122	$180	500
Demand(d_j)	450	200	300	300	

Total Capacity = 1550
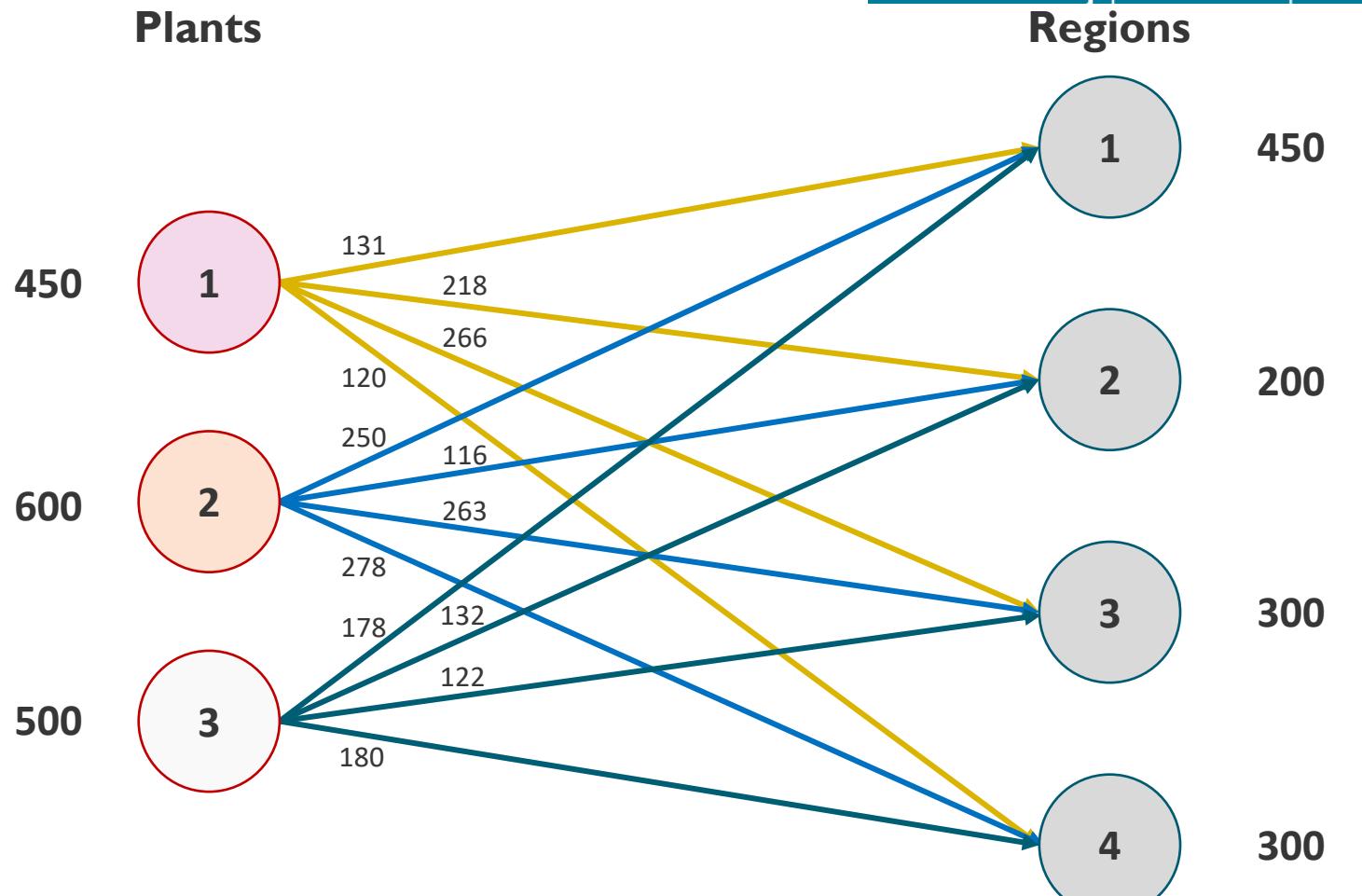
Total Demand = 1250

*Textbook example on pp. 221-232

Conceptual Model

Legend:
- Fixed inputs (constants)
- Decision variables (inputs)
- Calculated variables
- Output variable

Total shipped out ≤ Plant capacity

Amount shipped → Minimise total cost ← Unit shipping cost

Total shipped in ≥ Region demand

Network Model

	Region 1	Region 2	Region 3	Region 4	Capacity (s_i)
Plant 1	$131	$218	$266	$120	450
Plant 2	$250	$116	$263	$278	600
Plant 3	$178	$132	$122	$180	500
Demand(d_j)	450	200	300	300	

5.1 Algebraic Model for This Example

Decision variables: Let X_{ij} denote the number of units shipped from plant i to region j

where $i = 1, 2, 3; j = 1, 2, 3, 4$
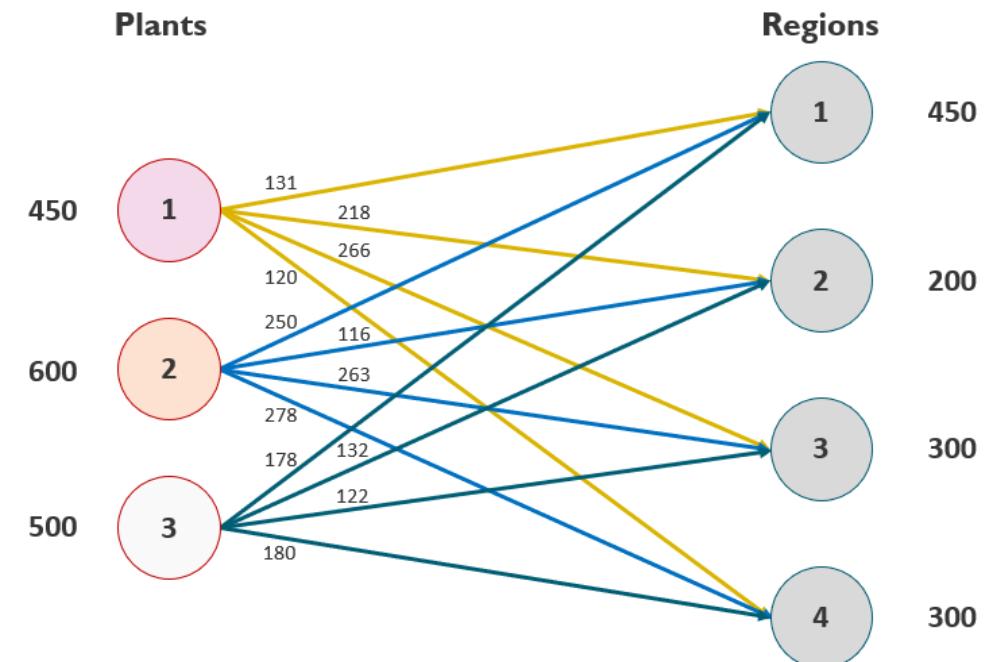
Objective: Minimise

$$131X_{11}+218X_{12}+266X_{13}+120X_{14}$$
$$+250X_{21}+116X_{22}+263X_{23}+278X_{24}$$
$$+178X_{31}+132X_{32}+122X_{33}+180X_{34}$$

Subject to constraints:

|	
$X_{11}+X_{12}+X_{13}+X_{14} \leq 450$	(capacity of plant 1)
$X_{21}+X_{22}+X_{23}+X_{24} \leq 600$	(capacity of plant 2)
$X_{31}+X_{32}+X_{33}+X_{34} \leq 500$	(capacity of plant 3)
$X_{11}+X_{21}+X_{31} \geq 450$	(demand of region 1)
$X_{12}+X_{22}+X_{32} \geq 200$	(demand of region 2)
$X_{13}+X_{23}+X_{33} \geq 300$	(demand of region 3)
$X_{14}+X_{24}+X_{34} \geq 300$	(demand of region 4)
$X_{ij} \geq 0$	(non-negativity)

General Algebraic Model

	Region 1	Region 2	Region 3	Region 4	Capacity (s_i)
Plant 1	c11	c12	c13	c14	s1
Plant 2	c21	c22	c23	c24	s2
Plant 3	c31	c32	c33	c34	s3
Demand(d_j)	d1	d2	d3	d4	

Decision variables:

X_{ij} = number of units shipped from origin i to destination j

Objective: Minimise total cost = $\sum c_{ij} X_{ij}$

where c_{ij} is the cost per unit of shipping from origin i to destination j

Constraints

Let s_i denote the capacity of origin i, and d_j denote the demand of destination j

Then

- $\sum_j X_{ij} \leq s_i$ for $i = 1, 2, \ldots, m$ (Capacity constraints)
- $\sum_i X_{ij} \geq d_j$ for $j = 1, 2, \ldots, n$ (Demand constraints)
- $X_{ij} \geq 0$ for all i and j

Solver Set-up & Optimal Solution

	Region 1	Region 2	Region 3	Region 4	Capacity (s_i)
Plant 1	$131	$218	$266	$120	450
Plant 2	$250	$116	$263	$278	600
Plant 3	$178	$132	$122	$180	500
Demand(d_j)	450	200	300	300	

Solver Parameters

Set Objective: $C$19

To: ○ Max   ● Min   ○ Value Of: [0]

By Changing Variable Cells:

$C$13:$F$15

Subject to the Constraints:

$C$24:$C$26 <= $E$24:$E$26
$C$28:$C$31 >= $E$28:$E$31

[Add] [Change] [Delete] [Reset All] [Load/Save]

☑ Make Unconstrained Variables Non-Negative

Select a Solving Method: Simplex LP   [Options]

Solving Method

Select the GRG Nonlinear engine for Solver Problems that are smooth nonlinear. Select the LP Simplex engine for linear Solver Problems, and select the Evolutionary engine for Solver problems that are non-smooth.

[Help]   [Solve]   [Close]

C19   =SUMPRODUCT(C5:F7,C13:F15)

	B	C	D	E	F	G	H
10	DECISION VARIABLES						
11							
12		Region 1	Region 2	Region 3	Region 4		
13	Plant 1	150	0	0	300		
14	Plant 2	100	200	0	0		
15	Plant 3	200	0	300	0		
16							
17	OBJECTIVE FUNCTION						
18							
19	Total cost	$176,050					
20							
21	CONSTRAINTS						
22							
23		Shipped			Capacity		
24	Plant 1	450	≤	450			
25	Plant 2	300	≤	600			
26	Plant 3	500	≤	500			
27							
28		Received			Demand		
29	Region 1	450	≥	450			
30	Region 2	200	≥	200			
31	Region 3	300	≥	300			
32	Region 4	300	≥	300			

Sensitivity Analysis

	Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
6	Variable Cells						
9	$C$13	Plant 1 Region 1	150	0	131	119	13
10	$D$13	Plant 1 Region 2	0	221	218	1E+30	221
11	$E$13	Plant 1 Region 3	0	191	266	1E+30	191
12	$F$13	Plant 1 Region 4	300	0	120	13	239
13	$C$14	Plant 2 Region 1	100	0	250	39	72
14	$D$14	Plant 2 Region 2	200	0	116	88	116
15	$E$14	Plant 2 Region 3	0	69	263	1E+30	69
16	$F$14	Plant 2 Region 4	0	39	278	1E+30	39
17	$C$15	Plant 3 Region 1	200	0	178	13	69
18	$D$15	Plant 3 Region 2	0	88	132	1E+30	88
19	$E$15	Plant 3 Region 3	300	0	122	69	194
20	$F$15	Plant 3 Region 4	0	13	180	1E+30	13

	Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
22	Constraints						
25	$C$24	Plant 1 Shipped	450	-119	450	100	150
26	$C$25	Plant 2 Shipped	300	0	600	1E+30	300
27	$C$26	Plant 3 Shipped	500	-72	500	100	200
28	$C$29	Region 1 Received	450	250	450	300	100
29	$C$30	Region 2 Received	200	116	200	300	200
30	$C$31	Region 3 Received	300	194	300	200	100
31	$C$32	Region 4 Received	300	239	300	150	100

- The top part shows the effect of a change in shipping costs
  - **E.g. if the unit cost from plant 2 to region 3 was reduced by more than $69 then that route would become attractive**
- The bottom part shows the effect of a change in capacity or demand
  - **E.g. if the capacity of plant 1 could be increased to 550 (450+100=550units), then that would decrease the total shipping cost by $11,900 (100*-119)**

What if Demand Exceeds Capacity?

- If the total demand exceeds the total capacity $\left(\sum_j d_j > \sum_i s_i\right)$ you can add a dummy origin with supply equal to the shortage amount. Assign a zero shipping cost per unit to the dummy origin. The amount "shipped" from the dummy origin (in the solution) will not actually be shipped

Capacity >= Demand

	Region 1	Region 2	Region 3	Region 4	Capacity (s_i)
Plant 1	$131	$218	$266	$120	450
Plant 2	$250	$116	$263	$278	600
Plant 3	$178	$132	$122	$180	500
Demand(d_j)	450	200	300	300	

Total Demand = 1250

Total Capacity = 1550

Capacity < Demand

	Region 1	Region 2	Region 3	Region 4	Capacity(sj)
Plant 1	$131	$218	$266	$120	450
Plant 2	$250	$116	$263	$278	600
Plant 3	$178	$132	$122	$180	500
Plant 4	$0	$0	$0	$0	100
Demand(dj)	450	600	300	300	
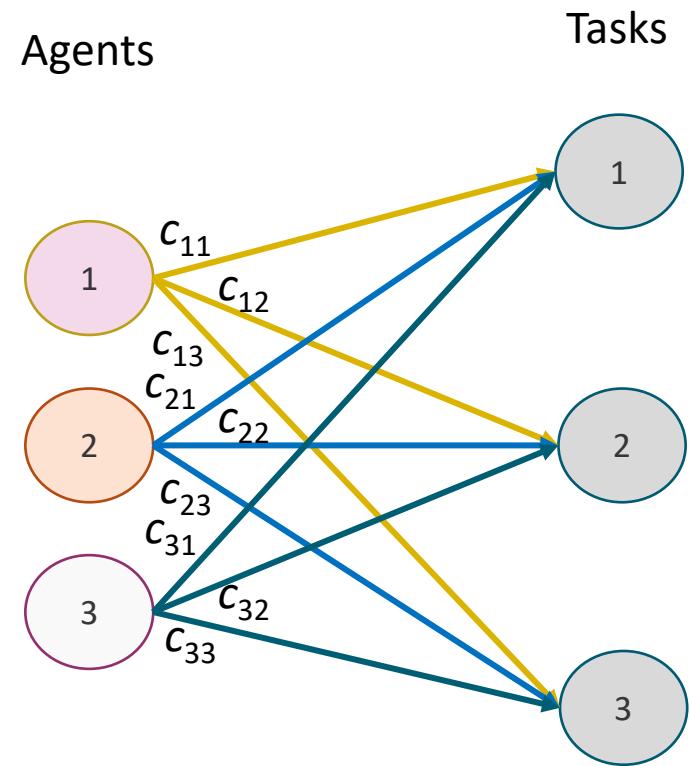
Total Demand = 1650

Total Capacity = 1550 + 100 =1650

*Textbook example on pp. 233-239

5.2 Assignment Models

- An assignment model seeks to minimise the total cost from assigning members of one set (agents) to members of another set (tasks), where the cost of agent i performing task j is c_{ij}

- A defining characteristic is that **each task is assigned to one and only one agent**

- An assignment model is a **special case of a transportation model** in which all demands are equal to 1; hence assignment models may be solved as an ILP model

Agents

Tasks

c_{11}
c_{12}
c_{13}
c_{21}
c_{22}
c_{23}
c_{31}
c_{32}
c_{33}

Network model for an assignment problem with three agents and three tasks

5.2 Assignment Example*

- A company has five machines and four jobs to be completed

- Each job is to be completed by one machine only

- The table below gives times for the machines to complete these jobs, as well as the capacity of each machine (i.e. number of jobs they can handle)
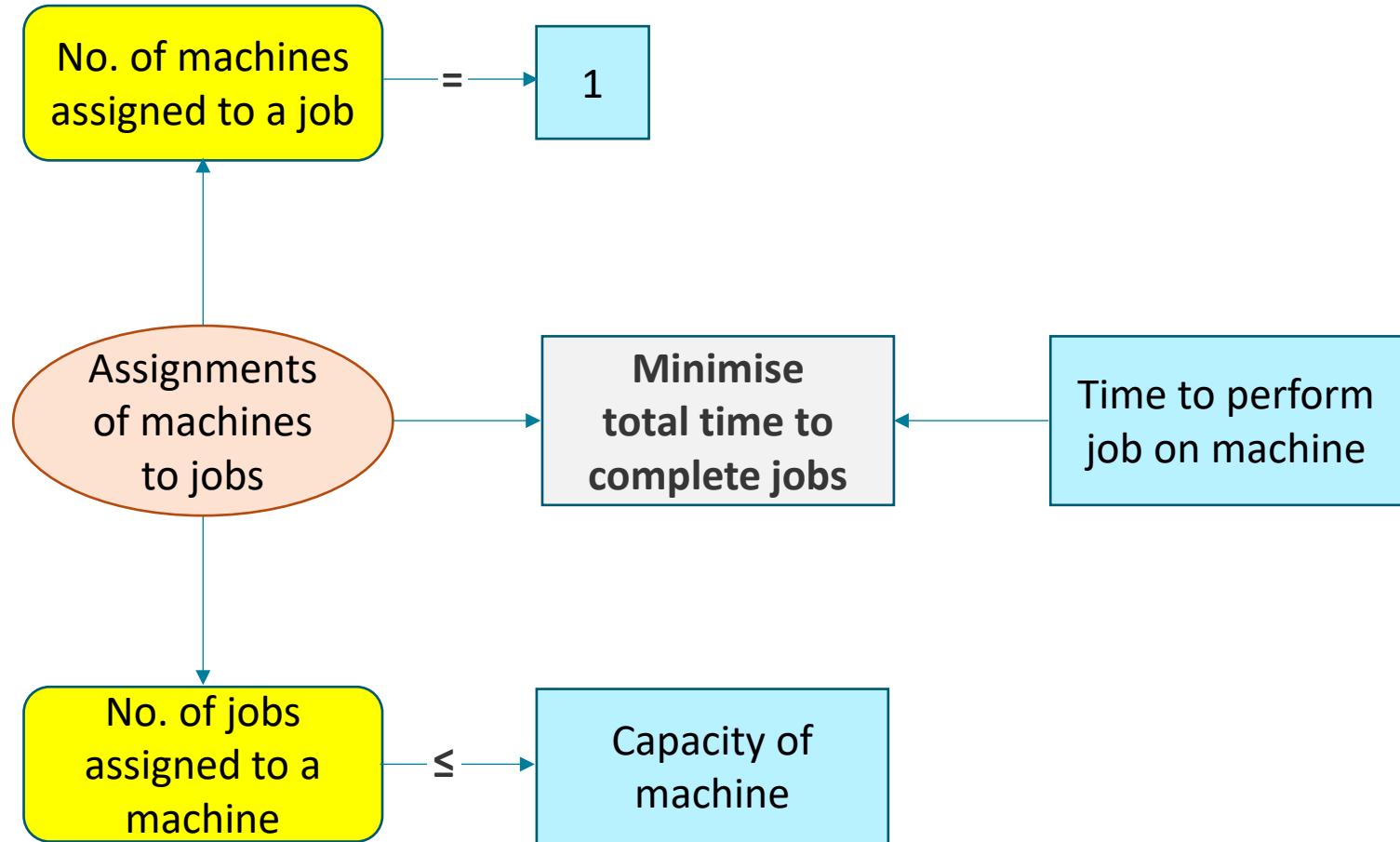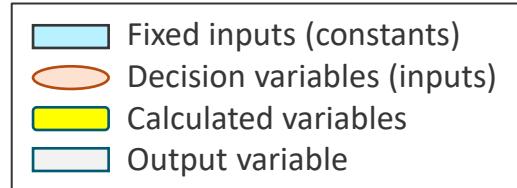
Machine	Job 1	Job 2	Job 3	Job 4	Capacity
1	14	5	8	7	1
2	2	12	6	5	2
3	7	8	3	9	1
4	2	4	6	10	2
5	5	5	4	8	1

If Capacity >= Machine

7>4

*Textbook example on pp. 233-235

Conceptual Model

Legend:
- Fixed inputs (constants)
- Decision variables (inputs)
- Calculated variables
- Output variable

No. of machines assigned to a job = 1

Assignments of machines to jobs

Minimise total time to complete jobs

Time to perform job on machine

No. of jobs assigned to a machine ≤ Capacity of machine

5.2 Algebraic Model for This Example

Decision variables: $X_{ij} = \begin{cases} 1, \text{if machine } i \text{ is assigned to job } j \\ 0, \text{otherwise} \end{cases}$

where $i = 1, 2, 3, 4, 5; j = 1, 2, 3, 4$

Objective: Minimise $\sum c_{ij} X_{ij}$, where c_{ij} is the time it takes machine i to complete job j

Subject to constraints:

$X_{11} + X_{12} + X_{13} + X_{14} \leq 1$ (capacity of machine 1)
$X_{21} + X_{22} + X_{23} + X_{24} \leq 2$ (capacity of machine 2)
$X_{31} + X_{32} + X_{33} + X_{34} \leq 1$ (capacity of machine 3)
$X_{41} + X_{42} + X_{43} + X_{44} \leq 2$ (capacity of machine 4)
$X_{51} + X_{52} + X_{53} + X_{54} \leq 1$ (capacity of machine 5)

Machine	Job 1	Job 2	Job 3	Job 4	Capacity
1	14	5	8	7	1
2	2	12	6	5	2
3	7	8	3	9	1
4	2	4	6	10	2
5	5	5	4	8	1

$X_{11} + X_{21} + X_{31} + X_{41} + X_{51} = 1$ (one machine is assigned to job 1)
$X_{12} + X_{22} + X_{32} + X_{42} + X_{52} = 1$ (one machine is assigned to job 2)
$X_{13} + X_{23} + X_{33} + X_{43} + X_{53} = 1$ (one machine is assigned to job 3)
$X_{14} + X_{24} + X_{34} + X_{44} + X_{54} = 1$ (one machine is assigned to job 4)

General Algebraic Model

Machine	Job1	Job2	Job3	Job4	Capacity
1	c11	c12	c13	c14	S1
2	c21	c22	c23	c24	S2
3	c31	c32	c33	c34	S3
4	c41	c42	c43	c44	S4
5	c51	c52	c53	c54	S5
	1	1	1	1	

Decision variables:

$$X_{ij} = \begin{cases} 1 \text{ if agent } i \text{ is assigned to task } j \\ 0 \text{ otherwise} \end{cases}$$

Objective: Minimise $\sum c_{ij} X_{ij}$

where c_{ij} is the cost (or time in this example) assigned to agent i performing task j

Constraints

Let s_i denote the capacity of agent i

Then

- $\sum_j X_{ij} \leq s_i$ for $i = 1, 2, \ldots, m$ (Capacity constraints for the agents)
- $\sum_i X_{ij} = 1$ for $j = 1, 2, \ldots, n$ (Task constraints)
- X_{ij} are binary for all i and j

Solver Set-up & Optimal Solution

Machine	Job 1	Job 2	Job 3	Job 4	Capacity
1	14	5	8	7	1
2	2	12	6	5	2
3	7	8	3	9	1
4	2	4	6	10	2
5	5	5	4	8	1

Solver Parameters

Set Objective: $C$22

To: ○ Max   ● Min   ○ Value Of: [0]

By Changing Variable Cells:
$C$14:$F$18

Subject to the Constraints:
$C$14:$F$18 = binary
$C$27:$C$31 <= $E$27:$E$31
$C$34:$C$37 = $E$34:$E$37

[Add] [Change] [Delete] [Reset All] [Load/Save]

☑ Make Unconstrained Variables Non-Negative

Select a Solving Method: [Simplex LP] [Options]

Solving Method
Select the GRG Nonlinear engine for Solver Problems that are smooth nonlinear. Select the LP Simplex engine for linear Solver Problems, and select the Evolutionary engine for Solver problems that are non-smooth.

[Help] [Solve] [Close]

C22   =SUMPRODUCT(C14:F18,C5:F9)

DECISION VARIABLES

	Job 1	Job 2	Job 3	Job 4
Machine 1	0	0	0	0
Machine 2	1	0	0	1
Machine 3	0	0	1	0
Machine 4	0	1	0	0
Machine 5	0	0	0	0

OBJECTIVE FUNCTION

Total time   14

CONSTRAINTS

	Jobs assigned		Capacity
Machine 1	0	≤	1
Machine 2	2	≤	2
Machine 3	1	≤	1
Machine 4	1	≤	2
Machine 5	0	≤	1

	Machines assigned		Required
Job 1	1	=	1
Job 2	1	=	1
Job 3	1	=	1
Job 4	1	=	1

What if the Number of Tasks Exceeds Capacity?
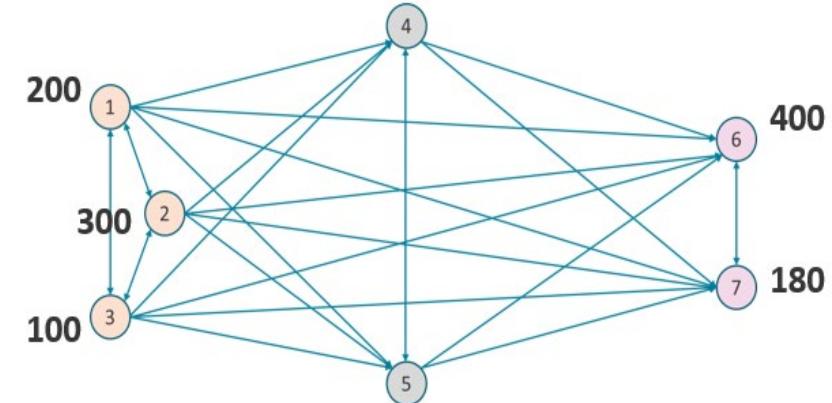
- If the number of tasks, n exceeds the total capacity across the agents $(n > \sum_i s_i)$ you can add a dummy agent with a capacity that equalises the total capacity with the number of tasks. Assign zero to coefficients c_{ij} for the dummy agent. The tasks assigned to the dummy agent (in the solution) will not actually be performed

Other Logistics Models

- Transshipment models extend transportation models by allowing shipments to move through intermediate nodes (known as **transshipment nodes**) before reaching their destination nodes

- The objective is to determine the number of units to ship over each arc in the network so that all destination demands are met with minimum overall transportation cost

5.3 Transshipment Example*

- A company produces tomato products at three plants. Product can be shipped directly to two customers, or via two warehouses. In the network diagram **the three plants are labelled 1, 2, 3**; the **two warehouses are labelled as 4 & 5;** and the **two customers are labelled as 6 & 7**

- Shipments are allowed between plants, customers and warehouses (where arrows are double-headed)

- It is assumed that at most 200 tonnes of product can be shipped between any two nodes

- There are capacity constraints of 200, 300, and 100 tonnes at plants 1, 2, and 3 respectively

- There are demand constraints of 400 tonnes and 180 tonnes from customers at nodes 6 and 7 respectively

Unit shipping costs ($'000 per tonne of product)

Origin	Destination						
	1	2	3	4	5	6	7
1		5	3	5	5	20	20
2	9		9	1	1	8	15
3	0.4	8		1	0.5	10	12
4					1.2	2	12
5				0.8		2	12
6							1
7						7	

*Textbook example on pp. 240-247

Conceptual Model

Legend:
- Fixed inputs (constants)
- Decision variables (inputs)
- Calculated variables
- Output variable

Route capacity

≤

Amounts sent on routes

Plant net outflow ≤ Plant capacity

Warehouse net outflow = 0

Customer net inflow ≥ Customer demand

Unit shipping cost → Minimise total cost

5.3 Algebraic Model for This Example

Decision variables: X_{ij} = amount shipped from node i to j

Objective: Minimise $5X_{12}+3X_{13}+5X_{14}+5X_{15}+20X_{16}+20X_{17}+9X_{21}+9X_{23}+X_{24}+X_{25}+8X_{26}+15X_{27}+$
$0.4X_{31}+8X_{32}+X_{34}+0.5X_{35}+10X_{36}+12X_{37}+1.2X_{45}+2X_{46}+12X_{47}+0.8X_{54}+2X_{56}+12X_{57}+X_{67}+7X_{76}$

Subject to constraints:

$X_{12}+X_{13}+X_{14}+X_{15}+X_{16}+X_{17}-X_{21}-X_{31} \leq 200$ (origin node 1)

$X_{21}+X_{23}+X_{24}+X_{25}+X_{26}+X_{27}-X_{12}-X_{32} \leq 300$ (origin node 2)

$X_{31}+X_{32}+X_{34}+X_{35}+X_{36}+X_{37}-X_{13}-X_{23} \leq 100$ (origin node 3)
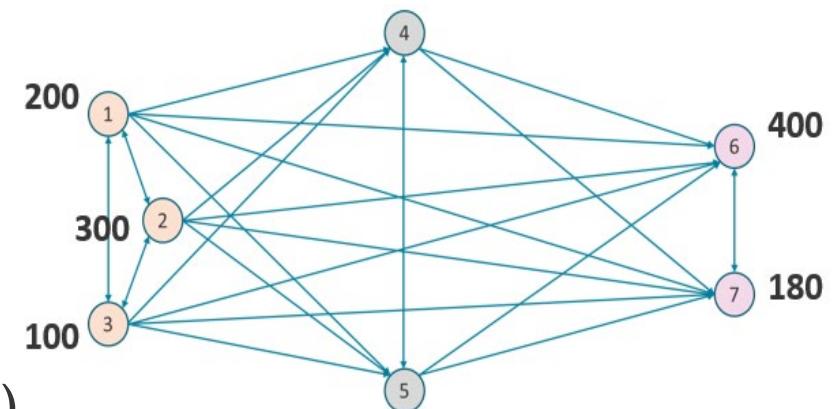
$X_{45}+X_{46}+X_{47}-X_{14}-X_{24}-X_{34}-X_{54} = 0$ (transshipment node 4)

$X_{54}+X_{56}+X_{57}-X_{15}-X_{25}-X_{35}-X_{45} = 0$ (transshipment node 5)

$X_{16}+X_{26}+X_{36}+X_{46}+X_{56}+X_{76}-X_{67} \geq 400$ (destination node 6)

$X_{17}+X_{27}+X_{37}+X_{47}+X_{57}+X_{67}-X_{76} \geq 180$ (destination node 7)

$X_{ij} \geq 0$ and $\max(X_{ij}) \leq 200$ for all $i, j = 1, 2, \ldots, 7$

Unit shipping costs ($'000 per tonne of product)

	Destination						
Origin	1	2	3	4	5	6	7
1		5	3	5	5	20	20
2	9		9	1	1	8	15
3	0.4	8		1	0.5	10	12
4					1.2	2	12
5				0.8		2	12
6							1
7						7	

General Algebraic Model

Decision variables:

X_{ij} = amount shipped from node i to j

Objective:

Minimise $\displaystyle\sum_{\text{all arcs}} c_{ij}X_{ij}$, where c_{ij} is the unit cost of shipping from node i to node j
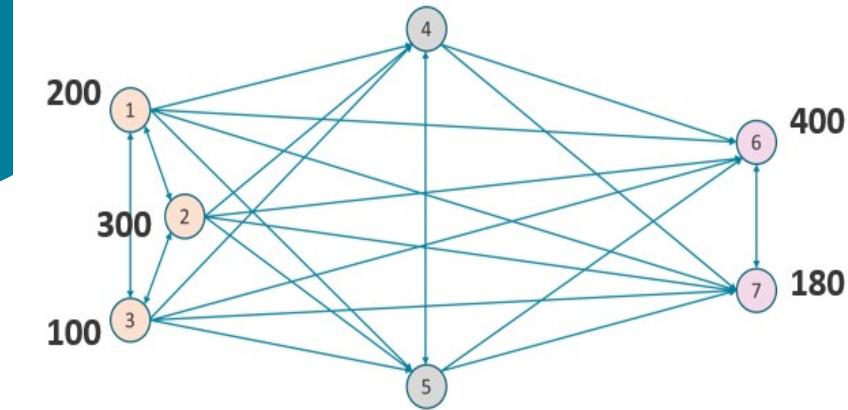
Constraints

$$\sum_{\text{arcs out}} X_{ij} - \sum_{\text{arcs in}} X_{ij} \leq s_i \qquad \text{(Origin nodes } i)$$
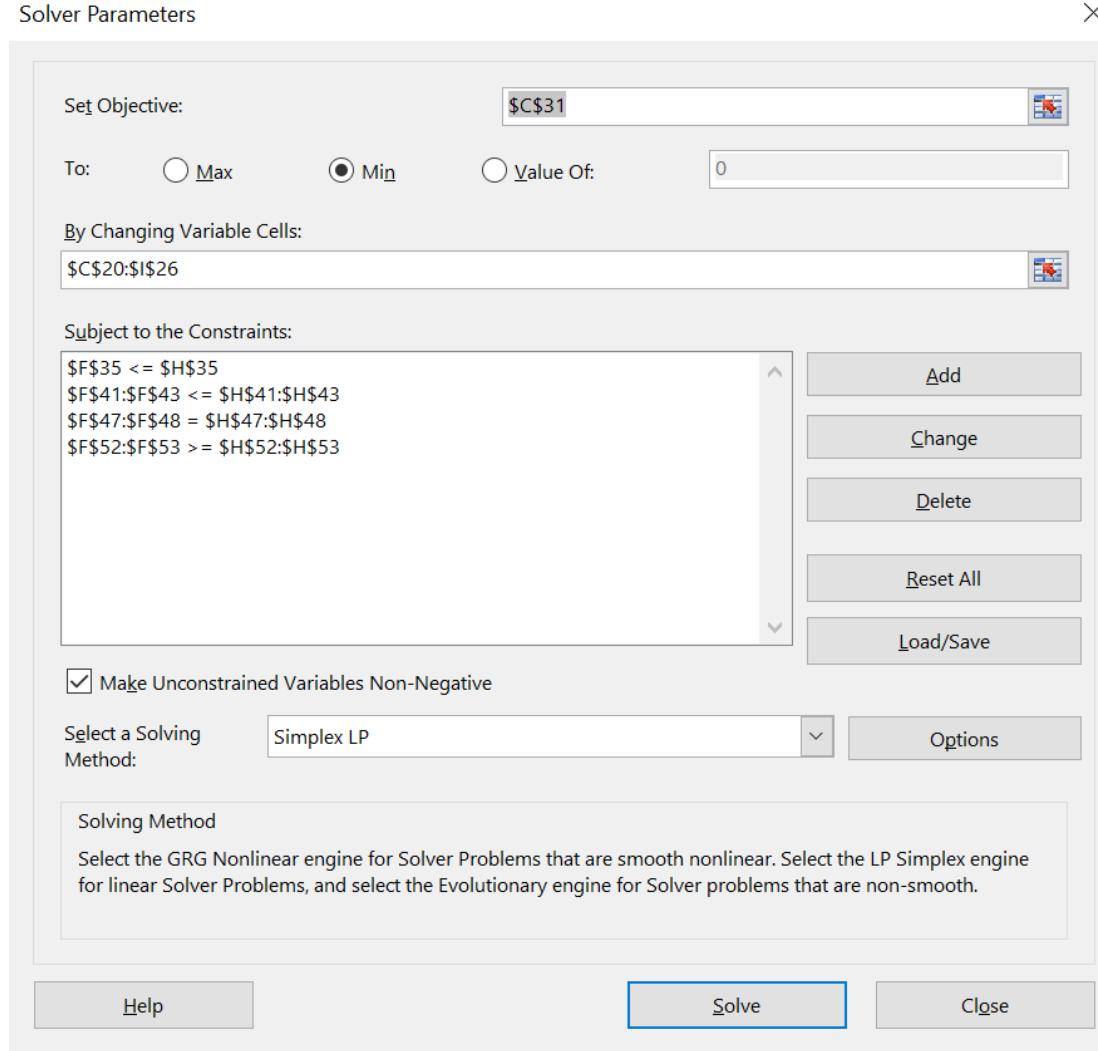
$$\sum_{\text{arcs out}} X_{ij} - \sum_{\text{arcs in}} X_{ij} = 0 \qquad \text{(Transshipment nodes)}$$

$$\sum_{\text{arcs in}} X_{ij} - \sum_{\text{arcs out}} X_{ij} \geq d_j \qquad \text{(Destination nodes } j\text{)}$$

$X_{ij} \geq 0$ and $\max(X_{ij}) \leq$ route capacity for all i and j

Solver Set-up

- Constraints need to satisfy the following:
  - **A maximum load (200 tonnes)**
  - **Net outflow from a plant cannot exceed its capacity**
  - **Total inflows must equal total outflows for each warehouse**
  - **Total inflows for each customer must at least equal their demand**

Optimal Solution

C31 | =SUMPRODUCT(C8:I14,C20:I26)
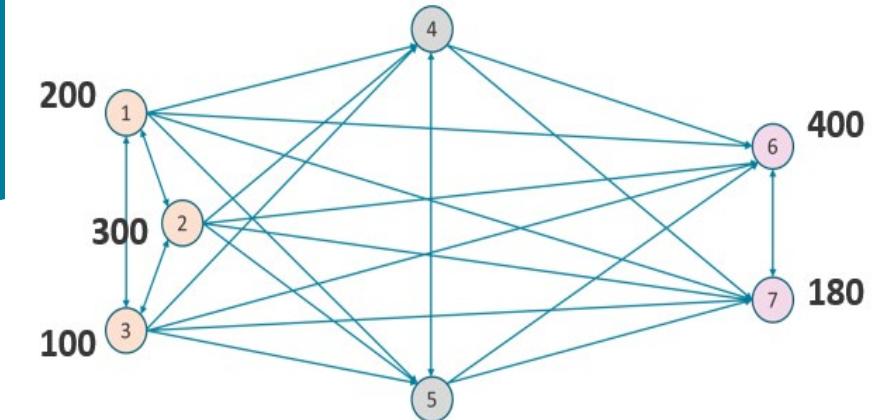
		Destination						
	DECISION VARIABLES							
		Destination						
	Origin	1	2	3	4	5	6	7
20	1	0	0	180	0	0	0	0
21	2	0	0	0	120	0	180	0
22	3	0	0	0	80	200	0	0
23	4	0	0	0	0	0	200	0
24	5	0	0	0	0	0	200	0
25	6	0	0	0	0	0	0	180
26	7	0	0	0	0	0	0	0

OBJECTIVE FUNCTION

Total cost $3,260

CONSTRAINTS

Max load carried on any route	200	≤	200	(Maximum capacity between any 2 nodes)

Node balance constraints

Plant constraints

Node	Net outflow		Plant capacity
1	180	<=	200
2	300	<=	300
3	100	<=	100

Warehouse constraints

Node	Net outflow		Required
4	0	=	0
5	0	=	0

Customer constraints

Node	Net inflow		Demand
6	400	>=	400
7	180	>=	180

Description of the Solution

- Plant 1 has a capacity of 200 tonnes. They ship 180 tonnes to plant 3, boosting its capacity from 100 to 280 tonnes

- Plant 2 ships 120 tonnes to warehouse 4 , and the remaining 180 tonnes of its capacity go directly to customer 6

- Plant 3 ships 80 tonnes to warehouse 4, and the remaining 200 tonnes to warehouse 5

- Warehouse 4 now holds 200 tonnes (120 + 80) which it ships to customer 6

- Warehouse 5 also ships 200 tonnes to customer 6. At this stage customer 6 has 580 tonnes, which is 180 tonnes more than their demand, so the tomato producing company ships this excess to customer 7, meeting their demand

Origin \ Destination	1	2	3	4	5	6	7
1	0	0	180	0	0	0	0
2	0	0	0	120	0	180	0
3	0	0	0	80	200	0	0
4	0	0	0	0	0	200	0
5	0	0	0	0	0	200	0
6	0	0	0	0	0	0	180
7	0	0	0	0	0	0	0
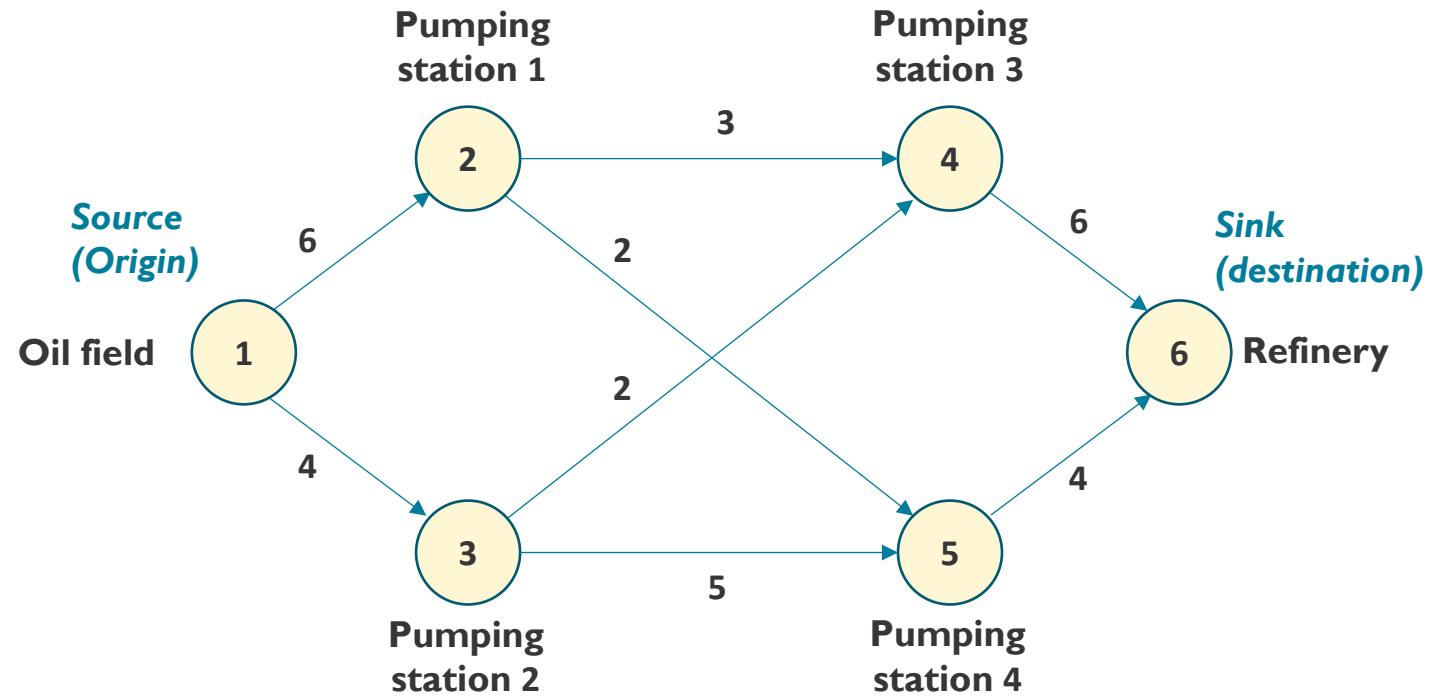
Maximal Flow Models

- In some networks the objective is to find the maximum flow through the network in a given period of time

- In these models we attempt to transmit flow through the arcs of the network as efficiently as possible

- The amount of flow is limited by the capacity restrictions on the arcs
  - **E.g., the amount of water that can flow through a network of pipes**
  - **E.g., the number of cars that can move through a network of streets**

- The upper limit on the flow in an arc is its **flow capacity**

5.4 Maximal Flow Example*
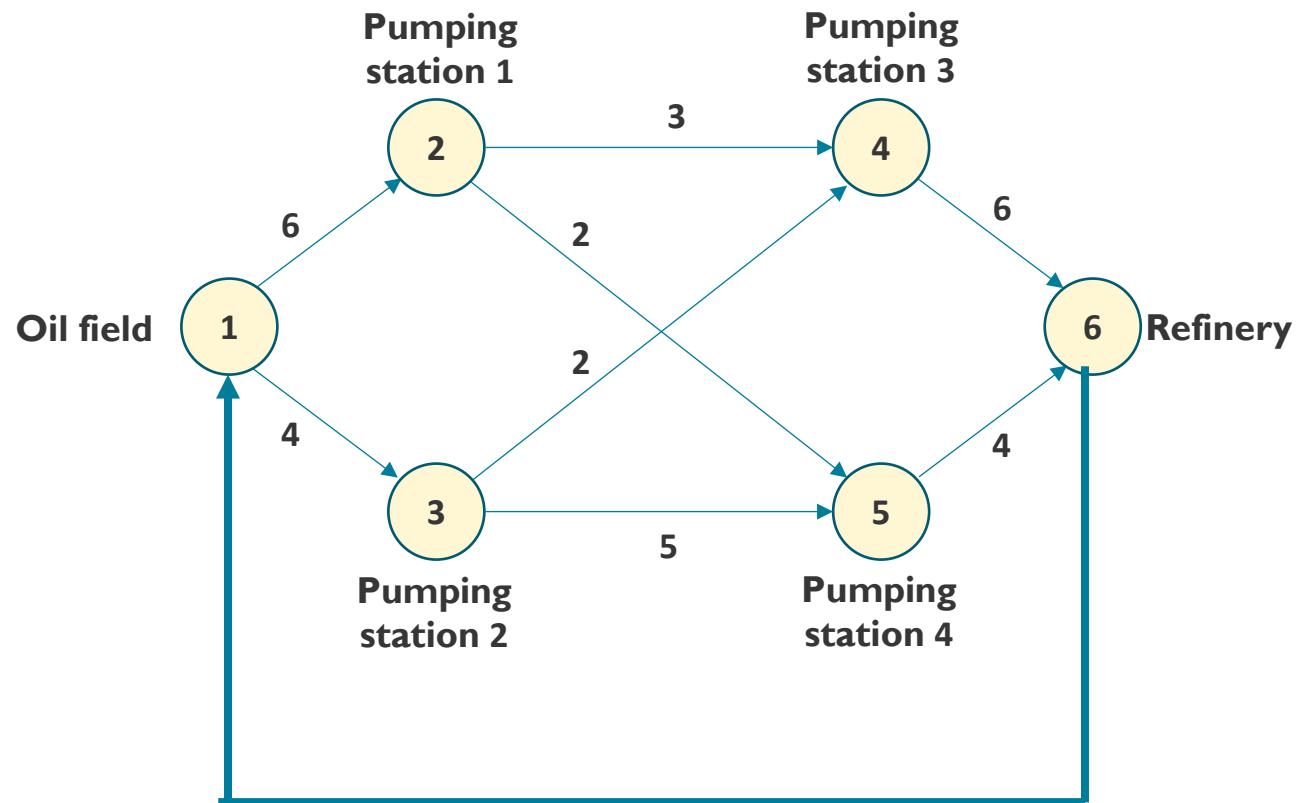
- The Northwest Petroleum Company has oil sent from an oil field through a network of pipes that flow through pumping stations along the way to a refinery.

- The numbers on the arcs represent the flow capacities (i.e. maximum flow that can be achieved on each section of the pipeline)

- One Source/ Origin and one Sink/ Destination

*This example is from Spreadsheet Modeling & Decision Analysis by C. Ragsdale

Maximal Flow Example

- This can be solved as a transshipment model:
  - **Add a dummy arc from the refinery to the oil field**

  - **Set the capacity at the origin and demand at the destination to zero**

  - Decision variables will be the amount of oil that flows along each arc. These flows cannot exceed the given flow capacities

5.4 Algebraic Model for This Example

Decision variables:

X_{ij} = number of units of oil shipped from node i to j

Objective:

Maximize X_{61}

Subject to constraints:

$X_{12} + X_{13} - X_{61} = 0$
$X_{24} + X_{25} - X_{12} = 0$
$X_{34} + X_{35} - X_{13} = 0$
$X_{46} - X_{24} - X_{34} = 0$
$X_{56} - X_{25} - X_{35} = 0$
$X_{61} - X_{46} - X_{56} = 0$
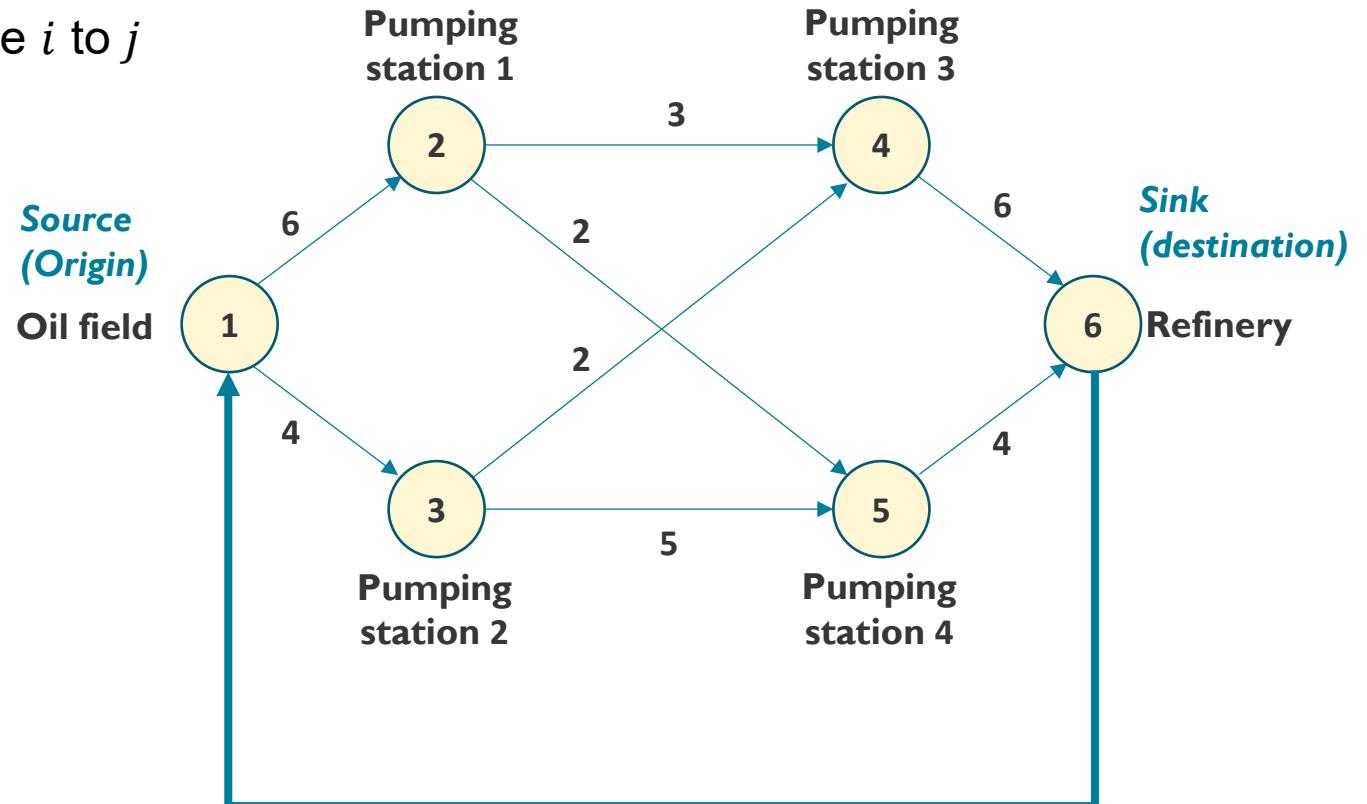
$X_{12} \leq 6, \quad X_{13} \leq 4, \quad X_{24} \leq 3, X_{25} \leq 2$

$X_{34} \leq 2, \quad X_{35} \leq 5, \quad X_{46} \leq 6, X_{56} \leq 4$

$X_{ij} \geq 0$ for all i, j.

Solver Set-up & Optimal Solution

Deakin University CRICOS Provider Code: 00113B

5.5 Shortest Path Models

- The shortest path model is concerned with finding the shortest path between two nodes in a network

- It is a special case of a transshipment problem where:
  - **There is one origin node with a "supply" of 1 unit**
  - **There is one destination node with a "demand" of 1 unit**
  - **All other nodes are transshipment nodes**

- Let X_{ij} denote the number of units that flow from node i to j. Because only one unit is to flow from the origin node to the destination node, the value of X_{ij} will be either 1 or zero, so all decision variables are binary

- The criterion to be minimised is not limited to distance even though the term "shortest" is used. Other criteria include time and cost

Shortest Path Example*
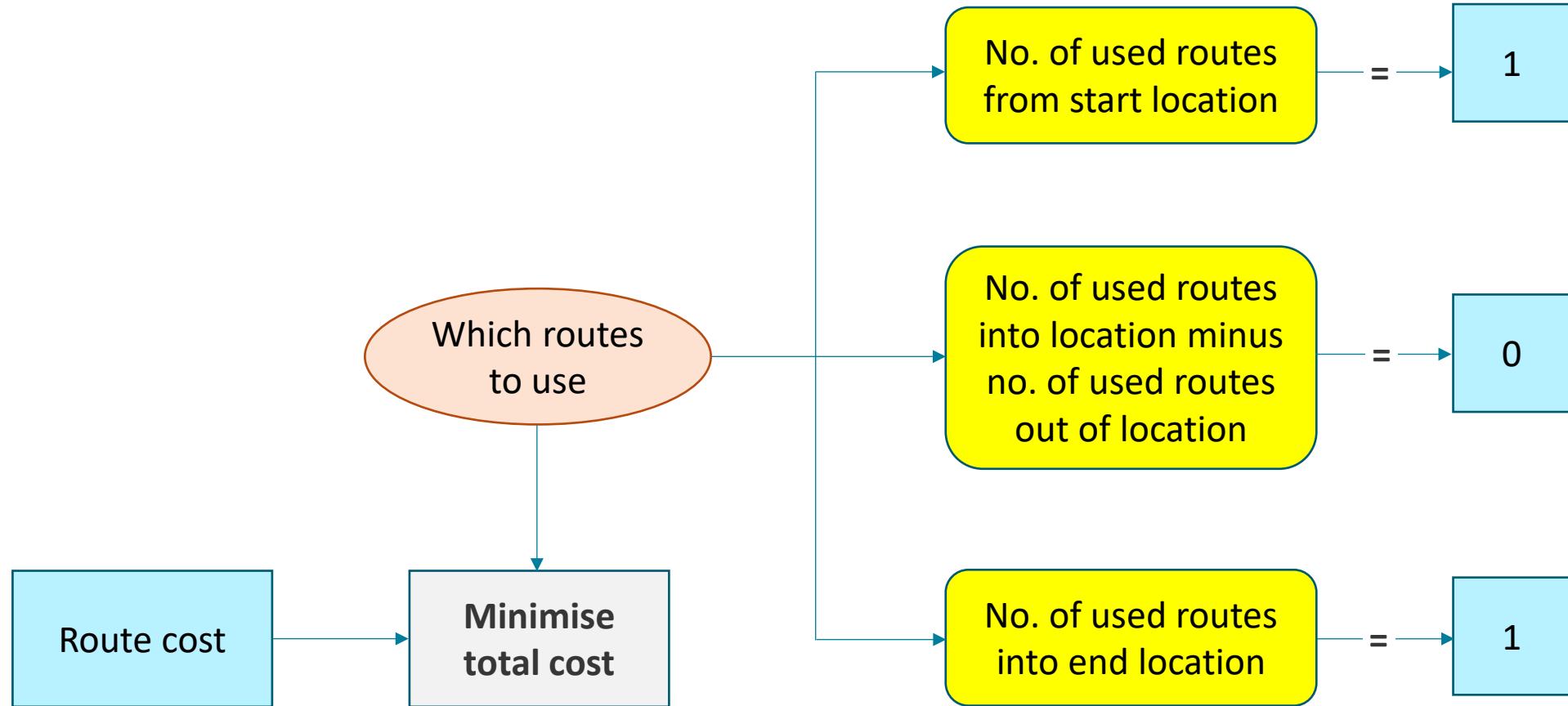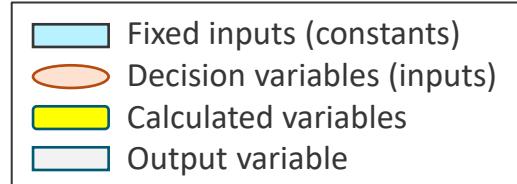
- A manager has an important business meeting in location B this evening. She has a number of alternate routes by which she can travel from the company headquarters in location A to location B.

- The network of alternate routes is shown

- For each section of the route, the transport mode, travel time, time cost (assuming she earns a wage of $15 per hour), ticket cost, and total cost appear below

Route	1-2	1-3	1-4	1-5	1-6	2-5	2-6	3-4	3-5	3-6	4-5	4-6	5-6
Transport mode	Train	Bus	Train	Plane	Taxi	Bus	Taxi	Taxi	Bus	Bus	Train	Bus	Train
Time (hrs:min)	4	2	3:20	1	6	3	3:20	1	4:40	6:20	2:20	4:40	1:20
Time cost	$60	$30	$50	$15	$90	$45	$50	$15	$70	$95	$35	$70	$20
Ticket cost	$20	$10	$30	$115	$90	$15	$50	$15	$20	$25	$15	$20	$10
Total cost	$80	$40	$80	$130	$180	$60	$100	$30	$90	$120	$50	$90	$30

Conceptual Model

5.5 Algebraic Model for This Example

Route	1-2	1-3	1-4	1-5	1-6	2-5	2-6	3-4	3-5	3-6	4-5	4-6	5-6
Total cost	$80	$40	$80	$130	$180	$60	$100	$30	$90	$120	$50	$90	$30

Decision variables:

$$X_{ij} = \begin{cases} 1, \text{if the arc from node } i \text{ to node } j \text{ is on the minimum cost route} \\ 0, \text{otherwise} \end{cases}$$

Objective: Minimise the total travel cost, given by

$$80X_{12} + 40X_{13} + 80X_{14} + 130X_{15} + 180X_{16} + 60X_{25} + 100X_{26} + 30X_{34} + 90X_{35} + 120X_{36}$$
$$+ 30X_{43} + 50X_{45} + 90X_{46} + 60X_{52} + 90X_{53} + 50X_{54} + 30X_{56}$$

Subject to constraints:

$X_{12}+X_{13}+X_{14}+X_{15}+X_{16} = 1$ (origin node 1)
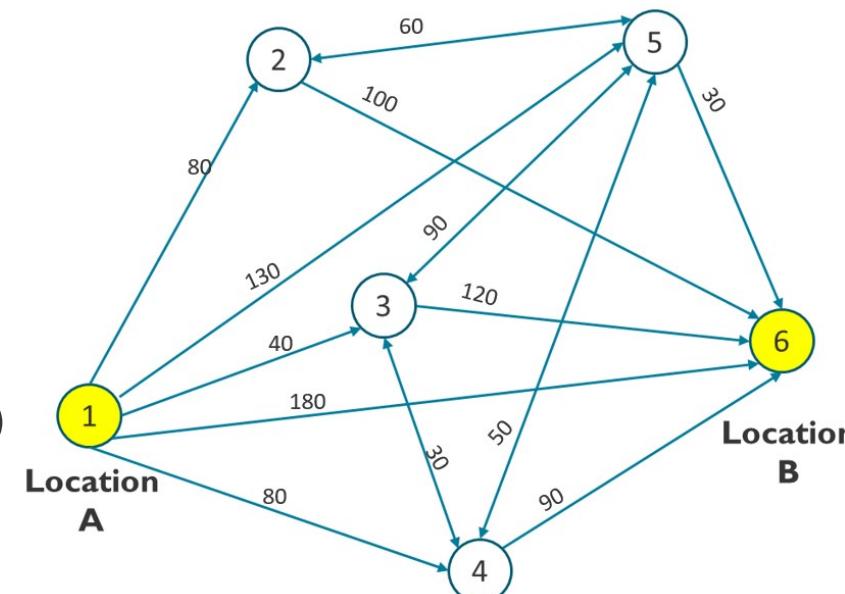
$X_{25}+X_{26} - X_{12} - X_{52} = 0$ (node 2)

$X_{34}+X_{35}+X_{36} - X_{13} - X_{43} - X_{53} = 0$ (node 3)

$X_{43}+X_{45}+X_{46} - X_{14} - X_{34} - X_{54} = 0$ (node 4)

$X_{52}+X_{53}+X_{54} + X_{56} - X_{15} - X_{25} - X_{35} - X_{45} = 0$ (node 5)

$X_{16}+X_{26}+X_{36}+X_{46}+X_{56} = 1$ (destination node 6)

General Algebraic Model

Let c_{ij} denote the measure (distance, time or cost) associated with the arc from node i to j

Decision variables:

$$X_{ij} = \begin{cases} 1, \text{if the arc from node } i \text{ to node } j \text{ is on the route that minimises the measure} \\ 0, \text{otherwise} \end{cases}$$

Objective: Minimise $\sum\limits_{\text{all arcs}} c_{ij} X_{ij},$

Constraints
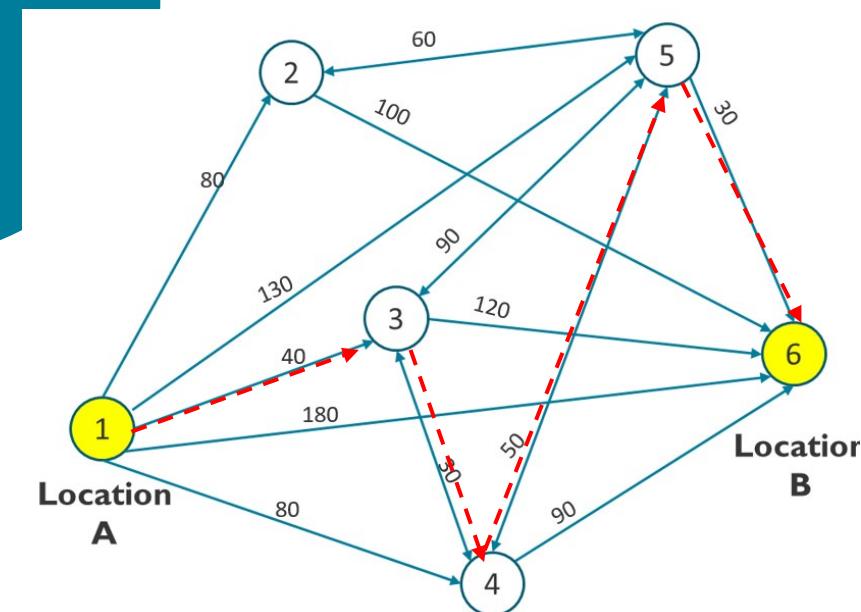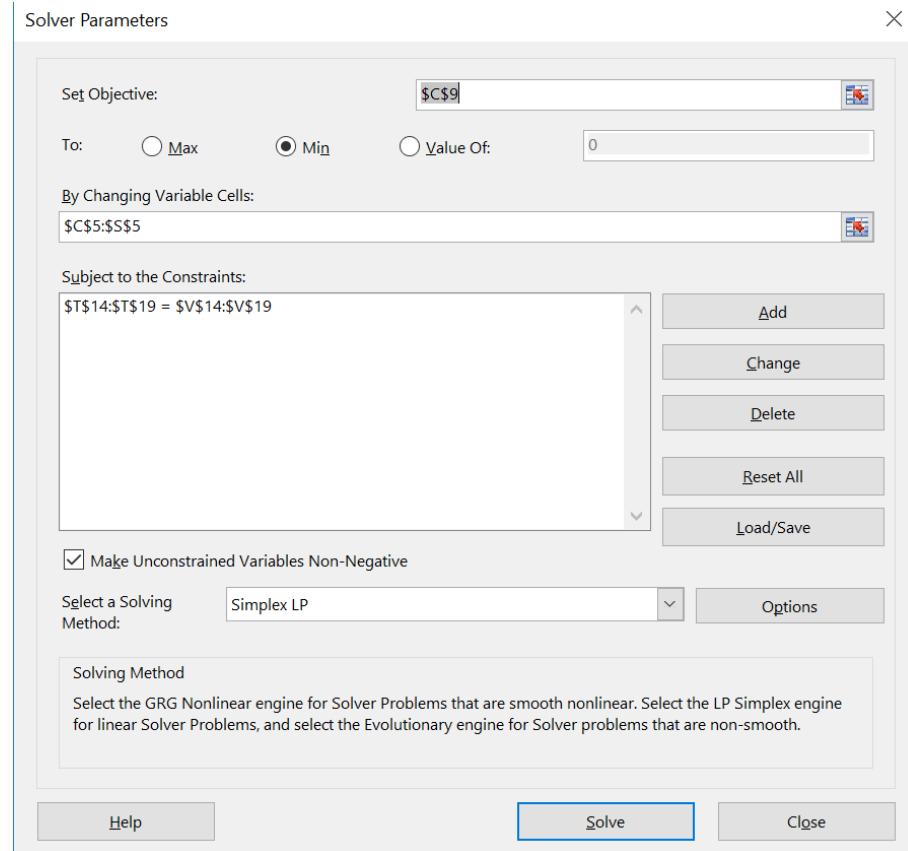
$$\sum_{\text{arcs out}} X_{ij} = 1 \qquad \text{(Origin node)}$$

$$\sum_{\text{arcs out}} X_{ij} - \sum_{\text{arcs in}} X_{ij} = 0 \qquad \text{(Transshipment nodes)}$$

$$\sum_{\text{arcs in}} X_{ij} = 1 \qquad \text{(Destination node)}$$

Solver Set-up & Optimal Solution

Summary

- A number of management science problems can be formulated as network models. Often these are logistics problems, that involve goods being shipped from one set of locations to another
  - **Transportation and transshipment models are useful when dealing with supply chain problems**
- Other network flow models covered:
  - **Assignment model, which is a special case of the transportation problem in which all demands are 1**
  - **Maximal flow model which can be used to determine the flows along the arcs of a network that will maximise the flow through the network**
  - **Shortest path model finds the shortest path (or least cost or quickest path) between two nodes. This can be expressed as a transshipment model where one unit is shipped from one origin to one destination**

Module 1 Learning Outcomes

- Understand the major business applications areas for LP, ILP and NLP
- Understand the basic assumptions, properties, and complexity of LP and its integer and nonlinear extensions
- Understand how to formulate LP, ILP and NLP problems from marketing, finance and operations management
- Understand the differences between LP, ILP and NLP models
- Understand network models
- Understand how to set-up a wide variety of LP, ILP and NLP models that arise in business in a spreadsheet, and solve them using Solver
- Carry out 'what if' analysis and sensitivity analysis on decision models and interpret computer output

Special Announcements

- **Assignment 1 is due on 9th April Wednesday at 8pm**

- Assignment one support session and student consultation time: **Mondays and Thursdays from 7 pm to 7:30 pm**

- Assignment 1 queries will only be answered via the Discussion forum from Monday to Friday

- Week 7, 18th April Good Friday is a public Holiday. No Seminar on that day.

- The online seminar will be on Thursday, 17th April at 7 pm

MIS313 – Strategic SCM Guest Lecture
"Sustainable Procurement and Sourcing"

Williams, Lisa. An accomplished C-suite executive and senior corporate services leader with over 25 years of international experience across procurement, commercial, and operations. She has worked in both the private and public sectors, including higher education, management consulting, energy, health, banking and financial services, and IT.

Currently serving as the **Executive Director, Strategic Procurement & Operations** at **Deakin University**, Lisa oversees the university's procurement strategy, expenditure, risk, supply chain management, and leveraged sourcing. She has held leadership roles across various industries, including three significant **Chief Procurement Officer (CPO)** positions, managing in expenditure portfolios and international supply chains.

Lisa is a former **international winner of the Chief Procurement Officer of the Year** award, which recognizes outstanding achievements in executive leadership, integrity, innovation, and technical expertise across the **Asia-Pacific** region.

Lisa Williams is recognized for her expertise in **strategic procurement, commercial leadership, operational transformation, and executive management,** making a significant impact across multiple sectors.

LISA WILLIAMS

**EXECUTIVE DIRECTOR,
STRATEGIC PROCUREMENT
& OPERATIONS
DEAKIN UNIVERSITY**

Date & Time : April 14th, Monday at 5 pm
Venue : Burwood Campus LT 1A, I 2.07
Meeting Link : https://deakin.zoom.us/j/86428536832?pwd=qa1f0s3zrQa36n5xb9FrmZhgO253pd.1
Meeting ID : 864 2853 6832
Password : 34025730

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