Assignment 1 is available

- Assignment one support session Timetable – Please see the Announcements area on the unit site
- Assignment 1 queries will only be answered via the Discussion forum from Monday to Friday during business hours
- To be fair to all students, we are unable to answer Assignment one queries via email.

Week 7 - Easter vacation/intra-trimester break:

Friday 18th April –Sunday 27th April Sunday (inclusive)

What's the Plan for Week 7?

- No F2F seminars or online seminars on Friday 18th April (Public Holiday)
- **Week 7 Online seminar will be on Thursday 17th April at 7 pm**
- **Friday On-Campus Seminar groups are very welcome to attend the week 7 online seminar**

MIS775 WEEK 4

MIS775 Decision Modelling for Business Analytics

TOPIC 3: Non-Linear Programming Models

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Recap

Optimization models find the most efficient way of using limited resources to achieve the objective set by an individual or a business

- **Linear Programming (LP)** seeks to maximise or minimise a **linear objective function**, subject to a set of **linear constraints**
- **Sensitivity Analysis** helps to answer questions about how sensitive an optimal solution is to changes in various coefficients or parameters in a model
- **Integer Linear Programming (ILP)** seeks to maximize or minimize a linear objective function, subject to a set of linear constraints, with the additional requirement that some or all decision variables must take integer values.

Non-Linear Programming (NLP)

- In real-world business problems, the objective function or a constraint may not be linear, in which case we can no longer use linear programming methods
- NLP models include at least one non-linear function, which could be the objective function or a constraint
- These models are formulated and implemented in almost the same way as linear models, but, in practice, they are difficult to solve and should be used with caution
- Main types of NLP models:
  - **Pricing models (section 7.3)**
  - **Advertising response and selection models (section 7.4)**
  - **Facility location models (section 7.5)**
  - **Portfolio optimisation models (section 7.7)**

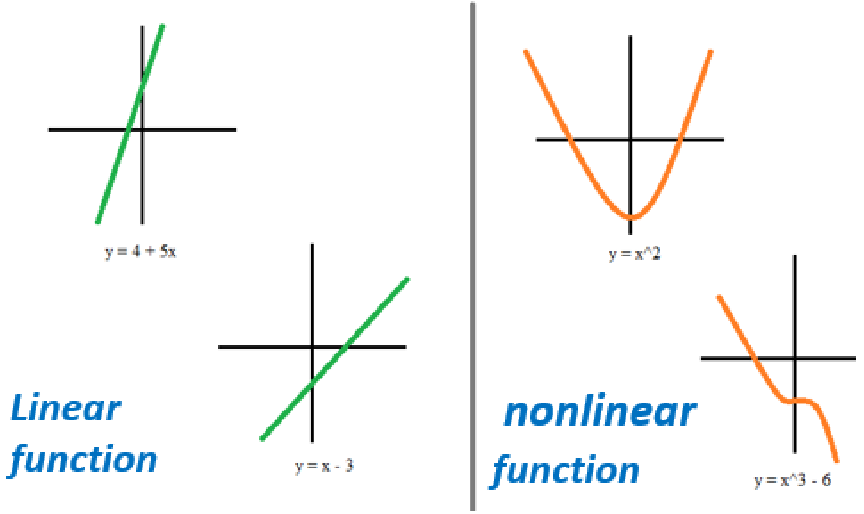
Learning Objectives

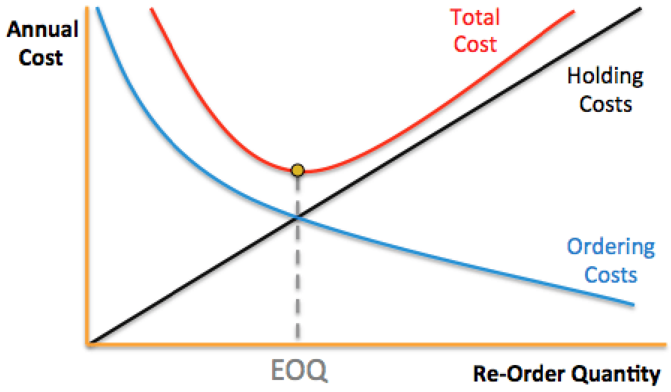
- Understand the differences between LP and NLP models
- Examine the reasons why NLP models are difficult to solve
- Understand the process of formulating, implementing and solving NLP models in a spreadsheet
- Understand how to set-up NLP problems in a spreadsheet and solve them using Excel's Solver

Textbook reading: Chapter 7 (7.3-7.5, 7.7)

Introduction

- Some business processes do not necessarily behave in a *linear* fashion but are more likely to behave in a *nonlinear* manner.





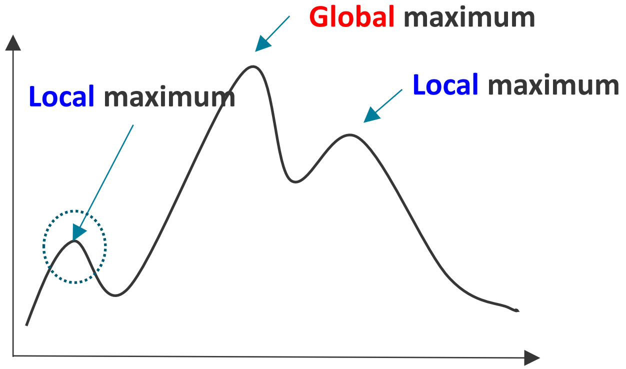
- For example, the marginal cost of production often decreases with the quantity produced, and the quantity demanded for a product is generally a nonlinear function of the price.

- Nonlinear optimization problem is an optimization problem in **which at least one term in the objective function *or* a constraint is nonlinear**.

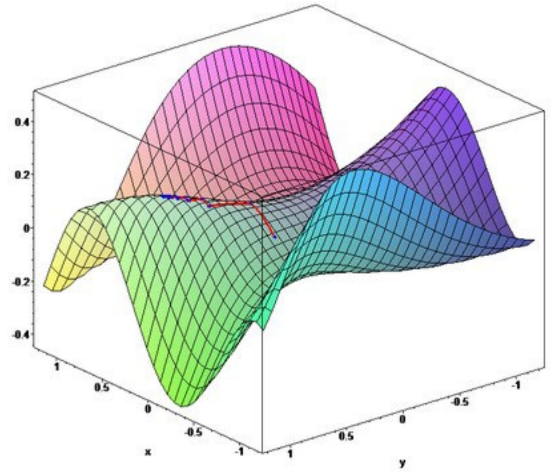
Why are NLP Problems Inherently much more Difficult to Solve?

1. Local optima vs Global optimum

As a feasible region may have more than one "peak" (if maximising) or "valley" (if minimising), Solver may get stuck at a local optima and be unable to find the global optimum



2. Solver's Multistart option is a way of tackling this problem. It uses many different initial points in the hope that one will lead to the global optimum



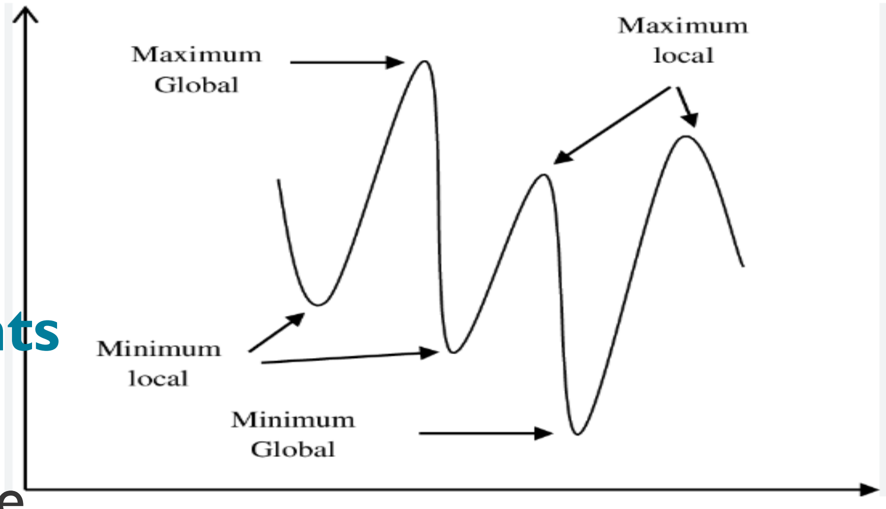
Why are NLP Problems Inherently much more Difficult to Solve?

3. It may be difficult to find a feasible starting point

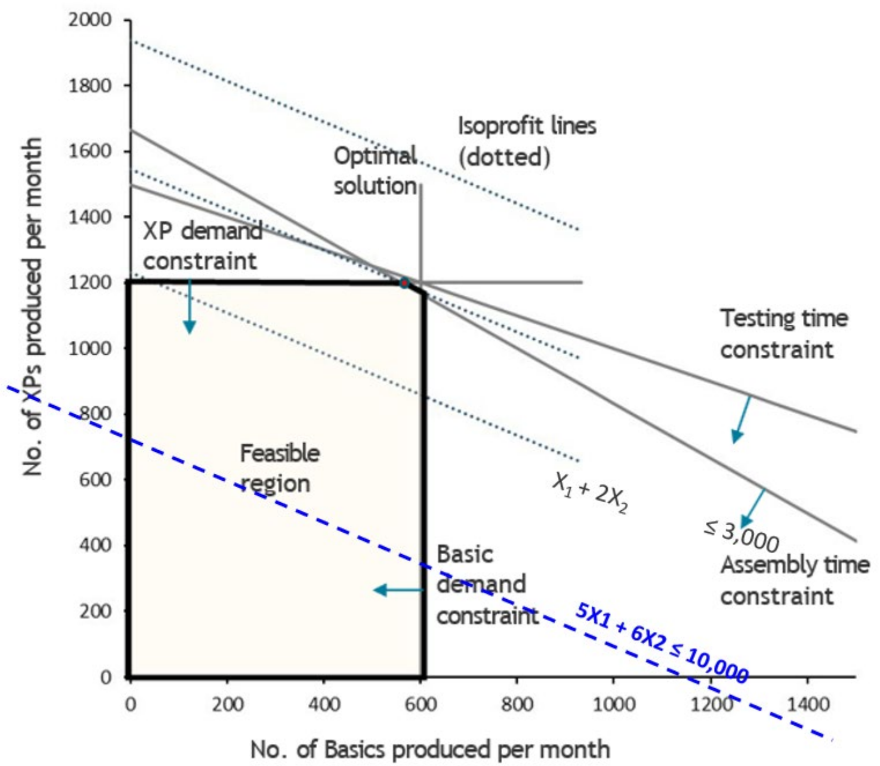
We may never find a feasible point to begin with. We would be left unsure as to whether the model is infeasible, or it is feasible, but we didn't start Solver in the right place

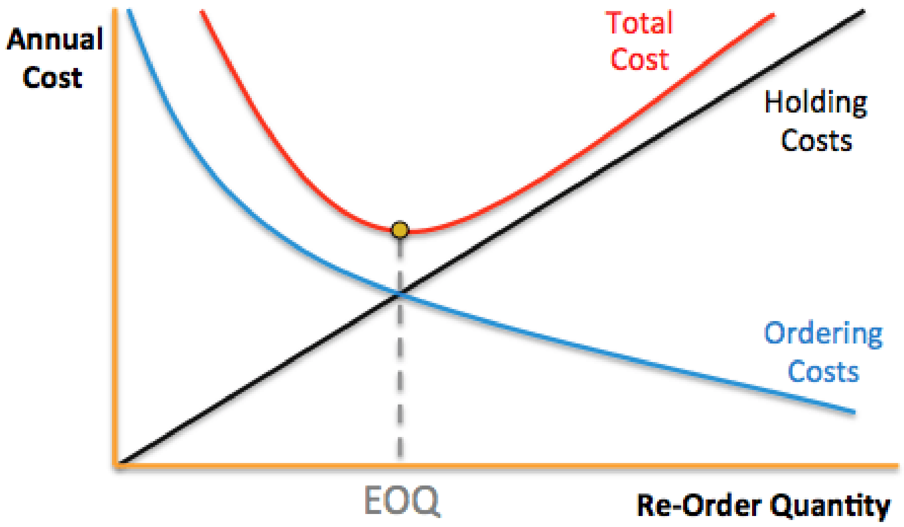
4. An optimum (local or global) solution is not restricted to the extreme (i.e. corner /vertex) points

In NLP problems, an optimum (local or global) could be anywhere: at an extreme point, along an edge of the feasible region, or in the interior of the feasible region



Ex : Linear Vs non-linear models

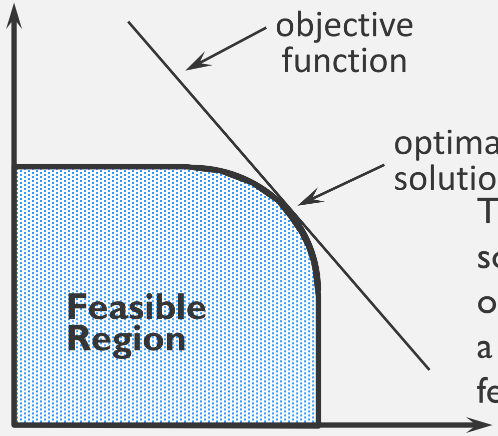




Linear

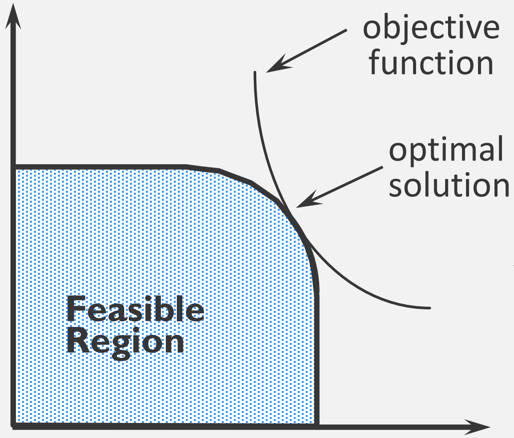
Non-Linear

Optimal Solution can be Anywhere!



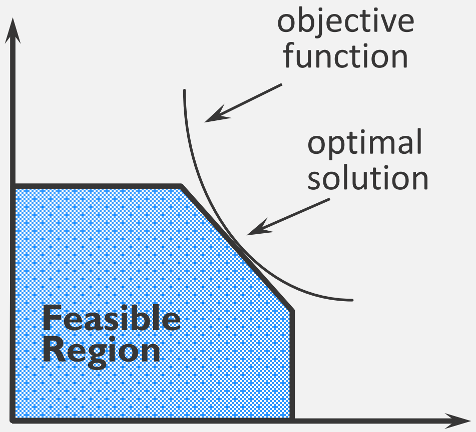
The unique optimal solution to this NLP model occurs at point that is **not** a corner point of the feasible region

linear objective,
nonlinear constraints



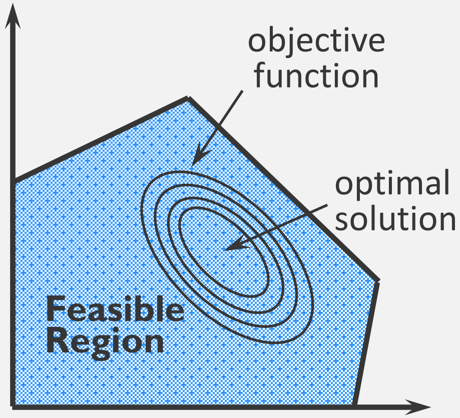
The optimal solution to this NLP model occurs at a point that is **not** a corner point of the feasible region

nonlinear objective,
nonlinear constraints



The optimal solution to this NLP model occurs at a point that is **not** a corner point of the feasible region, even though all constraints are linear

nonlinear objective,
linear constraints



The optimal solution to this NLP model occurs at a point in the **interior** of the feasible region

nonlinear objective,
linear constraints

Methods for Solving NLP problems

Solver has two methods for handling NLP problems:

Generalized Reduced Gradient (GRG) Algorithm

The GRG method is designed for NLP problems where the objective function and constraints are **smooth** – no jagged jumps. Graphically, a smooth function can be plotted as a single continuous line with no abrupt bends or breaks

Evolutionary Algorithm

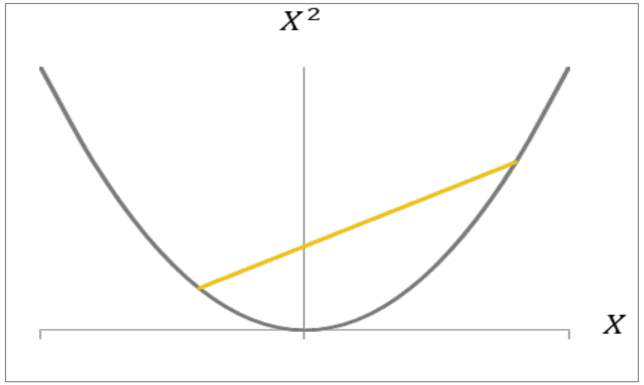
Best suited for **non-smooth** NLP problems. Graphically, a non-smooth function may have discontinuities/holes or may have sharp edges/angles

At best, the Evolutionary method will be able identify a good solution, but cannot guarantee that it is optimal

Problems that Solver will Always Solve Correctly

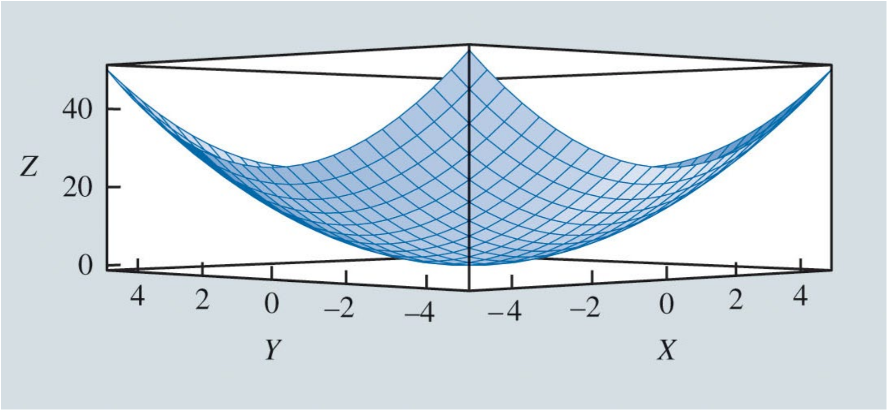
Solver will always find the global minimum (if it exists) when:

- The objective function is convex; and
- The constraints are linear



Convex objective function: A function that is **bowl-shaped up,** a line connecting *any* two points is above the curve.

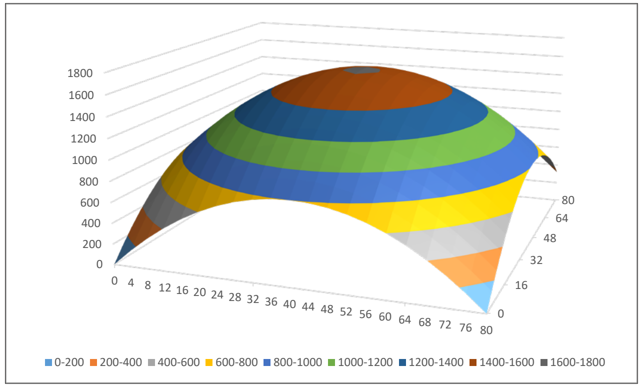
Example: Consider the function $f(X,Y) = X^2 + Y^2$; the shape of this function is



Problems that Solver will Always Solve Correctly

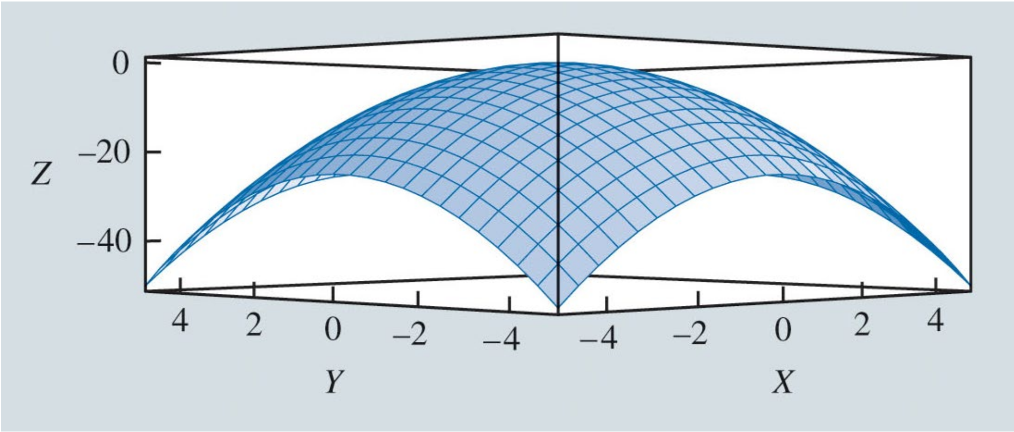
Solver will always find the global maximum (if it exists) when:

- The objective function is concave; and
- The constraints are linear



Concave objective function: A function that is **bowl-shaped down,** a line connecting *any* two points is below the curve

Example: Consider the function $f(X, Y) = -X^2 - Y^2$; the shape of this function is



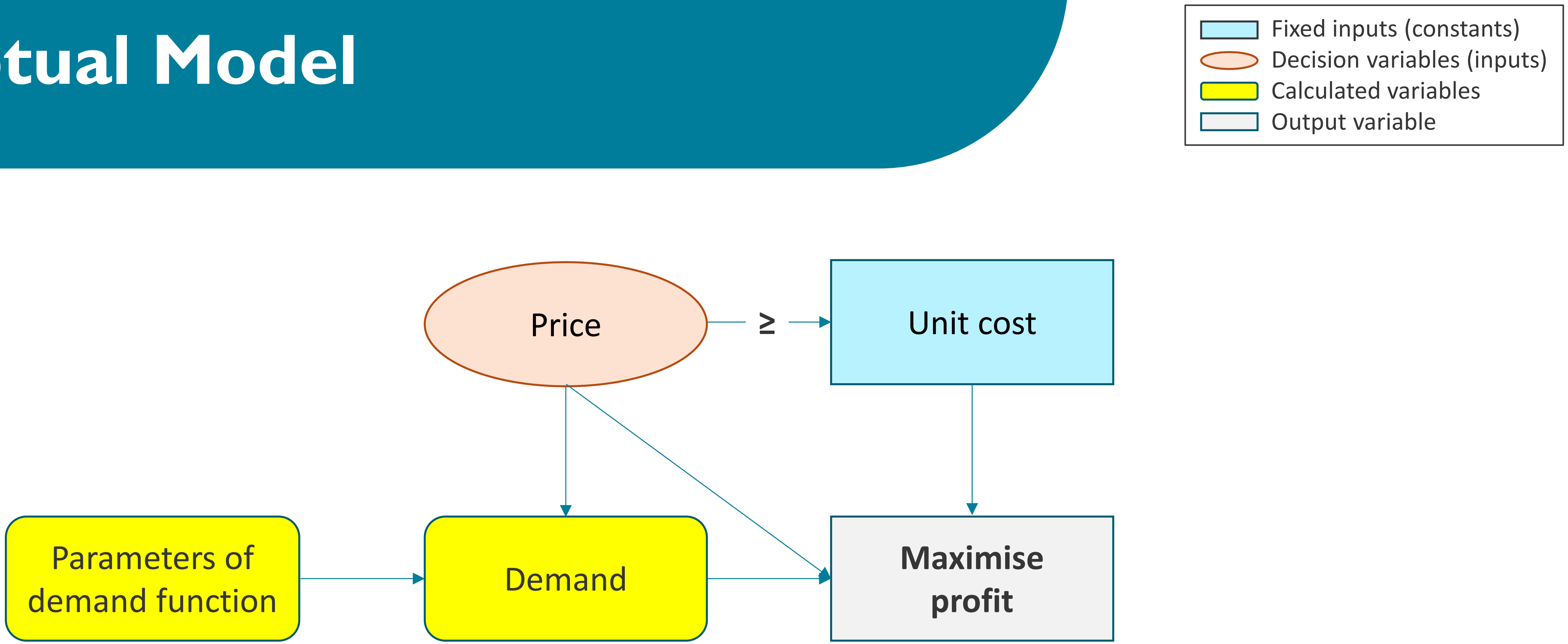
Ex1. Pricing models *

- A company manufactures and retails a certain product, and wants to determine the price at which profit will be maximised
- The cost of manufacturing and marketing the product is $50
- They estimate that they can sell 400 units at $70, but sales will fall to 300 units if the price is $80
- The only constraint is that they don't make a loss
- The company decides to employ a constant elasticity demand function: $\boxed{D = aP^b}$

Price	Demand
$70	400
$80	300

*Textbook example on pp. 347-352

Conceptual Model



- The parameters of the demand function a and b are unknown, so before we can optimise profit, we need to determine their values. **This can be done using Solver!**

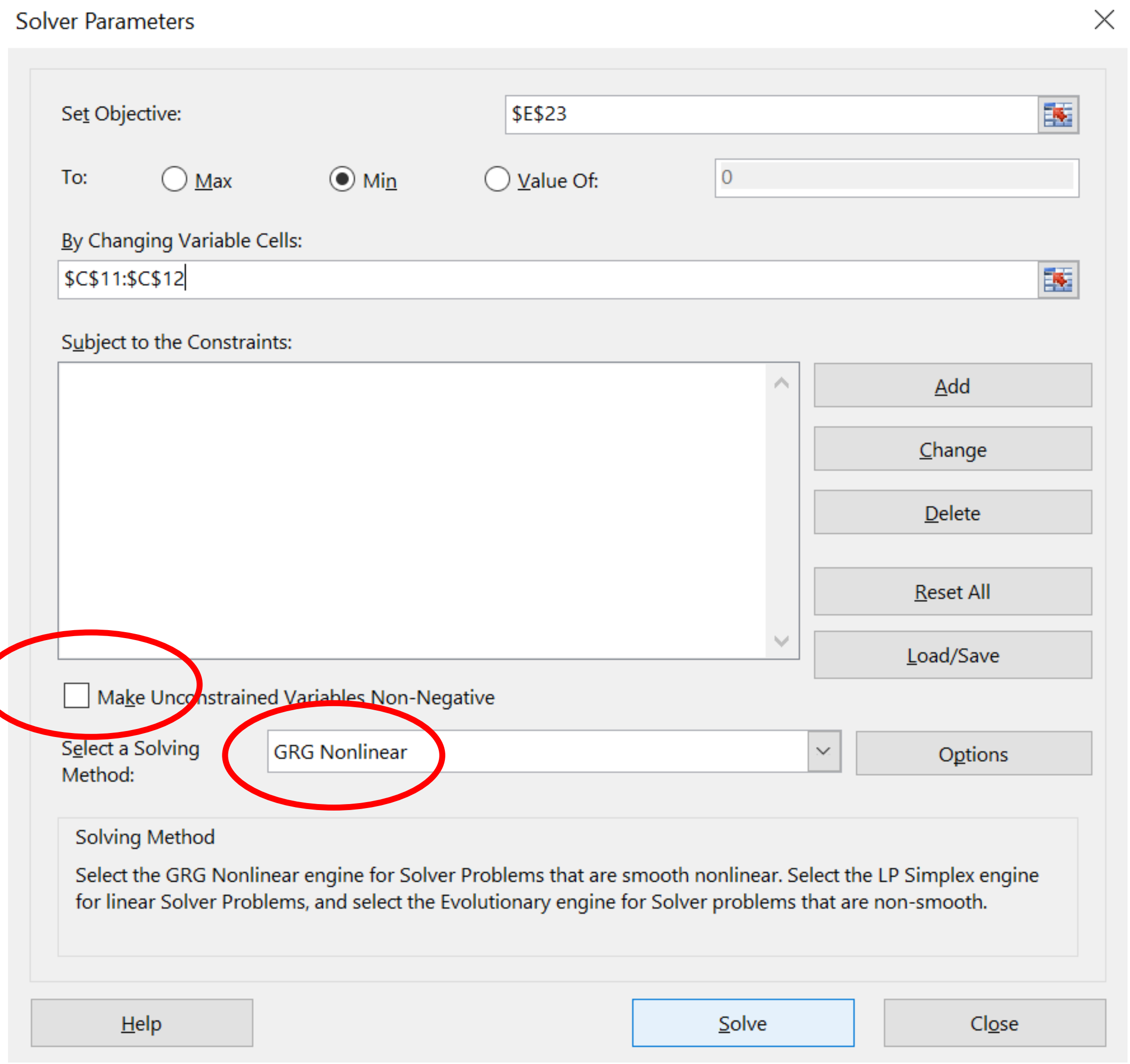
Algebraic Model for determining parameters a & b

Price	Demand
$70	400
$80	300

$D = aP^b$

- Set the parameters, a & b, as our decision variables
- **We wish to minimise the difference between the given demand and predicted demand**
- We set the objective function to be the sum of squared differences between the **given demands** and the **predicted demands** assuming a constant elasticity function
- In algebraic form, the objective function is $\left(400 - a70^b\right)^2 + \left(300 - a80^b\right)^2$
- Notice that the objective function is nonlinear (why?)
- We now use Solver to find values for *a* & *b* that **minimise the objective function** value (ideally making it zero)
- Note that no constraints are required

Solver Set-up



- We use the **GRG Nonlinear** solving method instead of Simplex LP whenever the model is nonlinear
- Also, note that in this example we allow the decision variables to be negative

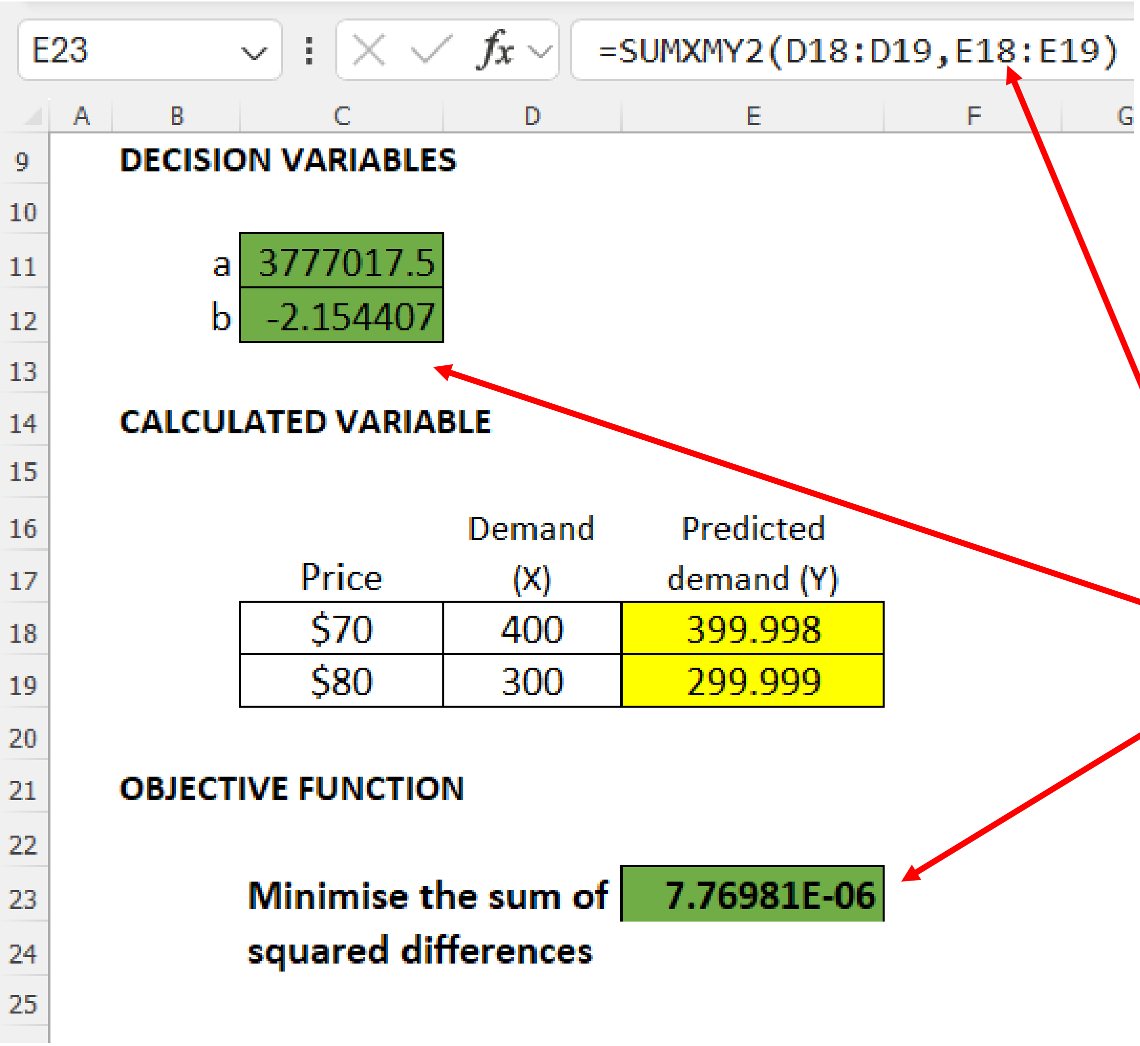
Optimal Parameter Values

X_1	Y_1
X_2	Y_2

$(x_1-y_1)^2 + (x_2-y_2)^2$

= SUMXMY2 ( )

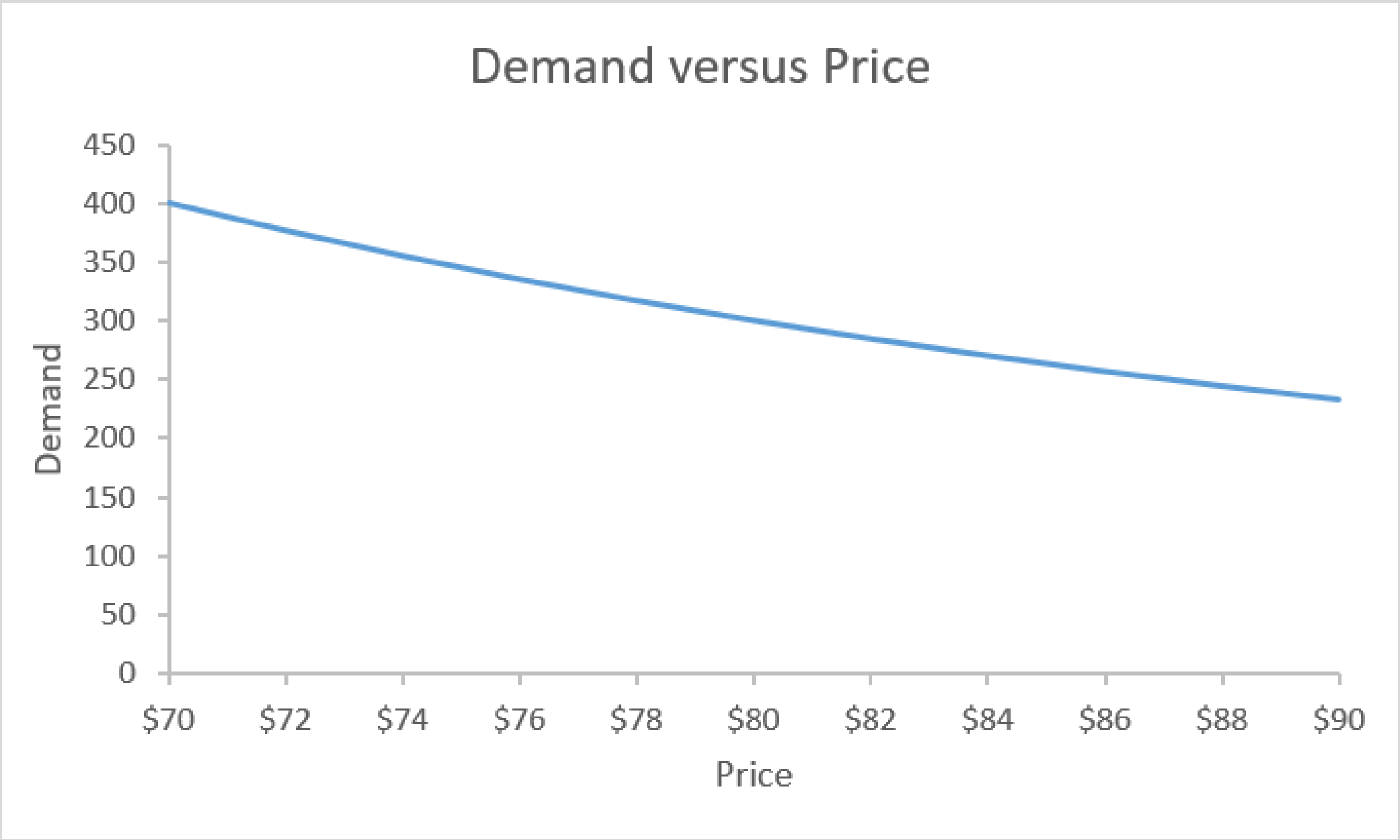
objective function is $\left(400 - a70^b\right)^2 + \left(300 - a80^b\right)^2$



- Excel built-in formula SUMXMY2 is used to calculate the sum of squared differences
- $a = 3{,}777017.5$ and $b = -2.1544$
- Notice that the objective function is close to zero (as required)

Constant Elasticity Demand Curve

Price	Demand
$70	400.0
$72	376.4
$74	354.9
$76	335.0
$78	316.8
$80	300.0
$82	284.4
$84	270.1
$86	256.7
$88	244.3
$90	232.7



Algebraic Model

- Decision variable:

  Let P denote the product price in dollars

- Calculate variable:

  Let D denote the demand at price P

  Then $D = aP^b$ where $a = 3,777017.5$and $b = -2.1544$

- Objective: Maximise profit ($) given by $(\boldsymbol{P} - \$\mathbf{50})\ \boldsymbol{D}$

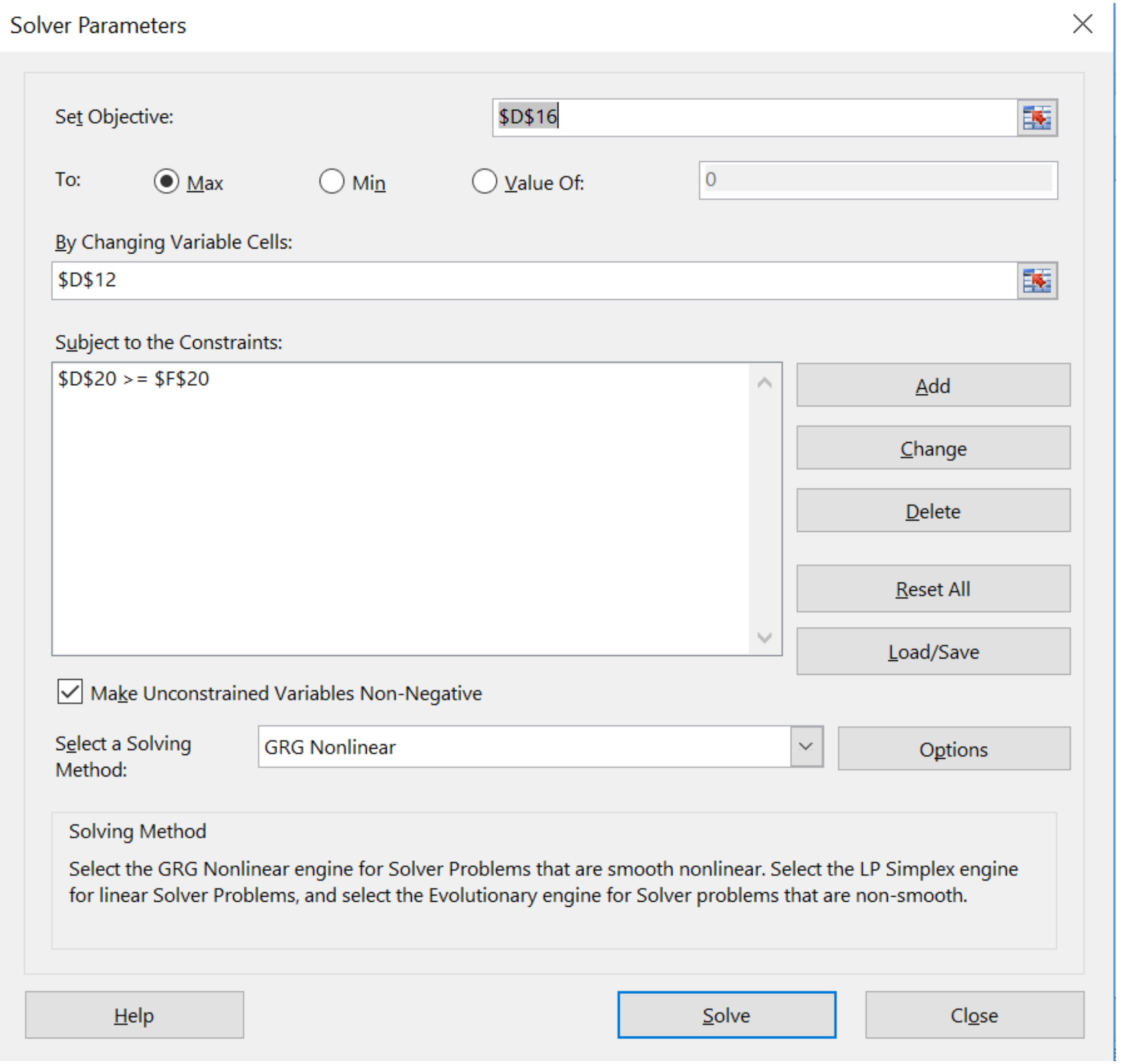
- Constraint: Profit ≥ 0

$D = aP^b$

Objective maximise= $(P - \$50)D$

= (P-$50)$aP^b$

Solver Set-up



- Note that we need to start with an initial positive value for price in the spreadsheet model, or Solver won't be able to start looking for an optimal solution

Optimal Solution

D16 =(D12-D5)*D7*D12^D8

PRICING MODELS

INPUT

Cost per unit	$50
Parameter values	
a	$3,777,018
b	-$2.154407

DECISION VARIABLE

Price	$93.31

OBJECTIVE FUNCTION

Maximise profit	$9,326.33

CONSTRAINT

LHS		RHS	
9326.3278	≥	0	(non-zero profit constraint)

a	3777017.5
b	-2.154407

$D = aPb$

Objective maximise= $(P - \$50)D$

= (P-\$50)aP^b

Ex 2. Advertising Response & Selection Models *

- In week 2 we discussed an advertising model, where a breakfast cereal company wanted to decide on the number of ads to place across three TV shows, that met certain minimum exposures across key female segments in its target market, and minimised the total cost. The algebraic model is as follows:
- X_A, X_B and X_C denote the number of ads on shows A, B and C
- Minimise total cost ($\$'000$) = $140X_A + 100X_B + 80X_C$

subject to:

$6X_A + X_B + 4X_C \geq 60$ (females aged $18 - 35$ constraint)

$4X_A + X_B + 2X_C \geq 60$ (females aged $36 - 55$ constraint)

$2X_A + X_B \geq 28$ (females aged over 55 constraint)

- These constraints assume that the number of exposures achieved has a *linear* relationship with the number of ads

***Modified textbook example on pp. 365-373**

Advertising Response & Selection Models

- But the effectiveness of ads is known to diminish over repeated exposures.

- A function which captures this type of behaviour is the modified exponential response model, which is of the form

$$Y = a\left(1 - e^{-bX}\right) \quad a, b > 0$$

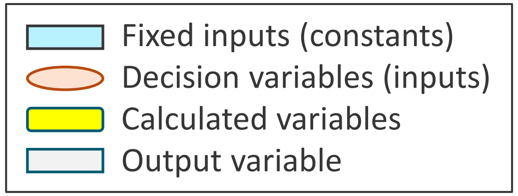
where X is the number of ads placed on a given show, and Y is the *actual* number of exposures achieved for a given segment, and a & b are constants determined for each combination of show and segment.

- Building this into our constraints makes the model nonlinear

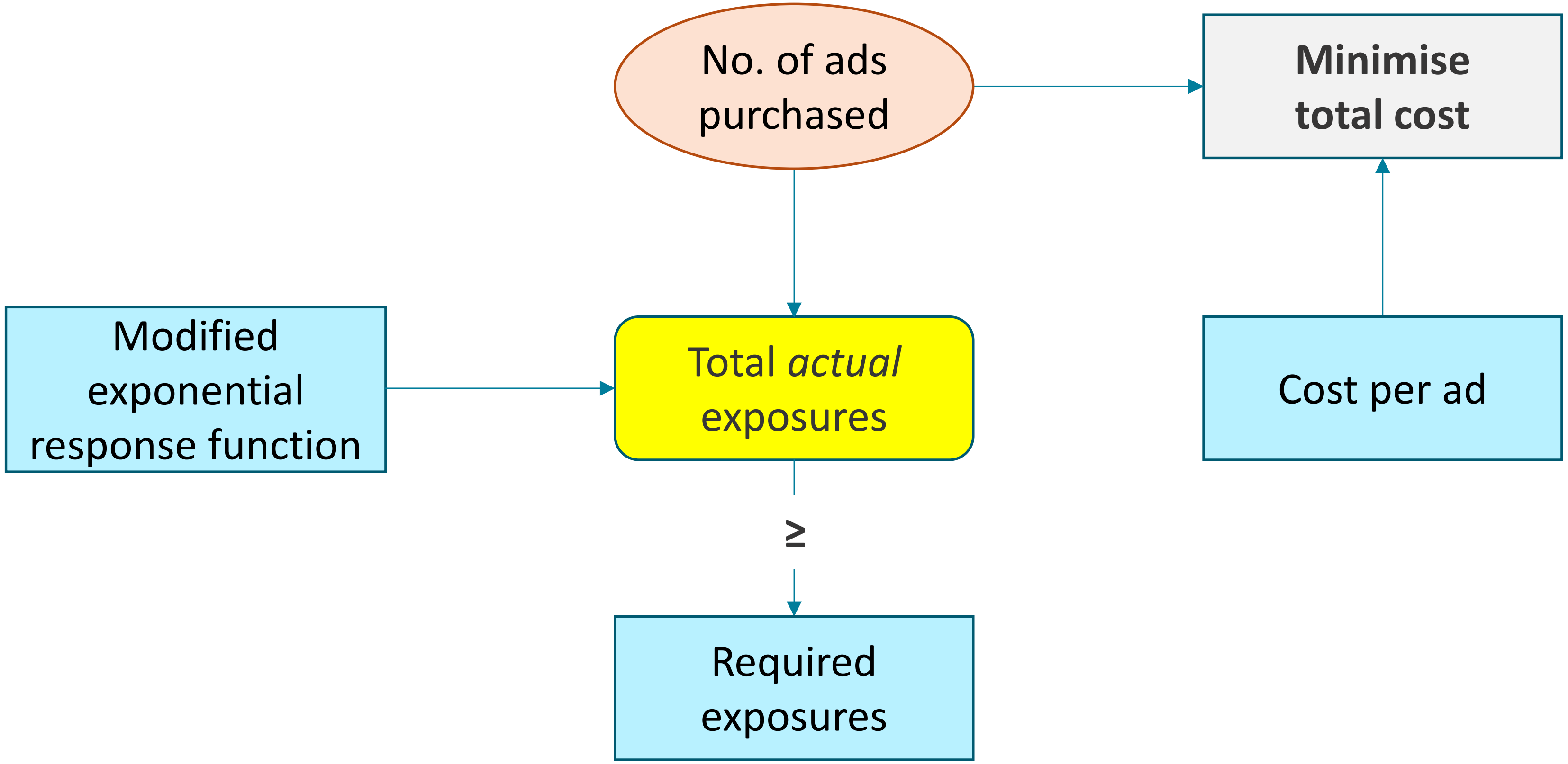
No. of ads on a given show	No. of exposures (in millions) for a given segment (Historical data)
1	4.7
8	22.1
20	48.7
50	90.3
100	130.5

*Modified textbook example on pp. 365-373

Conceptual Model



- Before we can solve the nonlinear model we need to determine the constants a & b that apply to each show and segment. **This can be done using Solver!**



How to Estimate a & b in the Response Function

- To begin with we need at least two data points (from historical company data)
- We model this in the same way that we did when estimating the parameters in a constant elasticity demand function, the only difference being the function is now given by

$$Y = a\left(1 - e^{-bX}\right), \text{ where } a, b > 0$$

- The objective is again to minimise the sum of squared differences between the actual exposures and the predicted exposures, this time using the modified exponential function

No. of ads on a given show	No. of exposures (in millions) for a given segment (Historical data)
1	4.7
8	22.1
20	48.7
50	90.3
100	130.5

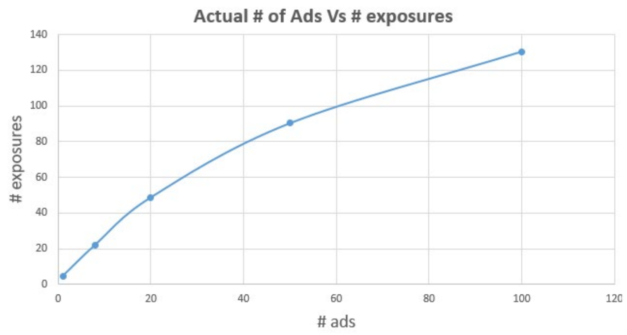
*Modified textbook example on pp. 365-373

Is the Solver Solution Optimal?

$$Y = a\left(1 - e^{-bX}\right) \quad a, b > 0$$

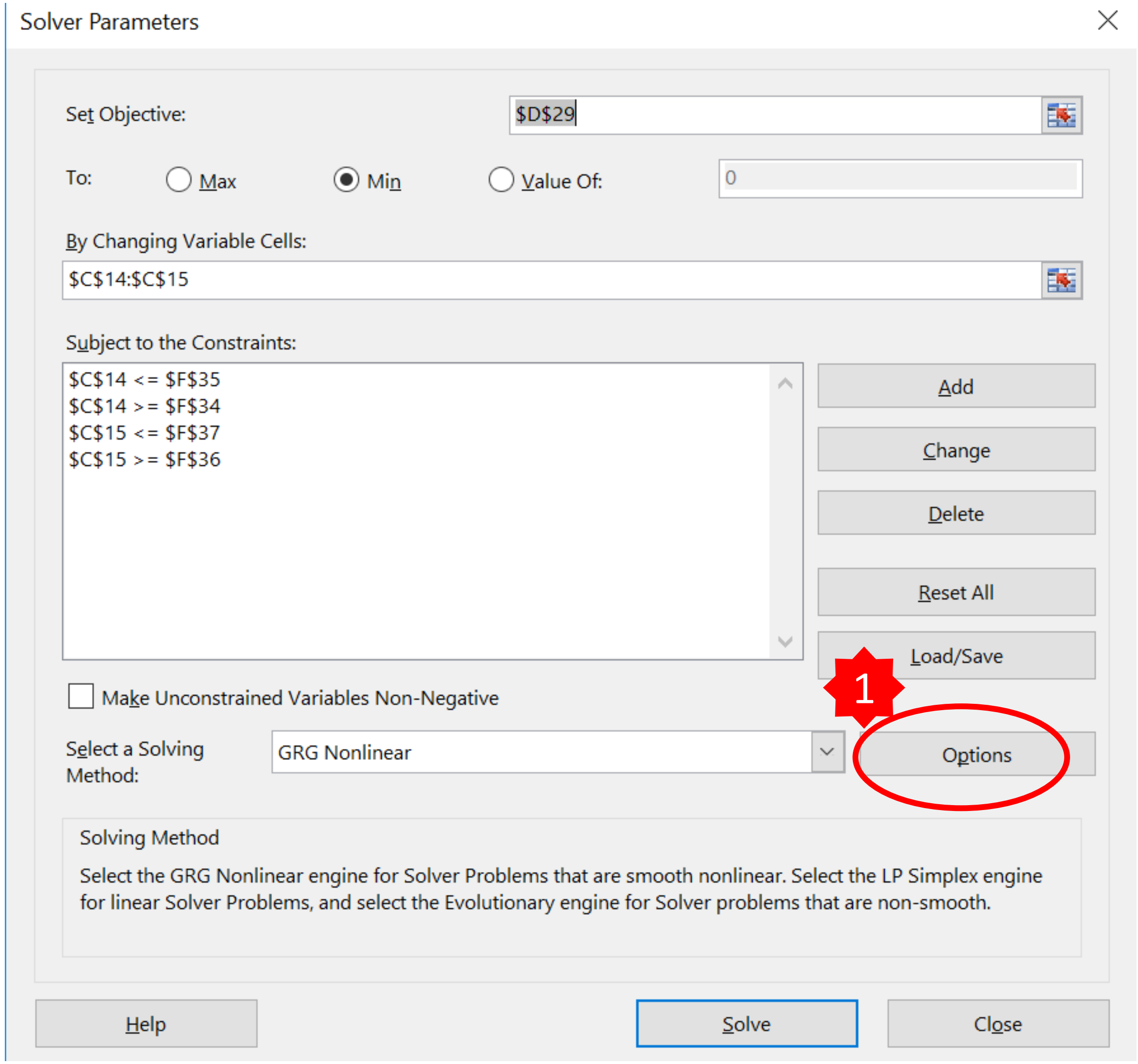
No. of ads on a given show	No. of exposures (in millions) for a given segment (Historical data)
1	4.7
8	22.1
20	48.7
50	90.3
100	130.5

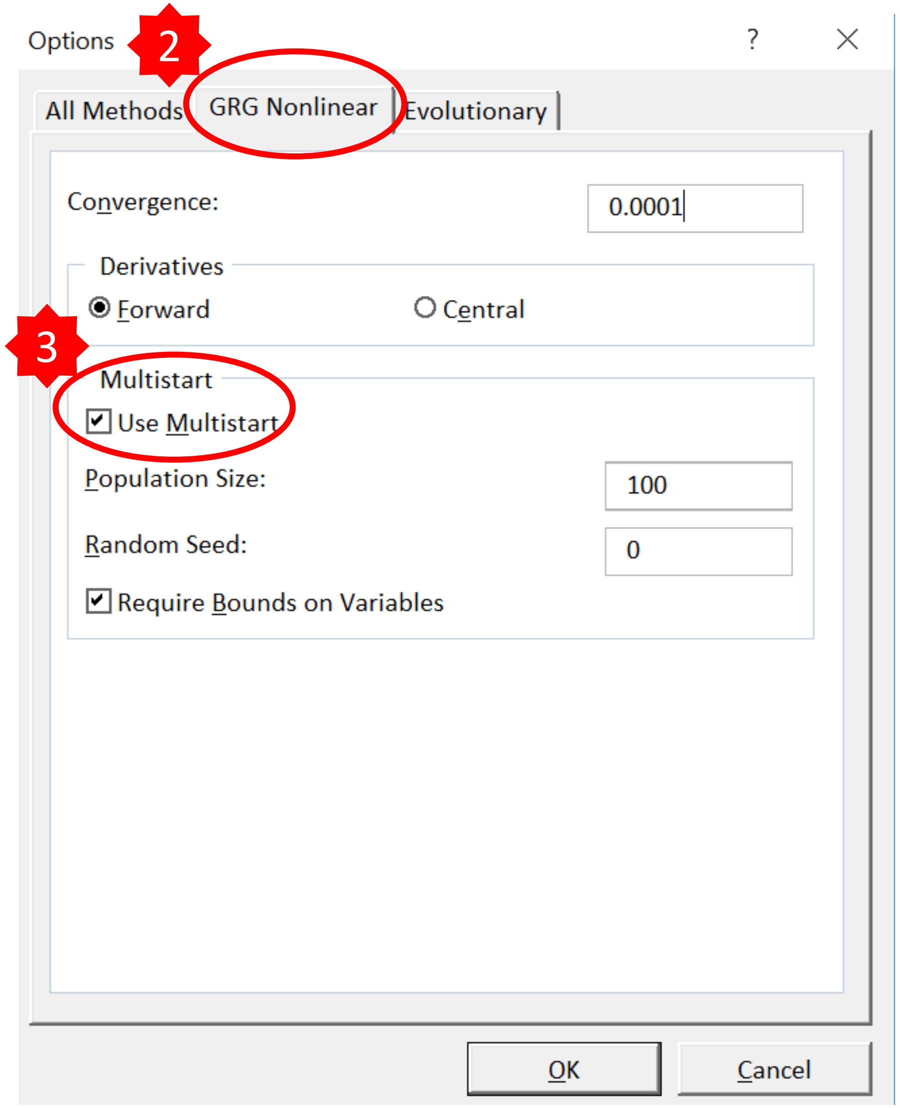
- Depending on the starting values of a and b Solver may stop at values that are not optimal
- For this reason, it is best to use Solver's **Multistart option**. This works only if there are upper and lower bounds on a and b, the tighter the better
- Parameter a represents a ceiling on the value of Y, so from the data it would appear that it is at least 130, and we will set an upper bound of twice this (260)
- Parameter b is probably small (as it is multiplied by the number of ads) so we choose lower and upper bounds of 0.0001 and 1



*Modified textbook example on pp. 365-373

Ex 2. Solver Set-up





Ex 2. Optimal Solution for a & b in the Response Function

D29 | =SUMXMY2(D21:D25,E21:E25)

DECISION VARIABLES

a	155.017447
b	0.01814301

CALCULATED VARIABLE

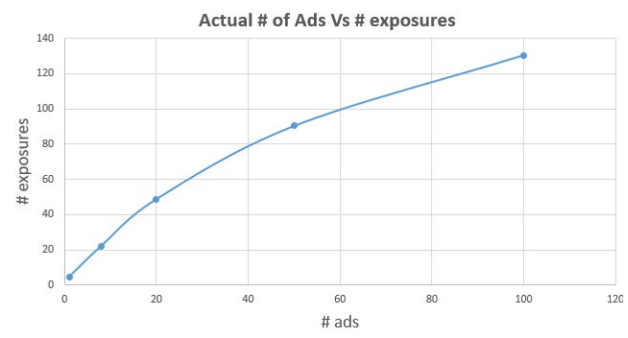
# ads	# exposures	Predicted # exposures
1	4.7	2.787
8	22.1	20.943
20	48.7	47.174
50	90.3	92.441
100	130.5	129.757

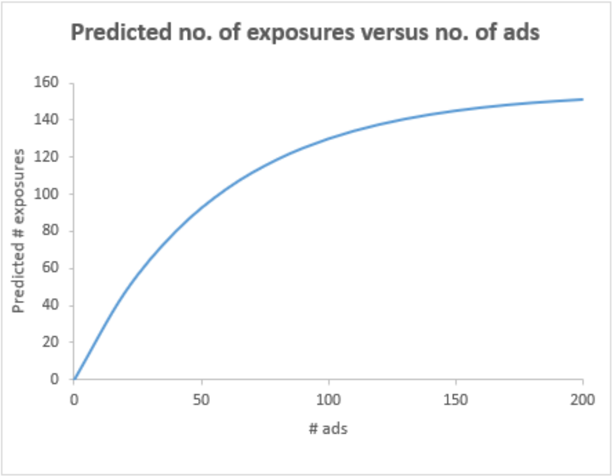
OBJECTIVE FUNCTION

Minimise the sum of squared differences: 12.461176

CONSTRAINTS

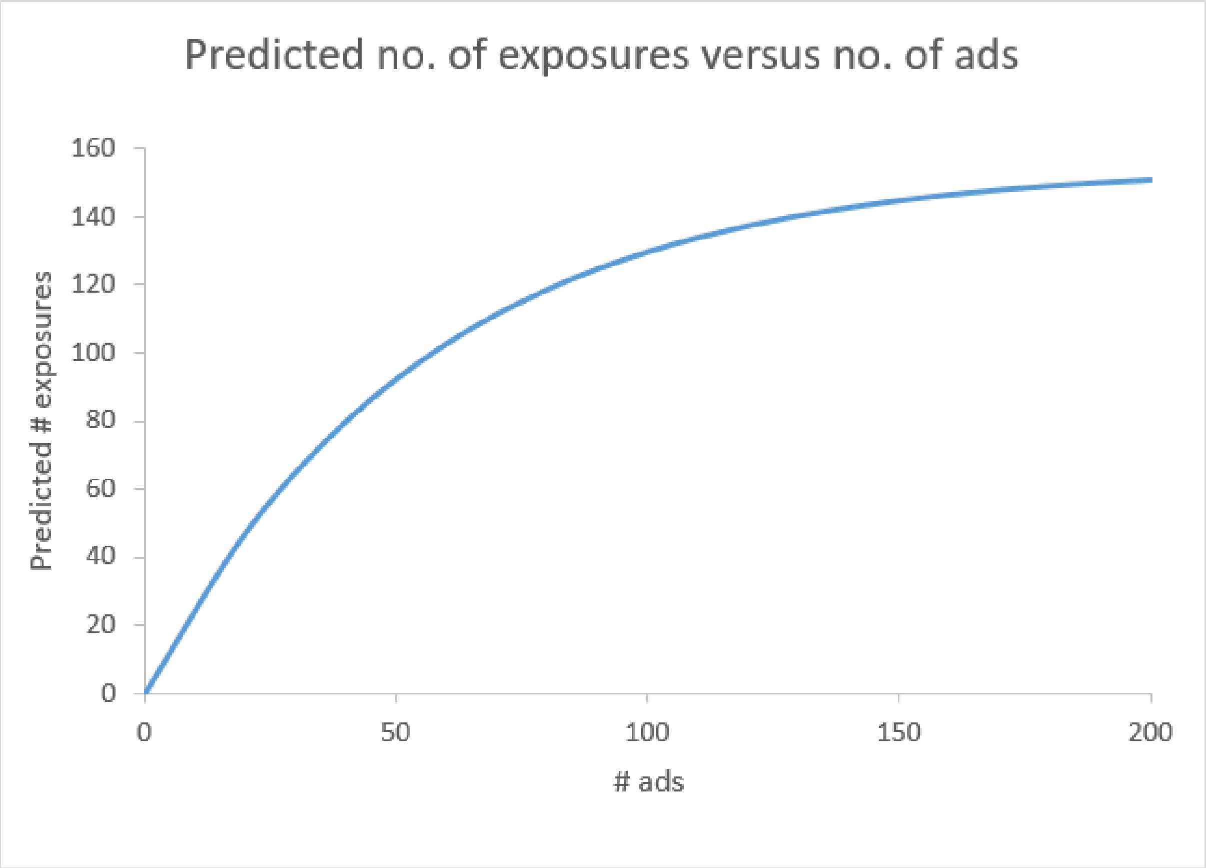
LHS		RHS	
155.017447	≥	130	Lower bound on a
155.017447	≤	260	Upper bound on a
0.01814301	≥	0.0001	Lower bound on b
0.01814301	≤	1	Upper bound on b





Ex 2. Shape of Modified Exponential Response Curve

# ads	Predicted # exposures
0	0.0
20	47.2
40	80.0
60	102.8
80	118.7
100	129.8
120	137.4
140	142.8
160	146.5
180	149.1
200	150.9



Ex2. Advertising Selection with Nonlinear Constraints

	B	C	D	E
3	**INPUTS**			
4	**Constant (a) in response function**			
5				
6	Segments	A	B	C
7	Women 18-35	105.803	40.113	66.998
8	Women 36-55	71.784	26.534	46.146
9	Women >55	56.828	17.209	8.887
10				
11	**Coefficient (b) of exponent in response function**			
12	Segments	A	B	C
13	Women 18-35	0.035	0.063	0.069
14	Women 36-55	0.089	0.057	0.061
15	Women >55	0.01	0.033	0.078

- In this example we will assume that the constant (a) and coefficient (b) in the exponent of the response function have already been determined for each show and segment

*Modified textbook example on pp. 365-373

Ex2. Advertising Selection with Nonlinear Constraints – Algebraic Model

- X_A, X_B and X_C denote the number of ads on shows A, B and C
- Minimise total cost ($'000) = $140X_A + 100X_B + 80X_C$

subject to: $Y_{A1} + Y_{B1} + Y_{C1} \geq 60$ (segment 1 - females aged $18 - 35$)

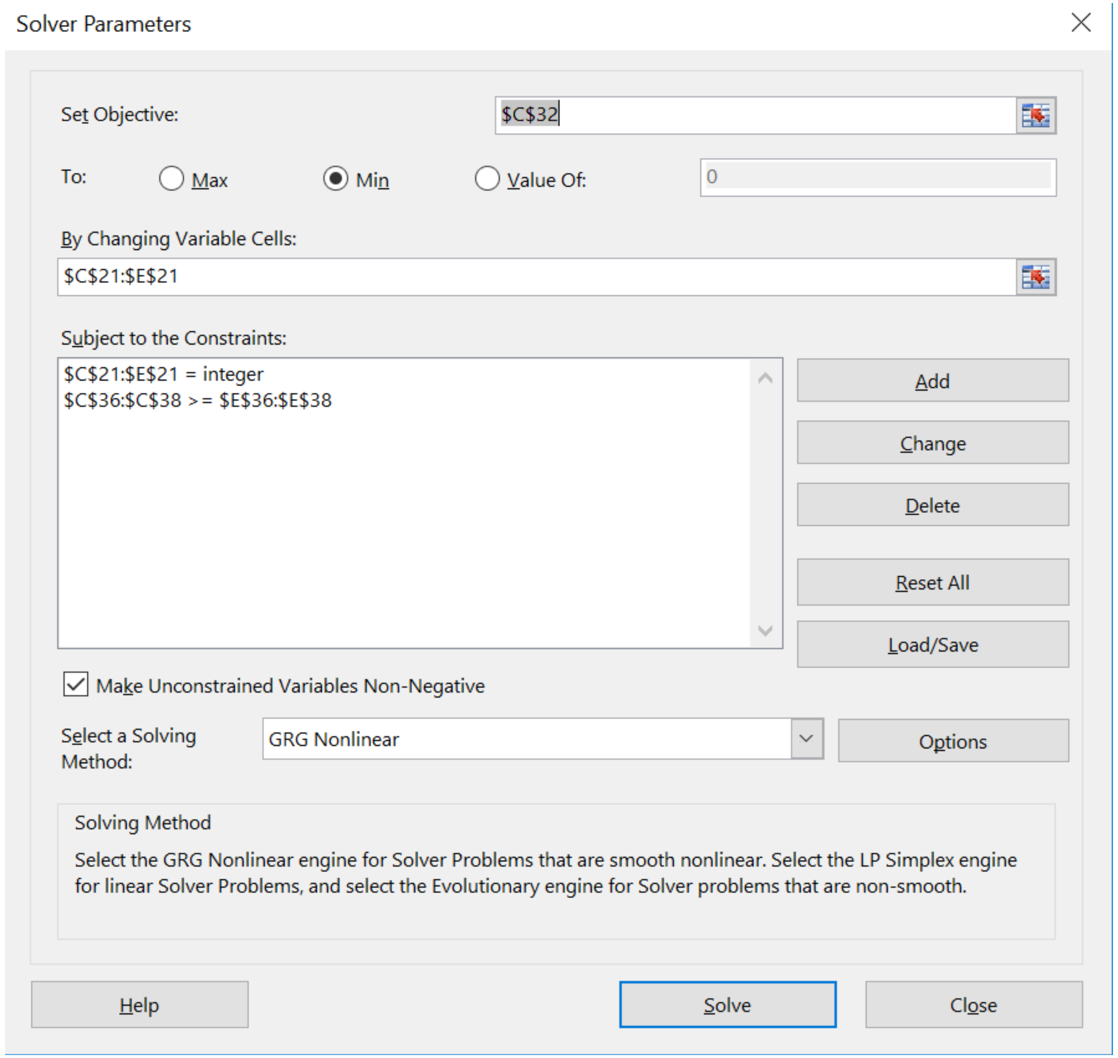
$Y_{A2} + Y_{B2} + Y_{C2} \geq 60$ (segment 2 - females aged $36 - 55$)

$Y_{A3} + Y_{B3} + Y_{C3} \geq 28$ (segment 3 - females aged over 55)

where $Y_{ij} = a_{ij}\left(1 - e^{-b_{ij}X_i}\right)$ for show i and segment j

*Modified textbook example on pp. 365-373

Ex 2. Solver Set-up



- We have added in a constraint that the solution needs to be integer-valued
- We could have also added in bounds for the decision variables so that the Multistart option could be used but *for this example* we would end up with the same optimal solution

Ex 2. Optimal Solution

C32 =SUMPRODUCT(C17:E17,C21:E21)

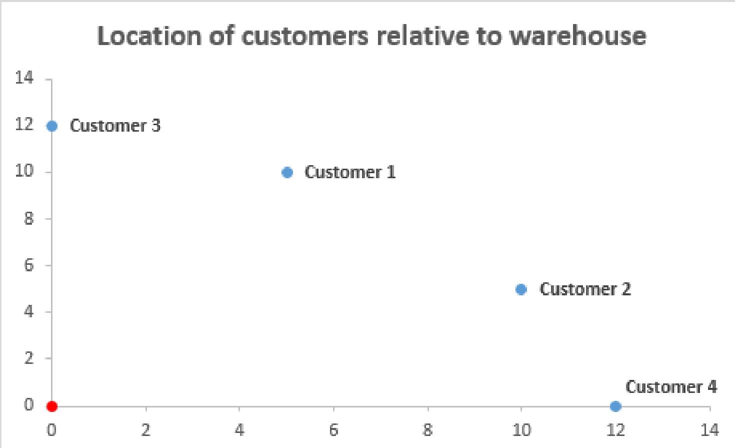
	B	C	D	E
17	Cost per ad	140	100	80
18				
19	**DECISION VARIABLES**			
20		A	B	C
21	No. ads	28	20	14
22				
23	**CALCULATED VARIABLES**			
24				
25	**Exposures to each group from each show**			
26	Segments	A	B	C
27	Women 18-35	66.094	28.735	41.498
28	Women 36-55	65.844	18.048	26.501
29	Women >55	13.878	8.315	5.905
30				
31	**OBJECTIVE FUNCTION**			
32	Total cost ($'000)	$7,040		
33				
34	**CONSTRAINTS**			
35	Segments	LHS		RHS
36	Women 18-35	136.327	≥	60
37	Women 36-55	110.393	≥	60
38	Women >55	28.098	≥	28

Week 2: we got the solutions as
A=15 B=0 ,C=0
total cost was $2100 or 2.1 Million

Ex 3. Facility Location Models*

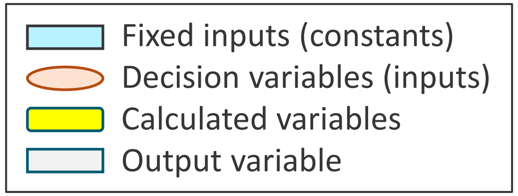
A company wants to locate a warehouse from which it will ship products to four customers. Coordinates are in km, relative to the point (0, 0). They want to position the warehouse, so it minimise the distance travelled over a year

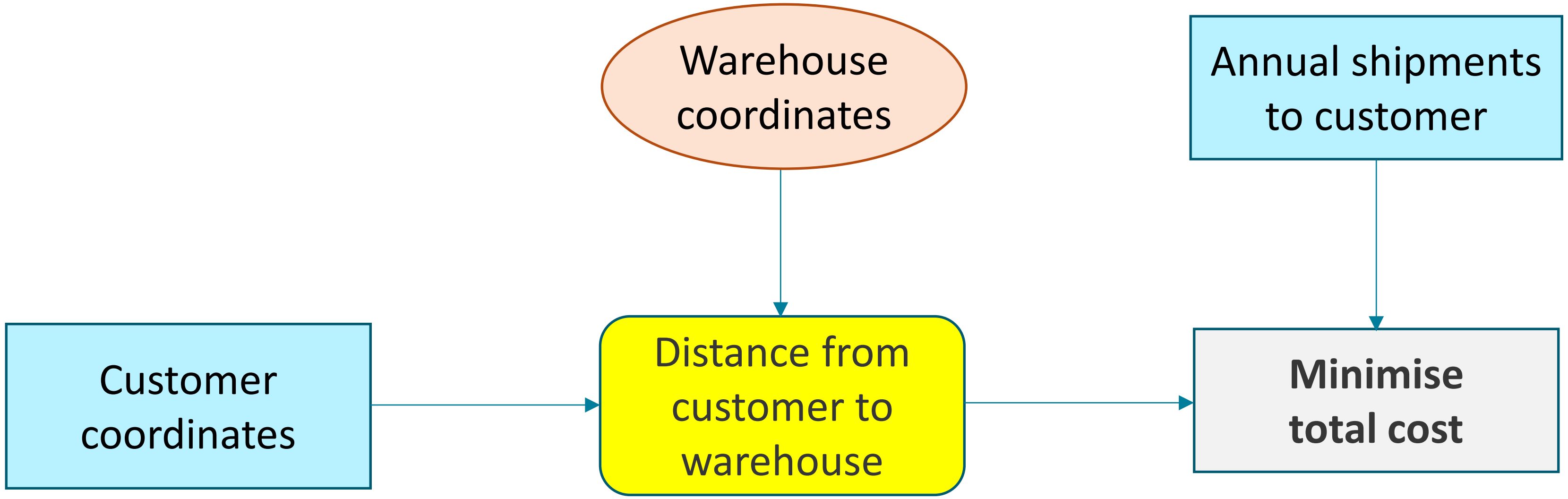
Customer	X-coordinate	Y-coordinate	Shipments per year
1	5	10	200
2	10	5	150
3	0	12	200
4	12	0	300



*Textbook example on pp. 374-378

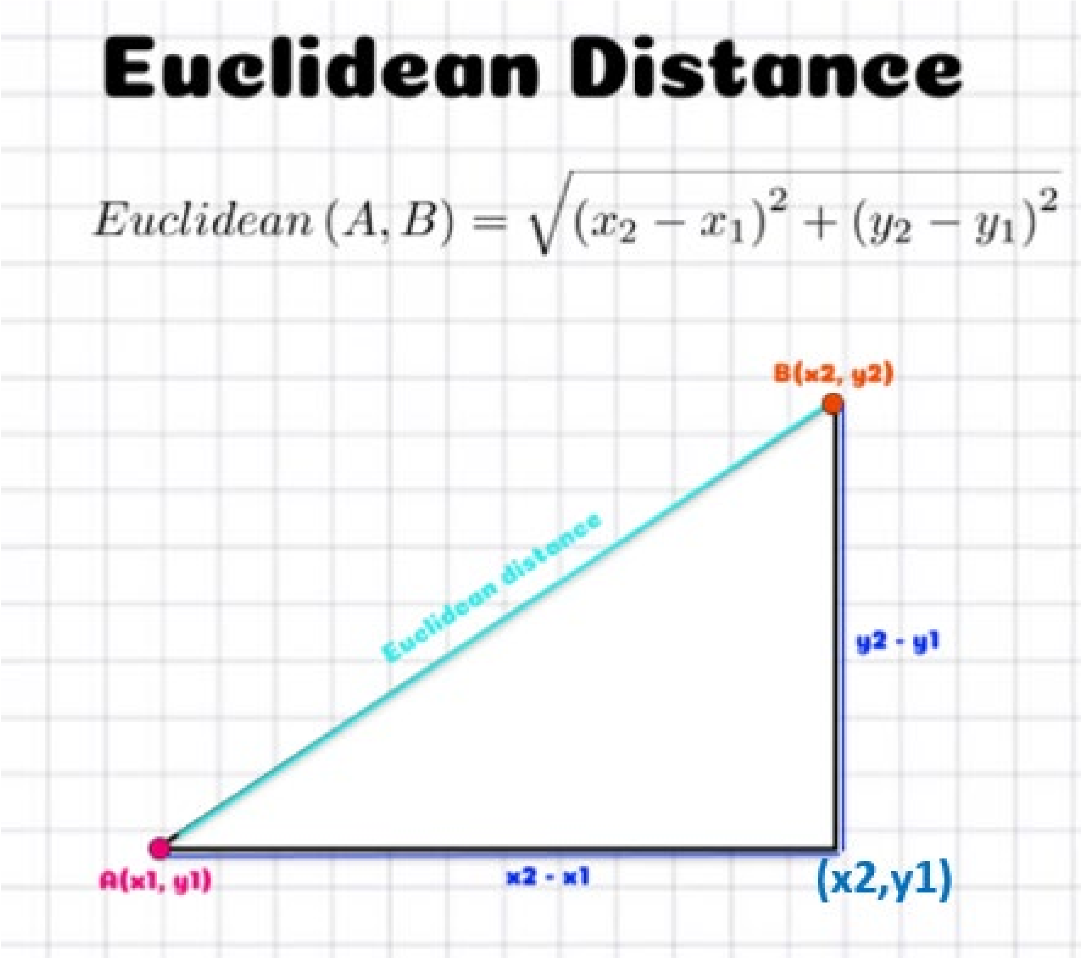
Conceptual Model





Euclidean Distance

How to find the distance between A and B?



Algebraic Model

- Let s_i denote the number of shipments per year from the warehouse to customer i
- Let (x_i, y_i) denote the coordinates of customer i
- **Decision variables**: Let (X, Y) denote the coordinates of the warehouse
- Let d_i denote the distance between customer i and the warehouse

  Then $d_i = \sqrt{(x_i - X)^2 + (y_i - Y)^2}$ (i.e. the Euclidean 'straight line' distance)
- **Objective**: *Minimise annual distance travelled from the warehouse*

  $= s_1 d_1 + s_2 d_2 + s_3 d_3 + s_4 d_4$

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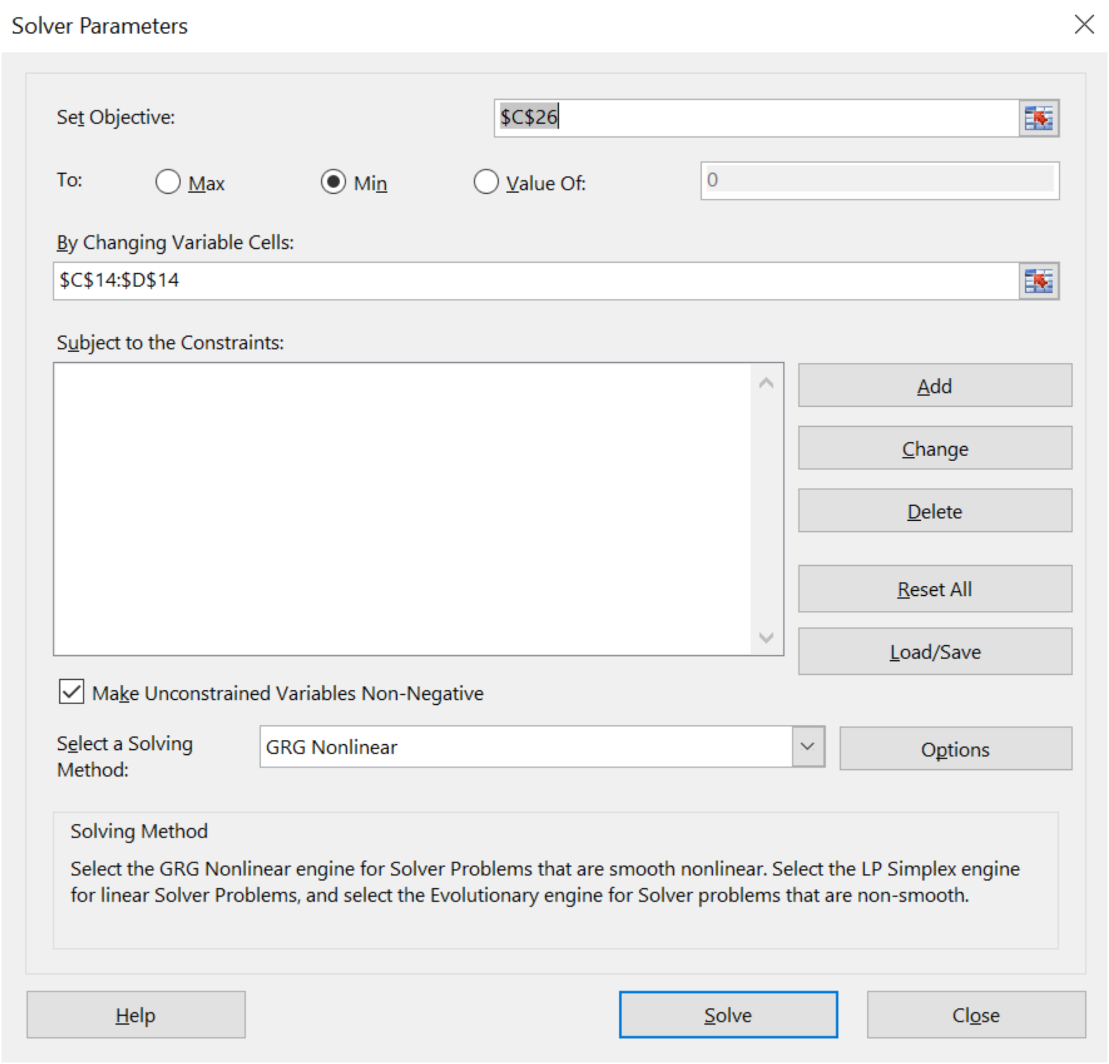
Solver Set-up



We could have also added in bounds for the decision variables so that the Multistart option could be used but *for this example* we would end up with the same optimal solution.

Optimal Solution

C26 =SUMPRODUCT(C19:C22,E6:E9)

INPUTS

Customer	X-coordinate	Y-coordinate	Shipments per year
1	5	10	200
2	10	5	150
3	0	12	200
4	12	0	300

DECISION VARIABLES

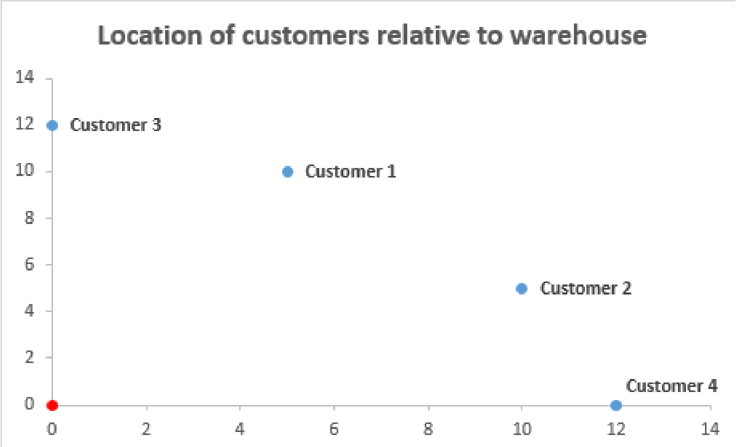
	X-coordinate	Y-coordinate
Warehouse	9.314	5.029

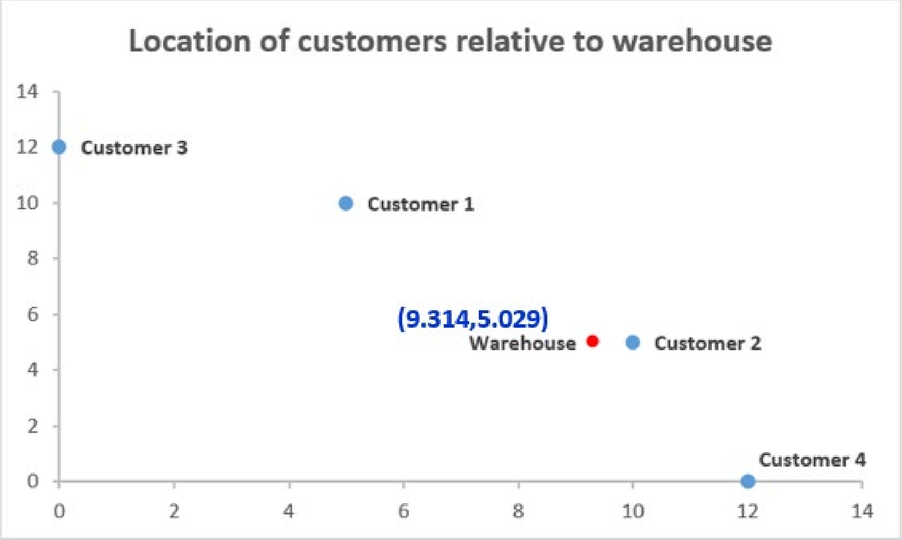
CALCULATED VARIABLES

Customer	Distance from warehouse
1	6.582
2	0.686
3	11.634
4	5.701

OBJECTIVE FUNCTION

Annual distance	5456.540





Portfolio Optimisation Models

- The optimisation method we use for these models is based on "Markowitz Mean-Variance" Portfolio optimisation (or Modern Portfolio Theory)

- There are three choices for optimising the objective function:
  1. **Minimise risk: subject to a minimum return that has to be achieved**

  2. **Maximise return: for a given level of risk**

  3. **Maximise the Sharpe Ratio. This is where we maximise the ratio of the return above the risk free rate relative to the amount of risk assumed**

  (We assume a risk free rate of 1.5%, the current cash rate)

Measuring Return

- The return of a stock over a time period is given by

$$\frac{Stock\ price\ at\ end\ of\ period - Stock\ price\ at\ start\ of\ period}{Stock\ price\ at\ start\ of\ period} \times 100$$

  (To keep things simple we ignore any dividends over the period)

  The time period could be a year or a month. In assignment 1 we will measure the expected (i.e. mean) monthly returns

- The **expected return on a portfolio** is the weighted average of expected returns from the individual stocks

  E.g. The expected return on a portfolio consisting of three stocks is the sum $w_1\mu_1 + w_2\,\mu_2 + w_3\,\mu_3$, where $\boldsymbol{w_i}$ **is the proportion** of the portfolio invested in stock i and μ_i is the expected return from stock i $(i = 1, 2, 3)$

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Measuring Risk

- Risk reflects the chance of the actual return differing from the expected (i.e. mean or average) return. It is usually measured by the standard deviation (or variance) of actual returns. The idea is that higher variability signals greater risk
- Risk may be based on recent history (i.e. recent returns for the stocks in question)
- The overall risk of a portfolio is NOT a linear combination of the risks of the individual components. It involves a more complex risk calculation based on how stocks move relative to each other (i.e. their covariances). E.g. the standard deviation of a portfolio consisting of three stocks is given by

$$\sqrt{w_1^2\sigma_1^2 + w_2^2\,\sigma_2^2 + w_3^2\,\sigma_3^2 + 2w_1w_2Cov(1,2) + 2w_1w_3Cov(1,3) + 2w_2w_3Cov(2,3)}$$

where σ_i^2 denotes the variance of stock i and $Cov(i,j)$ denotes the covariance between stocks i and j

Ex4.1 Portfolio Optimisation Models*

4.1 Minimise risk: subject to a minimum return that has to be achieved

An investment company intends to invest a given amount of money in three stocks. The means have been calculated from past returns, as well as the covariances between each pair of stocks

Stock	Mean (μ_i)
1	0.14
2	0.11
3	0.10

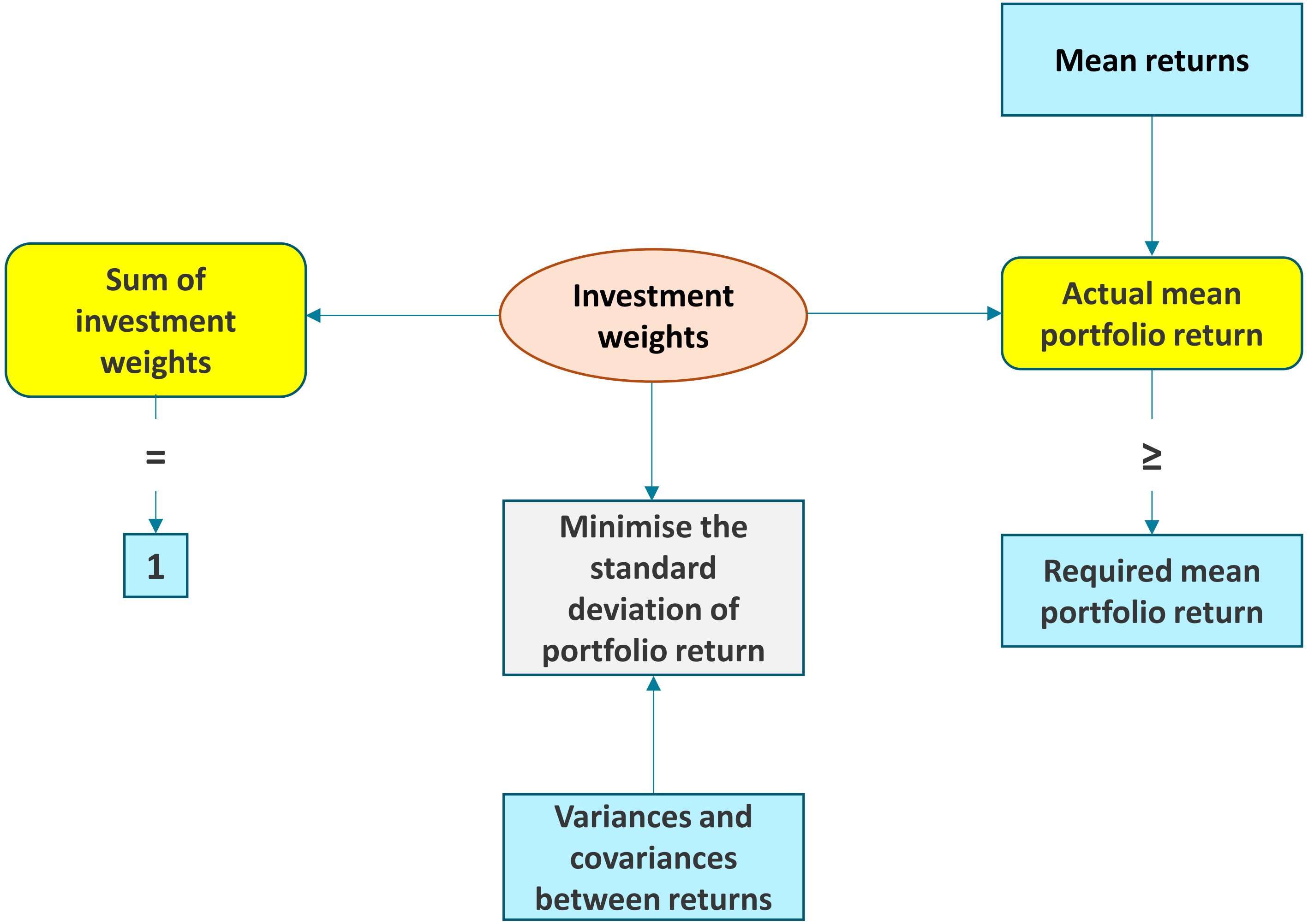
Covariance	1	2	3
1	0.0400	0.0180	0.0064
2	0.0180	0.0225	0.0084
3	0.0064	0.0084	0.0064

The company wants to find a **minimum-variance portfolio that yields an annual return of at least 12%.**

*Textbook example on pp. 384-392

Conceptual Model When Minimising Risk Subject to a Minimum Return

Fixed inputs (constants)
Decision variables (inputs)
Calculated variables
Output variable



Ex4.1 Portfolio Optimisation Models

Decision variables: $\boldsymbol{w_i}$ = investment proportions in the i^{th} investment

Stock	Mean (μ_i)
1	0.14
2	0.11
3	0.10

Objective is to minimise the risk

Standard deviation of a portfolio consisting of three stocks is given by = $\sqrt{w_1^2\sigma_1^2 + w_2^2\,\sigma_2^2 + w_3^2\,\sigma_3^2 + 2w_1w_2Cov(1,2) + 2w_1w_3Cov(1,3) + 2w_2w_3Cov(2,3)}$

Covariance	1	2	3
1	0.0400	0.0180	0.0064
2	0.0180	0.0225	0.0084
3	0.0064	0.0084	0.0064

Constraints:2

Expected Return = $w_1\mu_1 + w_2\,\mu_2 + w_3\,\mu_3 \;\geq\; 12\%$

Total % of investment proportions =100%

Ex 4.1 Optimal Solution

Objective is to minimise the risk

Standard deviation of a portfolio consisting of three stocks is given by = $\sqrt{w_1^2\sigma_1^2 + w_2^2\,\sigma_2^2 + w_3^2\,\sigma_3^2 + 2w_1w_2Cov(1,2) + 2w_1w_3Cov(1,3) + 2w_2w_3Cov(2,3)}$

Constraints:2

Expected Return = $w_1\mu_1 + w_2\,\mu_2 + w_3\,\mu_3 \geq 12\%$

Total % of investment proportions =100%

	A	B	C	D	E	F
5		Stock	1	2	3	
6		Mean	0.14	0.11	0.1	
7						
8		Cov	1	2	3	
9		1	0.0400	0.0180	0.0064	
10		2	0.0180	0.0225	0.0084	
11		3	0.0064	0.0084	0.0064	
12						
13		DECISION VARIABLES				
14			stock 1	stock 2	stock 3	
15		Investment weights	50%	0%	50%	

DECISION VARIABLES

	stock 1	stock 2	stock 3
Investment weights	50%	0%	50%

CALCULATED VARIABLES

		50%	0%	50%
	Cov	1	2	3
50%	1	0.0400	0.0180	0.0064
0%	2	0.0180	0.0225	0.0084
50%	3	0.0064	0.0084	0.0064

Portfolio variance terms	1	2	3
1	0.0100	0.0000	0.0016
2	0.0000	0.0000	0.0000
3	0.0016	0.0000	0.0016

Average portfolio return: 12.00%

OBJECTIVE FUNCTION

Standard deviation of portfolio return: 12.17%

CONSTRAINTS

LHS		RHS	
12%	≥	12%	(Minimum expected return of 12%)
100%	=	100%	(Portfolio = 100% of total investment)

Ex.4.2 Portfolio Optimisation Models*

2. Maximise the expected return: subject to a maximum risk that should not exceed

An investment company intends to invest a given amount of money in three stocks. The means have been calculated from past returns, as well as the covariances between each pair of stocks

Stock	Mean (μ_i)
1	0.14
2	0.11
3	0.10

Covariance	1	2	3
1	0.0400	0.0180	0.0064
2	0.0180	0.0225	0.0084
3	0.0064	0.0084	0.0064

The company wants to find the maximum expected return when the risk should not exceed more than **10%.**

*Textbook example on pp. 384-392

Ex4.2 Portfolio Optimisation Models

4.2 Maximise return: for a given level of risk

Decision variables: w_i = investment proportions in the i^{th} investment

Stock	Mean (μ_i)
1	0.14
2	0.11
3	0.10

Objective is to maximise the return

Expected Return = $w_1\mu_1 + w_2\,\mu_2 + w_3\,\mu_3$

Covariance	1	2	3
1	0.0400	0.0180	0.0064
2	0.0180	0.0225	0.0084
3	0.0064	0.0084	0.0064

Constraints:2

Standard deviation of a portfolio consisting of three stocks is given by = $\sqrt{w_1^2\sigma_1^2 + w_2^2\,\sigma_2^2 + w_3^2\,\sigma_3^2 + 2w_1w_2Cov(1,2) + 2w_1w_3Cov(1,3) + 2w_2w_3Cov(2,3)}$ <= 10%

Total % of investment proportions =100%

Ex4. 3 Portfolio Optimisation Models

4.3 Maximise the Sharpe Ratio. This is where we maximise the ratio of the return above the risk free rate relative to the amount of risk assumed
(We assume a risk free rate of 1.5%, the current cash rate)

The **Sharpe Ratio** is a measure used in investment analysis to assess the risk-adjusted return of an investment/ portfolio

$$\text{Sharpe Ratio} = \frac{R_p - R_f}{\sigma_p}$$

R_p = Portfolio return

R_f = Risk-free rate (e.g., government bond yield)

σ_p = Standard deviation of portfolio returns (risk)

- Should prefer investments that have a **Sharpe Ratio well above 1,** and provide a reasonable excess return over 1.5% while justifying the level of risk taken.
- **A Sharpe Ratio < 1** suggests lower efficiency in risk-adjusted returns.
- **A negative Sharpe** Ratio means the investment is underperforming the risk-free rate, implying it might not be worth the risk.

This will be explained during week 4 seminar

Summary

- Nonlinear models have a nonlinear objective function or a nonlinear constraint
- NLP models are difficult to solve (i.e. to find the optimal solution)
- A different solving method is used in Solver and, in general, the Multistart option is used to increase the chance of finding an optimal solution
- Four types of nonlinear models were covered:
  - **Pricing models**
  - **Advertising response and selection models**
  - **Facility location models**
  - **Portfolio optimisation models**
- Solver was also used to estimate parameters when a given functional form is fitted to data. **This important learning will also feature in module 2**

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Next Class

Topic 5: Network Modelling (Chapter 5 of the textbook)

- We will start by considering the basic ideas behind these models
- Then we will consider the main types of network models:
  - Transportation models (section 5.2)
  - Assignment models (section 5.3)
  - Other logistic models (section 5.4)
  - Shortest path models (section 5.5)