

**MIS775 Decision Modelling for Business Analytics**  
**Revision Questions with Solutions**



## **WEEK 2 QUESTIONS**

### **Ex 1. Marketing Application**

A marketing company specializes in evaluating consumer reaction to new products, services, and advertising campaigns. A client firm requested assistance in ascertaining consumer reaction to a recently marketed household product. During meetings with the client, they agreed to conduct door-to-door personal interviews to obtain responses from households with children and households without children. In addition, they agreed to conduct both day and evening interviews.

The client's contract requests to conduct 1000 interviews under the following quota guidelines:

1. Interview at least 400 households with children.
2. Interview at least 400 households without children.
3. The total number of households interviewed during the evening must be at least as great as the number of households interviewed during the day.
4. At least 40% of the interviews for households with children must be conducted during the evening.
5. At least 60% of the interviews for households without children must be conducted during the evening.

The estimates of the interview costs are as follows:

	<b>Interview cost</b>	
<b>Household</b>	<b>Day</b>	<b>Evening</b>
Children	\$20	\$25
No children	\$18	\$20

The solution of the model using Excel Solver is provided in the Sensitivity Report that follows.

## Sensitivity Report

Variable Cells						
Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
\$C\$7	with children-Day	240	0	20	5	4.666666667
\$D\$7	with children - Evening	160	0	25	1E+30	5
\$E\$7	without children -Day	240	0	18	2	1E+30
\$F\$7	without children - evening	360	0	20	4.666666667	2

Constraints						
Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
\$G\$22	HH sample size Used LHS	1000	19.2	1000	1E+30	200
\$G\$23	HHs with Children Used LHS	400	2.8	400	100	400
\$G\$24	HHs without Children Used LHS	600	0	400	200	1E+30
\$G\$25	HHs in evening Used LHS	40	0	0	40	1E+30
\$G\$26	HHs with Children in Even. Used LHS	0	5	0	240	20
\$G\$27	HHs without Children in Even. Used LHS	0	2	0	240	20

- Formulate the algebraic model.
- Write down the interview plan with the lowest cost, and determine the lowest cost.
- Assume they would like to interview at least 500 households without children, what is the total cost?
- If they decided to conduct a total of 800 interviews instead of the original plan of 1,000, what then would be the lowest cost?

## Quiz

1. Infeasibility means
  - The number of solutions to the linear programming models that satisfies all constraints is zero
  - The objective function can be increased to infinity
  - An optimal solution at some extreme point exists.
2. When the objective function can increase without ever contacting a constraint the LP model is said to be
  - Infeasible
  - Open-ended
  - Multi-optimal
  - Unbounded.

3. A facility produces two products and wants to maximize profit. The objective function to maximize is  $Z = 350X_1 + 300X_2$ . The optimal objective value is equal to 1200. This means:

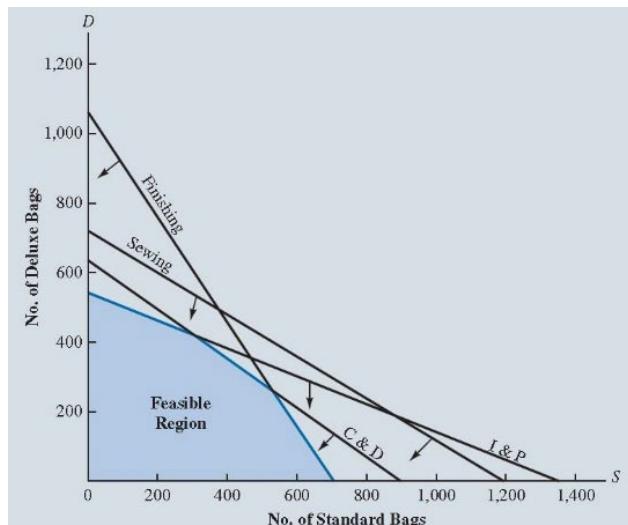
- a. One unit of product 2 contributes \$350 to the objective function
- b. A profit contribution of one unit of product 1 is equal to \$300
- c. This LP problem is bounded and feasible
- d. This LP problem has no constraints.

4. A redundant constraint is one which

- a. Plays no role in determining the feasible region of the problem
- b. Is parallel to the level curve
- c. Is added after the problem is already formulated
- d. Can only increase the objective function value.

5. Which constraint is redundant?

- a. Finishing
- b. Sewing
- c. C & P
- d. I & P.



6. A binding constraint is one that:

- a. Plays no role in determining the feasible region of the problem.
- b. Is redundant.
- c. Has an equality at the optimal solution
- d. Can only increase the objective function value.

## WEEK 2 SOLUTIONS

### Exercise 1a.

Formulate the algebraic model.

Decision variables:

Household	Day	Evening
Children	$X_1$	$X_2$
No children	$X_3$	$X_4$

Let:

- $X_1$  = Number of households with children interviewed during the day
- $X_2$  = Number of households with children interviewed during the evening
- $X_3$  = Number of households without children interviewed during the day
- $X_4$  = Number of households without children interviewed during the evening

**Objective Function (Minimize Cost):**

$$\text{Minimize } Z = 20x_1 + 25x_2 + 18x_3 + 20x_4$$

**Subjected to:**

$$X_1 + X_2 + X_3 + X_4 = 1000 \quad (\text{Total Interviews Must Be 1000})$$

$$X_1 + X_2 \geq 400 \quad (\text{At Least 400 Households with Children})$$

$$X_3 + X_4 \geq 400 \quad (\text{At Least 400 Households without Children})$$

$$X_2 + X_4 \geq X_1 + X_3 \quad (\text{Evening Interviews Must be At Least Equal to Day Interviews})$$

$$X_2 \geq 0.4(X_1 + X_2) \quad (\text{At Least 40\% of Interviews with Children Must Be in the Evening})$$

$$X_4 \geq 0.6(X_3 + X_4) \quad (\text{At Least 60\% of Interviews without Children Must Be in the Evening})$$

$$\text{Non-Negativity Constraint: } X_1, X_2, X_3, X_4 \geq 0$$

### Exercise 1b.

Write down the interview plan with the lowest cost, and determine the lowest cost.

5	Variable Cells						
6	Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
7	\$C\$7	with children day	240	0	20	5	4.666666667
8	\$D\$7	with children evening	160	0	25	1E+30	5
9	\$C\$8	without children day	240	0	18	2	1E+30
10	\$D\$8	without children evening	360	0	20	4.666666667	2

The interview plan with the lowest cost has 240 daytime interviews of households with children, 160 evening interviews of households with children, 240 daytime interviews of households without children, and 360 evening interviews of households without children. Minimum cost is therefore  $240 \times 20 + 160 \times 25 + 240 \times 18 + 360 \times 20 = \$20,320$ .

**Exercise 1c.**

Assume they would like to interview at least 500 households without children, what is the total cost?

14	Constraints	Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
15		\$C\$22	HH sample size Used	1000	19.2	1000	1E+30	200
16		\$C\$23	HHs with children Used	400	2.8	400	100	400
17		\$C\$24	HHs without children Used	600	0	400	200	1E+30
18		\$C\$25	Excess HHs in evening Used	40	0	0	40	1E+30
19		\$C\$26	HHs with children in evening Used	0	5	0	240	20
20		\$C\$27	HHs without children in evening Used	0	2	0	240	20
21								
22								
23								

The third constraint is not binding ( $240 + 360 \geq 400$ ), it means they are interviewing more people than the RHS.

Since the change in RHS for this constraint (from 400 to 500) is in allowable range, the optimal solution and its objective function value will not change.

**Exercise 1d. If they decided to conduct a total of 800 interviews instead of the original plan of 1,000, what then would be the lowest cost?**

14	Constraints	Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
15		\$C\$22	HH sample size Used	1000	19.2	1000	1E+30	200
16		\$C\$23	HHs with children Used	400	2.8	400	100	400
17		\$C\$24	HHs without children Used	600	0	400	200	1E+30
18		\$C\$25	Excess HHs in evening Used	40	0	0	40	1E+30
19		\$C\$26	HHs with children in evening Used	0	5	0	240	20
20		\$C\$27	HHs without children in evening Used	0	2	0	240	20
21								
22								
23								

The first constraint (Total interviews) is a binding constraint (e.g.  $240 + 160 + 240 + 360 = 1000$ ), which means it is satisfied in the final solution. The change in RHS for this constraint (from 1000 down to 800) is in the allowable range, so we have:

$$\begin{aligned} \text{Change in optimal Objective Function Value (OFV)} &= \text{Shadow price} \times \text{change in RHS} \\ &= 19.2 \times (1000 - 800) = \$3,840. \end{aligned}$$

Therefore the optimal OFV for 800 interviews is  $20,320 - 3,840 = \$16,480$ .

**Quiz**

- 1) a. 2) d. 3) c. 4) a. 5) b. 6) c.