DEAKIN BUSINESS SCHOOL
DEPARTMENT OF INFORMATION SYSTEMS AND BUSINESS ANALYTICS

MIS775 Decision Modelling for Business Analytics
Revision Questions with Solutions

WEEK 3 - ILP QUESTIONS

Exercise 1. A builder is faced with cutting quantities of material of various sizes from a certain pipe that comes in 25m lengths. The builder needs at least 200 sections of 6m pipe; 250 sections of 9m pipe; and 300 sections of 12m pipe and he decides to cut each 25m pipe by using 6 patterns.

Cutting Pattern	Number of 6m sections	Number of 9m sections	Number of 12m	Scrap (m)
1	4	0	0	1
2	2	1	0	4
3	2	0	1	1
4	1	2	0	1
5	0	1	1	4
6	0	0	2	1
Demand	200	250	300	

The builder wants to know how frequently each cutting pattern should be used so that the demands are met and the amount of scrap is minimized. Formulate the ILP for this problem.

Exercise 2. A company needs to hire workers to cover a 7 day work week. Employees work 5 consecutive days with 2 days off. The demand for workers by day of the week and the wages per shift are:

Days of Week	Minimum Workers Required
Sunday	54
Monday	50
Tuesday	36
Wednesday	38
Thursday	42
Friday	40
Saturday	48

Shift	Days off	Wage
1	Sun & Mon	800
2	Mon & Tue	900
3	Tue & Wed	900
4	Wed & Thurs	900
5	Thurs & Fri	900
6	Fri & Sat	800
7	Sat & Sun	700

Formulate the ILP for this problem to minimize cost.

Exercise 3. An investor is considering 7 different stocks: A, B, C, D, E, F, and G. The expected annual return for each stock is provided as follows:

Stock	Annual return
A	8.5%
B	7.0%
C	6.0%
D	9.0%
E	7.0%
F	10.0%
G	9.0%

The investor has imposed the following restrictions regarding the composition of the portfolio:

- The portfolio must include exactly one of the following stocks: either A or B.
- If stock B is selected, then stock F must also be selected.
- If stock D is selected, then stock E must be excluded.

The investor wants to know which stocks should be included in the portfolio to maximize annual return. Formulate the ILP for this problem.

Quiz

1. Let x_1 and x_2 be binary variables whose values indicate whether projects 1 and 2 are not done (0) or are done (1). Which answer below indicates that project 2 can be done only if project 1 is done?

    a. $x_1 + x_2 = 1$

    b. $x_1 + x_2 = 2$

    c. $x_1 - x_2 \leq 0$

    d. $x_1 - x_2 \geq 0$

2. Let x_1, x_2 and x_3 be binary variables whose values indicate whether the projects are not done (0) or are done (1). Which answer below indicates that at least two of the projects must be done?

    a. $x_1 + x_2 + x_3 \geq 2$

    b. $x_1 + x_2 + x_3 \leq 2$

    c. $x_1 + x_2 + x_3 = 2$

    d. $x_1 - x_2 = 0$

3. In a model, $x_1 \geq 0$ and integer, $x_2 \geq 0$, and $x_3 = \{0, 1\}$. Which solution would not be feasible?

    a. $x_1 = 5, x_2 = 3, x_3 = 0$

    b. $x_1 = 4, x_2 = .389, x_3 = 1$

    c. $x_1 = 2, x_2 = 3, x_3 = .578$

    d. $x_1 = 0, x_2 = 8, x_3 = 0$

4. Rounding the solution of an LP relaxation* of an ILP model to the nearest integer values provides

    a. A feasible but not necessarily optimal integer solution.

    b. An integer solution that is optimal.

    c. An integer solution that might be neither feasible nor optimal.

    d. An infeasible solution.

*See page 281 of the textbook.

WEEK 3 SOLUTIONS

Exercise 1

Let X_i be the number of times that cutting pattern "i" (i = 1, 2, …, 6) is used.

The ILP model for the builder's problem is given by:

Decision Variables: $X_1, X_2, X_3, X_4, X_5, X_6$.

Objective: Minimize the total amount of scrap: $X_1 + 4X_2 + X_3 + X_4 + 4X_5 + X_6$

Subject to:

$4X_1 + 2X_2 + 2X_3 + X_4 \geq 200$ (6m constraint)

$X_2 + 2X_4 + X_5 \geq 250$ (9m constraint)

$X_3 + X_5 + 2X_6 \geq 300$ (12m constraint)

$X_i \geq 0$ and integer, ($i = 1, \ldots, 6$).

Exercise 2

Let X_i denote the number of workers in each shift ($i = 1, 2, \ldots, 7$)

Shift	Sun	Mon	Tue	Wed	Thu	Fri	Sat
1			X_1	X_1	X_1	X_1	X_1
2	X_2			X_2	X_2	X_2	X_2
3	X_3	X_3			X_3	X_3	X_3
4	X_4	X_4	X_4			X_4	X_4
5	X_5	X_5	X_5	X_5			X_5
6	X_6	X_6	X_6	X_6	X_6		
7		X_7	X_7	X_7	X_7	X_7	
Total	54	50	36	38	42	40	48

Decision Variables: $X_1, X_2, X_3, X_4, X_5, X_6, X_7$.

Minimize: $800\,X_1 + 900\,X_2 + 900\,X_3 + 900\,X_4 + 900\,X_5 + 800\,X_6 + 700\,X_7$

Subject to:

$X_2 + X_3 + X_4 + X_5 + X_6 \geq 54$ (minimum workers required for Sunday)

$X_3 + X_4 + X_5 + X_6 + X_7 \geq 50$ (minimum workers required for Monday)

$X_4 + X_5 + X_6 + X_7 + X_1 \geq 36$ (minimum workers required for Tuesday)

$X_5 + X_6 + X_7 + X_1 + X_2 \geq 38$ (minimum workers required for Wednesday)

$X_6 + X_7 + X_1 + X_2 + X_3 \geq 42$ (minimum workers required for Thursday)

$X_7 + X_1 + X_2 + X_3 + X_4 \geq 40$ (minimum workers required for Friday)

$X_1 + X_2 + X_3 + X_4 + X_5 \geq 48$ (minimum workers required for Saturday)

$X_i \geq 0$ and integer, ($i = 1, \ldots, 7$).

Exercise 3

Decision variables:

$X_i = 1$ if the portfolio includes stock i, otherwise 0 (i = A, B, C, D, E, F & G)

Objective:

Maximize $0.085 X_A + 0.07 X_B + 0.06 X_C + 0.09 X_D + 0.07 X_E + 0.10 X_F + 0.09 X_G$

Subject to:

$X_A + X_B = 1$ (exactly one of A or B is included)

$X_B - X_F \leq 0$ (if B is selected, then stock F must also be selected)

$X_D + X_E \leq 1$ (if D is selected, then stock E must be excluded).

Logic tables for checking constraints:

Portfolio must include exactly one of A or B		
X_A	X_B	
0	0	×
0	1	✓
1	0	✓
1	1	×

If stock B is selected, then stock F must also be selected		
X_B	X_F	
0	0	✓
0	1	✓
1	0	×
1	1	✓

If stock D is selected, then stock E must be excluded		
X_D	X_E	
0	0	✓
0	1	✓
1	0	✓
1	1	×

$X_A + X_B = 1$ $X_B - X_F \leq 0$ $X_D + X_E \leq 1$

Quiz

1) d. 2) a. 3) c. 4) c