DEAKIN BUSINESS SCHOOL
DEPARTMENT OF INFORMATION SYSTEMS AND BUSINESS ANALYTICS

# MIS775 Decision Modelling for Business Analytics
# Revision Questions with Solutions

## WEEK 1 QUESTIONS

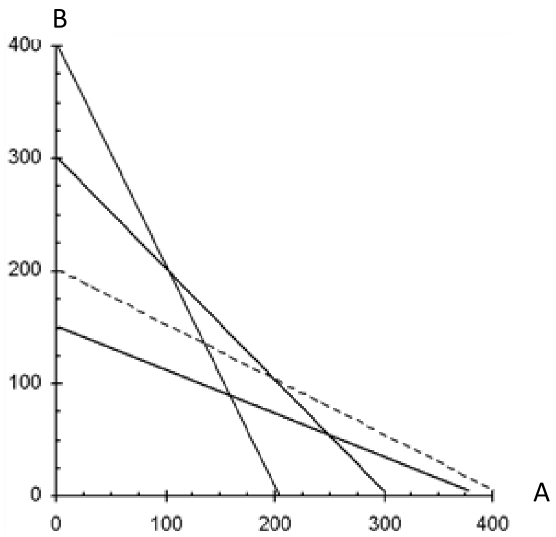
**Exercise 1.** An advertising campaign for a new snack chip will be conducted in a limited geographical area and can use TV time, radio time, and newspaper ads. Information about each medium is shown below.

| Medium | Cost per ad | # Reached | Exposure quality |
|---|---|---|---|
| TV | 500 | 10,000 | 30 |
| Radio | 200 | 3,000 | 40 |
| Newspaper | 400 | 5,000 | 25 |

The number of TV ads cannot exceed the number of radio ads by more than 4. The advertising budget is $10,000. **The company wants to maximise the number reached and achieve an exposure quality of at least 1,000 across all channels, in total**. Write down the algebraic formulation of the LP model.

**Exercise 2.** A Garden Shop mixes two types of grass seed into a blend. Each type of grass has been rated (per kg) according to its shade tolerance, ability to stand up to traffic, and drought resistance, as shown in the table. Type A seed costs $1 and Type B seed costs $2. The blend needs to score at least 300 points for shade tolerance, 400 points for traffic resistance, and 750 points for drought resistance,

| | Type A | Type B |
|---|---|---|
| Shade Tolerance | 1 | 1 |
| Traffic Resistance | 2 | 1 |
| Drought Resistance | 2 | 5 |




a. What is the linear programming model for this problem to minimise total cost.

b. How many kilograms of each seed should be in the blend? Use the graphical method to solve the problem.

**c.** Which targets will be exceeded?

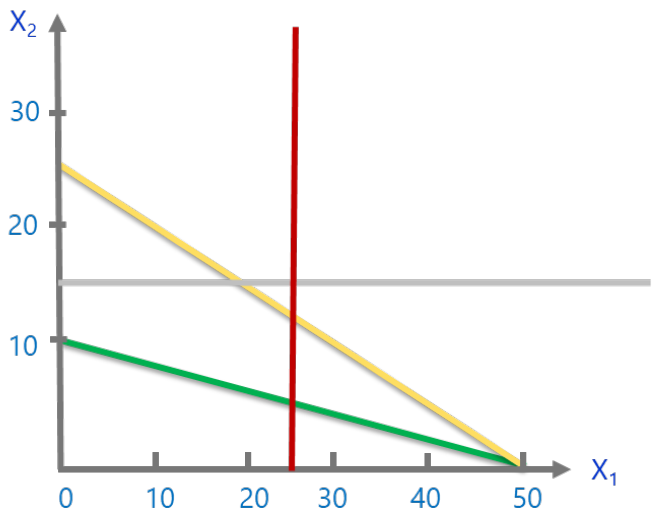
d. How much will the blend cost?

**Exercise 3.** A toy manufacturer makes two kinds of vehicles for young children: bikes and cars. Each week he has supplies of 100 wheels, and 50 kg of steel.

A bike requires 2 wheels, and 1 kg of steel, whereas a car requires 4 wheels and 5 kg of steel. A bike is sold for a profit of $40, and a car for a profit of $100. The market is such that at most 25 bikes can be sold each week, and at most 15 cars.

a) Formulate this as a LP problem for determining how many bikes and how many cars the manufacturer should make each week, in order to maximise profits.

b) A graphical representation of the constraint boundary lines follows:




Identify the feasible region, add in the objective function level curve, and locate the optimal solution.

c) What is the maximum profit that the manufacturer can achieve?

d) Identify the corner points of the feasible region. Evaluate the profit at each corner point and hence determine the optimal solution and the maximum profit.

**Exercise 4.** (Based on Winston & Albright: Question 26, p. 125)

A furniture company manufactures desks and chairs. Each desk uses 4 units of wood, and each chair uses 3 units of wood. A desk contributes $250 to profit, and a chair contributes $145. Marketing restrictions require that the number of chairs produced be at least four times the number of desks produced. There are 2000 units of wood available.

a) Write down the algebraic formulation of the LP model.

b) Use Solver to maximise the company's profit.

c) Confirm graphically that the solution in part (b) maximises the company's profit.

## Quiz

1. There is a fixed cost of $50,000 to start a production process. Once the process has begun, the variable cost per unit is $25. The revenue per unit is projected to be $45. Write an expression for total profit. x= the number of items to be produced.

   a. P(x) = 50000 + 25x
   b. P(x) = 45x
   c. P(x) = 45x − (50000 + 25x)
   d. P(x) = (50000 + 25x) – 45x

2. A furniture store has set aside 80 square meters to display its sofas and chairs. Each sofa utilizes 5 sq. m. and each chair utilizes 3 sq. m. At least four sofas and at least four chairs are to be displayed. Write down a mathematical constraint for the total number of pieces of furniture that can be displayed. s = the number of sofas; c = the number of chairs displayed.

   a. 5s + 3c ≤ 80
   b. 4s + 4c ≤ 80
   c. s ≤ 4
   d. c ≤ 4

3. Which of the following functions are linear?
   a. $x-8.9y-12.1$
   b. $x+7xy+3z+20$
   c. $\pi x$
   d. $A = x^2 + 2y$
   e. y
   f. $2^x$

4. Which of the following optimization problem(s) are **not** LP problem?
   a. MAX $7X_1 + 4X_2$ Subject to: $2X_1 + X_2 \leq 16$
   b. MIN $X_1 + 12.5X_2$ Subject to: $X_1 + 2X_2 \geq 5$ or $X_2 \geq 0$
   c. MAX $5X_1 + 6X_2$ Subject to: $X_1 + X_2 \geq 8$ and $X_2 \leq 0$
   d. MIN A + B Subject to: 20 A + 30 B ≥ 100

5. What are the three common elements of an optimisation problem?

   a. Objectives, resources, goals.
   b. Decisions, constraints, an objective.
   c. Decision variables, profit levels, costs.
   d. Decisions, resource, a profit function.

6. Limited resources are modeled in optimisation problems as
   a. An objective function
   b. Constraints
   c. Decision variables.

7. The first step in formulating a linear programming problem is
   a. Identify any upper or lower bounds on the decision variables
   b. State the constraints as linear combinations of the decision variables
   c. Identify the decision variables
   d. State the objective function as a linear combination of the decision variables.

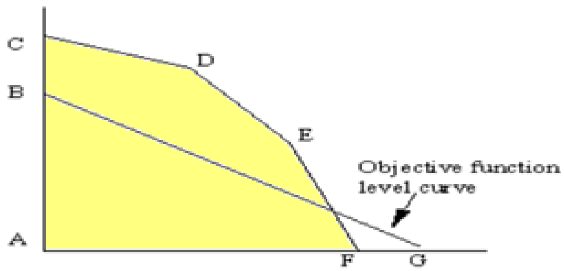
8. Let $x_1$ denote the number of ping-pong balls to be produced and $x_2$ denote the number of wiffle balls to be produced. If there is a maximum of 4,000 hours of labour available per month and 300 ping-pong balls or 125 wiffle balls can be produced per hour of labour, which of the following constraints reflects this situation?

   a. $300x_1 + 125x_2 \geq 4{,}000$
   b. $\frac{X1}{300} + \frac{X2}{125} \leq 4{,}000$
   c. $425(x_1 + x_2) \leq 4{,}000$
   d. $300x_1 + 125x_2 = 4{,}000$

9. The constraints of an LP model define the
   a. Feasible region
   b. Level curves
   c. Optimal region.

10. The constraint for resource 1 is $5X1 + 4X2 \leq 200$. If $X1 = 20$ and $X2 = 5$, how much of resource 1 is unused?
    a. 0
    b. 80
    c. 100
    d. 200

11. This graph shows the feasible region (defined by points ACDEF) and objective function level curve (BG) for a maximization problem. Which point corresponds to the optimal solution to the problem?




    a. A
    b. B
    c. C
    d. D
    e. E.

# WEEK 1 SOLUTIONS

## Exercise 1

Decision Variables
Let T = the number of TV ads
Let R = the number of radio ads
Let N = the number of newspaper ads

Objective function: MAX 10,000T + 3,000R + 5,000N

Constraints

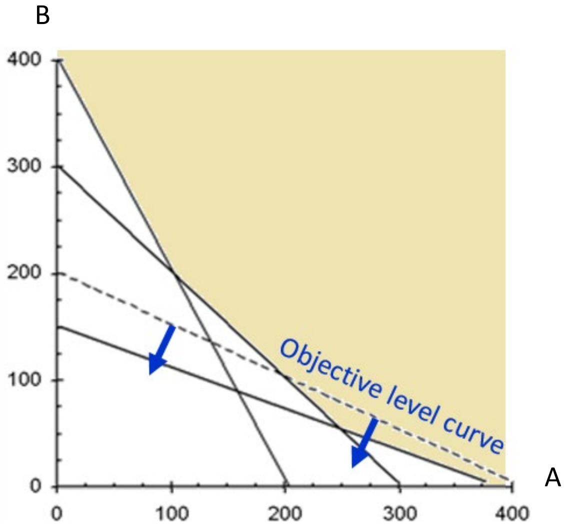
$$500T + 200R + 400N \leq 10{,}000$$
$$30T + 40R + 25N \geq 1{,}000$$
$$T - R \leq 4$$
$$T, R, N \geq 0$$

## Exercise 2

**a.** Let A = the kgs of Type A seed in the blend
Let B = the kgs of Type B seed in the blend

Min A + 2B
Subject to

$$A + B \geq 300$$
$$2A + B \geq 400$$
$$2A + 5B \geq 750$$
$$A, B \geq 0$$




**b.** The optimal solution is at A = 250, B = 50

**c.** Constraint 2 has a surplus value of 150

**d.** The cost is 350

## Exercise 3a

**Decision variables**

Let $X_1$ denote the number of bikes to be made each week

Let $X_2$ denote the number of cars to be made each week.

**Objective function**

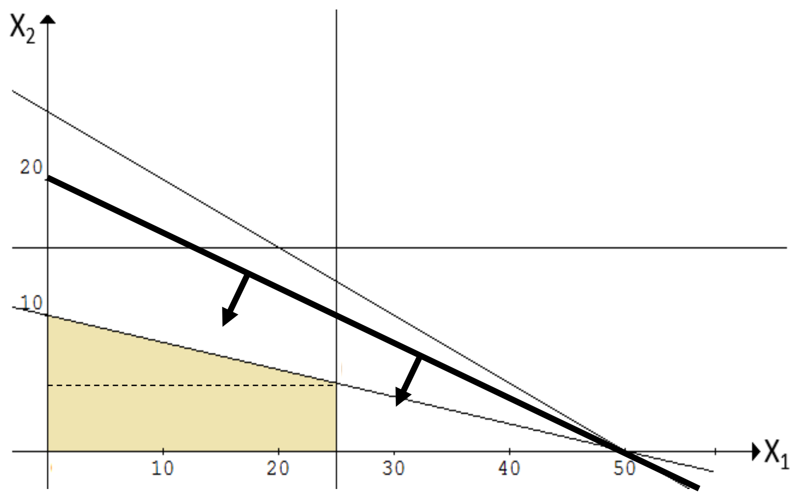
Maximise profit (\$) = $40X_1+100X_2$

**Subject to constraints**

| | |
|---|---|
| $2X_1 + 4X_2 \leq 100$ | (wheel constraint) |
| $X_1 + 5X2 \leq 50$ | (steel constraint) |
| $X_1 \leq 25$ and $X_2 \leq 15$ | (market constraints) |
| $X_1, X_2 \geq 0$ | (non-negativity constraints). |

## Exercise 3b

Notice that two of the constraints are redundant ($2X_1 + 4X_2 \leq 100$ and $X_2 \leq 15$). The feasible region is shaded below. The objective function level curve is shown as the thick black line that passes through (50, 0) and (0, 20). The optimal solution is at the point where the objective function level curve just touches the feasible region. This can be read off the chart as (25, 5).




## Exercise 3c

The maximum profit per week is 40×25 + 100×5 = $1,500.

## Exercise 3d

The corner points of the feasible region are shown in the following table, along with the objective function value. Again we see that the optimal solution is to build 25 bikes and 5 cars per week, for a weekly profit of $1,500.

| | | | | |
|---|---|---|---|---|
| Corner point | (0,0) | (25,0) | (25,5) | (0,10) |
| Objective function value | $0 | $1000 | $1500 | $1000 |

## Exercise 4

a. Decision Variables

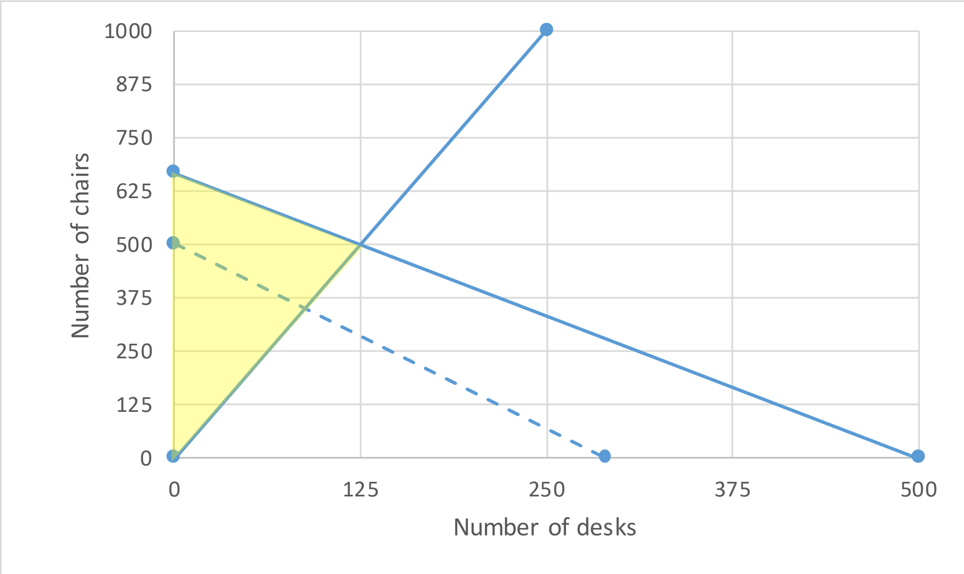
Let X = the number of desks
Let Y = the number of chairs

Objective function ($): MAX 250X + 145Y

Constraints
$4X - Y \leq 0$
$4X + 3Y \leq 2{,}000$
$X, Y \geq 0$

b. Refer to the Excel spreadsheet.

**c.** In the chart, we move the dashed line (i.e. objective function level curve, also known as the isoprofit line) up we see that the objective function value is maximised at the corner point of the shaded feasible region, where the number of desks is 125 and the number of chairs is 500.




## Quiz

1) c. 2) a. 3) a, c, e. 4) b, c. 5) b. 6) b. 7) c. 8) b. 9) a. 10) b. 11) d.