DEAKIN BUSINESS SCHOOL
DEPARTMENT OF INFORMATION SYSTEMS AND BUSINESS ANALYTICS

MIS775 Decision Modelling for Business Analytics
Week 11 Revision Questions

QUESTION 1

Formulate a linear programming model for this problem.

Bill's Grill is a popular restaurant that is famous for its hamburgers. The owner mixes fresh ground beef and pork in its hamburgers that have no more than 25% fat. The beef contains 80% meat and 20% fat, and costs $3 a kilo, while the pork contains 70% meat and 30% fat, and costs $2.5 a kilo. The owner wants to determine the minimum cost mixture of beef and pork that will have no more than 25% fat.

ANSWER

Decision variables

X = % of beef in the mixture

Y = % of pork in the mixture

Objective function

Cost per kilo ($) = 3X + 2.5Y

Minimise cost

Constraints

X + Y = 1

4X + 6Y - 5 <= 0 or equivalent

X, Y >= 0

QUESTION 2

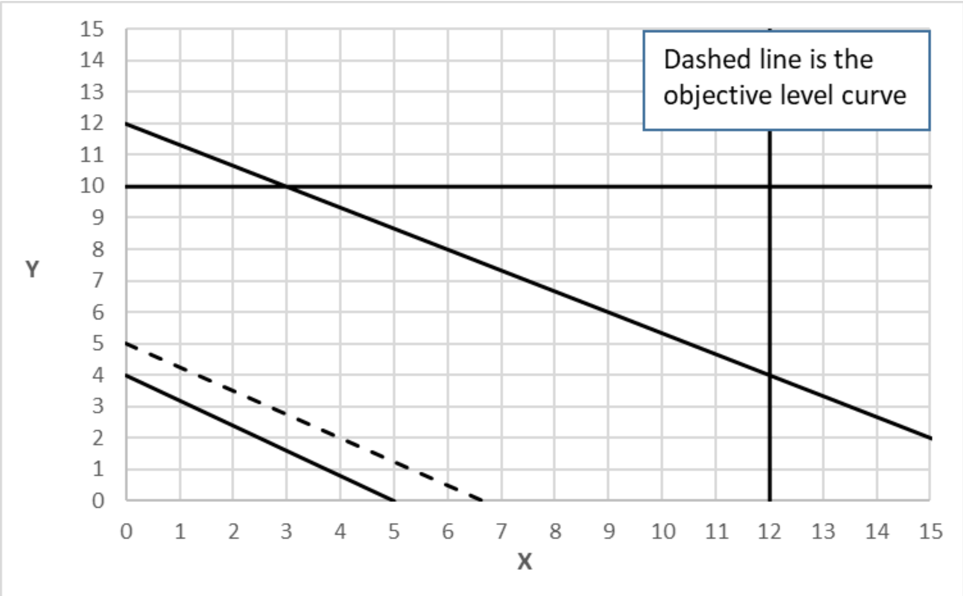
Consider the following linear programming model. Write down the coordinates of the feasible region, and derive the optimal solution.

Maximise $3X + 4Y$

Subject to:

$$X \leq 12$$
$$Y \leq 10$$
$$4X + 6Y \leq 72$$
$$4X + 5Y \geq 20$$
$$X, Y \geq 0.$$

A graphical representation follows:



ANSWER

Feasible region bounded by (0, 4), (0, 10), (3, 10), (12, 4), (12, 0) and (5, 0).

Evaluate OFV at corner points.

Otherwise move the objective function level curve up/right (12, 4) are optimal values

Max OFV = 52

QUESTION 3

Consider the following linear program:

Maximize total profit ($) = $10X_1 + 6.2X_2$

Subject to: $X_1 + X_2 \geq 1$

$X_2 \leq 5$

$X_1 \leq 6$

$7X_1 + 9X_2 \leq 63$

$X_1, X_2 \geq 0.$

The following solution output is provided:

6	Variable Cells						
7			Final	Reduced	Objective	Allowable	Allowable
8	Cell	Name	Value	Cost	Coefficient	Increase	Decrease
9	C5	X1	6	0	10	1E+30	5.178
10	D5	X2	2.3	0	6.2	6.657	6.2
11							
12	Constraints						
13			Final	Shadow	Constraint	Allowable	Allowable
14	Cell	Name	Value	Price	R.H. Side	Increase	Decrease
15	C12	Constraint 1 LHS	8.3	0	1	7.333	1E+30
16	C13	Constraint 2 LHS	2.3	0	5	1E+30	2.667
17	C14	Constraint 3 LHS	6	5.178	6	3	3.429
18	C15	Constraint 4 LHS	63	0.689	63	24	21
19							

a. What is the optimal solution, and the optimal value of the objective function?

b. Which constraints are non-binding? Explain your reasoning.

c. Suppose the profit per unit of X_1 is increased to $16, is the above solution still optimal?What is the value of the objective function when this unit profit is increased to $16? Show all working in obtaining the answer.

d. What is the optimal objective function value if the RHS of constraint 4 is increased to 73? Explain your reasoning and show all working in obtaining the answer.

a X1= 6 and X2 = 2.3, and OFV =$74.260 or exact answer $74.467

b First two constraints are non-binding since shadow price is zero (or LHS≠RHS)

c Optimal solution will not change since the change in profit of X1 is within allowable range.
The new optimal profit will equal to 16*6 + 6.2*2.3 = $110.26 or exact answer $110.467

d The constraint is binding . The change in RHS is within the allowable range
The new optimal value = $74.260+$6.89 = $81.150 or exact answer $81.356

QUESTION 4

Daylesford Manufacturing is planning its next production cycle. The company can produce three products, each of which must undergo machining, grinding, and assembly operations. The following table summarises the hours of machining, grinding, and assembly required by each unit of the product, the total hours of capacity available for each operation, and the maximum number of units that can be produced, given the total hours available.

	Hours required by			
Operation	**Product 1**	**Product 2**	**Product 3**	**Total hours available**
Machining	3	2	5	500
Grinding	4	3	7	300
Assembly	9	6	8	350
Maximum number of units	50	67	75	

The cost accounting department has estimated that each unit of product 1 manufactured and sold will contribute \$48 to profit, and each unit of products 2 and 3 contributes \$55 and \$50, respectively. However, manufacturing a unit of product 1 requires a setup operation on the production line that costs \$500. Similar setups are required for products 2 and 3 at costs of \$700 and \$600, respectively. The marketing department believes that it can sell all the products produced. Therefore, the management of Daylesford wants to determine the most profitable mix of products to produce.

a. Formulate the model by writing down the decision variables, objective function, and constraints.

b. Add a constraint to ensure that no more than two of the products are produced. Explain your reasoning.

a

Decision variables
Let X_i = the number of units made by product i (i=1,2,3)
Let W_i = 1 if product i is produced (requiring a set-up), = 0 otherwise

Objective function
Maximise $48X_1 + 55X_2 + 50X_3 – (500W_1 + 700W_2 + 600W_3)$

Constraints
$3X_1 + 2X_2 + 5X_3 \leq 500$
$4X_1 + 3X_2 + 7X_3 \leq 300$
$9X_1 + 6X_2 + 8X_3 \leq 350$
$X_1 \leq 50W_1$
$X2 \leq 67W2$
$X3 \leq 75W3$
$Xi \geq 0$ (i = 1, 2, 3) and non-negative integer-values

b W_i (i = 1, 2, 3) are binary or {0, 1} variables

Note: must include table (to show reasoning)

W_1	W_2	W_3	
0	0	0	√
0	0	1	√
0	1	0	√
1	0	0	√
1	1	0	√
0	1	1	√
1	0	1	√
1	1	1	x

$W_1 + W_2 + W_3 \leq 2$.

Question 5

For each of the scenarios below, choose an "appropriate" distribution, with its parameters, where possible, and justify your choice.

a. A company is about to develop and market a new product. It wants to build a simulation model for the entire process. The company experts believe the distribution of their development cost is somewhere between \$300,000 and \$600,000, but is most likely to be around \$500,000."

b. A queuing system has an arrival rate of 4 customers per hour and a service rate of 7 customers per hour. You decide to build a simulation model to describe the service time per customer.

c. The mean time between arrivals of cars at a service station is 4 minutes. We would like to know the probability distribution of the number of cars to arrive within an hour.

a Triangular Distribution with min \$300,000, mode \$500,000 and max \$600,000

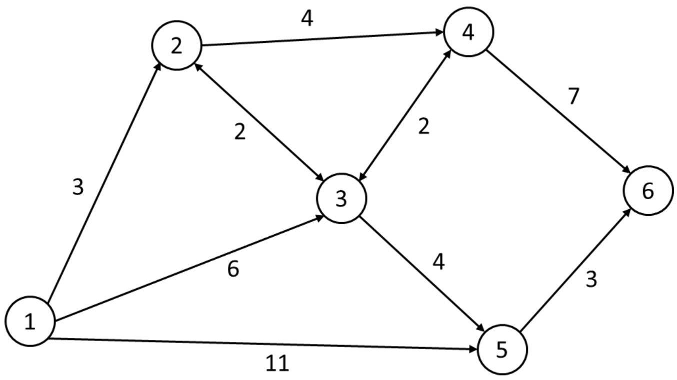
b Exponential.
Because can be used to represent time between events (or time to serve customer)
Parameter, λ=7

c Poisson.
Because Poisson is a discrete counting distribution for the number of times as event occurs in defined region of space or time
Parameter, λ=15

Question 6

Formulate the algebraic model for this problem by writing down the decision variables, objective function, and constraints.

Alternative travel times between six locations are as shown on the following diagram. A traveller wants to minimise the overall travel time between locations 1 and 6.



ANSWER

Decision variables: Xij = 1, if the arc from location i to location j is on the route that minimises the travel time between locations 1 and 6. 0 , otherwise.

Objective: Minimise

3X12 + 6X13 + 11X15 + 2X23 +4X24 + 2X32 + 2X34 + 4X35 + 2X43 + 7X46 + 3X56

Constraints:

X12 + X13 + X15 = 1

X24 + X23 - X12 – X32 = 0

X32 + X34 + X35 – (X13 + X23 + X43) = 0

X46 + X43 – (X24 + X34) = 0

X56 – (X15 + X35) = 0

X46 + X56 =1.

Note: In the exam, you would not have to write these constraints so that the RHS = 0

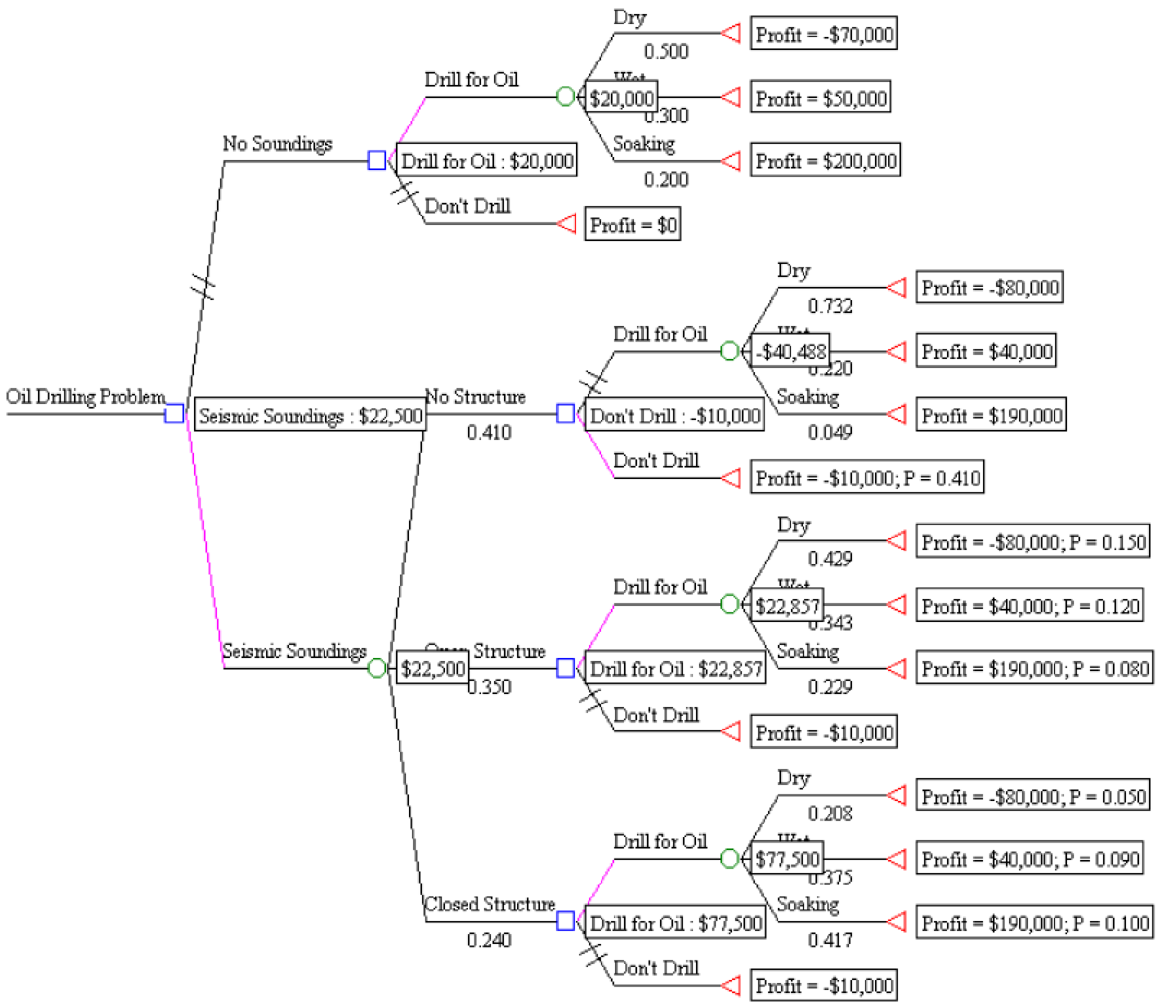
QUESTION 7

You have been invited to provide consulting advice to an oil exploration company. They have found a site and, after some discussions, they provided you with the following information.

The well will be Dry, Wet or Soaking, with associated probabilities 0.5, 0.3 and 0.2 respectively, and respective profits of -$70,000, $50,000 and $200,000. There is an option of conducting seismic soundings, at a cost of $10,000. The outcomes from the Soundings are either that there is No Structure, an Open Structure or a Closed Structure. They estimate that the probabilities of the eventual state of the well given these soundings are as follows:

	Dry	Wet	Soaking
No Structure	0.6	0.3	0.1
Open Structure	0.3	0.4	0.4
Closed Structure	0.1	0.3	0.5

The above information was entered into a Decision Analysis software package, resulting the decision tree shown below:



a. Provide management of the oil exploration company with a detailed interpretation of the decision tree. Note that, in this part of the question, you are not required to verify any of calculations.

There are two decisions required:
1) Whether to conduct seismic soundings
2) Whether to drill for oil.
The recommended course of action would be to conduct seismic soundings, for an expected profit of $22,500. This is $2,500 more than the expected profit if seismic soundings are not undertaken.
If no structure is found (41% chance) then don't drill for oil. This will result in a loss of $10k.
If an open structure is found (35% chance) then drill for oil, with an expected profit of $22,857, and a maximum loss of $80k if the well is found to be dry (42.9% chance).

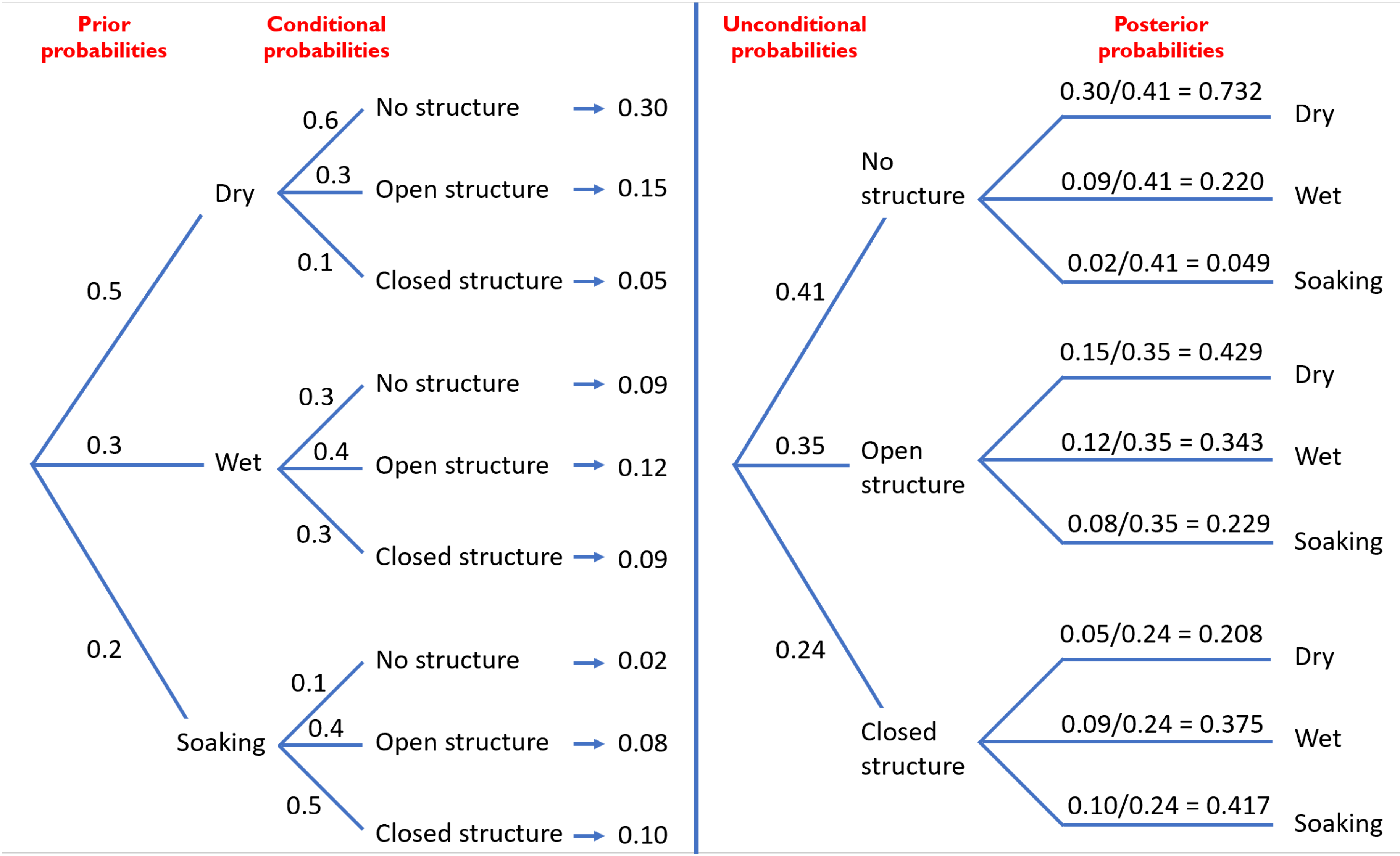
If a closed structure is found (24% chance) then drill for oil, with an expected profit of $77,500 and a maximum loss of $80k if the well is found to be dry (20.8% chance).

b. Find the posterior probabilities of all states of nature.

This can be answered by either:
listing all 9 probabilities. E.g. P(Dry|No structure) = 0.732, P(Wet|No structure) = 0.220, etc.

OR by arranging the probabilities in a table, e.g.

	Structure		
	No	Open	Closed
Dry	0.732	0.429	0.208
Wet	0.220	0.343	0.375
Soaking	0.049	0.229	0.417


Prior probabilities
Conditional probabilities
Unconditional probabilities
Posterior probabilities
0.5
Dry
0.6
No structure
0.30
0.3
Open structure
0.15
0.1
Closed structure
0.05
0.3
Wet
0.3
No structure
0.09
0.4
Open structure
0.12
0.3
Closed structure
0.09
0.2
Soaking
0.1
No structure
0.02
0.4
Open structure
0.08
0.5
Closed structure
0.10
0.41
No structure
0.30/0.41 = 0.732
Dry
0.09/0.41 = 0.220
Wet
0.02/0.41 = 0.049
Soaking
0.35
Open structure
0.15/0.35 = 0.429
Dry
0.12/0.35 = 0.343
Wet
0.08/0.35 = 0.229
Soaking
0.24
Closed structure
0.05/0.24 = 0.208
Dry
0.09/0.24 = 0.375
Wet
0.10/0.24 = 0.417
Soaking

c. Show how the figure -$40,488 is calculated.

Expected profit if you decide to drill for oil when seismic soundings showed no structure present: = 0.732*(-$80k) + 0.220*($40k) + 0.049*($190k) = -$40,488.

d. What is the most that you are willing to pay for the seismic soundings?

Currently paying $10k. Additional worth of soundings is $22.5k - $20k = $2.5k. If you were required to pay $10k + $2.5k = $12.5k then you would be indifferent to commissioning the soundings. This represents the most you would be willing to pay.

e. Determine the expected value of perfect information.

Calculate the difference between the expected profit with perfect information (EVPI) and the expected profit without any information (EV without PI).

EV with PI = 0.5*(0) + 0.3*($50k) + 0.2* ($200k) = $55k.
EV without PI = 0.5*(-$70k) + 0.3*($50k) + 0.2* ($200k) = $20k.
(OR you could simply read this number off the top part of the decision tree where there are no soundings. i.e. no information.)

EVPI = EV with PI - EV without PI = $55k - $20k = $35k.

Therefore, management should not pay more than $35k for any information about the state of nature.

f. What is the risk profile if seismic soundings are undertaken and the optimal drilling decision is made?

The risk profile table is given below.

Profit ($'000)	Probability
-80	20.0%
-10	41.0%
40	21.0%
190	18.0%

This could be deduced from the 'P' probabilities given in the tree diagram at the right, alongside the payoffs. For example, there is a 0.15 + 0.05 = 0.2 or 20% chance of a maximum loss of $80k if seismic soundings are undertaken and the optimal drilling decision is made.

Alternatively, the probability of each payoff is found by multiplying the probabilities for the branches from the chance nodes in the sequence.

For example, the company would lose $80k if the structure is open and the well is dry, which has a probability of 0.350×0.429 = 0.150. The company would also lose $80k if the structure is closed and the well is dry, which has a probability of 0.240×0.208 = 0.050. Adding these two probabilities gives the overall probability of 0.150 + 0.050 = 0.2 or 20% of the company losing $80k.