Application of Numerical Problem Solving in Chemical Engineering Coursework

Presenters: Robert P. Hesketh, Rowan University; Michael B. Cutlip, University of Connecticut

Polymath **Software** problem-solving capabilities include

- **Linear Equations** - up to **264** simultaneous equations
- **Nonlinear Equations** - up to **300** simultaneous nonlinear and 300 additional explicit algebraic equations
- **Differential Equations** - up to **300** simultaneous ordinary differential and 300 additional explicit algebraic equations
- **Data analysis and Regression** - up to 200 variables with up to 1000 data points for each, with capabilities for **linear, multiple linear, and nonlinear regressions** with extensive statistics plus polynomial and spline fitting with interpolation and graphing capabilities

Integration of POLYMATH with Fogler's chemical reaction engineering textbook

Example 6-2 Membrane Reactor

$$\frac{dF_A}{dV} = r_A$$
$$\frac{dF_B}{dV} = -r_A - k_c C_B$$
$$\frac{dF_C}{dV} = r_C$$
$$r_A = -k\left(C_A - \frac{C_B C_C}{K_{eq}}\right)$$

TABLE E6-2.1 POLYMATH PROGRAM

Differential equations
1 d(Fa)/d(V) = ra
2 d(Fb)/d(V) = -ra-kc*Cto*(Fb/Ft)
3 d(Fc)/d(V) = -ra

Explicit equations
1 Kc = 0.05
2 Ft = Fa+Fb+Fc
3 k = 0.7
4 Cto = 0.2
5 ra = -k*Cto*((Fa/Ft)-Cto/Kc*(Fb/Ft)*(Fc/Ft))
6 kc = 0.2

Calculated values of DEQ variables

	Variable	Initial value	Final value
1	Cto	0.2	0.2
2	Fa	10.	3.995179
3	Fb	0	1.832577
4	Fc	0	6.004821
5	Ft	10.	11.83258
6	k	0.7	0.7
7	Kc	0.05	0.05
8	kc	0.2	0.2
9	ra	-0.14	-0.0032558
10	V	0	500.

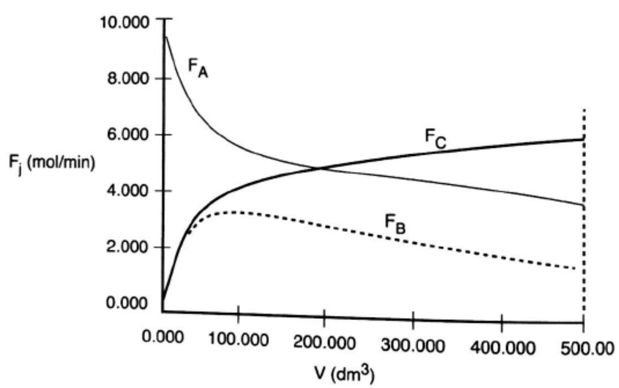
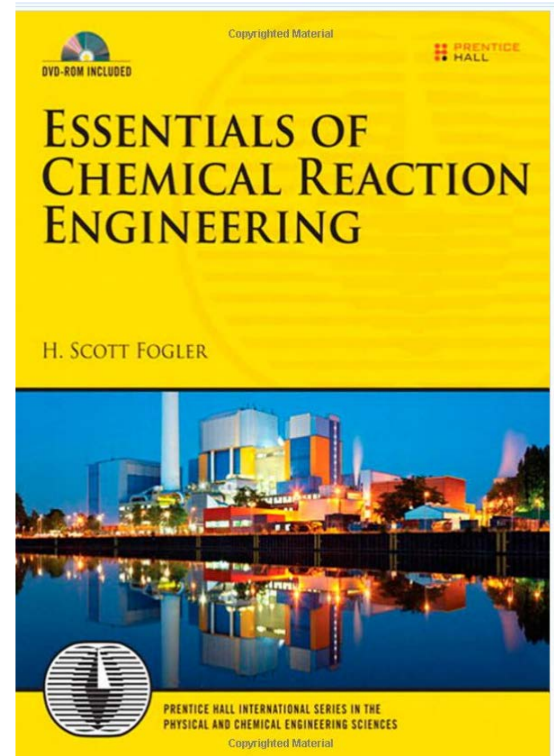


Figure E6-2.1 Polymath solution.

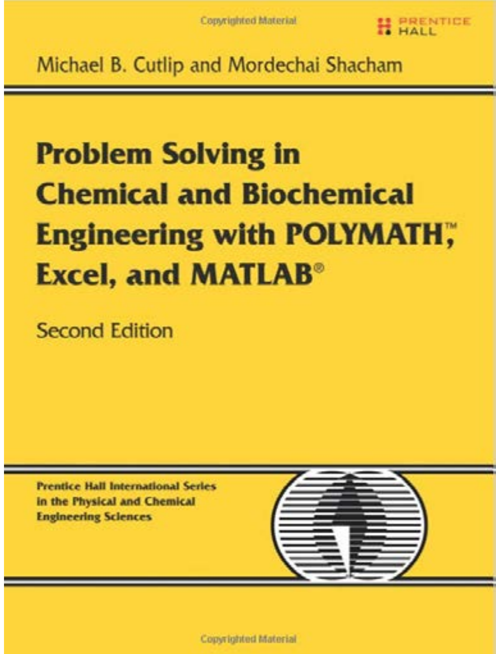


Integration already accomplished! No extra work required.

How do you integrate POLYMATH into your course?

Start with examples from POLYMATH Text

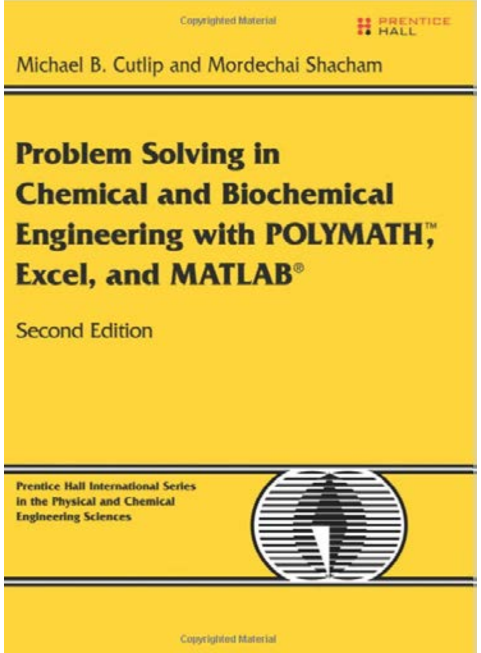
- Thermodynamics,
- Fluid Mechanics,
- Heat Transfer,
- Mass Transfer,
- Chemical Reaction Engineering,
- Phase Equilibria and Distillation,
- Process Dynamics and Control,
- Biochemical Engineering



Polymath Text: Fluids Course

Table I-3 Problems in Fluid Mechanics

NO.	PROBLEMS IN FLUID MECHANICS	PAGE
4.2	Excel—Calculation of the Flow Rate in a Pipeline	110
5.2	MATLAB—Calculation of the Flow Rate in a Pipeline	165
8.1	Laminar Flow of a Newtonian Fluid in a Horizontal Pipe	283
8.2	Laminar Flow of Non-Newtonian Fluids in a Horizontal Pipe	289
8.3	Vertical Laminar Flow of a Liquid Film	291
8.4	Laminar Flow of Non-Newtonian Fluids in a Horizontal Annulus	294
8.5	Temperature Dependency of Density and Viscosity of Various Liquids	297
8.6	Terminal Velocity of Falling Particles	299
8.7	Comparison of Friction Factor Correlations for Turbulent Pipe Flow	301
8.8	Calculations Involving Friction Factors for Flow in Pipes	303
8.9	Average Velocity in Turbulent Smooth Pipe Flow from Maximum Velocity	306
8.10	Calculation of the Flow Rate in a Pipeline	307
8.11	Flow Distribution in a Pipeline Network	309
8.12	Water Distribution Network	313
8.13	Pipe and Pump Network	315
8.14	Optimal Pipe Length for Draining a Cylindrical Tank in Turbulent Flow	317
8.15	Optimal Pipe Length for Draining a Cylindrical Tank in Laminar Flow	320
8.16	Baseball Trajectories as a Function of Elevation	322
8.17	Velocity Profiles for a Wall Suddenly Set in Motion—Laminar Flow	325
8.18	Boundary Layer Flow of a Newtonian Fluid on a Flat Plate	328
10.15	Diffusion and Reaction in a Falling Laminar Liquid Film	438



21 Problems in fluids

Example Schedule of Topics for ChE Fluids (2 credit hour)

Chemical Engineering Fluid Mechanics Schedule of Topics → Spring 2016 → Revised 3/18/2016

TOPIC LIST & COURSE SCHEDULE (TENTATIVE)
Tuesday 08:00 AM – 10:45 AM ROW 340 (Double Period)
Friday 08:00 AM – 9:15 AM ROW 340 (Single Period)
All Chapter and section references are to the de Nevers text unless referenced otherwise.

Polymath: Nonlinear Equation Solver (NLE)

Polymath: Differential Equation Solver (DEQ) & COMSOL

Date	Topics
January 19 Tuesday	**Introduction to Course, Objectives, Syllabus** Team Problem Solving, Inductive Topic Order Chemical Engineers $\rho g = \gamma$ Mechanical Engineers (eqn 2.7) **Fluids Lab 1: Introduction to Fluids Experiments** **Chapter 2** Fluid Statics Sections 2 - 2.2, 2.6, 2.7 and **Chapter 5 Elementary Fluid Dynamics** (Also review Felder & Rousseau Section 3.1-3.4 Fluid Pressure, Hydrostatic Head, Manometers)
22 Friday	**Fluid Flow without accounting for friction** **Review of Intro to Fluids lab** **Chapter 2 & Chapter 5** – The Bernoulli Equation - Neglecting Friction! Felder & Rouseau 7.7 Mechanical Energy Balances, eqn 7.7-1 3.5 Unsteady-State Mass Balances
26 Tuesday	3.5 Unsteady-State Mass Balances (continued) 3.4.1 Average Velocity Applications of Unsteady-State Mass Balances and Bernoulli's Equation Tank Drainage Problem **Fluids Lab 2: Tank Drainage & Siphon Experiments**
29 Friday	Applications of Bernoulli's Equation continued: 5.5 Diffusers and Sudden Expansions, ;
February 2 Tuesday	**Fluids Lab 3:** **F1-15 Bernoulli's Theorem – venturi** **F1-17 Orifice and Free Jet Flow** **Pressure Drop in Pipes**: Hampden **Computer Lab: Introduction to POLYMATH Laboratory**
5 Friday	5.8.3 Venturi, and Restrictions on the Use of the Bernoulli Equation 5.8.1 & 5.8.2 Pitot tube
9 Tuesday	**Chapter 6 Viscous Flow in Pipes** Incompressible Flow in Pipes and Channels Figure 6.10: Friction Factor Chart 6.1 Reynolds Number (*Re*) and viscosity μ Cutlip & Shacham 8.7 Comparison of Friction Factor Correlations for Turbulent Pipe Flow Cutlip & Shacham 8.8 Calculations Involving Friction Factors for Flow in Pipes
12 Friday	6.5 Pipe Flow Problems – fanning friction factor Example problems: **simple piping** Standard Steel Pipe Properties: Appendix A.2 page 598, Standard Tube Properties: Cutlip & Shacham p699, Chemical Engineer's Handbook has both
16 Tuesday	**Fluids Lab 4:** **Pressure Drop in Pipeline Elements: Hampden** **F1-22 Energy Losses in Bends and Fittings** **Osborne-Reynolds Demonstration** **Computer Lab - Excel**
19 Friday	6.8 & 6.9 Minor Pressure Losses. Frictional Losses in Pipeline Elements Perry's p6-16 (See Table 6-4 for turbulent, Table 6-5 for laminar,) Review for Exam 1 Cutlip & Shacham 8.10 Calculation of the Flow Rate in a Pipeline Cutlip & Shacham 8.14 Optimal Pipe Length for Draining a Cylindrical Tank in Turbulent Flow Cutlip & Shacham 8.14 Optimal Pipe Length for Draining a Cylindrical Tank in Laminar Flow
23 Tuesday	6.13 Terminal Velocities Solid Objects and Spheres **Fluids Lab 5:** **Lab: Measurement of Terminal Velocities**
26 Friday	**Exam 1: Chapters 2 and 5**
March 1 Tuesday	Terminal Velocity Continued
4 Friday	6.2 Laminar and Turbulent flow 6.3 Laminar Flow Velocity Profile Entrance Region and Fully Developed Flow
8 Tuesday	6.10.3 Tubulent Flow in Noncircular Channels 6.10.2 Seal Leaks 6.12 Economic Pipe Diameter, Economic Velocity
11 Friday	**Introduction to Pipe Flow Rate Measurements: orifice, venturi and rotameter** Permanent and Temporary Pressure Loss How to purchase a Flowmeter 5.8: Bernoulli Equation Perry's 10-6 to 10-20 Measurement of Flow
14-19	**Spring Break**
22 Tuesday	**Chapter 7: Mass, Energy, and Momentum Balances** 7.2 Momentum Balance Typical Forces: gravity, Pressure and Wall Shear Stress **Fluids Lab 6: Flowmeters:** **Rotameter – Variable Area Flowmeter, Venturi. Orifice, Pitot tube**
25 Friday	Good Friday: No Classes
29 Tuesday	7.3 Momentum Balances Applications: Flow Through a Nozzle (Example 7.5), U-Bend in piping, Reducing Elbow, Jet Ejector Pump (7.3.5) Macroscopic Control Volume: Pressure drop and Wall Stress
April 1 Friday	Microscopic Control Volume – Derivation of laminar flow velocity profile Examples of the Momentum Balances: Alpha term in Bernoulli Equation & Diameter of a Free Jet
5 Tuesday	Examples of the Momentum Balance Continued: Impinging jet, Orifice Plate, Sudden Expansion, 7.4 Relative velocities & Trolley Example Review for Exam, **Impact of a Jet Videos** (See the Force of water)
8 Friday	**Exam 2: Chapter 6 and 5.8: Pipe Flow, Fittings & Valves, and Flowmeters**
12 Tuesday	Examples of the Momentum Balance Continued : Rotameter (also see Chapter 6 in Denn), 6.10.3 Turbulent flow in Noncircular Channels **Fluids Lab 7: Aspirator laboratory**
15 Friday	7.5 Starting and Stopping Flows: Water Hammer
19 Tuesday	7.7 Introduction to Angular Momentum
22 Friday	**Chapter 9: Dimensionless Numbers and Dimensional Analysis**
26 Tuesday	Chapter 9: Dimensionless Numbers and Dimensional Analysis (continued) **(3rd floor computer lab not available)**
29 Friday	Review for Comprehensive Final Exam
May	Final Exam 3 May 2016 CHEM ENGINEER FLUID MECHNICS Hesketh, Robert Paul T 0800 1000 ROWAN 340 (Exam) Go out and design a fluid transport system for your parent's fountain and pond

Adv. ChE Fluids (2 cr)

Tentative Schedule of Topics

Process Fluid Transport CHE06 309 2 2016

Polymath: Nonlinear Equation Solver (NLE)

Polymath: Differential Equation Solver (DEQ) & COMSOL

Date:	Proposed Topics for Section 1: **Wednesday (double period)** - Friday (single period)
September 9/2/16 Friday	Course Introduction **Review of Fluid Mechanics**: Statics and Bernoulli Chapter 3.3 Pumps and Gas-Moving Equipment – Geankoplis Chapter 10: Centrifugal Pumps - FMChE
9/7/16 Wednesday	Centrifugal Pumps (continued)
9/9/16 Friday	Centrifugal Pumps (continued)
9/14/16 Wednesday	Centrifugal Pumps: NPSH Complex Flow Networks C&S2nd 8.11 and **FMChE** pages 213-214
9/16/16 Friday	Complex Flow Networks C&S2nd 8.11 and **FMChE** pages 213-214 (continued)
9/21/16 Wednesday	Single Pump Lab: Standard Pump Curve POLYMATH – C&S 6.1 & 6.5 (If new to POLYMATH review POLYMATH Introduction)
9/23/16 Friday	Chapter 10: Introduction to Positive Displacement Pumps (Syringe and Squirt Gun) – **FMChE**
9/28/16 Wednesday	Review Chapter 7 The Momentum Balance sections through 7.2 and Geankoplis 2.8 Macroscopic Momentum Balance Pipe Flow Laminar Flow Between Parallel Plates Geankoplis 2.9C
9/30/16 Friday	Laminar Flow Between Parallel Plates Geankoplis 2.9C (continued) Momentum Balance Derivation for Laminar flow in a pipe C&S2nd 8.1 Geankoplis 2.9B Chapter 20 Computational Fluid Dynamics- **FMChE**
October 10/5/16 Wednesday	Comsol Fluids Computer Lab – Introduction Flow Between Parallel Plates Exam 1: Pumps and Complex Flow Networks
10/7/16 Friday	Momentum Balance Derivation for Laminar flow in a pipe (continued)
10/12/16 Wednesday	C&S2nd 8.3 Vertical Laminar Flow of a Liquid Film – Newtonian fluid Geankoplis 2.9C Comment on Laminar Flow in an Annulus **Navier Stokes Equations**: Geankoplis 3.6 – 3.7, 3.8B and Chapter 15: Two and Three Dimensional Fluid Mechanics - FMChE Flow Between two coaxial Cylinders, Fluid flow in a rotating cylinder, Geankoplis 3.8C Flow Between two coaxial Cylinders, Fluid flow in a rotating cylinder, Geankoplis 3.8C
10/14/16 Friday	Comsol Fluids Computer Lab – Rotational Flows (Bring your LAPTOP to class)
10/19/16 Wednesday	Geankoplis 3.5 Non-Newtonian Fluids **Non-Newtonian Fluids** – Flow between parallel plates – power law fluid & Bingham Plastics **Non-Newtonian Fluids** – Flow in a horizontal pipe – power law fluid & Bingham Plastics C&S2nd 8.2 Non-Newtonian laminar flow in a horizontal pipe Geankoplis 3.5H Non-Newtonian laminar flow in a horizontal pipe

10/21/16 Friday	Non-Newtonian Fluid Flow Continued Chapter 13 Non-Newtonian Fluid Flow in Circular Pipes – **FMChE** C&S2nd 8.3 Vertical Laminar Flow of a Liquid Film – Non-Newtonian fluid C&S2nd 8.4 Laminar Flow of Non-Newtonian Fluids in a Horizontal Annulus
10/26/16 Wednesday	Geankoplis 3.5E Laminar Flow of time-Independent Non-Newtonian fluids **Comsol Fluids Computer Lab-Non Newtonian Flows**
10/28/16	Geankoplis 3.1C Flow in Packed Beds
	h 13.3,
	n fluid video
	n fluid video
	e it to other uct is the ing this demo.
Wednesday	**Fluidized Bed Experiment**
December, 12/2/16 Friday	Gas-Liquid Flows Continued Compressible Gas Flows Chapter 8 **FMChE** and Geankoplis 2.11 Nozzle Choking, 8.3 **FMChE**
12/7/16 Wednesday	Mixing Geankoplis 3.4 Agitation and Mixing of Fluids and Power Requirements and Chapter 19 Mixing – **FMChE** **POLYMATH and COMSOL Quizzes**
12/9/16 Friday	Geankoplis 3.4 Agitation and Mixing of Fluids and Power Requirements and Chapter 19 Mixing – **FMChE** **continued** Evaluations Review for final
12/14/16 Wednesday	**Comprehensive Final Exam 8:00 10:00 AM ROWAN 340**
Finals Week	14-20 December
	Go out and make your holiday process fluid transport toys – They make great gifts!

Polymath: Nonlinear Equation Solver (NLE)

Polymath: Differential Equation Solver (DEQ) & COMSOL

Typical Fluids Problems

ChE Course	Problem Name	Numerical Method Illustrated	Equations
Fluids	Unsteady-state tank drainage using a siphon tube (similar to POLYMATH text 8.14) *C&S8-14soln.pdf*	Solution of an first order ordinary differential equation (DEQ)	$\frac{dh_T}{dt} = v_{out}\frac{A_{out}}{A_{tank}}$ $v_{out} = f(h_T)$
	Calculations involving Friction Factors for Flow in Pipes (POLYMATH Text 8.7) and *pipeflow homework frictionfactorcalcsoln.pdf* *Excel Tutorial Solver Add-Ins rev4.pdf*	Solution of a system of simultaneous nonlinear algebraic equations (NLE)	$\frac{\Delta P}{\Delta L} = 2f_F\frac{\rho v^2}{D}$ $f_F = f(\varepsilon/D\,, Re)$ $Re = \rho v D/\mu$
Advanced Fluids	NonNewtonian fluid flow through a pipe (POLYMATH Text 8.2c) *NonNewtonian C&S 8.2 solutions & comsol.pdf* NonNewtonian fluid flow through an annulus (POLYMATH Text 8.4) *NonNewtonian C&S8.4 polymath&comsol & 3.8-8 solutions 2017.pdf*	Solution of 2 simultaneous first order ordinary differential equations with split boundary value conditions and comparison with solution using COMSOL which is an advanced finite element program	$\frac{d(r\tau_{rx})}{dr} = -\frac{dP}{dx}r$ $\tau_{rx} = -K\left(\frac{dv_x}{dr}\right)\left(\left\|\frac{dv_x}{dr}\right\|\right)^{(n-1)}$

What about models that are formulated as integrals?

- Previous *state of the art* numerical methods where based on evaluating integrals
  - Trapezoidal rule
  - Simpson’s Rule
- Many textbooks present models only as integrals

Packed Towers: Gas Absorption
Traditional Approach using integrals

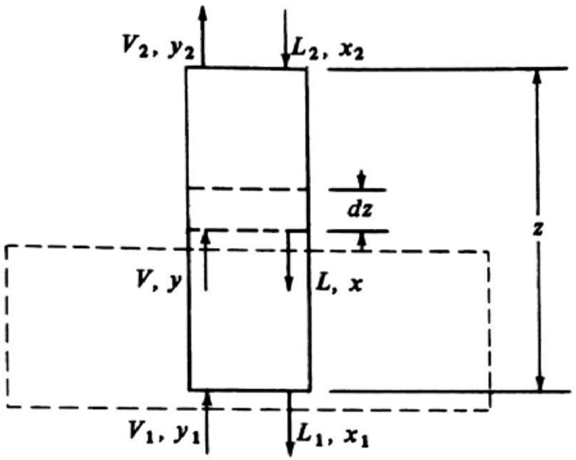


FIGURE 10.6-9. *Material balance for a countercurrent packed absorption tower.*

5. *Design method for packed towers using mass-transfer coefficients.* For absorption of A from stagnant B, the operating-line equation (10.6-5) holds. For the differential height of tower dz in Fig. 10.6-9, the moles of A leaving V equal the moles entering L:

$$d(Vy) = d(Lx) \tag{10.6-10}$$

where V = kg mol total gas/s, L = kg mol total liquid/s, and $d(Vy) = d(Lx)$ = kg mol A transferred/s in height dz m. The kg mol A transferred/s from Eq. (10.6-10) must equal the kg mol A transferred/s from the mass-transfer equation for N_A. Equation (10.4-8) gives the flux N_A using the gas-film and liquid-film coefficients:

$$N_A = \frac{k'_y}{(1-y_A)_{iM}}(y_{AG} - y_{Ai}) = \frac{k'_x}{(1-x_A)_{iM}}(x_{Ai} - x_{AL}) \tag{10.4-8}$$

where $(1-y_A)_{iM}$ and $(1-x_A)_{iM}$ are defined by Eqs. (10.4-6) and (10.4-7). Multiplying the left-hand side of Eq. (10.4-8) by dA and the two right-side terms by $aS\,dz$ from Eq. (10.6-9),

$$N_A\,dA = \frac{k'_y a}{(1-y_A)_{iM}}(y_{AG} - y_{Ai})S\,dz = \frac{k'_x a}{(1-x_A)_{iM}}(x_{Ai} - x_{AL})S\,dz \tag{10.6-11}$$

where $N_A\,dA$ = kg mol A transferred/s in height dz m (lb mol/h).

Equating Eq. (10.6-10) to (10.6-11) and using y_{AG} for the bulk gas phase and x_{AL} for the bulk liquid phase,

Dropping the subscripts A, G, and L and integrating, the final equations are as follows using film coefficients:

$$\int_0^z dz = z = \int_{y_2}^{y_1} \frac{V\,dy}{\dfrac{k'_y aS}{(1-y)_{iM}}(1-y)(y-y_i)} \tag{10.6-17}$$

$$\int_0^z dz = z = \int_{x_2}^{x_1} \frac{L\,dx}{\dfrac{k'_x aS}{(1-x)_{iM}}(1-x)(x_i-x)} \tag{10.6-18}$$

Derive model using Plug Flow Assumption: Create Differential Equations

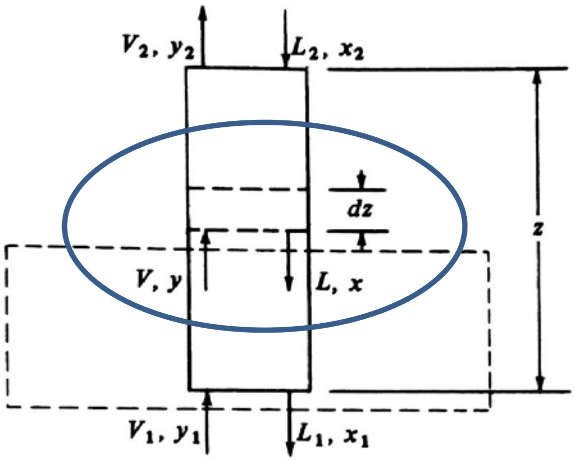


FIGURE 10.6-9. *Material balance for a countercurrent packed absorption tower.*

$$\frac{d(Vy_{AG})}{dz} = -\frac{k_y' aS}{(1-y_A)_{iM}}(y_{AG} - y_{Ai})$$

$$\frac{d(Lx_{AG})}{dz} = -\frac{k_x' aS}{(1-x_A)_{iM}}(x_{Ai} - x_{AL})$$

5. *Design method for packed towers using mass-transfer coefficients.* For absorption of A from stagnant B, the operating-line equation (10.6-5) holds. For the differential height of tower dz in Fig. 10.6-9, the moles of A leaving V equal the moles entering L:

$$d(Vy) = d(Lx) \qquad \textbf{(10.6-10)}$$

where V = kg mol total gas/s, L = kg mol total liquid/s, and $d(Vy) = d(Lx)$ = kg mol A transferred/s in height dz m. The kg mol A transferred/s from Eq. (10.6-10) must equal the kg mol A transferred/s from the mass-transfer equation for N_A. Equation (10.4-8) gives the flux N_A using the gas-film and liquid-film coefficients:

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where $(1-y_A)_{iM}$ and $(1-x_A)_{iM}$ are defined by Eqs. (10.4-6) and (10.4-7). Multiplying the left-hand side of Eq. (10.4-8) by dA and the two right-side terms by $aS\,dz$ from Eq. (10.6-9),

$$N_A\,dA = \frac{k_y' a}{(1-y_A)_{iM}}(y_{AG} - y_{Ai})S\,dz = \frac{k_x' a}{(1-x_A)_{iM}}(x_{Ai} - x_{AL})S\,dz \qquad \textbf{(10.6-11)}$$

where $N_A\,dA$ = kg mol A transferred/s in height dz m (lb mol/h).

Equating Eq. (10.6-10) to (10.6-11) and using y_{AG} for the bulk gas phase and x_{AL} for the bulk liquid phase,

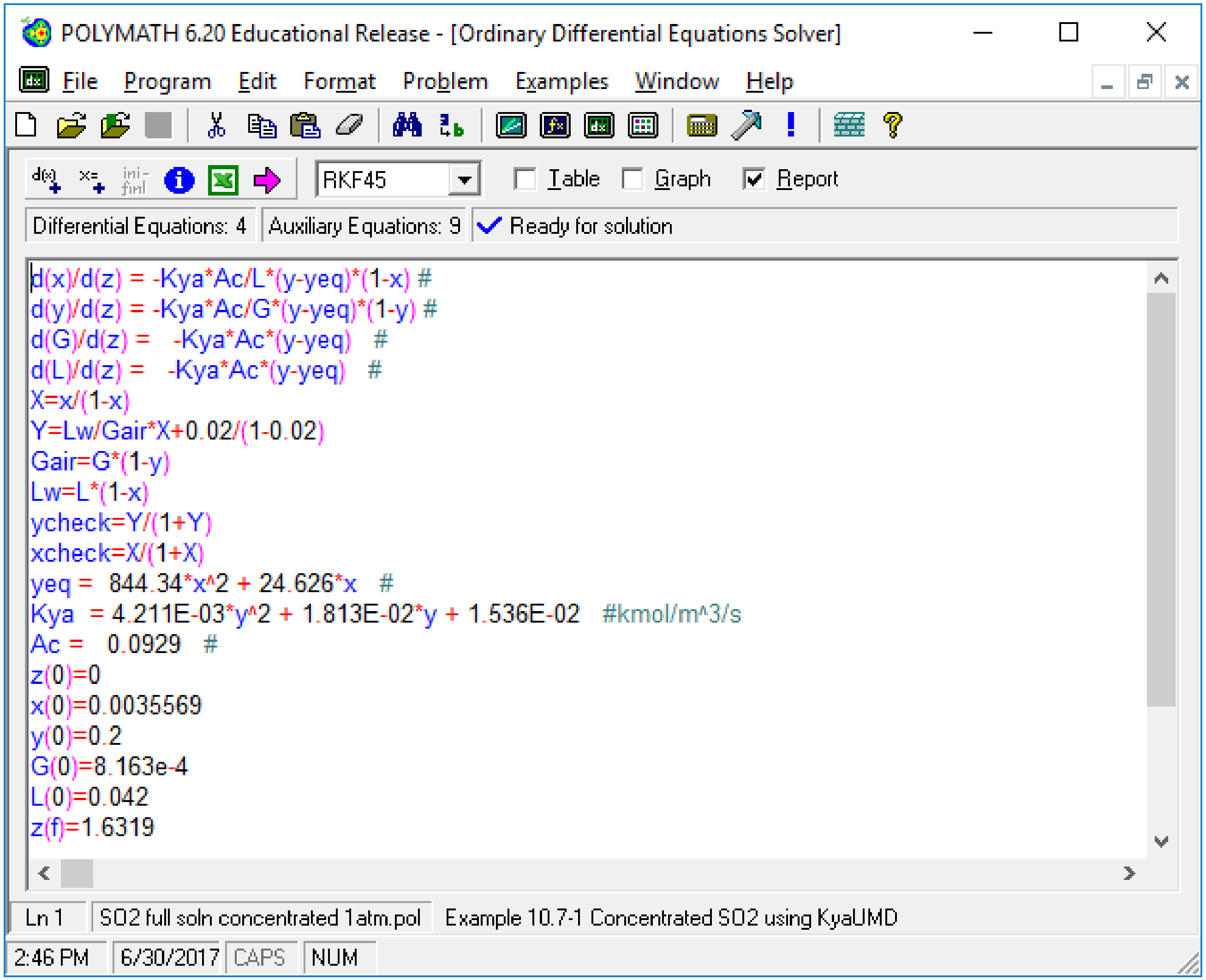
$$d(Vy_{AG}) = \frac{k_y' a}{(1-y_A)_{iM}}(y_{AG} - y_{Ai})S\,dz \qquad \textbf{(10.6-12)}$$

$$d(Lx_{AL}) = \frac{k_x' a}{(1-x_A)_{iM}}(x_{Ai} - x_{AL})S\,dz \qquad \textbf{(10.6-13)}$$

Since $V' = V(1-y_{AG})$ or $V = V'/(1-y_{AG})$,

$$d(Vy_{AG}) = d\left(\frac{V'}{(1-y_{AG})}y_{AG}\right) = V'\,d\left(\frac{y_{AG}}{1-y_{AG}}\right) = \frac{V'\,dy_{AG}}{(1-y_{AG})^2} \qquad \textbf{(10.6-14)}$$

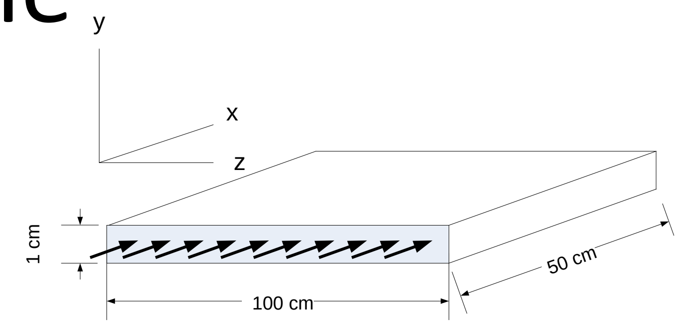
Polymath Absorber Model

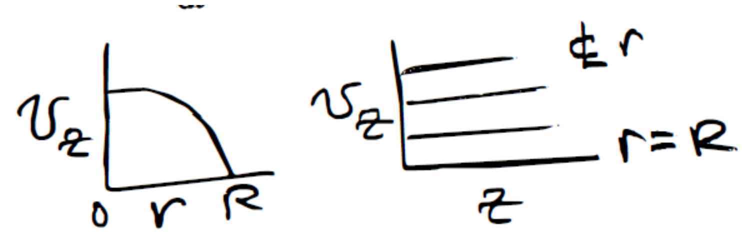


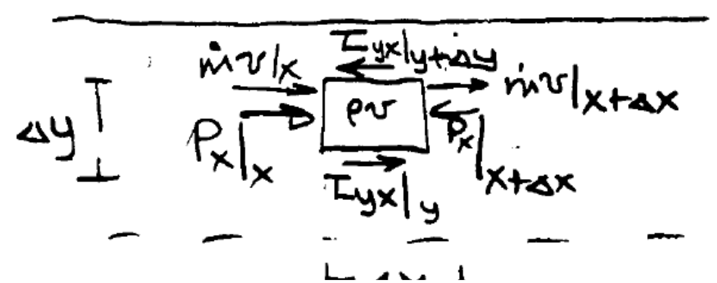
POLYMATH is a learning tool

- Students need to derive the model equations
- Then enter them into POLYMATH
- POLYMATH is not a “canned” program in which the equations are hidden such as in COMSOL and ASPEN
- The next slides give an example of using POLYMATH with a problem in Fluids

Newtonian Fluid Flow Between Parallel Plates Example







- Figures showing flow
- Graphs with expected behavior
- Control Volume – shell balance
- Derivation
- Simplifications: steady-state etc.

$$\cancelto{0}{\frac{\partial(\rho v)}{\partial t}} = -\cancelto{0}{\frac{\partial(\rho v v)}{\partial x}} - \frac{\partial P_x}{\partial x} - \frac{\partial(\tau_{yx})}{\partial y}$$

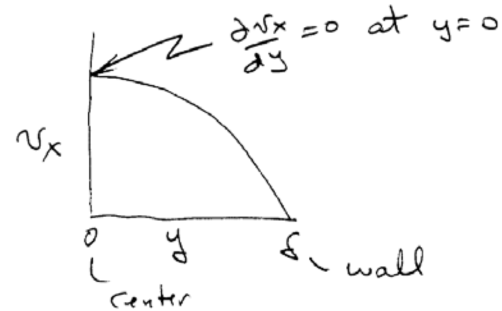
Analytical Solution

- Newtonian Fluid

$$\tau_{yx} = -\mu \frac{\partial v_x}{\partial y}$$

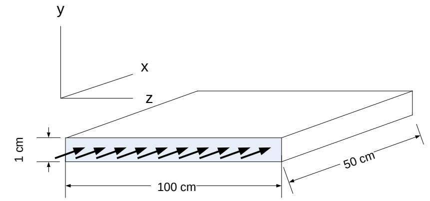
$$\frac{\partial(\tau_{yx})}{\partial y} = \underbrace{-\frac{\partial P_x}{\partial x}}_{\text{this is a constant}}$$

- Boundary Conditions
  - ➢ $y = 0 \quad v_x = max \quad \tau_{yx} = 0$
  - ➢ $y = wall \; v_x = 0 \quad \tau_{yx} = \max$



- Integrate Twice: Analytical Solution
  - ➢ $v_x = -\frac{dP}{dx}\left(\frac{\delta^2}{2\mu}\right)\left[1 - \left(\frac{y}{\delta}\right)^2\right]$

Numerical Solution



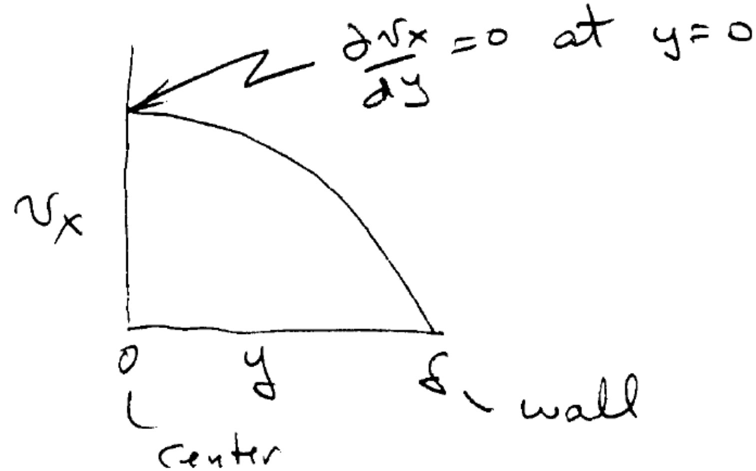
- Newtonian Fluid

$$\tau_{yx} = -\mu \frac{\partial v_x}{\partial y}$$

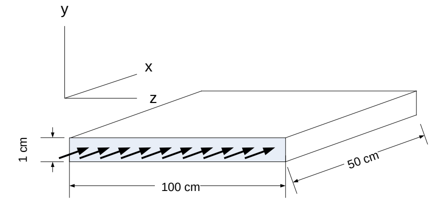
$$\frac{\partial (\tau_{yx})}{\partial y} = -\underbrace{\frac{\partial P_x}{\partial x}}$$

this is a constant

- Two coupled ODE's
  Split Boundary Condition
- Then manipulate two ODE's so they can be solved using the POLYMATH Differential Equation Solver (DEQ)



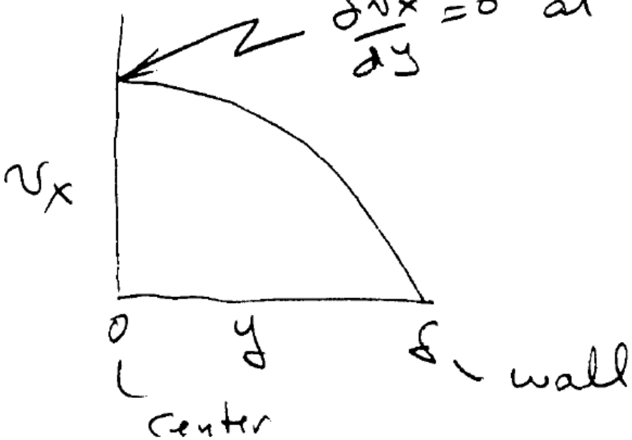
Required manipulation to solve 2 ODE's with split Boundary conditions



$$\tau_{yx} = -\mu \frac{\partial v_x}{\partial y} \quad \Rightarrow \quad \left(\frac{\partial v_x}{\partial y}\right) = \frac{\tau_{yx}}{-\mu}$$

$$\frac{\partial(\tau_{yx})}{\partial y} = -\underbrace{\frac{\partial P_x}{\partial x}}_{\text{this is a constant}} \quad \Rightarrow \quad \left(\frac{\partial \tau_{yx}}{\partial y}\right) = -\frac{dP}{dx}$$

$\frac{\partial v_x}{\partial y} = 0$ at $y = 0$

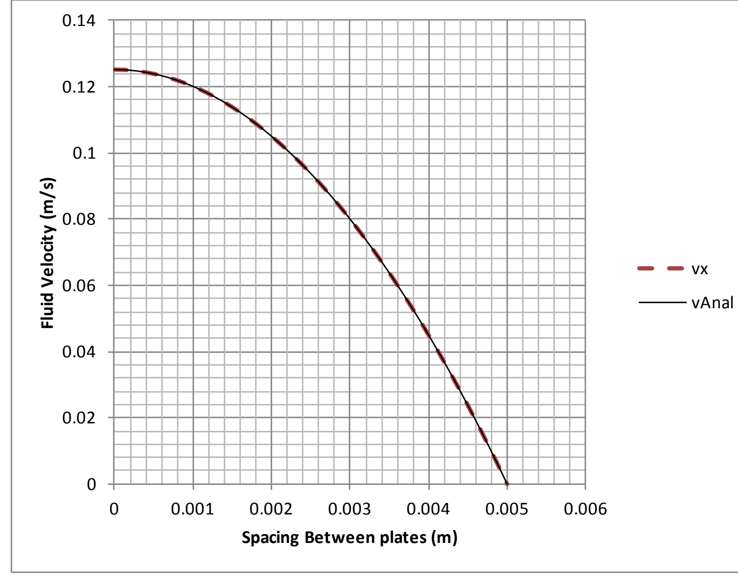


$y = 0 \quad v_x = \max \quad \tau_{yx} = 0$

$y = wall \ v_x = 0 \quad \tau_{yx} = \max$

Integration starts at y=0 and both initial conditions must be known! Solution is to guess v_x at $y = 0$ until at y =wall v_x=0

Solution



Differential Equations Solution #6

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POLYMATH Report Newtonian Fluid Flow Between 2 parallel plates
Ordinary Differential Equations 30-Jun-2017

Calculated values of DEQ variables

	Variable	Initial value	Minimal value	Maximal value	Final value
1	delta	0.005	0.005	0.005	0.005
2	dPdx	100.	100.	100.	100.
3	mu	0.01	0.01	0.01	0.01
4	Re	62.5	1.9396E-13	62.5	1.9396E-13
5	rho	1000.	1000.	1000.	1000.
6	Tau	0	0	0.5	0.5
7	vAnal	0.125	0	0.125	0
8	vx	0.125	3.8793E-16	0.125	3.8793E-16
9	y	0	0	0.005	0.005

Differential equations

1 d(Tau)/d(y) = dPdx

2 d(vx)/d(y) = -Tau/mu

Explicit equations

1 rho = 1000

2 mu = 0.01

3 delta = 0.01/2

4 Re = rho*delta*vx/mu

5 dPdx = 100

6 vAnal = dPdx*delta^2/2/mu*(1-(y/delta)^2)

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RKF45 Table Graph Report

Differential Equations: 2 | Auxiliary Equations: 6 | Ready for solution

```
d(Tau) / d(y) = dPdx
d(vx) / d(y) = -Tau/mu
rho=1000
Re = rho*delta*vx/mu
delta = 0.01/2
vAnal = dPdx*delta^2/2/mu*(1-(y/delta)^2)
dPdx = 100
mu = 0.01
y(0) = 0
y(f) = 0.005
Tau(0) = 0
vx(0) = 0.125
```

Ln 6 | parallel plates newtonian.pol | Newtonian Fluid Flow Between 2 parallel plates

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Trials using interpolation after 3rd guess

v at center	v at wall
0.1	-0.025
0.5	0.375
0.1250	3.88E-16

Students are Confused

- **Question**
  Why do I have to do trial & error for the initial velocity? Why not just plug-in the maximum velocity from the analytical solution?
- **Answer**
  Your goal is always to compare a numerical solution to a simple analytical problem solution. This shows that the numerical solution method is correct.
- Then give students a more complex problem with one of the plates heated resulting in a temperature profile in the liquid. Now they must do the trial and error procedure.

Temperature Profile in liquid:

$$T = 5000\frac{K}{m}y + 293.15K$$

$$\mu = \frac{196.99\mathrm{kg}}{m\ s}\exp\left(-\frac{0.033}{K}\mathrm{T}\right)$$

Heated Plates

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RKF45

Differential Equations: 2 Auxiliary Equations: 7 Re

```
d(Tau) / d(y) = dPdx
d(vx) / d(y) = -Tau/mu
rho=1000
Re = rho*delta*vx/mu
delta = 0.01/2
mu = 196.99*exp(-0.033*T)
T= 5000*y + 293.15
vAnal = dPdx*delta^2/2/mu*(1-(y/delta)^2)
dPdx = 100
y(0) = 0
y(f) = 0.005
Tau(0) = 0
vx(0) = 0.1781
```

Trials using interpolation after 3rd guess

v at center	v at wall
0.125	-0.05306
0.12	-0.05806
0.180	0.001939
0.17806	3.95E-05

New

Differential Equations Solution #11

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POLYMATH Report Newtonian Fluid Flow Between 2 parallel heated plates with viscosity a function of T

Ordinary Differential Equations 30-Jun-2017

Calculated values of DEQ variables

	Variable	Initial value	Minimal value	Maximal value	Final value
1	delta	0.005	0.005	0.005	0.005
2	dPdx	100.	100.	100.	100.
3	mu	0.01239085	0.0054301	0.01239085	0.0054301
4	Re	71.867551	0.03636595	88.835003	0.03636595
5	rho	1000.	1000.	1000.	1000.
6	T	293.15	293.15	318.15	318.15
7	Tau	0	0	0.5	0.5
8	vAnal	0.1008809	0	0.11817159	0
9	vx	0.1781	0.00003949	0.1781	0.00003949
10	y	0	0	0.005	0.005

Differential equations

1 d(Tau)/d(y) = dPdx
2 d(vx)/d(y) = -Tau/mu

Explicit equations

1 rho = 1000
2 T = 5000*y + 293.15
3 delta = 0.01/2
4 mu = 196.99*exp(-0.033*T)
5 Re = rho*delta*vx/mu
6 dPdx = 100
7 vAnal = dPdx*delta^2/2/mu*(1-(y/delta)^2)

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