DEAKIN UNIVERSITY
FACULTY OF BUSINESS AND LAW
SAMPLE EXAMINATION SOLUTIONS

Unit Code: MIS775
Unit Name: Decision Modelling for Business Analytics

Instructions for candidates:

The final assessment will be an Open-book CloudDeakin Online Exam Quiz.

Late submissions will not be marked.

Answer all the questions in this exam.

Please note that the exam carries a total of 100 marks and constitutes 50% of your assessment in this unit.

You will have 2.5 hours to complete and submit the quiz.

It is anticipated that the quiz will involve approximately 2 hours of working time although you will have 2.5 hours to complete and submit your work. The additional 30 minutes allow for time taken to read through the quiz questions and provide time to address any minor technical issues you may experience during the quiz.

If you encounter any technical issues with CloudDeakin, please contact the IT Service Desk online or via phone (1800 463 888; +61 5227 8888 if calling from outside Australia) and record your ticket number as evidence of technical issues during the examination period.

Please do not contact the Unit team during the examination period for any reason. Academic staff are not allowed to communicate with students sitting an exam (this includes both phone calls and emails).

QUESTION 1 *(10 marks)*

A chemical manufacturer produces three chemicals: A, B and C. These chemicals are produced via two production processes: 1 and 2. Running process 1 for an hour costs $4 and yields 3 units of A, 1 unit of B and 1 unit of C. Running process 2 for an hour costs $1 and produces 1 unit of A and 1 unit of B. To meet customer demands, at least 10 units of A, 5 of B, and 3 of C must be produced daily.

The manufacturer wants a daily production plan that minimises the cost of meeting the chemical's daily demands. Assume a daily production plan is for 7 hours of working.

Formulate the algebraic model for this problem by writing down the decision variables, objective function, and constraints.

Decision Variables
X1 = Number of hours per day of running process 1
X2 = Number of hours per day of running process 2

Objective function
Let Z = Cost of meeting daily demands for chemicals
= Cost of operating processes 1 and 2

Minimise Z = 4X1 + X2

Constraints
For chemical A: 3X1 + X2 ≥10
For chemical B: X1 + X2 ≥ 5
For chemical C: X1 ≥ 3
X1 + X2 ≤ 7
Non-negativity constraints: X1, X2 ≥ 0.

QUESTION 2 ***(7 + 3 = 10 marks)***

Consider the following linear program problem.

Minimise 2X + 3Y subject to the following constraints:

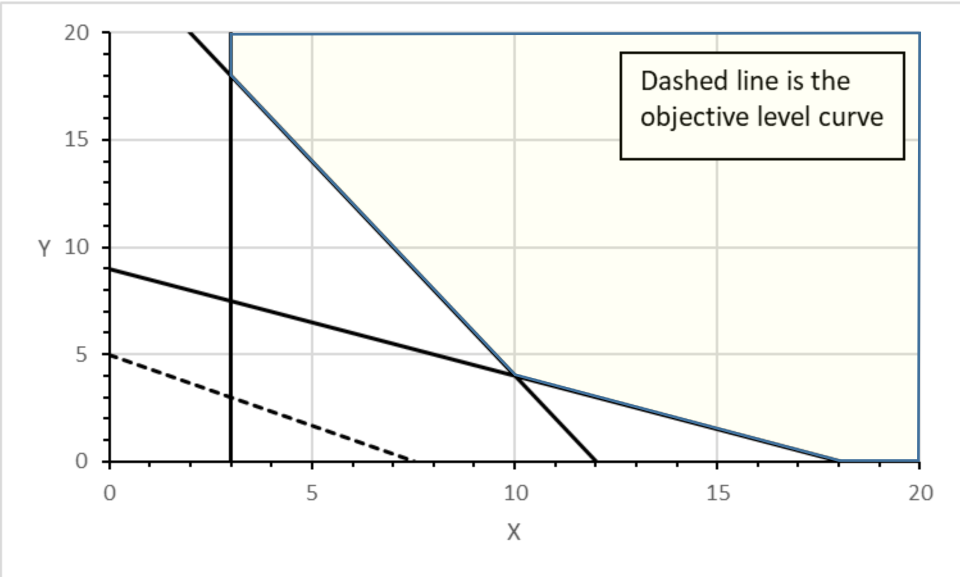
5X + 10Y ≥ 90 (constraint 1)

4X + 2Y ≥ 48 (constraint 2)

0.5X ≥ 1.5 (constraint 3)

X, Y ≥ 0.

A graphical representation follows:



a. Solve the problem. Show all working in obtaining the answer. ***(7 marks)***

(Start by identifying the feasible region, shown above in yellow. You can do this by identifying the corner points of the region shown.)

(3, 20), (20, 20), (20, 0), (18, 0), (10, 4), (3, 18).

(You don't need to explain how you arrived at this, but here is how I did it.

Consider constraint 1. Identify the line 5X + 10Y = 90 on the graph by setting Y = 0. This gives X = 18.

Consider constraint 2. Identify the line 4X + 2Y = 48 on the graph by setting Y = 0. This gives X = 12.

Consider constraint 3. Rewrite as X ≥ 3.

All inequalities need to be satisfied, giving the region with the corner points as shown.)

(Now you need to find the optimal solution. There are two ways of locating the minimum OFV. You need only provide one way.)

SOLUTION 1

Find the corner point of the feasible region with the minimum objective function value (OFV).

(You can ignore the points with X or Y equal to 20 since the feasible region extends beyond the region shown.)

X	Y	2X + 3Y
3	18	6 + 54 = 60
10	4	20 + 12 = 32 ←Minimum
18	0	36 + 0 = 36

The optimal point is (10, 4) which has a minimum OFV of 32.

SOLUTION 2

Move the objective level curve up/right until it just touches the feasible region.

(10, 4) is the first point it touches, for a minimum OFV of 2(10) + 3(4) = 32.

b. Find the slack, or surplus, or both, for each of the three labelled constraints. ***(3 marks)***

Constraint 1 is binding (no slack or surplus) because at the optimal solution (10, 4) the LHS = 5(10) + 10(4) = 90 = RHS

Constraint 2 is binding (no slack or surplus) because at the optimal solution (10, 4) the LHS = 4(10) + 2(4) = 48 = RHS

Constraint 3 is non-binding because at the optimal solution (10, 4) the LHS = 0.5(10) = 5 ≥ RHS. So there is a surplus of X = 7.

QUESTION 3 ***(5 + 5 + 5 + 5 = 20 marks)***

Consider the following problem:

Maximize total profit ($) = $2X_1 + 4X_2$
Subject to constraints:

$X_1 + 3X_2 \leq 10$ (constraint 1)
$2X_1 + X_2 \leq 8$ (constraint 2)
$X_1 \leq 3$ (constraint 3)
$X_1, X_2 \geq 0$.

The following solution output is provided:

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
C3	X1	2.8	0	2	6	0.67
D3	X2	2.4	0	4	2	3

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
E9	Constraints LHS	10	1.2	10	14	1
E10	LHS	8	0.4	8	0.33	4.67
E11	LHS	2.8	0	3	1E+30	0.2

a. What is the optimal solution, including the optimal value of the objective function? ***(5 marks)***

Optimal solution: X1 = 2.8 & X2 = 2.4. Optimal objective function value = (2×2.8) + (4×2.4) = $15.2.

b. Suppose the profit on X_1 is increased to $7. Is the above solution still optimal? What is the value of the objective function when this unit profit is increased to $7? ***(5 marks)***

Optimal solution will not change. An increase of $5 (from $2 to $7) is within the allowable increase of $6. Optimal profit = 15.2 + (5×2.8) = 15.2 + 14 = $29.2. OR (7×2.8) + (4×2.4) = 19.6 + 9.6 = $29.2.

c. If the unit profit on X_2 was $2 instead of $4, would the optimal solution change? ***(5 marks)***

Optimal solution won't change. A decrease of $2 (from $4 to $2) is within the allowable decrease of $3. Optimal profit = 15.2 – (2×2.4) = $10.4. OR (2×2.8) + (2×2.4) = 5.6 + 4.8 = $10.4.

d. Assume the new capacity for constraint 1 is increased to 12, what is the value of objective function? ***(5 marks)***

This constraint is binding (as (1×2.8) + (3×2.4) = 10), and the increase of 2 in the RHS is within the allowable increase of 14. The change in optimal objective function value = shadow price × change in RHS = 1.2× (12-10) = 1.2×2 = 2.4. New optimal value = 15.2 + 2.4 = $17.6.

QUESTION 4 ***(15 + 5 = 20 Marks)***

A Company has a contract to produce 10,000 garden hoses for a large discount chain. This company has four different machines that can produce this kind of hose. Because these machines are from different manufacturers and use differing technologies, their specifications are not the same.

Machine	Fixed Cost to Set Up Production Run	Variable Cost Per Hose	Capacity
1	750	1.25	6000
2	500	1.50	7500
3	1000	1.00	4000
4	300	2.00	5000

The company wants to minimise the total cost.

a. Formulate the algebraic model by writing down the decision variables, objective function, and constraints. ***(15 marks)***

Decision variables

Let Xi = the number of hoses produced by machine *i*, where *i* = 1, 2, 3, 4.

Binary decision variables:

$$\text{Let } Wi = \begin{cases} 1 \text{ if machine i is used, where } i = 1, 2, 3, 4. \\ 0 \text{ otherwise} \end{cases}$$

Objective function

We want to minimise the objective function, given by:

1.25X1 + 1.5X2 + X3 + 2X4 + 750W1 + 500W2 + 1000W3 + 300W4

Constraints

X1 ≤ 6000W1 (capacity constraint of machine 1)

X2 ≤ 7500W2 (capacity constraint of machine 2)

X3 ≤ 4000W3 (capacity constraint of machine 3)

X4 ≤ 5000W4 (capacity constraint of machine 4)

X1 + X2 + X3 + X4 = (or ≥) 10,000 (total demand constraint)

Xi = integer (*i* = 1, 2, 3, 4)

Wi = binary OR {0, 1} variables (*i* = 1, 2, 3, 4).

b. Add a constraint to ensure that if machine 1 is not used, then machines 2 is used. ***(5 marks)***

(Notice that machine 2 is contingent on machine 1 not being used. If machine 1 is not used, then machine 2 is used. Start by setting up a table of values for W1 and W2 and then Put a tick where the pair is allowed. Otherwise put a cross.)

W1	W2	
0	0	✗
0	1	✓
1	0	✓
1	1	✓

The constraint is therefore W1 + W2 ≥ 1.

QUESTION 5 ***(15 + 5 = 20 Marks)***

a. A transportation company is to move goods from three factories to three distribution centres. Information about the move is given below.

Source	Supply	Destination	Demand
A	200	X	250
B	100	Y	125
C	150	Z	125

Shipping costs are:

	Destination		
Source	X	Y	Z
A	3	2	5
B	9	10	--
C	5	6	4

(Source B cannot ship to destination Z)

The transportation company is interested in scheduling its material flow at the minimum possible cost.

Formulate the algebraic model for this problem by writing down the decision variables, objective function, and constraints. ***(15 marks)***

Decision Variables

Let Nij denote the amount shipped from source i to destination j, (i = A, B, C; j = X, Y, Z).

Objective function

Minimise 3NAX + 2NAY + 5NAZ + 9NBX + 10NBY + 5NCX + 6NCY + 4NCZ

Constraints

Supply constraints

NAX + NAY + NAZ = 200

NBX + NBY = 100

NCX + NCY + NCZ = 150

Demand constraints

NAX + NBX + NCX ≤ 250

NAY + NBY + NCY ≤ 125

NAZ + NCZ ≤ 125

and Nij ≥ 0 (i = A, B, C; j = X, Y, Z) integer-values.

There is an alternative solution to this part that involves including a dummy origin node, which we label as D. The reason for this is that the problem is not balanced since the total demand (500) exceeds the total capacity (450). If you give this solution, then there are two versions, depending on the inequalities used.

<u>Decision Variables</u>

Let Nij denote the amount shipped from source i to destination j, (i = A, B, C, D; j = X, Y, Z).

<u>Objective function</u>

The objective function doesn't change since the shipping costs from the dummy origin are all zero.)

Minimise 3NAX + 2NAY + 5NAZ + 9NBX + 10NBY + 5NCX + 6NCY + 4NCZ

<u>Constraints</u>

Supply constraints

(There is a new supply constraint added with its RHS = 50 so that the total capacity equals the total demand)

NAX + NAY + NAZ = (or ≤) 200

NBX + NBY = (or ≤) 100

NCX + NCY + NCZ = (or ≤) 150

NDX + NDY + NDZ = (or ≤) 50

Demand constraints

(The three demand constraints need to include the number of units "shipped" from node D to each of X, Y, and Z)

NAX + NBX + NCX + NDX = (or ≥) 250

NAY + NBY + NCY + NDY = (or ≥) 125

NAZ + NCZ + NDZ = (or ≥) 125

and Nij ≥ 0 (i = A, B, C; j = X, Y, Z) integer-values.

b. A Skateboard manufacturer produces two models of skateboards, the *FX* and the *ZX*. Assume that the manufacturer can sell every skateboard it produces.

Let PF denote the price charged for an *FX* and PZ denote the price charged for a *ZX*. The demand for *FX* skateboards per week, DF, and the demand for *ZX* skateboards per week, DZ, are

DF = 1,000 – 5PF

DZ = 500 – 2PZ

The cost of producing an *FX* is $75 and the cost of producing a *ZX* is $100.

The company has 80 labour-hours available per week in its workshop. Each *FX* skateboard requires 2 labour-hours and each *ZX* skateboard requires 3 labour-hours.

The manufacturer wants to maximise the total profit per week. Formulate the algebraic model for this production planning problem by writing down the decision variables, objective function, and constraints. In your answer, express the objective function in its simplest form, containing only the decision variables. ***(5 marks)***

<u>Decision variables</u>

DF, DZ.

<u>Objective function</u>

We want to maximise the manufacturer's total revenue ($), given by

DF(PF - 75) + DZ(PZ - 100)

= DF(200 – 0.2DF - 75) + DZ(250 – 0.5DZ - 100)

= DF(125 – 0.2DF) + DZ(150 – 0.5DZ) **OR** 125DF – 0.2DF^2 + 150 DZ – 0.5DZ^2

<u>Constraints</u>

2DF + 3DZ ≤ 80

DF, DZ ≥ 0 **OR** DF, DZ integer-values.

QUESTION 6 ***(6 + 2 + 2 = 10 Marks)***

Sales of a new product are expected to have the following distribution:

Units Sold	Probability
600	0.35
800	0.45
1000	0.20

a. Use the random numbers 0.513, 0.977, 0.587, 0.221, and 0.163 to generate five simulation trials from this distribution. ***(6 marks)***

Units sold	**Probability**	**Cumulative probability**	**Interval of random numbers**
600	0.35	0.35	0 up to 0.35
800	0.45	0.80	0.35 up to 0.80
1000	0.20	1.00	0.80 and greater

Trial	**1**	**2**	**3**	**4**	**5**
Random number	0.513	0.977	0.587	0.221	0.163
Units sold	800	1000	800	600	600

b. Calculate the average demand based on the simulated sales from part (a), and the expected demand based on the probability distribution. ***(2 marks)***

Average from simulated data = 760

Expected demand from sales distribution = 0.35×600 + 0.45×800 + 0.2×1000 = 770.

c. Should the two answers in part (b) be the same? Discuss why or why not. ***(2 marks)***

Would not expect the same answer, but the number of simulated trials increase we would expect the average of the simulated data to become closer and closer to the expected demand.

QUESTION 7 ***(2 + 2 + 2 + 2 + 2 = 10 Marks)***

The reference desk at a university library receives requests for assistance. Assume that a Poisson probability distribution with an arrival rate of 10 requests per hour can be used to describe the arrival pattern and that service times follow an exponential probability distribution with a service rate of 12 requests per hour.

a. What is the probability that no requests for assistance are in the system? ***(2 marks)***

(Here is the formula we use: $Pr(n\ cust.\ in\ system) = (1-\rho)\rho^n$ where $\rho = \lambda/\mu$.)
Probability of no requests (i.e. $n = 0$) is 1 – (10/12) = 0.167 or 16.7%.

b. What is the average number of requests that will be waiting for assistance? ***(2 marks)***

(Here is the formula we use: $L_Q = \lambda W_Q$. We would need to complete part (**c**) first to find $W_Q = 0.4167$.)

The average number of requests waiting for assistance = 10×0.4167 = 4.167.

c. What is the average waiting time in minutes before service begins? ***(2 marks)***

(We use two formulas here: $W_Q = W - W_S = L/\lambda - 1/\mu$ and $L = \frac{\lambda}{\mu-\lambda}$)

The average number in the system is L = 10/(12-10) = 5.

So the average wait time in the queue = (5/10) – (1/12) = 0.4167 hours (or 25 mins).

d. What is the average time at the reference desk in minutes (waiting time plus service time)? ***(2 marks)***

(We use formula for W again: $W = L/\lambda$)

Average total time is W = 5/10 = 0.5 hour (or 30 mins).

e. What is the probability that a new arrival has to wait for service? ***(2 marks)***

(If a new arrival is waiting for service, then the server must be busy. We can deduce this answer from our answer to part (**a**))

Pr(Busy) = 1 – Pr(0 customers in the system) = 1 – 0.167 = 0.833 (or 83.3%).