3.5 Heat Transfer Correlations from Dimensional Analysis

3.5.1 Concepts Demonstrated

Correlation of heat transfer data using dimensionless groups.

3.5.2 Numerical Methods Utilized

Linear and nonlinear regression of data with linearization and use of transformation functions.

3.5.3 Excel Options and Functions Demonstrated

An important tool in the correlation of engineering data is the use of dimensional analysis. This treatment leads to the determination of the independent dimensionless numbers, which may be important for a particular problem. Linear and nonlinear regression can be very useful in determining the correlations of dimensionless numbers with experimental data.

A treatment of heat transfer within a pipe has been considered by Geankoplis[3] using the Buckingham method, and the result is that the Nusselt number is expected to be a function of the Reynolds and Prandtl numbers.

$$Nu = f(Re, Pr) \quad \text{or} \quad \frac{hD}{k} = f\left(\frac{Dv\rho}{\mu}, \frac{C_p\mu}{k}\right) \tag{3-19}$$

A typical correlation function suggested by Equation (3-19) is

$$Nu = aRe^b Pr^c \tag{3-20}$$

where a, b, and c are parameters that can be determined from experimental data.

A widely used correlation for heat transfer during turbulent flow in pipes is the Sieder-Tate[4] equation:

$$Nu = 0.023Re^{0.8} Pr^{1/3} (\mu/\mu_w)^{0.14} \tag{3-21}$$

in which a dimensionless viscosity ratio has been added. This ratio (μ/μ_w) is the viscosity at the mean fluid temperature to that at the wall temperature.

Table B–15 gives some of the data reported by Williams and Katz[5] for heat transfer external to 3/4-inch outside diameter tubes where the Re, Pr, (μ/μ_w) and Nu dimensionless numbers have been measured.

(a) Use multiple linear regression to determine the parameter values of the functional forms of Equations (3-20) and (3-21) that represent the data of Table B–15.
(b) Repeat part (a) using nonlinear regression.
(c) Which functional form and parameter values should be recommended as a correlation for this data set? Justify your selection.

3.5.4 Problem Definition

For convenience, let the functional forms of Equations (3-20) and (3-21) be written as

$$Nu = a_i Re^{b_i} Pr^{c_i} \tag{3-22}$$

$$Nu = d_i Re^{e_i} Pr^{f_i} Mu^{g_i} \tag{3-23}$$

where $i = 1$ indicates parameter values from linear regression and $i = 2$ indicates parameter values from nonlinear regression. Mu represents the viscosity ratio. The POLYMATH *Polynomial, Multiple Linear and Nonlinear Regression Program* can be used to carry out the linear and nonlinear regressions.

(a) A linear regression of either Equation (3-22) or (3-23) requires a transformation into a linear form. This is easily accomplished by taking the ln (natural logarithm) of each side of each equation. The resulting transformed equations are

$$\ln Nu = \ln a_1 + b_1 \ln Re + c_1 \ln Pr \tag{3-24}$$

$$\ln Nu = \ln d_1 + e_1 \ln Re + f_1 \ln Pr + g_1 \ln Mu \tag{3-25}$$

The data of Table B–15 can be entered directly into POLYMATH under columns defined as Re, Pr, Mu, and Nu. Additional columns can be created that provide the needed ln's for the linear regression by using lnRe = ln(Re), lnPr = ln(Pr), lnMu = ln(Mu) and lnNu = ln(Nu). The results of the linear regressions are summarized in Table 3–5. Note that the reported values for a_1 and d_1 have been calculated from the regression results that give the ln values of these parameters. The calculated values from the current linear regression can be automatically entered into the data sheet by pressing "s" from the Display Option menu, which gives the regression results.

The linear regression results in Table 3–5 for Equations (3-24) and (3-25) are consistent when comparing parameters a with d, b with e, and c with f. The 95% confidence intervals are all relatively small except for a, d and g, which have large intervals that include zero.

Table 3–5 Summary of Parameter Values from Regressions

Parameter	Linear Regression Equations (3-24) and (3-25)		Nonlinear Regression Equations (3-22) and (3-23)	
	Value	95% Confidence Interval	Value	95% Confidence Interval
a	0.6623	3.878	0.1655	0.009780
b	0.5395	0.1160	0.6636	0.005149
c	0.2454	0.1139	0.3414	0.01418
d	0.5347	5.634	0.1491	0.001117
e	0.5588	0.1507	0.6733	0.0007076
f	0.2524	0.1230	0.3286	0.003538
g	−0.06772	0.3164	−0.1778	0.03055

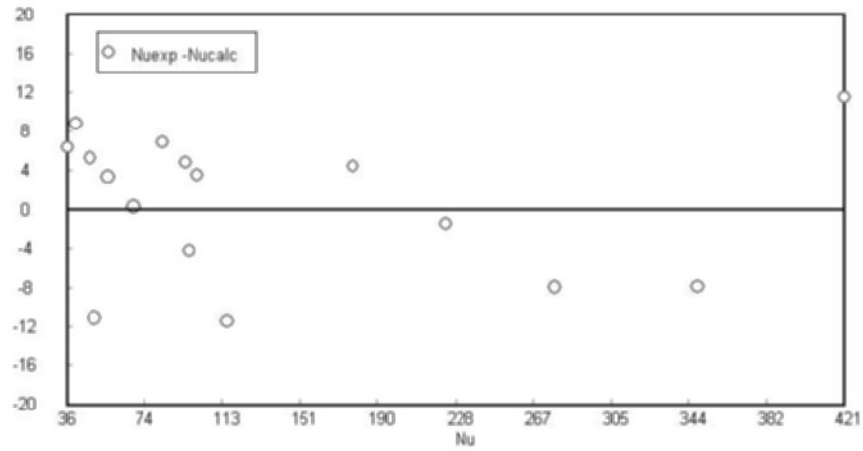
(b) Direct nonlinear regressions of both Equations (3-22) and (3-23) can be completed where the converged values from the linear regressions of part (a) can be used as initial parameter estimates. The results are also summarized in Table 3–5. Again the results are consistent when comparing parameters a with d, b with e, and c with f. The 95% confidence intervals are all relatively small except for g which has a large interval that includes zero.

(c) There are a number of considerations when selecting the most appropriate correlation.

Confidence Intervals A major indicator is the 95% confidence interval of each parameter. When the confidence interval is very large relative to the parameter, this suggests that the parameter may not be important in the correlation and perhaps should be set to zero. This seems to be the case in *both* the linear and nonlinear regressions carried out in parts (a) and (b) for the parameter g, which is the exponent of Mu. This is an indication that Mu should not be included in the correlation. An alternate explanation is that Mu may be dependent upon other variables in the regression. An examination of this will be considered later in this section.

Residual Plots It is always very useful to examine residual plots of regressions to determine if there are any obvious trends, as the errors should be random. A typical residual plot is shown in Figure 3–15 for the nonlinear regression of Equation (3-22). This residual plot demonstrates that the error is randomly distributed. Since there are no unusual error patterns in any of the linear and nonlinear regressions considered here, the residual plots are not helpful in selecting a correlation in this case.

Comparison of Variances A comparison of variances for these correlations deserves special attention because the dependent variable in the linear regression is $\ln(Nu)$ while in the nonlinear regression it is Nu. It is necessary to



Model: Nu = d2*Re^e2*Pr^f2*Mu^g2

Variable	Initial guess	Value	95% confidence
d2	0.53	0.149055	0.0011173
e2	0.56	0.6733455	0.0007076
f2	0.25	0.3286108	0.0035381
g2	-0.067	-0.177765	0.0305463

Figure 3–15 Residual Plot for Heat Transfer Data Using Nonlinear Regression with Equation (3-22)

use a variance based on the same variable for comparisons. The variance based on the Nusselt number for this problem is defined as

$$\sigma^2 = \frac{1}{\nu}\sum_{i=1}^{N}(Nu_{(\text{obs})} - Nu_{(\text{calc})})^2 \tag{3-26}$$

where ν is the degrees of freedom, which is equal to the number of data points less the number of model parameters, $(N - m)$. A relative error variance for this problem can also be defined as

$$\sigma_r^2 = \frac{1}{\nu}\sum_{i=1}^{N}\left(\frac{Nu_{(\text{obs})} - Nu_{(\text{calc})}}{Nu_{(\text{obs})}}\right)^2 \tag{3-27}$$

These variances can be calculated within POLYMATH after the various regressions have been completed by defining columns to evaluate the terms in the summation functions. There is a convenient option within POLYMATH that can sum individual columns. The resulting calculations are summarized in Table 3–6.

An examination of the *variance column* in Table 3–6 indicates that both correlations obtained with nonlinear regression are superior to both correlations

Table 3–6 Calculated Variances for Heat Transfer Correlations

Regression Equation	Variance σ^2	Relative Variance σ_r^2
Linear Regression Equation (3-24)	265.6	0.01028
Linear Regression Equation (3-25)	234.7	0.01135
Nonlinear Regression Equation (3-22)	68.28	0.01684
Nonlinear Regression Equation (3-23)	66.68	0.01477

obtained with linear regression. This is because higher *Nu* values have a greater influence on the regression. Consideration of the *relative variance column* suggests that both linear regressions result in the lowest relative variance. This is because the ln(*Nu*) is used as the dependent variable in the regression, which lessens the effect of data points with larger *Nu* values. This indicates the general conclusion that *logarithmic transformations are useful if relative errors are to be minimized*.

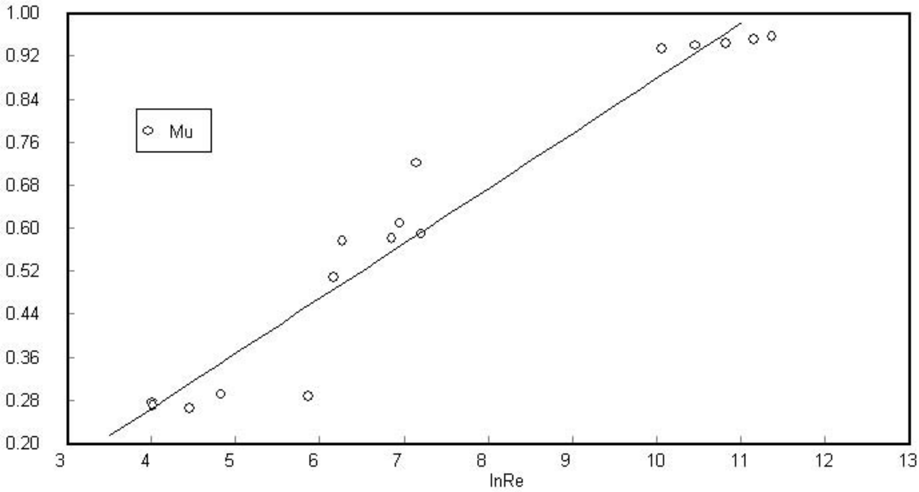
Thus the selection of the regression equation depends on the experimental errors as to whether they are relative or proportional to the measured *Nu* values. Since this information is not known about this data set, the selection cannot be made on the variance or relative variance calculations.

Possible Interdependency of Variables The linear and nonlinear regressions assume that the independent variables do not depend on each other. Possible dependency between assumed "independent" variables can be examined by plotting one variable with another. In this case, the large confidence interval on *Mu,* which includes zero, is an indication that *Mu* may be related to other variables. A regression of *Mu* versus ln(*Re*) shows definite dependence, as shown in Figure 3–16. Since the viscosity ratio apparently was not changed independently of *Re* during the experiments, its effect on *Nu* cannot be isolated. The conclusion with regard to the power of the viscosity ratio in Equation (3-23) is that the data of Katz and Williams are insufficient for determining an exponent for this ratio.

Final Correlation Since the nonlinear regression gives the *Nu* directly and the variance as calculated by Equation (3-26) is usually employed in regression, the recommended correlation for the data set is given by

$$Nu = (0.1655 \pm 0.00978) Re^{(0.6636 \pm 0.005149)} Pr^{(0.3414 \pm 0.01418)} \quad \textbf{(3-28)}$$

More experiments should be performed to investigate further the *Mu* ratio and to determine the best variance to use in the regression of this data set.



Model: Mu = a0 + a1*lnRe

Variable	Value	95% confidence
a0	-0.1419525	0.116736
a1	0.1024107	0.0150536

Figure 3–16 Plot of the Viscosity Ratio (μ/μ_w) versus ln of Reynolds Number $\ln(Re)$

The problem solution file is found in directory Chapter 3 with file names **P3-05AB1.POL** and **P3-05AB2.POL**.