MIS775 Decision Modelling for Business Analytics

TOPIC 3:
Integer Linear Programming (ILP) Models

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MIS775 WEEK 3

Assignment 1 is available

- Assignment one support session Timetable – Please see the Announcements area on the unit site
- Assignment 1 queries will only be answered via the Discussion forum from Monday to Friday during business hours
- To be fair to all students, we are unable to answer Assignment one queries via email.

Week 7 - Easter vacation/intra-trimester break:
Friday 18th April –Sunday 27th April Sunday (inclusive)

What's the Plan?

- No f2f seminars or online seminars on Friday 18th April (Public Holiday)
- Week 7 Online seminar will be on Thursday 17th April at 7 pm
- **Friday On-Campus students are very welcome to attend this seminar**

Recap

- Optimization models find the most efficient way of using limited resources to achieve the objective set by a business
- Linear Programming (LP) seeks to maximise or minimise a **linear objective function**, subject to a set of **linear constraints**
- **Sensitivity analysis** helps to answer questions about how sensitive an optimal solution is to changes in various coefficients or parameters in a model

Motivation for Considering Integer Linear Programming (ILP) Models

- Many business situations require integer solutions to LP problems
  - **e.g. an optimal solution of 5.37 doesn't make sense in the context of the number of flights to operate between Melbourne and Sydney; and**
  - **rounding off doesn't necessarily yield an optimal (or even feasible) integer solution**
- ILP solves LP problems in which some or all of the decision variables are constrained to take non-negative integer values, i.e. $\{0, 1, 2, 3, \dots\}$
- Some LP problems involve Yes/No decisions. We use binary variables (take only the values 0 and 1) to solve them as ILP problems. Then it is called Binary IntegerLnear Programming (BILP) problem

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Learning Objectives

- Understand the basic ideas of integer linear programming and the main techniques for formulating ILP models
- Solve optimisation models in which decision variables take only integer values
- Understand how to set-up ILP models in a spreadsheet and solve with Excel's Solver
- Become familiar with three standard types of ILP models:
  - **Capital budgeting models (textbook section 6.3)**
  - **Fixed cost models (textbook section 6.4)**
  - **Cutting stock models (textbook section 6.6)**

Textbook reading: Chapter 6 (6.3-6.4, 6.6)

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What is Capital Budgeting?

- Capital budgeting is a process of deciding which projects are worth funding
- Capital funds are limited, so typically not all projects can be funded
- The objective is to **maximise the total Net Present Values (NPV)**
- A complication with these models is that any project must be fully funded or not funded at all - it cannot be funded to a partial extent

Ex. 1) Capital Budgeting Models*

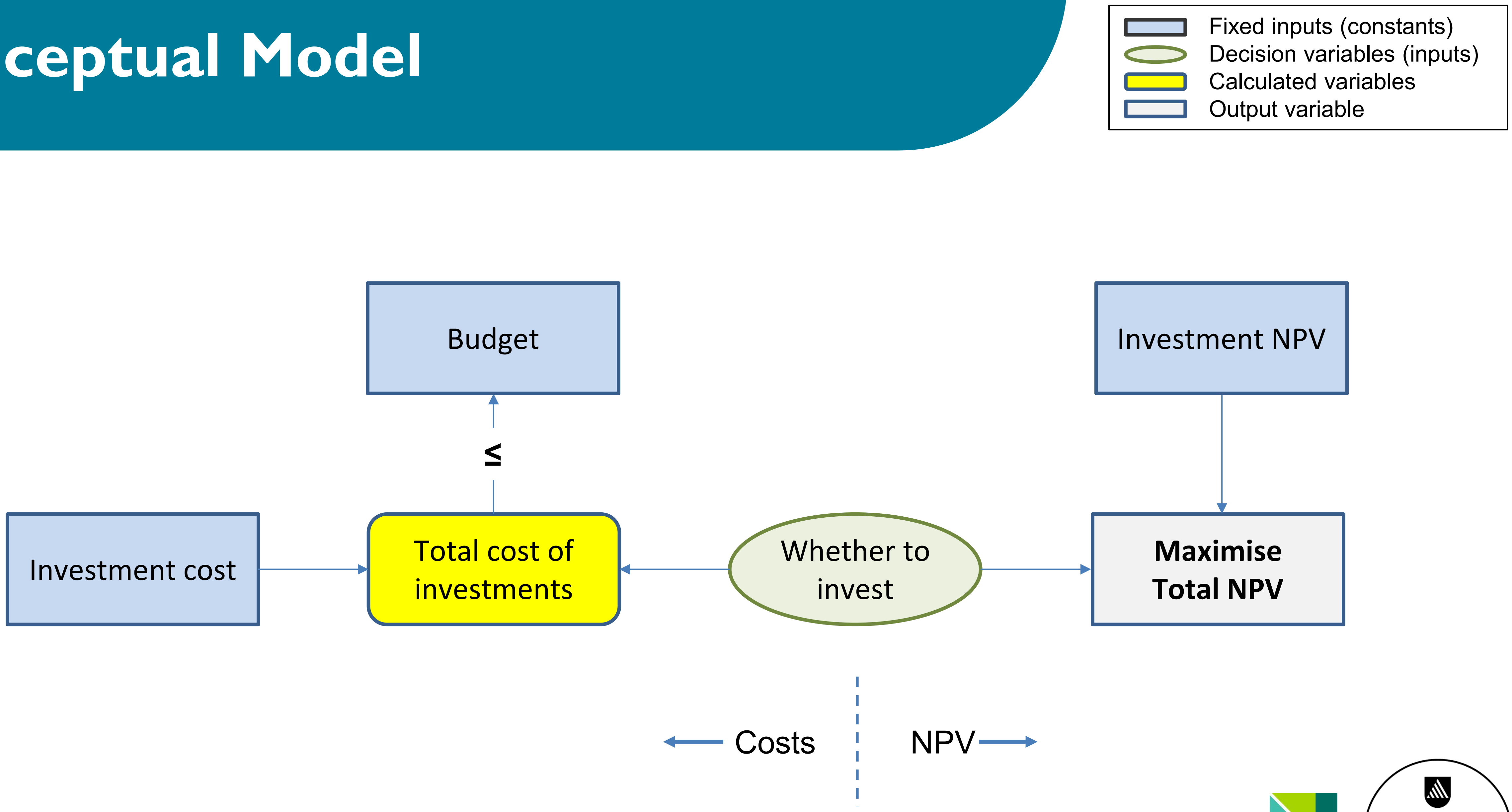
- A company is considering seven investments. The cash required for each investment and the **Net Present Value (NPV)** each investment in $ million is shown below
- The **Return on Investment (ROI)** is given by $\left(\frac{NPV - Cash\ Required}{Cash\ Required}\right)$
- The budget available for investing is $15 million
- The company wants an investment policy that maximises the total NPV
- However, partial investments are not allowed

ROI = $\left(\frac{NPV - Cash\ Required}{Cash\ Required}\right)$
=(5.6 - 5) / 5
=0.6/5
=0.12 or 12%

Investment	1	2	3	4	5	6	7
Cash required	$5.0	$2.4	$3.5	$5.9	$6.9	$4.5	$3.0
NPV	$5.6	$2.7	$3.9	$6.8	$7.7	$5.1	$3.3
ROI	12.0%	12.5%	11.4%	15.3%	11.6%	13.3%	10.0%

*Textbook example on pp. 288-289

Conceptual Model



Algebraic Model

- Decision variables:

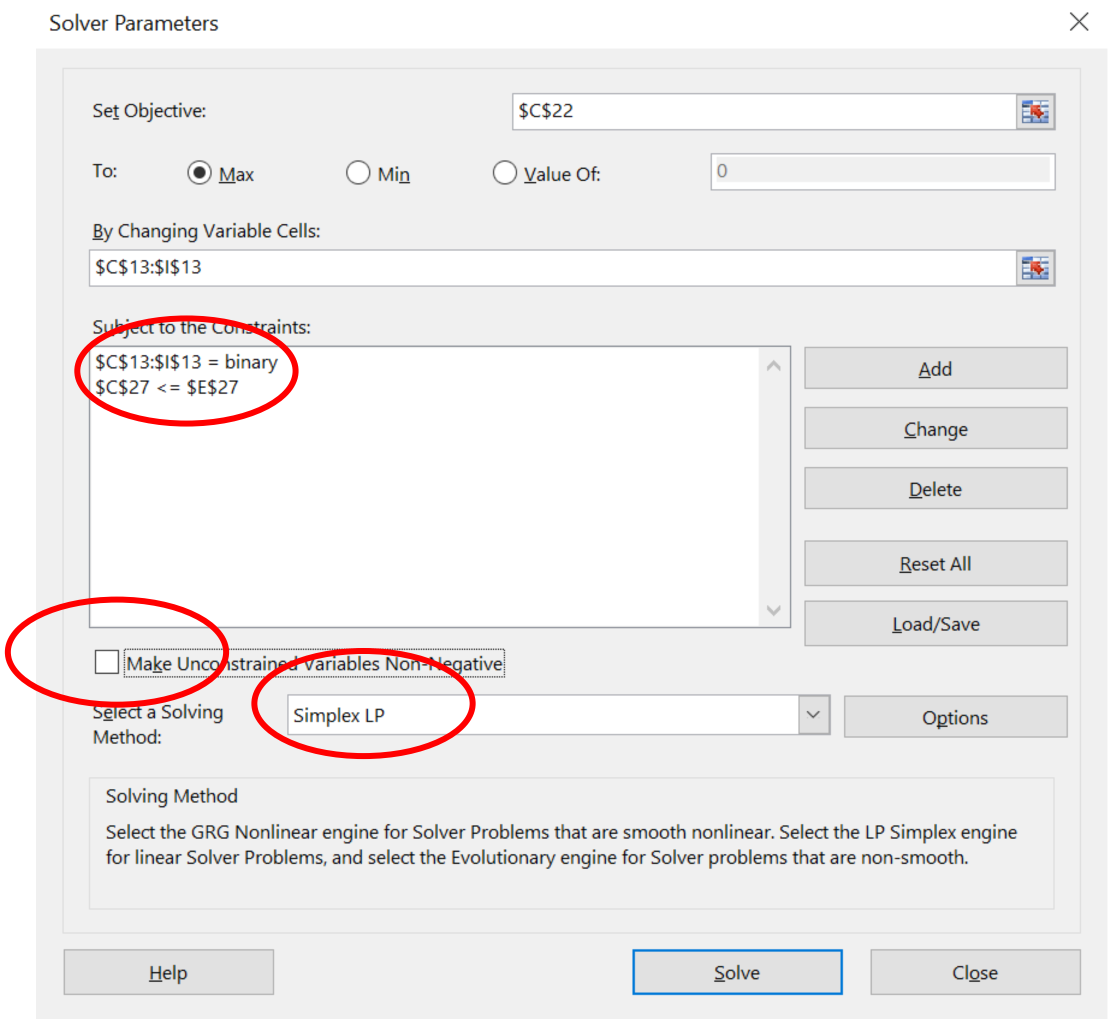
  Define binary decision variables $W_i = \begin{cases} 1, \text{if company invests in investment } i \\ 0, \text{otherwise} \end{cases}$

  and let c_i denote the cost of investment i, where $i = 1, 2, \dots, 7$

- Objective: Maximise total NPV = $\sum NPV_i \times W_i$

- Constraint: $\sum c_i W_i \leq \$15$ million

Solver Set-up



- Need to add binary constraint for the decision variables
- No need to tick the box to make unconstrained variables non-negative, since binary variables are automatically non-negative
- Remember to use the Simplex LP method. Even though ILP models don't satisfy the divisibility property, Solver still considers them to be linear

Optimal Solution

C22 | =SUMPRODUCT(C7:I7,C13:I13)

	B	C	D	E	F	G	H	I
3	**INPUTS**							
4								
5	Investment	1	2	3	4	5	6	7
6	Cash required	$5.0	$2.4	$3.5	$5.9	$6.9	$4.5	$3.0
7	NPV	$5.6	$2.7	$3.9	$6.8	$7.7	$5.1	$3.3
8	ROI	12.0%	12.5%	11.4%	15.3%	11.6%	13.3%	10.0%
9								
10	**DECISION VARIABLES**							
11								
12	Investment	1	2	3	4	5	6	7
13	Invest?	**0**	**1**	**1**	**1**	**0**	**0**	**1**
14	(1=yes, 0=no)							
15								
16	**CALCULATED VARIABLES**							
17	Total cost of	**14.8**						
18	investments							
19								
20	**OBJECTIVE FUNCTION**							
21								
22	Total NPV	**$16.70**						
23	($ million)							
24								
25	**CONSTRAINTS**							
26		**LHS**		**RHS**				
27		**$14.80**	≤	**$15**		(budget constraint)		

*Textbook has another solution that is also optimal

- This is one of three possible solutions*, all with the same Total NPV
- Notice that the optimal solution does not include investing in investment 6, which has the second largest ROI

The answer is that the company should invest in investments 2, 3, 4 and 7 for a total NPV of $16.7 million

Logic Constraints

- Decisions about which projects to fund may depend on one another
- Consider projects $\boldsymbol{i}$ and $\boldsymbol{j}$ with binary decision variables W_i and W_j
- There are three primary logical considerations:
  - **Project i is contingent on project j (constraint $\boldsymbol{W_i \leq W_j}$ or $\boldsymbol{W_i - W_j \leq 0}$)**
  - **Project i is a co-requisite for project j (constraint $\boldsymbol{W_i = W_j}$ or $\boldsymbol{W_i - W_j = 0}$)**
  - **Projects i and j are mutually exclusive (constraint $\boldsymbol{W_i + W_j \leq 1}$)**
- Check these constraints with a logic table, where 1 means we invest in a project and 0 means we don't
- Put a tick (✓) where the pair is allowed. Otherwise put a cross (**X**)

$\boldsymbol{W_i}$	$\boldsymbol{W_j}$
0	0
0	1
1	0
1	1

Logic Constraints Explained

- If project i is contingent on project j then this means that project i can't go ahead unless project j goes ahead. The inequality $\boldsymbol{W_i \leq W_j}$ means that $\boldsymbol{W_j}$ = 0 forces $\boldsymbol{W_i}$ = 0 (i.e. project i cannot go ahead if project j doesn't go ahead). But note that if project j does go ahead, then there is no requirement that project i goes ahead or not

$\boldsymbol{W_i}$	$\boldsymbol{W_j}$	
0	0	✓
0	1	✓
1	0	X
1	1	✓

- If project i is a co-requisite for project j then either both are approved or both rejected, i.e. $\boldsymbol{W_i = W_j}$

$\boldsymbol{W_i}$	$\boldsymbol{W_j}$	
0	0	✓
0	1	X
1	0	X
1	1	✓

- If project i and project j are mutually exclusive then they can't both go ahead. The constraint is therefore $\boldsymbol{W_i + W_j \leq 1}$

$\boldsymbol{W_i}$	$\boldsymbol{W_j}$	
0	0	✓
0	1	✓
1	0	✓
1	1	X

Ex 2a) Two-Period Capital Budgeting Model

- The investment example can be modified to cover the case where the cash required is split over two years, and there are budget constraints of $9 million for year 1 and $6 million for year 2.
- We leave this as an exercise to complete in the seminar. The answer is that the company should invest in **investments 1, 2 and 4 for a total NPV of $15.1 million**

Investment	1	2	3	4	5	6	7
Year 1 cost	$4.0	$2.0	$1.0	$3.0	$3.9	$1.5	$0.0
Year 2 cost	$1.0	$0.4	$2.5	$2.9	$3.0	$3.0	$3.0
NPV	$5.6	$2.7	$3.9	$6.8	$7.7	$5.1	$3.3
ROI	12.0%	12.5%	11.4%	15.3%	11.6%	13.3%	10.0%

*Textbook example on pp. 283-289

Ex 2b) Two-Period Capital Budgeting Model

- Same investment in Ex 2a) is further modified that **you must choose between Investment 1 and Investment 2; you cannot invest in both**.
- We leave this also as an exercise to complete in the seminar. The answer is that the company should invest in **investments $4, and\ 5$ for a total NPV of $14.5 million**

Investment	1	2	3	4	5	6	7
Year 1 cost	$4.0	$2.0	$1.0	$3.0	$3.9	$1.5	$0.0
Year 2 cost	$1.0	$0.4	$2.5	$2.9	$3.0	$3.0	$3.0
NPV	$5.6	$2.7	$3.9	$6.8	$7.7	$5.1	$3.3
ROI	12.0%	12.5%	11.4%	15.3%	11.6%	13.3%	10.0%

*Textbook example on pp. 283-289

Ex 3. Fixed-Cost Models*

A textile company manufactures shirts, shorts, pants, and skirts. The **machinery needed to make each type of clothing must be rented at weekly rates shown**. The table also gives the amount of cloth and labour required per unit, as well as the selling price and the unit variable cost.

In a given week, **4,000 hours of labour** and **4,500 square metres of cloth** are available. Find the solution that maximises profit.

	Rental Cost (fi)	Hrs of Labour (li)	Cloth (sq metres)(mi)	Selling Price (Si)	Unit Variable Cost (Vi)
Shirts	\$1500	2	3.0	\$35	\$20
Shorts	\$1200	1	2.5	\$40	\$10
Pants	\$1600	6	4.0	\$65	\$25
Skirts	\$1500	4	4.5	\$70	\$35
		4,000	4,500		

*Modified textbook example on pp. 290-298

Fixed-Cost Models

- This is an example of a fixed-cost model due to the rental cost, which only applies if a machine is rented
- Any cost which is independent of the level of an activity, and is only applied **"if"** the activity takes place, is referred to as a fixed cost
- A clever use of binary variables enables us to convert these types of problems into linear models
- To start with, we determine the capacity for each type of item

How to Determine Capacities

	Rental Cost (fi)	Hrs of Labour (li)	Cloth (sq metres)(mi)	Selling Price (Si)	Unit Variable Cost (Vi)
Shirts	$1500	2	3.0	$35	$20
Shorts	$1200	1	2.5	$40	$10
Pants	$1600	6	4.0	$65	$25
Skirts	$1500	4	4.5	$70	$35
		4,000	4,500		

product	Maximum capacity (ci)
Shirts	1500
Shorts	1800
Pants	666.6667
Skirts	1000

- There are two resource constraints to consider 1) amount of labour hrs and cloth(sq meters)
- We illustrate with shirts. Consider labour. Shirts require 2 hrs of labour. As there are 4,000 hrs of labour available, the most number of shirts that can be produced is 2,000
- Now consider cloth. Shirts require 3 sq metres of cloth. As there is 4,500 sq metres of cloth available, the most number of shirts we could produce is 1,500
- As BOTH constraints need to be satisfied, the capacity is the smaller of 2,000 and 1,500, i.e. 1,500, is the most number of shirts that could be produced

Use of Binary Variables

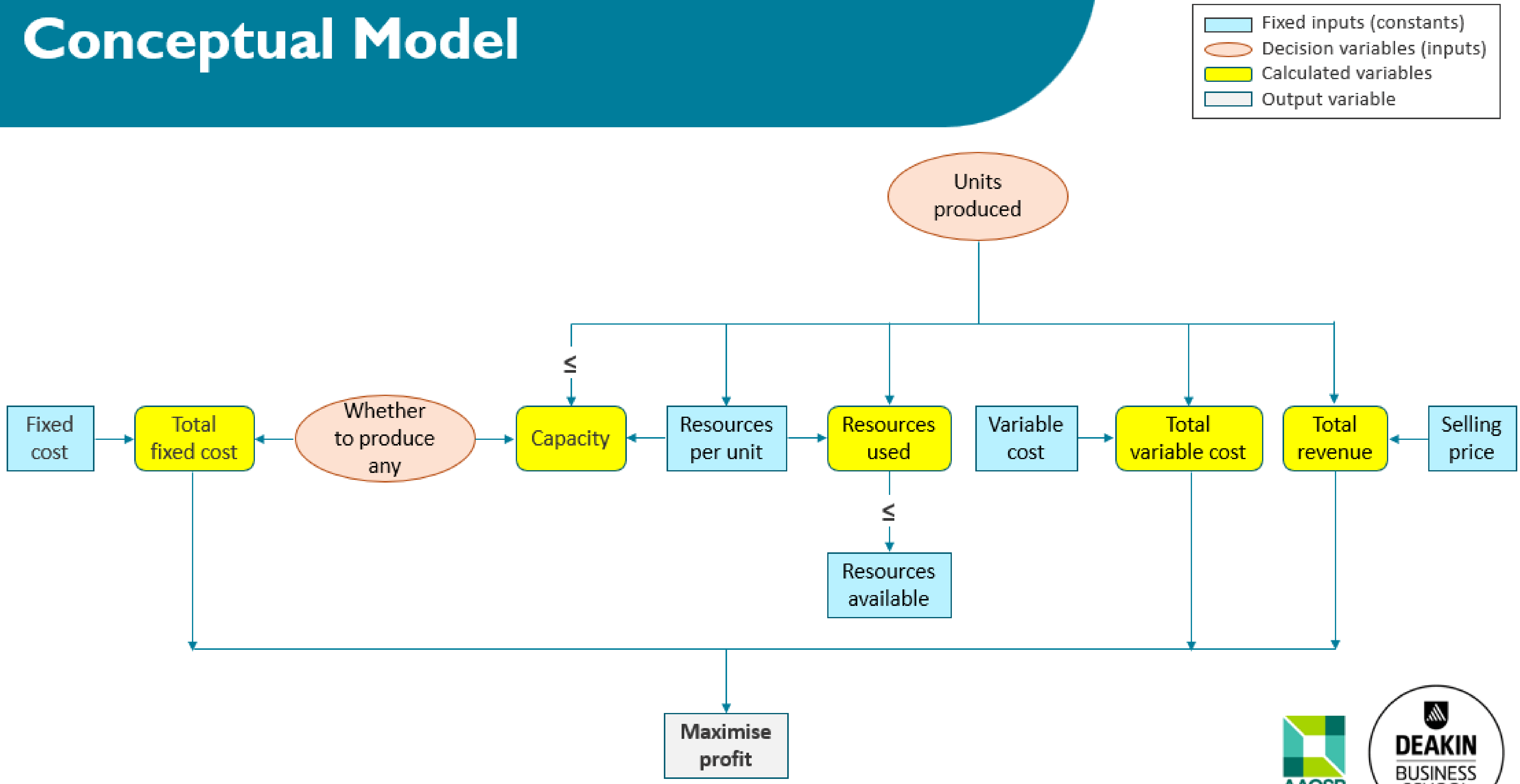
- We introduce the Binary Variables to consider the **"if"** options

- If $\begin{cases} 1, \text{if company manufactures product type } i \\ 0, \text{otherwise} \end{cases}$

- We introduce a binary variable for each type of product, where the variable is 1 if the produced is manufactured, and 0 otherwise

- These binary variables are used in the objective function AND in the RHS of constraints for each type of product

Conceptual Model



Algebraic Model

	Rental Cost (fi)	Hrs of Labour (li)	Cloth (sq metres)(mi)	Selling Price (Si)	Unit Variable Cost (Vi)
Shirts	$1500	2	3.0	$35	$20
Shorts	$1200	1	2.5	$40	$10
Pants	$1600	6	4.0	$65	$25
Skirts	$1500	4	4.5	$70	$35
		4,000	4,500		

Denote shirts, shorts, pants, and skirts as products of type $1, 2, 3$, and 4 resp.

Decision variables:

- Let X_i denote the number of units manufactured of type i
- Define binary decision variables $W_i = \begin{cases} 1, \text{if company manufactures product type } i \\ 0, \text{otherwise} \end{cases}$

Fixed inputs:

- f_i denotes the fixed (rental) cost for product type i
- l_i denotes the hours of labour per unit of product type i
- m_i denotes the amount of material (cloth) required per unit of product type i
- s_i denotes the selling price per unit of product type i
- v_i denotes the variable cost per unit of product type i
- c_i denotes the capacity of product type i

Algebraic Model (Continued)

Let X_i denote the number of units manufactured of type i

Define binary decision variables $W_i = \begin{cases} 1, \text{if company manufactures product type } i \\ 0, \text{otherwise} \end{cases}$

	Rental Cost (fi)	Hrs of Labour (li)	Cloth (sq metres)(mi)	Selling Price (Si)	Unit Variable Cost (Vi)	Maximum capacity (ci)
Shirts	\$1500	2	3.0	\$35	\$20	1500
Shorts	\$1200	1	2.5	\$40	\$10	1800
Pants	\$1600	6	4.0	\$65	\$25	666.6667
Skirts	\$1500	4	4.5	\$70	\$35	1000
		4,000	4,500			

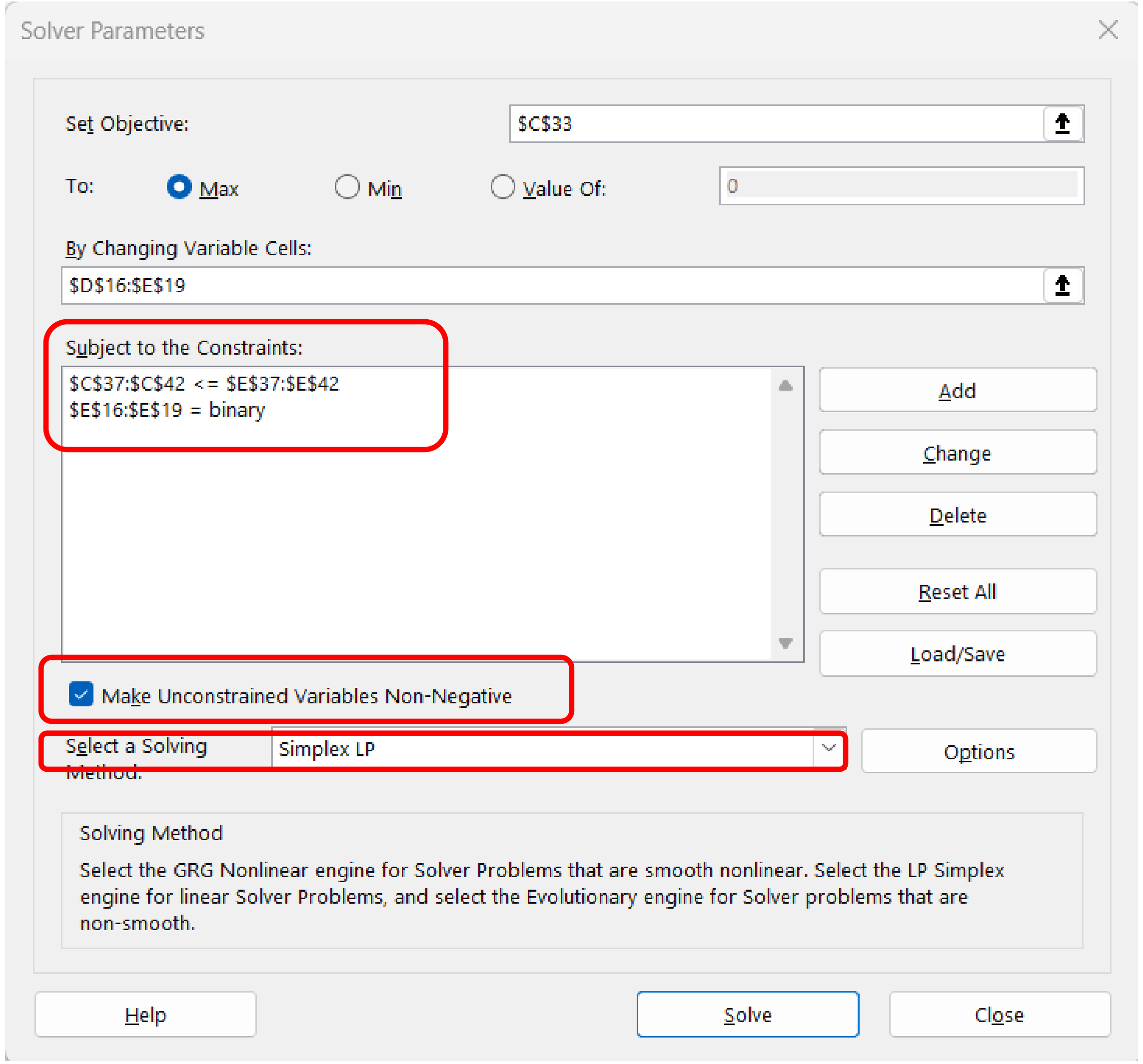
Objective Function:

- Maximise $\sum s_i X_i - \sum v_i X_i - \sum f_i W_i$

Subject to constraints:

- $\sum l_i X_i \leq 4{,}000$ (labour constraint)
- $\sum m_i X_i \leq 4{,}500$ (cloth constraint)
- $X_1 \leq 1500_1 W_1$ (constraint on number of shirts)
- $X_2 \leq 1800 W_2$ (constraint on number of shorts)
- $X_3 \leq 666.67 W_3$ (constraint on number of pants)
- $X_4 \leq 1000 W_4$ (constraint on number of skirts)

Solver Set-up



- These models require integer decision variables
- Need to tick the box to make unconstrained variables non-negative since we are working with binary variables and non integer variables
- Remember to use the Simplex LP method

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Optimal Solution

	B	C	D	E	F
22	CALCULATED VARIABLES				
23					
24			Capacity		
25		Shirts	1500		
26		Shorts	1800		
27		Pants	667		
28		Skirts	1000		
29					
30					
31	OBJECTIVE FUNCTION				
32					
33		$52,800			
34					
35	CONSTRAINTS				
36		LHS		RHS	
37		1800.0	≤	4000	(Labour constraint)
38		4500.0	≤	4500	(Cloth constraint)
39		0.0	≤	0	(Shirts constraint)
40		1800.0	≤	1800	(Shorts constraint)
41		0.0	≤	0	(Pants constraint)
42		0.0	≤	0	(Skirts constraint)

	B	C	D	E
13	DECISION VARIABLES			
14				
15			No. to produce	Binary variables
16		Shirts	0	0
17		Shorts	1800	1
18		Pants	0	0
19		Skirts	0	0

Cutting Stock Models*

This week the orders

Width	12	15	20	24	30	40
Quantity	48	19	22	32	14	7

- A company produces a standard roll of paper that is 60 inches wide and 200 yards long. Customers can have any of the following widths: 12, 15, 20, 24, 30, or 40 inches and the length of all pieces are 200 yards. In a given week they wait for orders and then decide how to cut their 60-inch rolls.
- There are 26 different ways of cutting patterns identified.
- If this week the orders are as shown in the table, find the most economical way to meet their orders

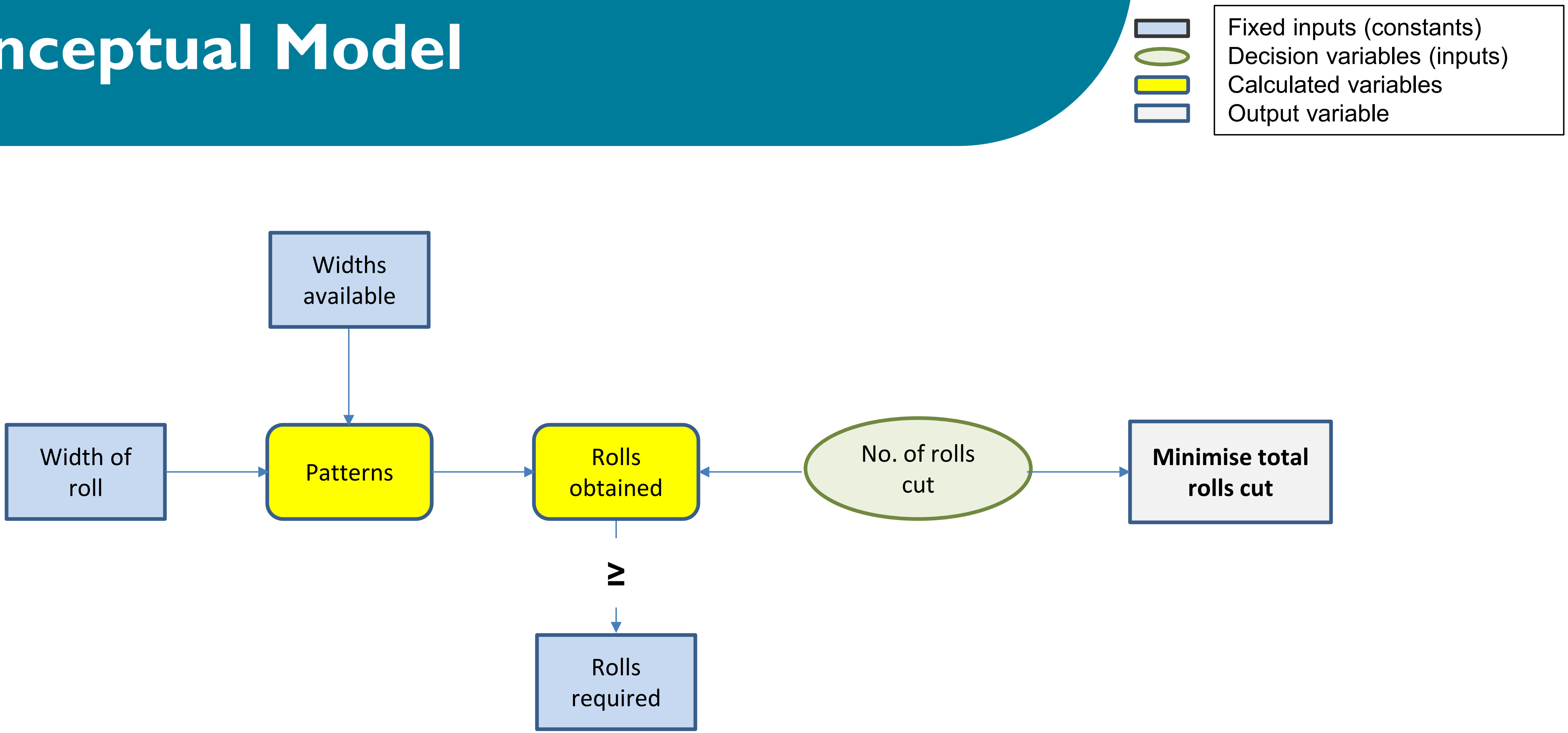
*Textbook example on pp. 320-324

Different Patterns (or Ways) of Cutting up a Roll

Pattern	12	15	20	24	30	40	Waste
	Width						
1	5	0	0	0	0	0	0
2	3	1	0	0	0	0	9
3	3	0	1	0	0	0	4
4	3	0	0	1	0	0	0
5	2	2	0	0	0	0	6
6	2	1	1	0	0	0	1
7	2	0	0	0	1	0	6
8	1	3	0	0	0	0	3
9	1	1	0	1	0	0	9
10	1	1	0	0	1	0	3
11	1	0	2	0	0	0	8
12	1	0	1	1	0	0	4
13	1	0	0	2	0	0	0
14	1	0	0	0	0	1	8
15	0	4	0	0	0	0	0
16	0	2	1	0	0	0	10
17	0	2	0	1	0	0	6
18	0	2	0	0	1	0	0
19	0	1	2	0	0	0	5
20	0	1	1	1	0	0	1
21	0	1	0	0	0	1	5
22	0	0	3	0	0	0	0
23	0	0	1	0	1	0	10
24	0	0	1	0	0	1	0
25	0	0	0	1	1	0	6
26	0	0	0	0	2	0	0

- A 60 inch wide roll can be cut into one of 26 possible patterns, for example
  - Five12 inch rolls, with no residual waste
  - Three 12 inch rolls + a 15 inch roll, with a 9 inch roll remaining as waste
- There's no simple way to produce this list, but the company only needs to do it once
- The company then needs to decide the number of rolls to cut into each of these patterns in order to fulfil the demands at minimum cost

Conceptual Model



Algebraic Model

Decision variables:

- Let X_i denote the number of rolls used to produce pattern type i $(i = 1, 2, \dots, 26)$

Fixed inputs:

- Initially all rolls have a standard width of 60
- d_j denotes the demand for rolls of width j $(j = 12, 15, 20, 24, 30, 40)$
- f_{ij} denotes the number of rolls of width j produced by pattern i

Calculated variables:

- $w_i = 60 - \sum_j j f_{ij}$ denotes the waste per standard roll from using pattern i
- Total waste = $\sum_i w_i X_i + \sum_j j\left(\sum_i f_{ij} X_i - d_j\right) = 60 \sum_i X_i - \sum_j j d_j$

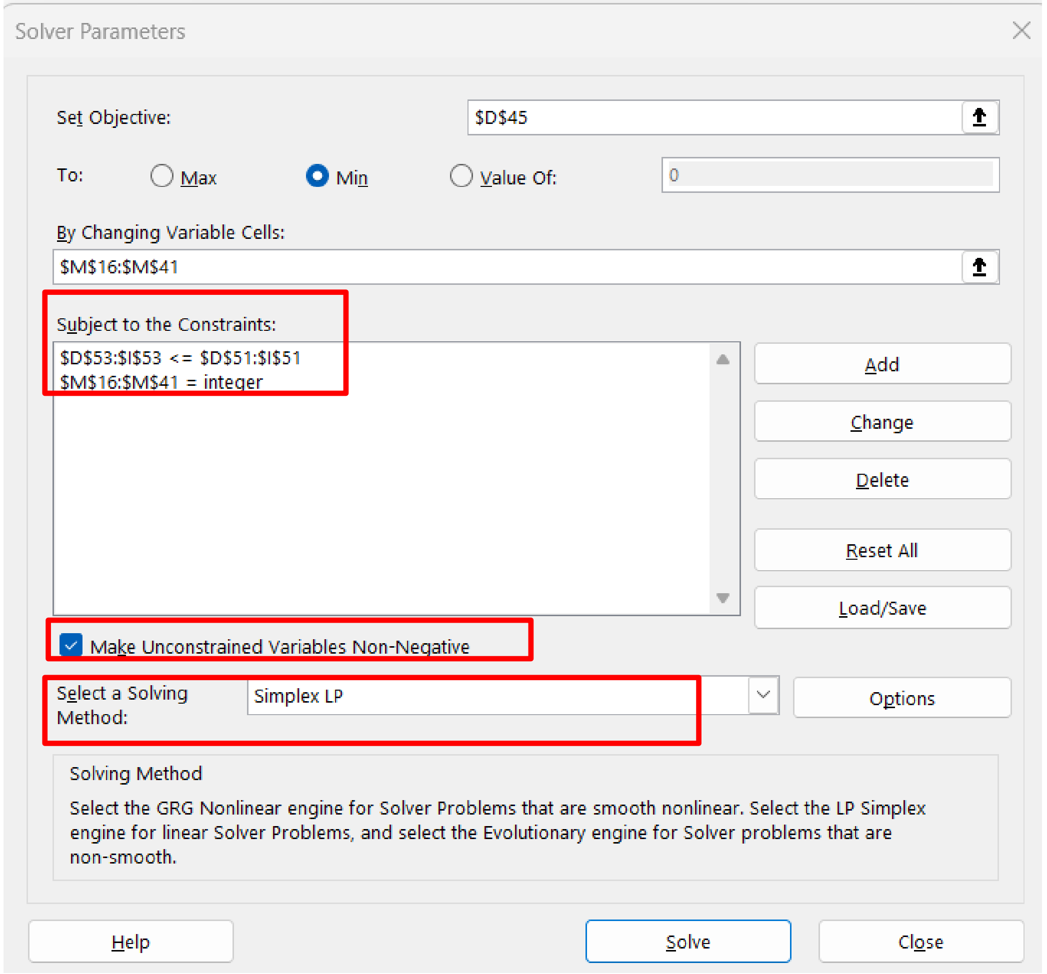
Objective function:

- Minimise the total number of rolls used, i.e. $\sum_i X_i$. (Equivalent to minimising the total waste)

Constraints:

- $\sum_i f_{ij} X_i \geq d_j$ (demand constraints) and X_i are integer variables

Solver Set-up



- These models require integer decision variables.
- Need to tick the box to make unconstrained variables non-negative since we aren't working with binary variables
- Remember to use the Simplex LP method

Optimal Solution

D45 =SUM(M16:M41)

Feasible ways of cutting up a roll

Pattern	Width 12	15	20	24	30	40	Waste
1	5	0	0	0	0	0	0
2	3	1	0	0	0	0	9
3	3	0	1	0	0	0	4
4	3	0	0	1	0	0	0
5	2	2	0	0	0	0	6
6	2	1	1	0	0	0	1
7	2	0	0	0	1	0	6
8	1	3	0	0	0	0	3
9	1	1	0	1	0	0	9
10	1	1	0	0	1	0	3
11	1	0	2	0	0	0	8
12	1	0	1	1	0	0	4
13	1	0	0	2	0	0	0
14	1	0	0	0	0	1	8
15	0	4	0	0	0	0	0
16	0	2	1	0	0	0	10
17	0	2	0	1	0	0	6
18	0	2	0	0	1	0	0
19	0	1	2	0	0	0	5
20	0	1	1	1	0	0	1
21	0	1	0	0	0	1	5
22	0	0	3	0	0	0	0
23	0	0	1	0	1	0	10
24	0	0	1	0	0	1	0
25	0	0	0	1	1	0	6
26	0	0	0	0	2	0	0

DECISION VARIABLES

Pattern	Rolls cut
1	6
2	0
3	0
4	0
5	0
6	0
7	0
8	1
9	0
10	0
11	1
12	0
13	16
14	0
15	4
16	0
17	0
18	0
19	0
20	0
21	0
22	4
23	1
24	7
25	0
26	7

OBJECTIVE FUNCTION

Total rolls cut 47 Waste 51

CONSTRAINTS

	Width 12	15	20	24	30	40
No. obtained	48	19	22	32	15	7
	≥	≥	≥	≥	≥	≥
No. required	48	19	22	32	14	7
Excess	0	0	0	0	1	0

- There are multiple solutions with the same optimal value (47 rolls to be cut)
- We can optimise either the total number or rolls or the total waste (this follows from the algebraic model)
- **If you have the integer optimality set to 0, Solver seems to run forever (you can always cancel Solver by pressing Cntl+Alt+Delete if it is taking too long).**
- The textbook authors suggest setting integer optimality to 2%

Summary

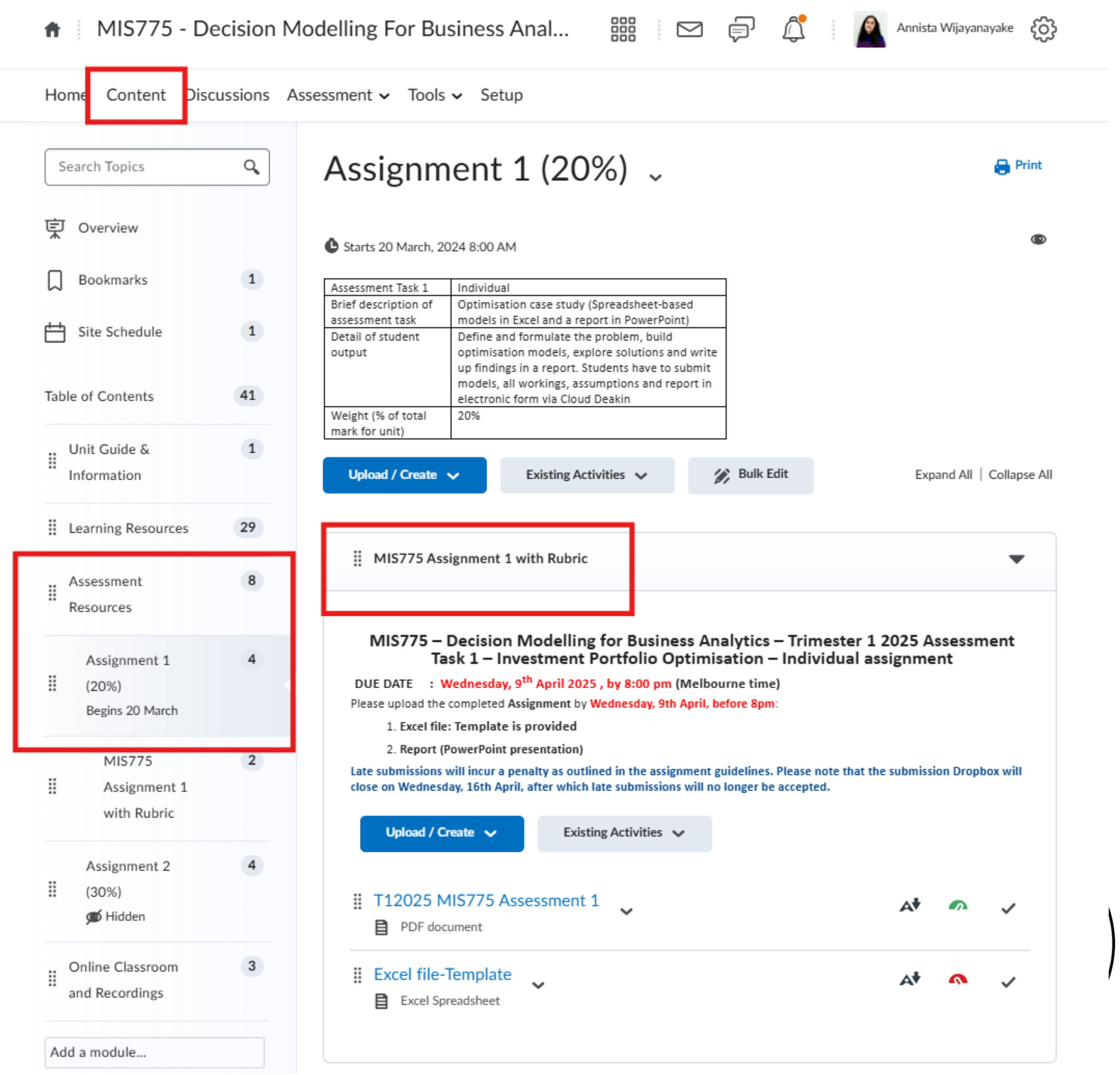
- ILP solves LP problems in which some or all of the decision variables are constrained to take non-negative integer values, i.e. {0, 1, 2, 3, …}
- Some LP problems require the use binary variables

- We considered three types of ILP models:
  - **In capital budgeting models the decision variables are all binary and these models may include logic constraints**
  - **With fixed-cost models, we learned that binary variables can be used to solve complex real-world problems by converting them into an ILP model**
  - **With cutting stock models we learned how a manufacturing company could use ILP to minimise waste**

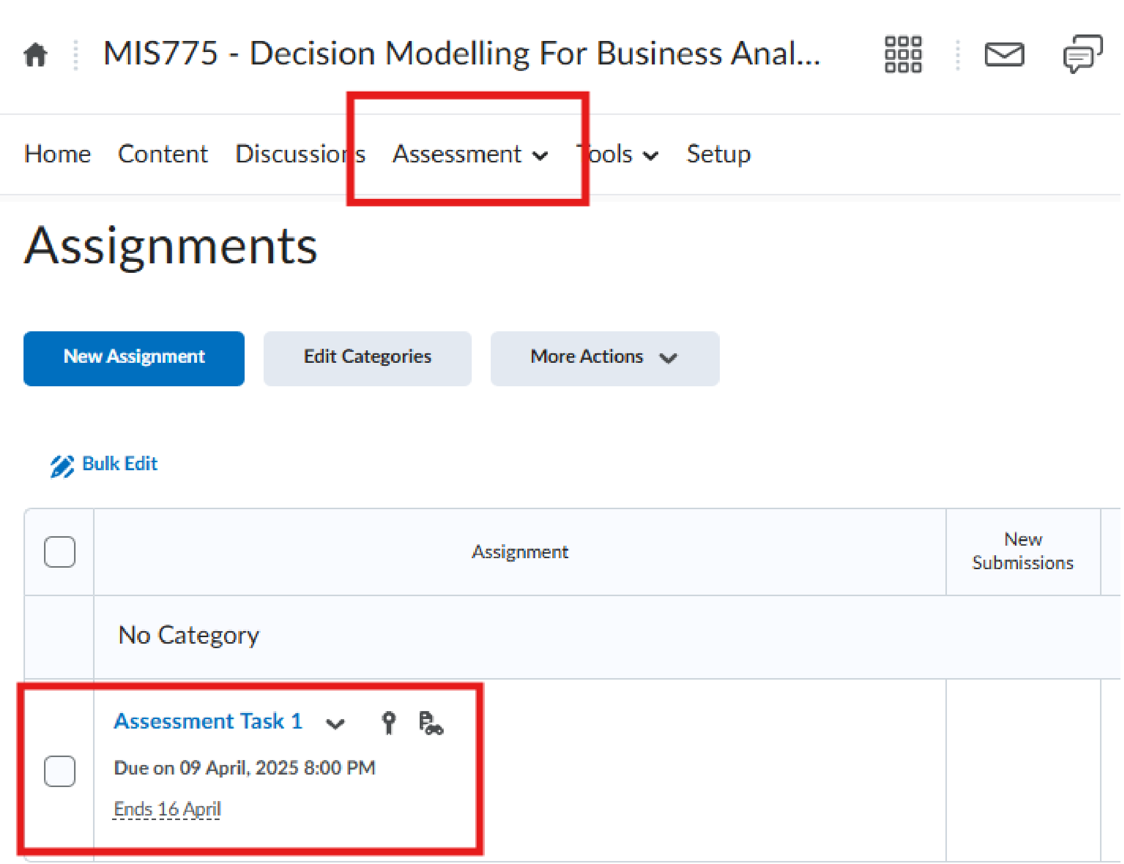
Assignment 1 has been released

MIS775 – Decision Modelling for Business Analytics – Trimester 1 2025
Assessment Task 1 – Investment Portfolio Optimisation – Individual

DUE DATE : **Wednesday, 9th April 2025 , by 8:00 pm** **(Melbourne time)**
PERCENTAGE OF FINAL GRADE: 20%



Assignment 1 has been released



📌 Submission Method:

Online submission only via the CloudDeakin Unit site as shown below.

Submission Requirements:

To complete your submission, please upload the completed assignment documents by **Wednesday, 9th April, before 8:00 PM**:

1. **Excel File** – Template provided
2. **Report** – PowerPoint presentation format

If you have any questions, please reach out to your Tutor in your upcoming seminars.

For further clarification, you can also post your queries in the discussion forum.

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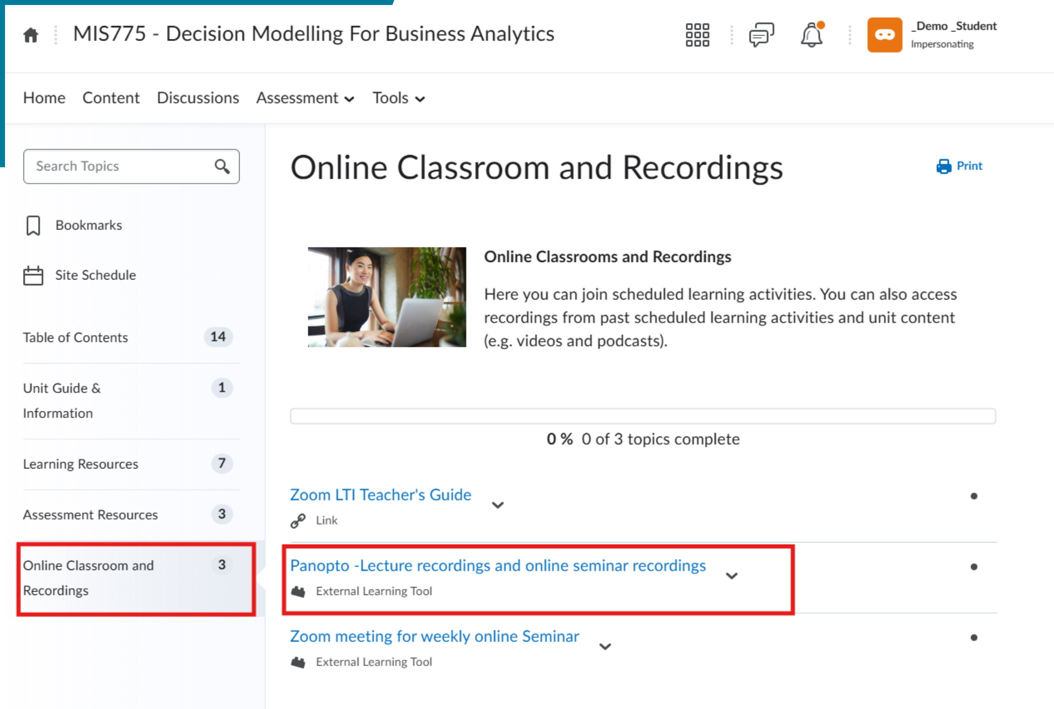
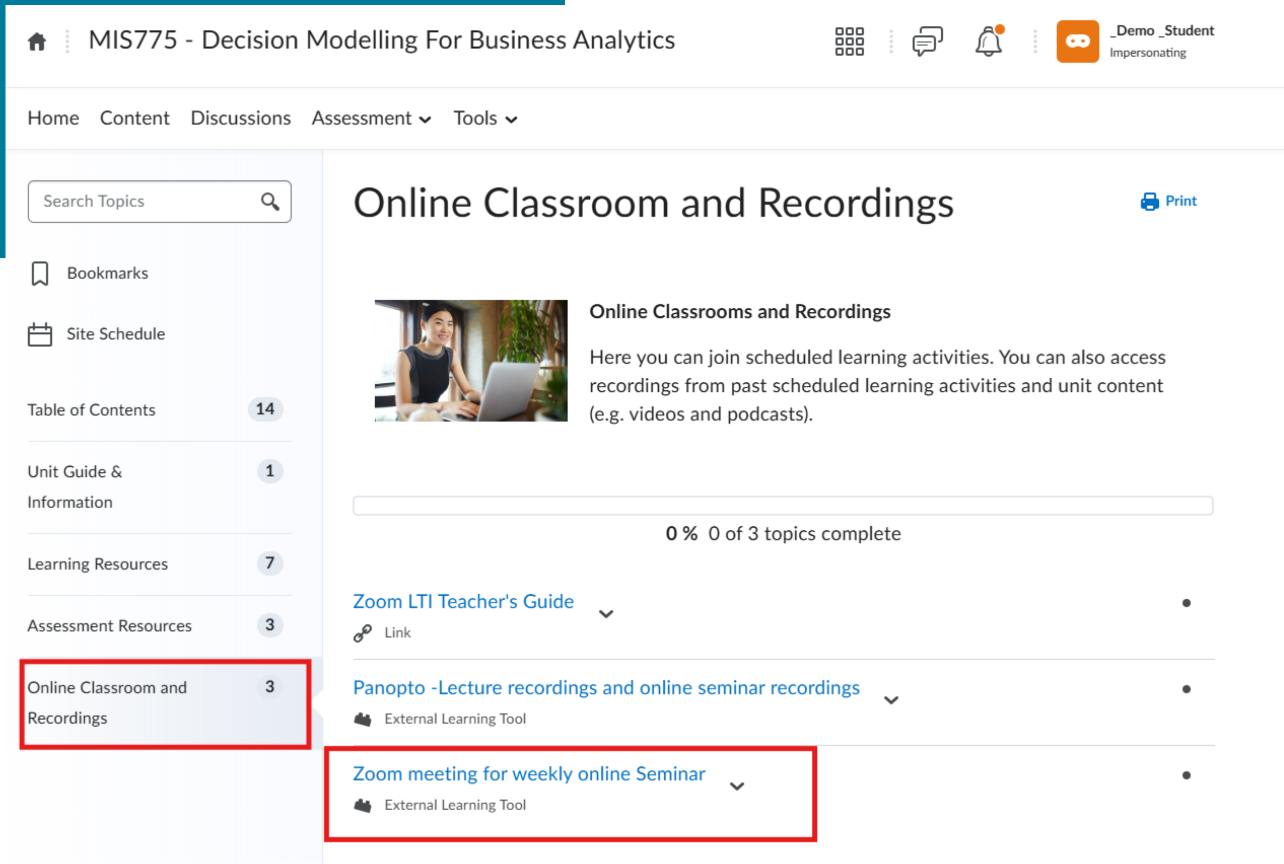
Lecture and Seminar recordings



- Please go through the lecture materials and exercises before attending the seminar.
- Online students are also encouraged to attend in-person seminars at Burwood campus.
- **The weekly on-campus lectures and online seminars are recorded and available on the CloudDeakin unit site, as outlined below.**
- **Cloud Deakin unit site > Content > Online Classroom and Recordings > Panopto -Lecture recordings and online seminar recordings**

Online seminar links



For online students, a live online seminar is scheduled every Friday from 6:00 PM to 7:00 PM. Please use the relevant link provided below to join the session.

Cloud Deakin unit site > Content > Online Classroom and Recordings > Zoom meeting for weekly online Seminar

Next Class

Topic 4: Non-Linear Programming (NLP) Models (Chapter 7 of the textbook)

We will start by considering the basic ideas behind nonlinear optimisation
Then we will consider four standard types of NLP models:

- **Pricing models (section 7.3)**
- **Advertising response and selection models (section 7.4)**
- **Facility location models (section 7.5)**
- **Portfolio optimisation models (section 7.7)**