MIS775 Decision Modelling for Business Analytics

TOPIC 2: Sensitivity Analysis & Linear Programming Applications

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Recap

- Last week we've developed an understanding of decision models and their importance to managerial decision-making
- We've learnt that there are two types of decision models – deterministic and stochastic
- For LP models we now know:
  - **Their general structure and properties**
  - **The steps to finding the optimal value and optimal solutions**
  - **How to apply a graphical solution when there are two decision variables**
  - **The optimal solution is always located at a corner point of the feasible region**

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Motivation for Sensitivity Analysis

- In solving an LP model we assume that the values of all model parameters are known with certainty, but such certainty rarely exists
- Sensitivity analysis helps us to answer questions about how sensitive an optimal solution is to changes in the values of parameters in a model
- Solver provides useful 'what if' analysis, once an optimal solution has been found
  - **E.g. 1. What would be the effect on the optimal solution if an additional unit of one of the resources could be found?**
  - **E.g. 2. What would be the effect on the optimal value of the objective function if one of its coefficients was decreased by two units?**

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Learning Objectives

- How to perform a sensitivity analysis using Excel's Solver routine
- How to interpret the output of a sensitivity analysis
- How to build some standard types of LP models:
  1. Advertising models
  2. Employee scheduling models
  3. Aggregate planning models

Textbook reading: Chapter 3 (3.4), 4 (4.1-4.4)

Sensitivity Analysis

- Sensitivity analysis is the analysis of how changes in the parameters of an LP model might affect the optimal solution
- **Why is sensitivity analysis important:**
  - **It's unlikely that the values of parameters are known exactly. Values used are often only likely estimates of the true numbers**
  - **Dynamic environment may cause changes in the values of parameters**
  - **'What-if' analysis may provide useful economic and operational information**
- We could always change the values of parameters and solve the new model from scratch, but is it possible to do this without producing a new solution?

Types of Sensitivity Analysis

- Sensitivity analysis can be undertaken using Solver when considering:
  1. **Change to a single coefficient in the objective function**
  2. **Change to the RHS (Right Hand Side) of a single constraint**
- A change to a coefficient in the LHS of a constraint is difficult to analyse, so it is usually more effective to simply solve the new model using the altered coefficient
- If changes in two or more coefficients need to be considered at the same time, then a full solution of the new model must be computed*

*This assumes that we can only access the standard Excel package. There are some free Excel add-ins and other licensed software (e.g. Frontline Solvers) that provide more robust sensitivity analyses

E.g. Two-variable Product Mix Model

- X_1 and X_2 are the number of Basics and XPs produced per month
- Objective is to maximise profit ($) = **80X1 + 129X2**

1. Change to a single coefficient in the objective function

What if the unit margin on a Basic model increased to $107 … or $108?

Subject to constraints:

$5X1 + 6X2 \leq 10{,}000$ (assembly time)

$X1 + 2X2 \leq 3{,}000$ (testing time)

$X1 \leq 600$ (monthly demand for Basics)

$X2 \leq 1200$ (monthly demand for XPs)

$X1, X2 \geq 0$ (non-negativity)

What if the available testing time could be increased by 10%?

What if the demand for XPs fell to 1,000?

2. Change to the RHS (Right Hand Side) of a single constraint

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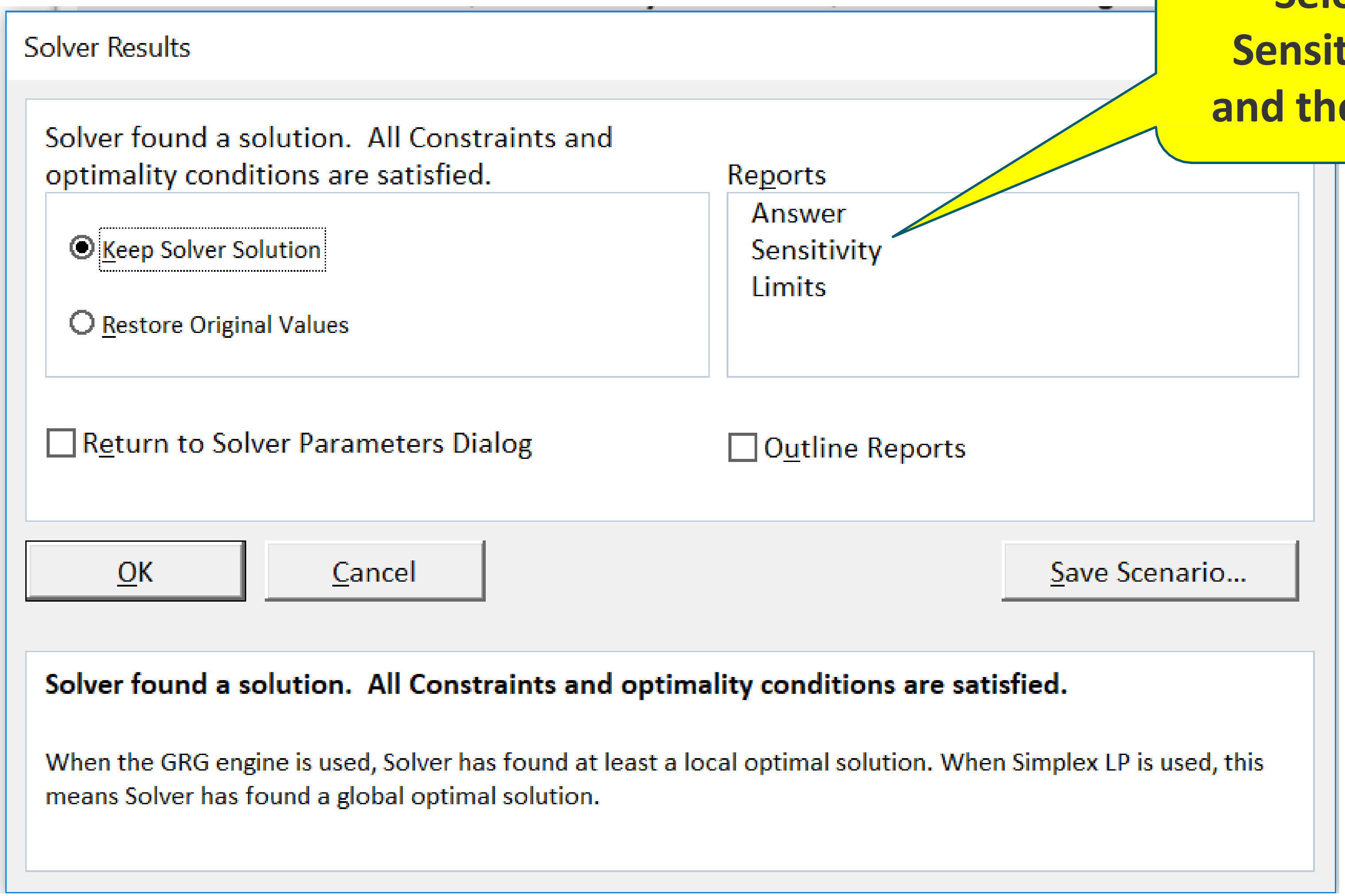
How to Access Solver's Sensitivity Report

Select Sensitivity and then OK



Sensitivity Report for the Two-variable Product Mix Model

Optimal solutions
$X1=560$ and $X2=1200$

Optimal Value $= 80X1 + 129X2$
$= 80*560+129*1200$
$=199600$

From Solver results, select Sensitivity Report:

	A	B	C	D	E	F	G	H
1	**Microsoft Excel 16.0 Sensitivity Report**							
2	**Worksheet: [MIS775_Topic_1_Workings.xlsx]Spreadsheet model1**							
3	**Report Created: 3/03/2024 11:40:59 AM**							
4								
5						Solutions		
6	Variable Cells							
7				**Final**	**Reduced**	**Objective**	**Allowable**	**Allowable**
8		**Cell**	**Name**	**Value**	**Cost**	**Coefficient**	**Increase**	**Decrease**
9		\$C\$17	No. to produce Basic	560	0	80	27.5	80
10		\$D\$17	No. to produce XP	1200	0	129	1E+30	33
11								
12	Constraints							
13				**Final**	**Shadow**	**Constraint**	**Allowable**	**Allowable**
14		**Cell**	**Name**	**Value**	**Price**	**R.H. Side**	**Increase**	**Decrease**
15		\$E\$28	Assembly time Used	10000	16	10000	200	2800
16		\$E\$29	Testing time Used	2960	0	3000	1E+30	40
17		\$E\$30	Market demand for Basics Used	560	0	600	1E+30	40
18		\$E\$31	Market demand for XPs Used	1200	33	1200	50	33.33333333
19								

Variable Cells section. Each row indicates how the optimal solution changes if one of the objective function coefficients change

Constraints section. Each row indicates how the optimal solution changes if one of the constraint R.H. Sides change

1. Change to a single coefficient in the objective function

2. Change to the RHS of a single constraint

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Sensitivity Report - Variable Cells

Optimal Value = 100X1 + 129X2
= 100*560+129* 1200
=210800

Optimal Value = 100X1 + 129X2
= 80*560+129*1200
=199600

6	Variable Cells						
7			Final	Reduced	Objective	Allowable	Allowable
8	Cell	Name	Value	Cost	Coefficient	Increase	Decrease
9	C15	No. to produce Basic	560	0	80	27.5	80
10	D15	No. to produce XP	1200	0	129	1E+30	33

- Final value gives the current optimal solution (560 Basics and 1200 XPs)
- Reduced cost – we'll discuss this later
- **Objective coefficient** gives the current unit margin values
- Allowable increase and decrease from the given unit margins give the range in which the current optimal solution (560, 1200) holds
  - **Unit margin of a Basic computer can range between $0 - $107.5 without changing the optimal** *solution (however, the optimal value change accordingly)*
  - **Unit margin of an XP computer can range anywhere from (129-33=96) $96 and above without changing the optimal solution**

If the unit margin on a Basic increases beyond $107.5 (80+27.5) , the optimal solution might change, so Solver would need to be re-run with the new unit margin

Sensitivity Report - Variable Cells

6	Variable Cells						
7			Final	Reduced	Objective	Allowable	Allowable
8	Cell	Name	Value	Cost	Coefficient	Increase	Decrease
9	C15	No. to produce Basic	560	0	80	27.5	80
10	D15	No. to produce XP	1200	0	129	1E+30	33

12	Constraints						
13			Final	Shadow	Constraint	Allowable	Allowable
14	Cell	Name	Value	Price	R.H. Side	Increase	Decrease
15	E26	Assembly time Used	10000	16	10000	200	2800
16	E27	Testing time Used	2960	0	3000	1E+30	40
17	E28	Market demand for Basics Used	560	0	600	1E+30	40
18	E29	Market demand for XPs Used	1200	33	1200	50	33.33333333

We are already meeting market demand for XPs, so no matter how high the unit margin is, we wouldn't produce any more than the demand.

This explains why there is no upper limit to the unit margin for XPs

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Sensitivity Report - Constraints

Maximise profit ($) = $80X1 + 129X2$

Subject to:

$5X1 + 6X2 \leq 10{,}000$

$X1 + 2X2 \leq 3{,}000$

$X1 \leq 600$

$X2 \leq 1200$

$X1, X2 \geq 0$

LHS | RHS

12	Constraints						
13			Final	Shadow	Constraint	Allowable	Allowable
14	Cell	Name	Value	Price	R.H. Side	Increase	Decrease
15	E26	Assembly time Used	10000	16	10000	200	2800
16	E27	Testing time Used	2960	0	3000	1E+30	40
17	E28	Market demand for Basics Used	560	0	600	1E+30	40
18	E29	Market demand for XPs Used	1200	33	1200	50	33.33333333

- We call a constraint **binding** when the final value of the LHS = RHS
- **Slack value** is the unused capacity for each '≤' constraint (Slack = RHS - LHS)
- **Surplus value** is the excess quantity from minimum requirement of each '≥' constraint (Surplus = LHS - RHS)

Sensitivity Report - Constraints

12	Constraints						
13			Final	Shadow	Constraint	Allowable	Allowable
14	Cell	Name	Value	Price	R.H. Side	Increase	Decrease
15	E26	Assembly time Used	10000	16	10000	200	2800
16	E27	Testing time Used	2960	0	3000	1E+30	40
17	E28	Market demand for Basics Used	560	0	600	1E+30	40
18	E29	Market demand for XPs Used	1200	33	1200	50	33.33333333

- A change in the RHS of a constraint might affect the feasible region and perhaps cause a change in the optimal solution
- The change in the **objective function value** per unit increase in the RHS of a constraint is called the **shadow price** (or dual price)
- **Only binding constraints have non-zero shadow prices**
- Shadow prices only apply within the allowed ranges for the RHS

Sensitivity Report – Some Scenarios

Allowable range for assembly time
Is 7200 to 10200

12	Constraints						
13			Final	Shadow	Constraint	Allowable	Allowable
14	Cell	Name	Value	Price	R.H. Side	Increase	Decrease
15	E26	Assembly time Used	10000	16	10000	200	2800
						10000+200=10200	10000-2800=7200

Use the Sensitivity Report to determine the impact that each of the following changes would have on profit

a) Management are wondering whether to increase assembly time by a further 200 hours per month (from 10000 to 10200 hrs)

b) Available assembly time is to be cut to 8,000 hours

c) Available assembly time is to be increased by 300 hours

d) The assembling cost is to be increased from $11 to $12 per hour

e) The Basic computer can now be reconfigured so it only requires 4 hours of assembly, down from 5 hours

Some Scenarios – Answers

Allowable range for assembly time
Is s 7200 to 10200

12	Constraints						
13			Final	Shadow	Constraint	Allowable	Allowable
14	Cell	Name	Value	Price	R.H. Side	Increase	Decrease
15	E26	Assembly time Used	10000	16	10000	200	2800

What impact would each of the following have on profit?

a) Management are wondering whether to increase assembly time by a further 200 hours per month. (from 10000 to 10200 hrs)

If the RHS of assembly time used is increased to 10,200

(an allowable increase of 200) monthly profit would increase by 200 × $16 = $3,200.

The company can now weigh up whether to pursue this increase in assembly time

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Some Scenarios – Answers

Allowable range for assembly time
Is s 7200 to 10200

12	Constraints						
13			Final	Shadow	Constraint	Allowable	Allowable
14	Cell	Name	Value	Price	R.H. Side	Increase	Decrease
15	E26	Assembly time Used	10000	16	10000	200	2800

What impact would each of the following have on profit?

b) Available assembly time is to be cut to 8,000 hours.

If the RHS is decreased to 8,000 (an allowable decrease of 2,000) monthly profit would then decrease by 2,000 × \$16 = \$32,000

c) Available assembly time is to be increased by 300 hours.

This is beyond the allowable increase. We need to rerun Solver with the RHS set to 10,300

Some Scenarios – Answers

Maximise profit (\$) = $80X1 + 129X2$

Subject to:

$5X1 + 6X2 \leq 10{,}000$ (assembly time constraint)

$X1 + 2X2 \leq 3{,}000$ (testing time constraint)

$X1 \leq 600$ (monthly demand for Basics)

$X2 \leq 1200$ (monthly demand for XPs)

$X1, X2 \geq 0$ (non-negativity conditions)

12 Constraints

Row	Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
15	\$E\$26	Assembly time Used	10000	16	10000	200	2800

What impact would each of the following have on profit?

d) The assembling cost is to be increased from \$11 to \$12 per hour.

This would change the unit margins of both computer models, but the Sensitivity Report can only deal with a change to one coefficient of the objective function.We therefore need to rerun Solver with this change

e) The Basic computer can now be reconfigured so it only requires 4 hours of assembly, down from 5 hours.

A Change in the LHS of a constraint coefficient cannot be handled by the Sensitivity Report.We therefore need to rerun Solver with this change

Some Standard Types of LP Models

1. Advertising models
2. Employee scheduling models
3. Aggregate planning models

1. Advertising Models*

A breakfast cereal company wants to purchases 30 second TV ads to achieve set minimum exposures (i.e. when a person has an opportunity to see an ad) across key female segments in its target market. They want to minimise the total cost

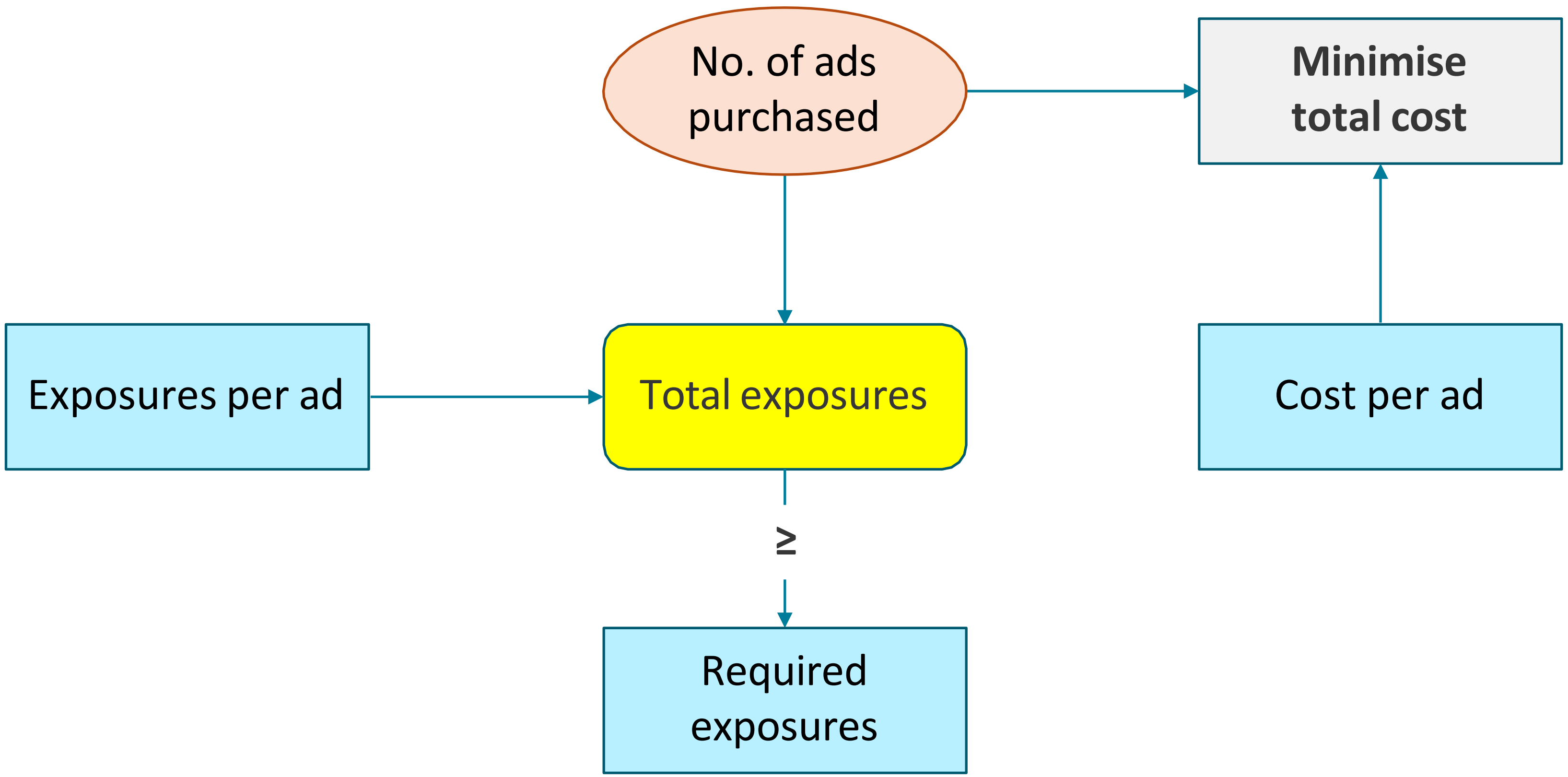
Segment	Millions of exposures per TV show			Minimum required exposures
	Show A	Show B	Show C	
18-35	6	1	4	60
36-55	4	1	2	60
Over 55	2	1	0	28
Cost per ad	**$140,000**	**$100,000**	**$80,000**	

The objective is to minimise the total cost subject to exposure constraints per segment

***Scaled down version of textbook examples on pp. 137-147**

Conceptual Model

Fixed inputs (constants)	
Decision variables (inputs)	
Calculated variables	
Output variable	



Algebraic Model

Segment	Millions of exposures per TV show			Minimum required exposures
	Show A	Show B	Show C	
18-35	6	1	4	60
36-55	4	1	2	60
Over 55	2	1	0	28
Cost per ad	\$140,000	\$100,000	\$80,000	

- Let X_A, X_B and X_C denote the *decision variables* (number of 30 second ads on shows A, B and C)
- The objective is to *minimise total cost* subject to exposure constraints per segment.
- Minimise total cost (\$'000) = $140X_A + 100X_B + 80X_C$

Subject to:

$6X_A + X_B + 4X_C \geq 60$ (females aged 18-35 constraint)

$4X_A + X_B + 2X_C \geq 60$ (females aged 36-55 constraint)

$2X_A + X_B \geq 28$ (females aged over 55 constraint)

$X_A, X_B, X_C \geq 0$

- After setting this up in Excel and running Solver we find that the optimal solution occurs when $X_A = 15$, $X_B = 0$ and $X_C = 0$ for a minimum cost of \$2.1 million (140*15+ 100*0+80*0 = 2100 thousands).

  This raises an interesting question: suppose the marketing manager wanted to have an ad run on show B. What impact would this have on the optimal solution?
- This is exactly the kind of question that **reduced cost** was designed to answer

Sensitivity Report – Interpretation of Reduced Cost

- Reduced cost represents the amount that the objective function will worsen if one unit of a decision variable is forced into the solution
- **The reduced cost value is only non-zero when the optimal value of a variable is zero**

Segment	Millions of exposures per TV show		
	Show A	Show B	Show C
18-35	6	1	4
36-55	4	1	2
Over 55	2	1	0
Cost per ad	**$140,000**	**$100,000**	**$80,000**

Optimal value= 15*140 = 2100

- If the manager decides to run one ad on show B then the total cost will worsen (i.e. increase) by $65,000; two ads on B and the total cost will worsen by $2 \times \$65,000 = \$130,000$

  (100-65=35) 35,000 < B advertisement cost < infinity

- The report also tells us that the optimal solution (15, 0, 0) holds provided that the cost of an ad on show B should be at least $35,000

6	Variable Cells						
7			Final	Reduced	Objective	Allowable	Allowable
8	Cell	Name	Value	Cost	Coefficient	Increase	Decrease
9	C14	No. ads A	15	0	140	20	140
10	D14	No. ads B	0	65	100	1E+30	65
11	E14	No. ads C	0	10	80	1E+30	10
12							
13	Constraints						
14			Final	Shadow	Constraint	Allowable	Allowable
15	Cell	Name	Value	Price	R.H. Side	Increase	Decrease
16	C22	Women 18-35 LHS	90	0	60	30	1E+30
17	C23	Women 36-55 LHS	60	35	60	1E+30	4
18	C24	Women >55 LHS	30	0	28	2	1E+30

Alternative Advertising Model*

Suppose instead that the breakfast cereal company has a budget of $2.1 million for purchasing the TV ads. What is the **maximum exposure** it can achieve in its target market, while still meeting the minimum exposures required per segment?

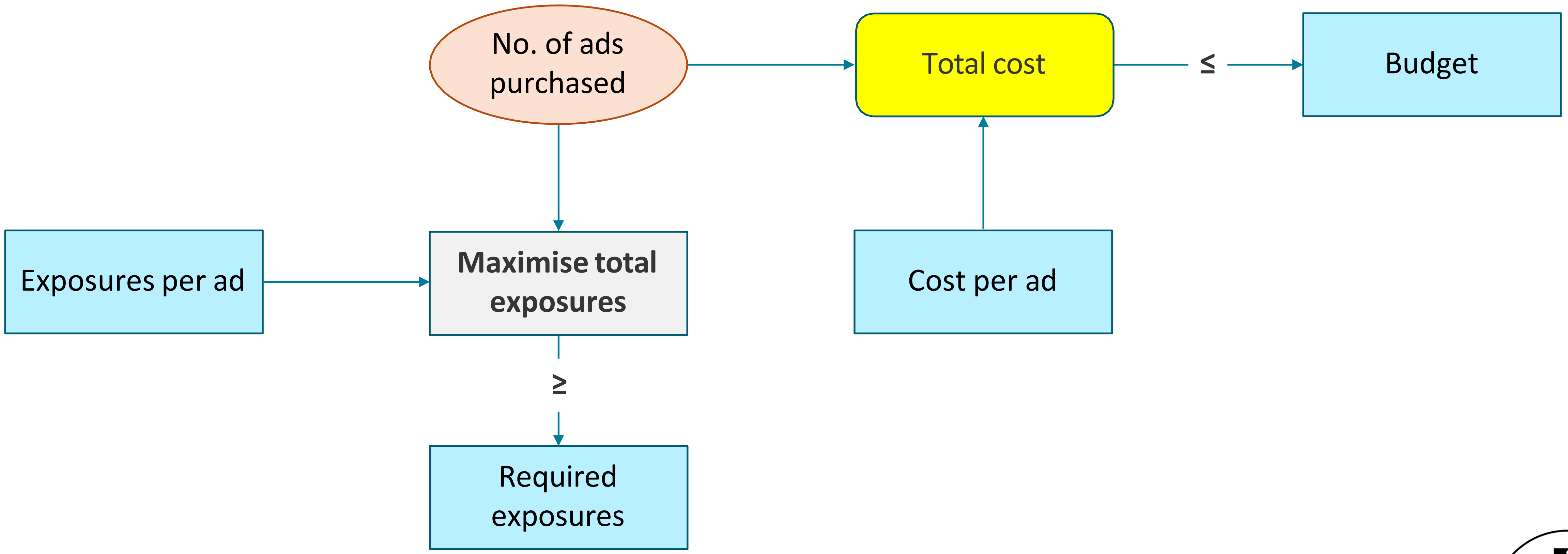
Its worth recognising that this alternative model and the one before stem from the fact that the company really has two objectives: maximising total exposures and minimising total cost. But since these are contradictory objectives they need to constrain one while optimising the other, hence the two possible versions.

The conceptual and mathematical models for this alternative follow

*Scaled down version of textbook examples on pp 137-147

Conceptual Model

Fixed inputs (constants)
Decision variables (inputs)
Calculated variables
Output variable



Algebraic Model

Segment	Millions of exposures per TV show		
	Show A	Show B	Show C
18-35	6	1	4
36-55	4	1	2
Over 55	2	1	0
Cost per ad	\$140,000	\$100,000	\$80,000

- Let X_A, X_B and X_C denote the *decision variables* (number of 30 second ads on shows A, B and C)
- Objective is to *maximise total number of exposures* subject to exposure constraints per segment and a budget constraint
- **Maximise total number of exposures** (in millions) = $12X_A + 3X_B + 6X_C$

Subject to:

$6X_A + X_B + 4X_C \geq 60$ (females aged 18-35 constraint)

$4X_A + X_B + 2X_C \geq 60$ (females aged 36-55 constraint)

$2X_A + X_B \geq 28$ (females aged 55 and over constraint)

$140X_A + 100X_B + 80X_C \leq 2{,}100$ (budget constraint)

$X_A, X_B, X_C \geq 0$

- After setting this up in Excel and running Solver we find that the total number of exposures is maximised when $X_A = 15$, $X_B = 0$ and $X_C = 0$, at a cost of \$2.1 million

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2. Employee Scheduling Models*

- A small business requires different numbers of full-time employees on different days of the week, as follows

Day of week	Mon	Tues	Wed	Thurs	Fri	Sat	Sun
Minimum no. of employees required	17	13	15	19	14	16	11

- Company policy is that full-time employees must work for 5 consecutive days followed by two days off. For example, an employee who works Monday to Friday must be off on Saturday and Sunday.

- The Company wants to meet its daily requirements using only full-time employees. Its objective is to minimize the number of full-time employees on its payroll.

*Textbook example on pp. 147-154

Employee Scheduling Models*

Day of week	Mon	Tues	Wed	Thurs	Fri	Sat	Sun
Minimum no. of employees required	17	13	15	19	14	16	11

	Start day of 5 day shift								
Day of week	Mon	Tue	Wed	Thurs	Fri	Sat	Sun		Requirment
Mon	X1	0	0	X4	X5	X6	X7	≥	**17**
Tue	X1	X2	0	0	X5	X6	X7	≥	**13**
Wed	X1	X2	X3	0	0	X6	X7	≥	**15**
Thurs	X1	X2	X3	X4	0	0	X7	≥	**19**
Fri	X1	X2	X3	X4	X5	0	0	≥	**14**
Sat	0	X2	X3	X4	X5	X6	0	≥	**16**
Sun	0	0	X3	X4	X5	X6	X7	≥	**11**

Algebraic Model

- Decision variables: Let X_i denote the number of employees starting their 5 day shift on day i (i = 1, …, 7) where Monday is day 1, …, Sunday is day 7.
- Objective:

Minimise number of full-time employees = $X_1 + X_2 + X_3 + X_4 + X_5 + X_6 + X_7$

Subject to:

$$
\begin{array}{llll}
X_1 + & X_4 + X_5 + X_6 + X_7 & \geq 17 & \text{(Monday day 1 constraint)} \\
X_1 + X_2 & + X_5 + X_6 + X_7 & \geq 13 & \text{(Tuesday day 2 constraint)} \\
X_1 + X_2 + X_3 & + X_6 + X_7 & \geq 15 & \text{(day 3 constraint)} \\
X_1 + X_2 + X_3 + X_4 & + X_7 & \geq 19 & \text{(day 4 constraint)} \\
X_1 + X_2 + X_3 + X_4 + X_5 & & \geq 14 & \text{(day 5 constraint)} \\
X_2 + X_3 + X_4 + X_5 + X_6 & & \geq 16 & \text{(day 6 constraint)} \\
X_3 + X_4 + X_5 + X_6 + X_7 & & \geq 11 & \text{(day 7 constraint)} \\
X_i \geq 0 & & & \text{(non-negativity constraint)}
\end{array}
$$

X_i are integers (i.e. they can only take integer values)

Optimal Solution when Integer Constraints are not Included

EMPLOYEE SCHEDULING MODEL

INPUT
Company policy

Day of week	Start day of 5 day shift: Mon	Tue	Wed	Thurs	Fri	Sat	Sun
Mon	1	0	0	1	1	1	1
Tue	1	1	0	0	1	1	1
Wed	1	1	1	0	0	1	1
Thurs	1	1	1	1	0	0	1
Fri	1	1	1	1	1	0	0
Sat	0	1	1	1	1	1	0
Sun	0	0	1	1	1	1	1

DECISION VARIABLES

	Mon	Tue	Wed	Thurs	Fri	Sat	Sun
No. of employees starting 5 day shift on this day	6.33	3.33	2.00	7.33	0.00	3.33	0.00

OBJECTIVE FUNCTION

No. of employees on payroll	22.33

CONSTRAINTS

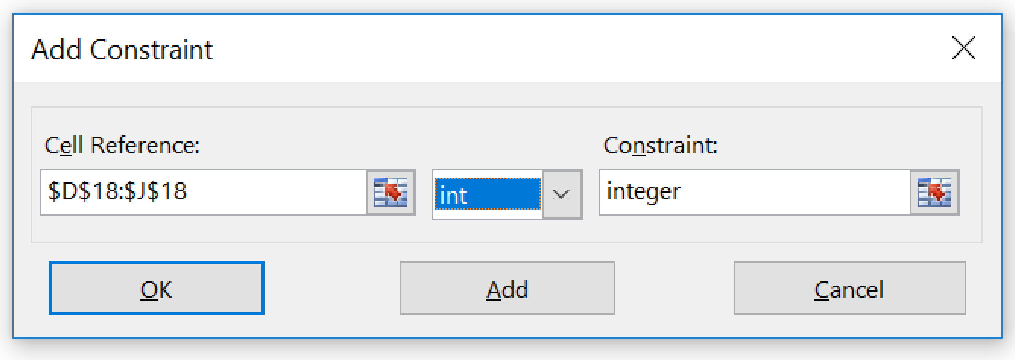
Day of week	Total employee		Minimum required	difference
Mon	17.00	≥	17	0
Tue	13.00	≥	13	0
Wed	15.00	≥	15	0
Thurs	19.00	≥	19	0
Fri	19.00	≥	14	5
Sat	16.00	≥	16	0
Sun	12.67	≥	11	2

- If we don't specify that the decision variables are integers, the optimal solution could get fractional employees, which is unrealistic

- This is easily addressed by including integer constraints on the decision variables

Optimal Solution when Integer Constraints are Included

- Adding integer constraints is easy:



- Optimal solution is now integer valued
- Again this solution isn't unique either. This gives management a choice so they can better satisfy other possible criteria

EMPLOYEE SCHEDULING MODEL

INPUT

Company policy

Day of week	Start day of 5 day shift: Mon	Tue	Wed	Thurs	Fri	Sat	Sun
Mon	1	0	0	1	1	1	1
Tue	1	1	0	0	1	1	1
Wed	1	1	1	0	0	1	1
Thurs	1	1	1	1	0	0	1
Fri	1	1	1	1	1	0	0
Sat	0	1	1	1	1	1	0
Sun	0	0	1	1	1	1	1

DECISION VARIABLES

	Mon	Tue	Wed	Thurs	Fri	Sat	Sun
No. of employees starting 5 day shift on this day	6.00	3.00	3.00	7.00	0.00	3.00	1.00

OBJECTIVE FUNCTION

No. of employees on payroll	23.00

CONSTRAINTS

Day of week	Total employee		Minimum required	difference
Mon	17.00	≥	17	0
Tue	13.00	≥	13	0
Wed	16.00	≥	15	1
Thurs	20.00	≥	19	1
Fri	19.00	≥	14	5
Sat	16.00	≥	16	0
Sun	14.00	≥	11	3

Sensitivity Analysis by Rerunning Solver

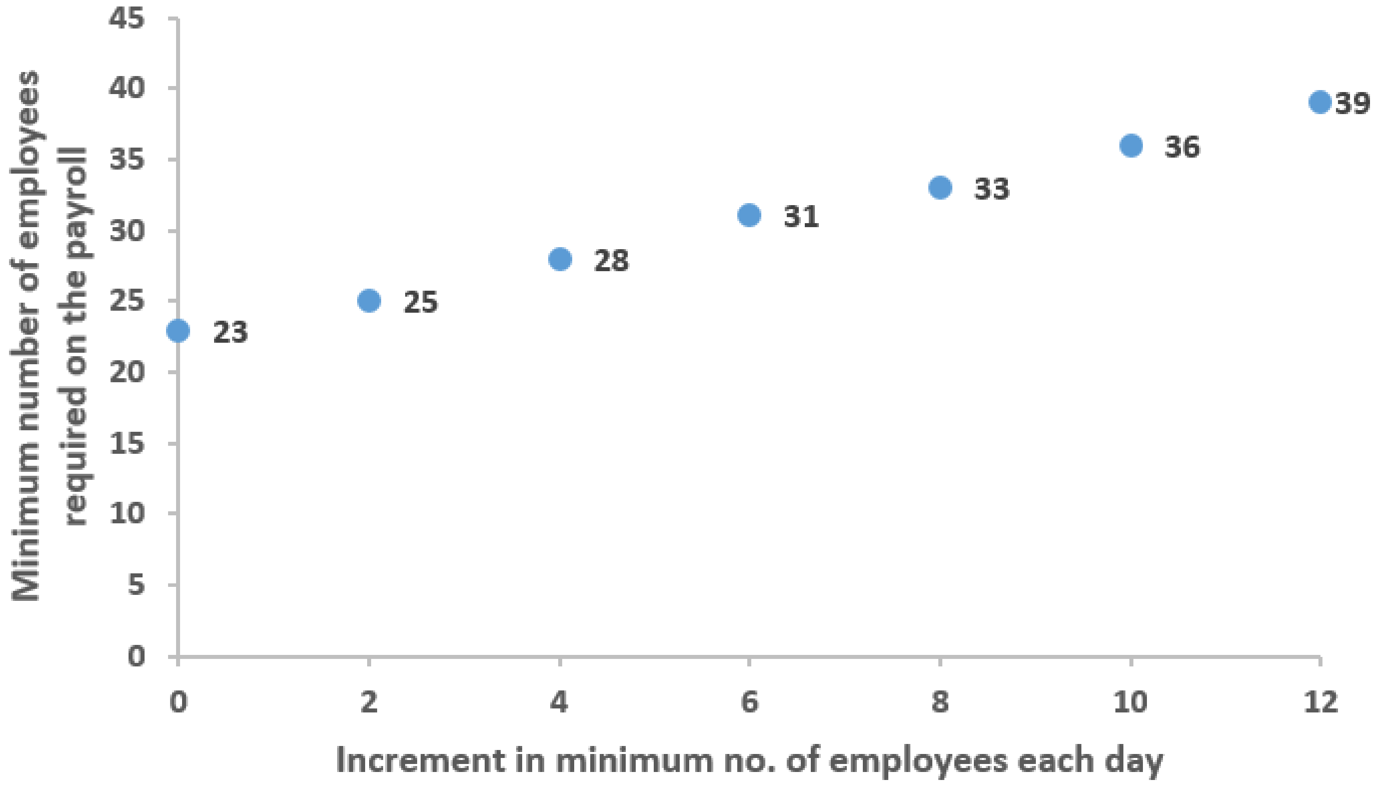
- **Solver doesn't produce a Sensitivity Report when there are integer constraints** but we can rerun Solver under varying conditions to get information on the model's sensitivity to changes in parameter values.

- E.g., to measure the impact from increasing the minimum number of employees on each day by 2, 4, …, 12, we introduce an increment cell in our model so we can easily update the minimum required each day

	B	C	D	E	F	G	H	I	J	K	L	M
16	**DECISION VARIABLES**											
17			**Mon**	**Tue**	**Wed**	**Thurs**	**Fri**	**Sat**	**Sun**			
18	No. of employees		7	9	1	11	0	7	4			
19	starting 5 day shift											
20	on this day											
21												
22	**OBJECTIVE FUNCTION**											
23	No. of employees on		39									
24	payroll											
25												
26	**CONSTRAINTS**											
27		**Day of**	**Total**		**Minimum**		**Base**		**Increment**		**Extra**	**Total**
28		**week**	**employees**		**required**		**required**		12		**required**	**employees**
29		Mon	29	≥	29		17				0	23
30		Tue	27	≥	25		13				2	25
31		Wed	28	≥	27		15				4	28
32		Thurs	32	≥	31		19				6	31
33		Fri	28	≥	26		14				8	33
34		Sat	28	≥	28		16				10	36
35		Sun	23	≥	23		11				12	39

Sensitivity Analysis by Rerunning Solver

- The chart shows a close to linear relationship between the minimum number of employees required on the payroll and the size of the increment in the minimum number required each day



3. Aggregate Planning Models*

- These models are used to determine workforce levels and production schedules over a multi-period horizon. Consider a shoe manufacturer over four months

- During the next four months the Company must meet (on time) the following

Month	1	2	3	4
Demand	3,000	5,000	2,000	1,000

- Determine the optimal production schedule to minimize the cost and labour policy, given forecast:

*Textbook example on pp. 155-166

3. Aggregate Planning Models

Month	1	2	3	4
Demand	3,000	5,000	2,000	1,000

- At the beginning of month 1, 500 pairs of shoes are on hand, and has 100 workers.
- A worker is paid $1500 per month. Each worker can work up to 160 hours a month before he or she receives overtime.
- A worker can work up to 20 hours of overtime per month and is paid $13 per hour for overtime labor.
- It takes 4 hours of labor and $15 of raw material to produce a pair of shoes.
- At the beginning of each month, workers can be hired or fired. Each hired worker costs $1600, and each fired worker costs $2000.
- At the end of each month, a holding cost of $3 per pair of shoes left in inventory is incurred. Production in a given month can be used to meet that month's demand.
- Use LP to determine its optimal production schedule and labor policy.

INPUT

No. of workers at start	100
Pairs of shoes in stock at start	500
Cost of hiring a new worker	$1,600
Cost of firing a worker	$2,000
Cost of worker for month	$1,500
Hourly rate for overtime	$13
Cost of materials per pair	$15
Holding cost of shoes left in inventory	$3
Regular hours per worker	160
Maximum hrs of overtime per worker	20
No. hours to produce a pair	4

Inputs, objective and variable

INPUT

No. of workers at start	100
Pairs of shoes in stock at start	500
Cost of hiring a new worker	$1,600
Cost of firing a worker	$2,000
Cost of worker for month	$1,500
Hourly rate for overtime	$13
Cost of materials per pair	$15
Holding cost of shoes left in inventory	$3
Regular hours per worker	160
Maximum hrs of overtime per worker	20
No. hours to produce a pair	4

Total Cost of Production

1. Total cost of hiring
2. Total cost of firing
3. Total cost of regular salary
4. Total cost of overtime
5. Total cost of materials
6.Total cost of inventory

Variables

1. No. of workers hired
2. No. of workers fired
3. No. of hours of overtime
4.No. pairs of shoes produced

Other Inputs

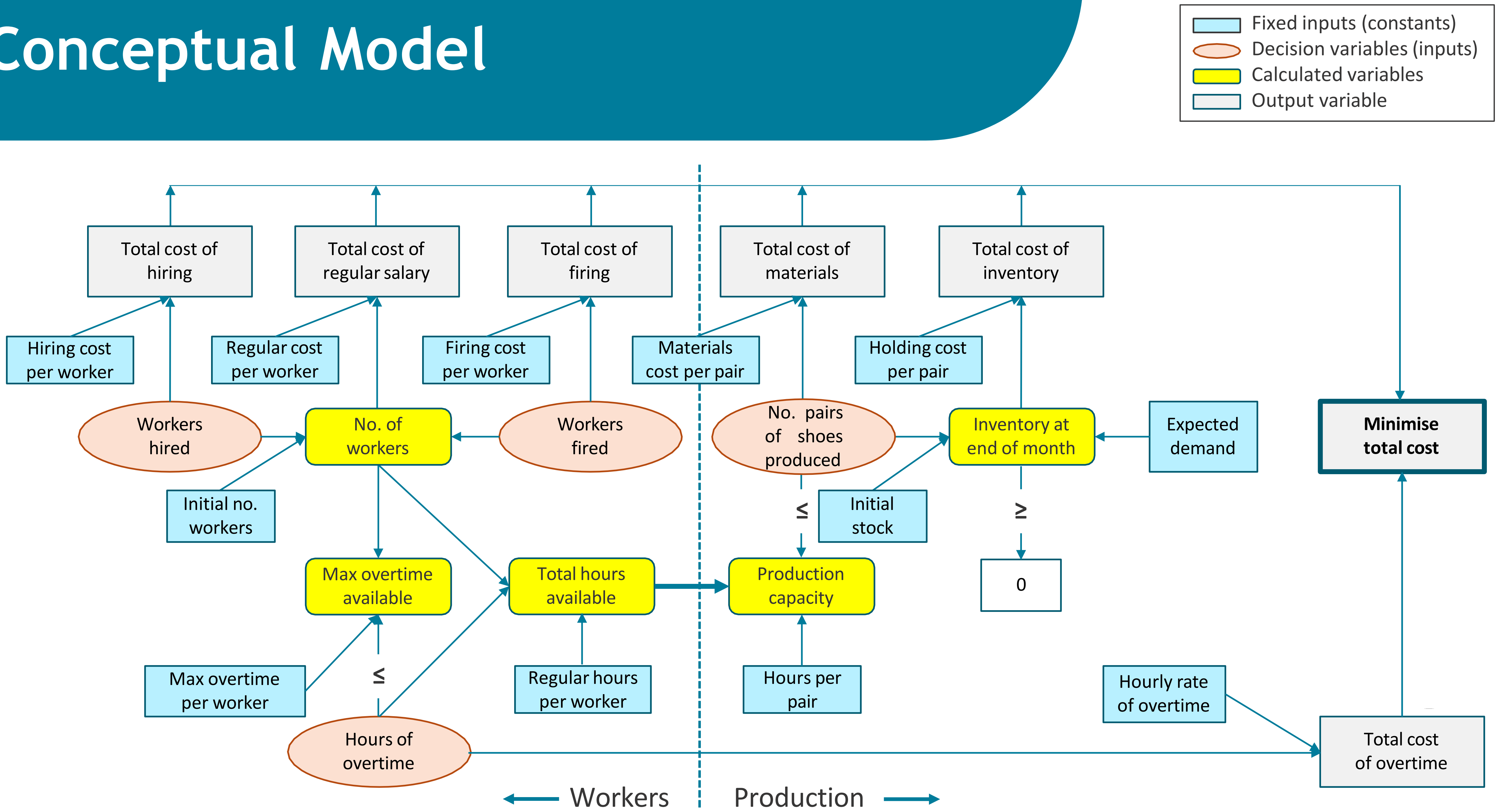
Workers available after hiring and firing

Hours of labour excluding overtime

Total hours of labour per month

Ending inventory

Conceptual Model



Algebraic Model

- <u>Decision variables</u>:

  Let H_i and F_i denote the number of workers hired and fired, respectively, at the start of month i (i = $1, 2, 3, 4$)

  Let O_i denote the number of hours of overtime in month i

  Let S_i denote the number of pairs of shoes to produce in month i

- <u>Calculated variables</u>:

  Let W_i denote the number of workers in month i after hiring and firing.

  Then $W_i = W_{i-1} + H_i - F_i$ where $W_0 = 100$ (given).

  Let I_i denote the number of pairs of shoes left in inventory at the end of month i.

  Then $I_i = I_{i-1} + S_i - d_i$ where $I_0 = 500$ (given) and d_i is the demand in month i.

  Let T_i denote the time available in hours for month i. Then $T_i = 160W_i + O_i$

- <u>Objective:</u> Minimise total cost (\$) given by $1{,}600\sum H_i + 2{,}000\sum F_i + 1{,}500\sum W_i + 13\sum O_i + 15\sum S_i + 3\sum I_i$

- Constraints:

  $I_i \geq 0$ (inventory constraint)

  $O_i \leq 20\,W_i$ (overtime constraint)

  $S_i \leq 0.25T_i$ (production constraint)

Optimal Solution

Use LP to determine its optimal production schedule and labor policy

DECISION VARIABLES

Month	1	2	3	4
No. of workers hired	0	0	0	0
No. of workers fired	6	1	43	0
No. of hours of overtime	0	80	0	0
No. pairs of shoes produced	3760	3740	2000	1000

Labour policy

Optimal production schedule

CALCULATED VARIABLES

Month	0	1	2	3	4
Workers available after hiring and firing	100	94	93	50	50
Hours of labour excl. overtime		15040	14880	8000	8000
Total hours of labour per month		15040	14960	8000	8000
Ending inventory	500	1260	0	0	0

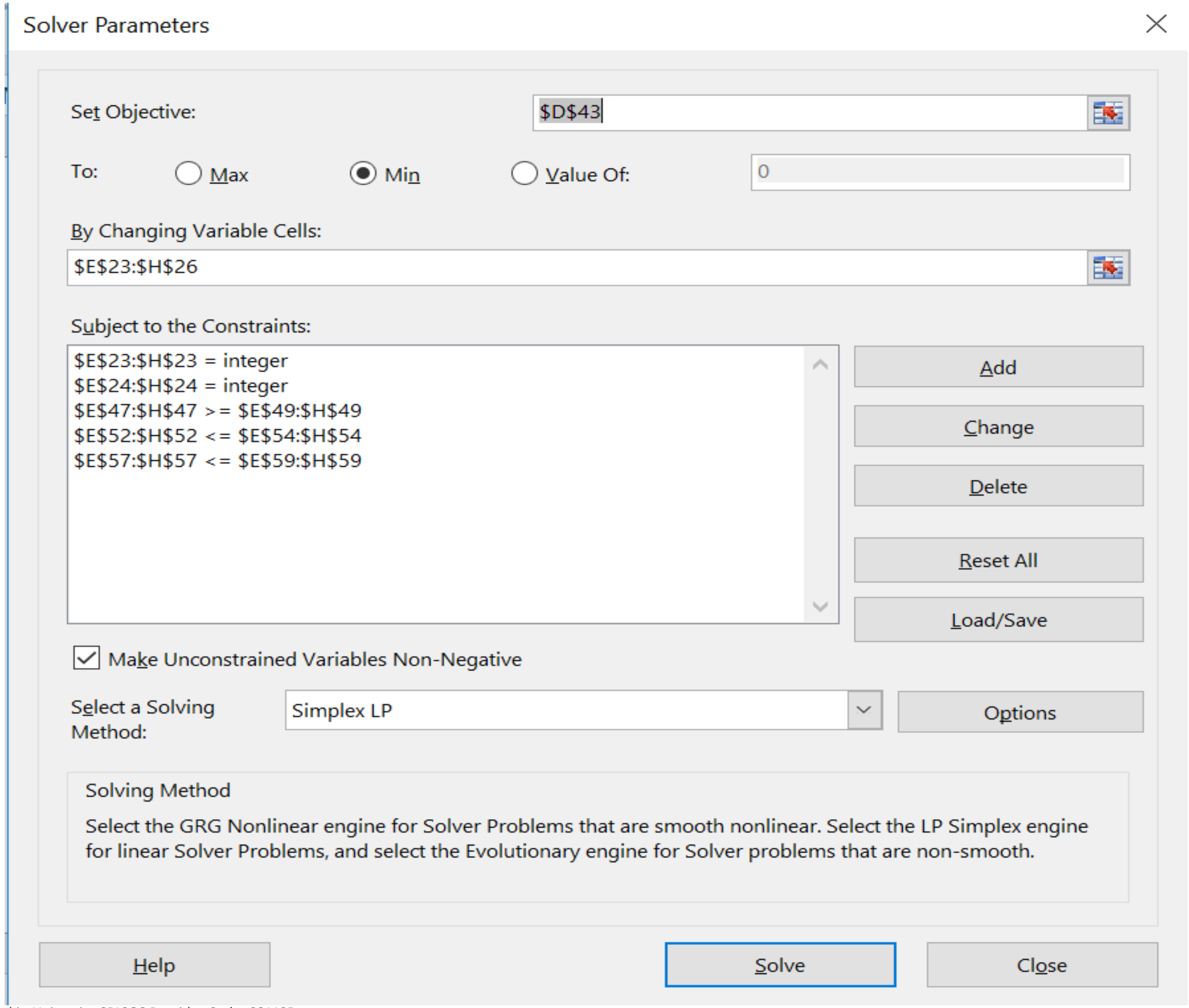
Total cost of hiring	$0
Total cost of firing	$100,000
Total cost of regular salary	$430,500
Total cost of overtime	$1,040
Total cost of materials	$157,500
Total cost of inventory	$3,780

OBJECTIVE FUNCTION

Minimise total cost	$692,820

Total cost

Solver Set-up



- Note the two integer constraints – these are required to ensure that Solver produces whole numbers (i.e. integers) for the number of workers hired and fired each month

Modelling Flexibility Provided by binary Variables

Binary variables are also useful in modeling a number of logical conditions. When x_i and x_j represent binary variables designating whether projects i and j have been completed, the following special constraints may be formulated:

At most *k out of n* projects will be completed:

$$\boxed{x_1 + x_2 + \ldots\ldots + x_n \leq k}$$

Exactly *k out of n* projects will be completed:

$$\boxed{x_1 + x_2 + \ldots\ldots + x_n = k}$$

Multiple-Choice Constraint requires that the sum of two or more 0-1 variables *equals* 1. Thus, any feasible solution makes a choice of which variable to set equal to 1.

$$\boxed{x_1 + x_2 + \ldots\ldots + x_n = 1}$$

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Modelling Flexibility Provided by 0-1 Variables

Project *j* is conditional on project *i (*project *j* can be completed only if project *i* has been completed, or if product *j* is completed then project *i* must be completed*)*:

$$\boxed{x_j - x_i \leq 0}$$

Project *i* is a co-requisite for project *j*

Either both project *i* and project *j* are completed, or neither of them is completed.

$$\boxed{x_j - x_i = 0}$$

Projects *i* and *j* are mutually exclusive

Project *i* and project *j* cannot go hand in hand.

$$\boxed{x_i + x_j \leq 1}$$

Logical constraints

Logical constraints	Constraint Model
out of project 1,2 and 3 ,at most one should be selected	X1+X2+X3 <=1
out of project 1,2 and 3, no more than one should be selected	X1+X2+X3 <=1
out of project 1,2 and 3, only exactly one should be selected	X1+X2+X3 = 1
project 2 cannot be selected unless project 1 is selected	X2<=X1
if project 2 is chosen then project 1 should be chosen	X2<=X1
If project 1 then not project 2 (either but not both)	X1 + X 2 <=1
if not project 1 then project 2 (at least one of that)	X1 + X2 >= 1
if project 1 then project 2 and project 3	X2 + X3 >= 2X1
if project 1 and 2 selected then project 3 must be selected	X3 + 1 >= X1 + X2

Summary

- We learnt how to perform sensitivity analysis in Excel using Solver
- We saw how to interpret the output from a sensitivity analysis, and learnt some important concepts along the way, such as:
  - **The difference between binding and non-binding constraints**
  - **The shadow price for the RHS of a constraint**
  - **The reduced cost of a decision variable cell**
- We also reviewed three standard types of LP models

Next Class

Topic 3: Integer Linear Programming Models (Chapter 6 of the textbook)

- We will start by considering the basic ideas behind these models
- Then we will consider three standard types of ILP models:
  - **Capital budgeting models (section 6.3)**
  - **Fixed cost models (section 6.4)**
  - **Cutting stock models (section 6.6)**